REINFORCED CONCRETE AND MASONRY STRUCTURES

REINFORCED CONCRETE AND MASONRY STRUCTURES

COMPILED BY A STAFF OF SPECIALISTS

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PREFACE

This volume is one of a series designed to provide the engineer and the student with a reference work covering thoroughly the design and construction of the principal kinds and types of modern civil engineering structures. An effort has been made to give such a complete treatment of the elementary theory that the books may also be used for home study.

The titles of the six volumes comprising this series are as follows:

Foundations, Abutments and Footings
Structural Members and Connections
Stresses in Framed Structures
Steel and Timber Structures
Reinforced Concrete and Masonry Structures
Movable and Long-span Steel Bridges

Each volume is a unit in itself, as references are not made from one volume to another by section and article numbers. This arrangement allows the use of any one of the volumes as a text in schools and colleges without the use of any of the other volumes.

Data and details have been collected from many sources and credit is given in the body of the books for all material so obtained. A few chapters, however, throughout the six volumes have been taken without special mention, and with but few changes, from Hool and Johnson's Handbook of Building Construction.

The Editors-in-Chief wish to express their appreciation of the spirit of cooperation shown by the Associate Editors and the Publishers. This spirit of cooperation has made the task of the Editors-in-Chief one of pleasure and satisfaction.

G. A. H.
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May, 1923.

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REINFORCED CONCRETE AND MASONRY STRUCTURES

SECTION 1

PREPARATION AND PLACING OF CONCRETE

PREPARATION OF CONCRETE

By G. M. Williams

1. The Problem.—It is the problem of the field engineer to produce concrete of the quality specified and assumed in design with available materials. Generally some definite compressive strength value must be attained, but in some cases the qualities of watertightness or resistance to abrasion and wear are of first importance. Present knowledge does not permit a definite statement regarding the relation of the important factors—namely: Compressive strength, watertightness and resistance to wear—but most available data indicate that in general the two latter qualities increase as strength increases, considering concrete made of materials from the same source of supply.

In practice definite compressive strength values are assumed in design, and proportions of cement and aggregate are often specified regardless of whether concrete of the assumed quality can be attained with the materials available. Such a procedure will in many localities result in a concrete having not more than 25 to 50 per cent of the specified strength, but owing to lack of inspection and tests of field concrete this condition is not generally known. For example, it is commonly assumed that a concrete composed of one part cement, two parts fine aggregate and four parts coarse aggregate will have a compressive strength of 2,000 lb. per sq. in. at 28 days and a safe working stress of 650 lb. per sq. in. is permitted in design. With given materials this value may be exceeded in the laboratory where comparatively dry consistencies are employed, but in the field where wetter consistencies must be used the strength may not exceed 1,000 lb. per sq. in. Too often the quality of cement and aggregate is specified in great detail, sometimes barring the use of available materials, while no attention is given to verifying the quality of the concrete actually placed.

Production of concrete of definite quality can be assured only by a proper method of inspection and tests. No rules can be stated which will result in concrete of definite quality for the various types and grades of aggregates and qualities of cement which may be encountered. Their concrete making qualities can be determined only by compressive strength tests of concrete in

which the particular cement and aggregate are used. Experience with material of any particular type will permit of fairly close estimates to be made of the value of aggregates of the same type and characteristics from other sources, but on work of importance judgment should be verified by the results of tests. In general each locality has its own problem and the efficient use of available material is dependent upon a knowledge of the characteristics of the material as indicated by tests of mortars and concretes.

2. Concrete Materials.

2*a*. Cement.—On all work of importance the cement should meet the specification requirements for Portland cement as drawn up by the American Society for Testing Materials. It is not sufficient to specify cement by some brand name which has been found satisfactory on previous work, but each lot or shipment should be tested to insure that it complies with the specification. All well-established mills are able to produce cement which fully meets the specification requirements, and likewise all mills may occasionally turn out an inferior product for short periods. It is for the purpose of detecting such occasional inferior lots that a systematic system of inspection and testing should be employed.

Cement is the one strength furnishing material in a mortar or concrete. Strength is affected by the physical properties of the aggregate and processes employed, but cement is the essential component in the mass. Neat cement in compression is generally weaker than the individual particles of the aggregate. Contrary to common understanding, aggregate is not employed to strengthen concrete, but only for purposes of economy and in a small percentage of cases for added resistance to abrasion. Any addition of aggregate to neat cement reduces both compressive strength and watertightness. Cement usually costs from eight to ten times as much as aggregate per unit volume and the addition of the latter reduces cost and results in a material which still retains requisite strength for structural purposes. Regrading and special treatments of aggregate, keeping the cement content constant, may in some cases be a means of slightly increasing compressive strength, but in general strength can be most economically increased, when required, by an increase in cement content.

The tendency of Portland cement to deteriorate when stored under usual conditions for a considerable period of time, or under abnormal conditions for only a short period, requires that proper consideration be given to storage conditions. Cement protected from the moisture and carbon dioxide of the air may be kept indefinitely without loss of strength, but exposure in the usual type of storage sheds for only a few months, may, depending upon climatic conditions, result in strength reductions of 50 per cent or more when used in a concrete. Shipments received direct from the mill and used within a few weeks will require no attention in this respect but cement packed in bags which may be held over from one construction season to the next should be re-tested before use. The usual routine physical tests of the cement laboratory do not fully indicate the reduction of a cement's concrete making qualities and such tests should preferably be made on the concrete where facilities are available. Cements which have deteriorated due to storage conditions will usually have an appreciable higher "loss on ignition" which may in some cases be used as a criterion of the amount of deterioration. Concretes in which old storage cements are used are very slow

hardening and of inferior ultimate strength and where exposed to severe climatic conditions, abrasion or water velocities are likely to require early replacement or extensive repairs. To attain the specified strength with such cements it is customary to increase the proportion of cement in the mix, but it has not been well established by tests that the abrasive and watertight qualities are improved proportionally.

2b. Aggregates.—(1) *Types and Their Characteristics.*—Aggregates are chemically inert, volume constant materials used with cement and water to produce mortar and concrete. They serve as adulterants or extenders with neat cement to reduce cost. The quantity of aggregate which may be mixed with a given amount of cement is mainly dependent upon the compressive strength required and varies somewhat with the physical characteristics of the particular aggregate used. It is desirable that the aggregate particles have a higher compressive strength than the concrete although this is not essential in rich mixtures high in cement content. Most types of material used as aggregates easily meet this requirement. The qualities of hardness and toughness are required in aggregates which will be exposed to abrasion and wear. Materials such as shale, which expands and disintegrates when saturated, and sandstone which may contain a binder, weak and easily disintegrated by exposure to moisture, should be avoided.

Aggregates in common use are of several types—such as river or bank gravel, crushed limestone, granite or trap and blast furnace slag. The gravels usually contain varying percentages of silicious and calcareous material together with small quantities of clay, loam, and organic impurities which may necessitate removal before use in concrete. Where available, gravel is the most economical material but in many localities it is necessary to employ crushed rock of limestone granite or trap. Blast furnace slag is sometimes used as a coarse aggregate in the regions of steel production. In cases where granulated slag is used as a fine aggregate, it is usually necessary to add some sand also, otherwise the mixture will be too harsh working and porous. In 1918, the need for an aggregate which would produce a light weight concrete resulted in the production, under the supervision of the Concrete Ship Section of the U. S. Shipping Board, of a burnt shale aggregate[1] which furnished a concrete equal in strength to that in which gravel or stone is used, and having a weight per cubic foot of 110 lb. as compared with 145 lb.

It is not possible to state in detail the characteristics which an aggregate should have without barring from use materials which would be found satisfactory in tests. Much expense has been incurred in some localities and concrete producers have been given wrong ideas of aggregate quality by specifications based upon fineness or quantities passing certain sieves. Such standards have usually been derived from the results of mortar tests which, as usually conducted, are of no value as a means of judging the worth of an aggregate for use in concrete. The presence of loam and clay in appreciable quantities is objectionable and can usually be detected by visual inspection. Test results show that finely divided clays up to 6 or 8 per cent may be beneficial when the aggregate is deficient in fine particles, especially in lean mixtures. The old requirement that the aggregate must be sharp has been eliminated from most specifications since strength tests have indicated that round particles do not produce an inferior concrete.

[1] *Engineering News-Record*, April 24, 1919.

There is also greater ease in chuting and in compactly placing concrete containing rounded aggregate particles. So far as gradation is concerned, aggregates conforming to a straight line gradation or having approximately equal quantities of the various sized particles is desirable but what may be described as freak gradations, such as that of beach sand, may often be economically used. It is not possible to draw definite conclusions as to the superiority of any one type of aggregate. As a class there is little choice between limestones, granites or gravels and decision in any case when each are available should be based upon compressive strength tests. It is stated in Technologic Paper 58, of the U. S. Bureau of Standards that, "No type of aggregate such as granite, gravel or limestone can be said to be generally superior to all other types. There are good and poor aggregates of each type."

(2) *Tests of Aggregates.*—While testing of aggregates has not been fully standardized, certain tests are commonly employed which singly or together are of some assistance in determining the concrete making qualities of aggregates. In general these tests are valuable in identifying or classifying material, and calculating proportions, rather than for definitely establishing absolute value.

Visual Inspection.—Visual inspection alone may be sufficient to permit a fairly accurate classification of an aggregate, especially when it is similar in type and appearance to material from another source which has proven satisfactory. Mineral constituents may be identified, gradation of particles, quantity of silt or impalpable powder and its condition whether free or adhering may be noted. Visual inspection will not result in detection of organic material or other injurious matter and should therefore be used with caution in judging material from new sources of supply.

Organic Impurities.—The presence of organic impurities in aggregate has been found to reduce the strength of the mortar and concrete or even cause disintegration. The Abrams-Harder Colormetric Test for organic impurities in sands furnishes a simple and efficient means of detecting organic impurities. This test, which may be readily carried out in the field, is described in detail in Appendix *B*, p. 726.

Silt.—Finely divided particles of aggregate, clay and loam are known as silt. Tests have shown that silt composed of rock particles or clay may be beneficial up to 6 or 8 per cent, especially in a coarse graded aggregate. Its effect can best be determined by tests of mortar or concrete in comparison with the same aggregate from which the silt has been removed. Excess silt can be removed by washing. Proportion of silt in an aggregate may be determined in the field by shaking a definite volume in a graduated vessel, such as a glass graduate, with water. After the liquid becomes clear the silt will be seen in a layer just above the coarse aggregate and the percentage is the thickness of the silt layer divided by the height of the aggregate column. This result is only approximate. More accurate determinations, in the laboratory, can be made as specified by the American Society for Testing Materials (see Appendix *C*, p. 727).

Specific Gravity, Voids and Weight per Cubic Foot.—These related physical properties of aggregate while of interest in comparing test results and of value in accurate proportioning, especially when transposing from weight to volume proportions, bear little relation to quality of concrete. For the same

aggregate gradings or the same fineness, weight of aggregate and consequently weight of concrete per cubic foot is proportional to the specific gravity of the aggregate. For any one aggregate, weight per cubic foot is inversely proportional to the percentage of voids. Percentage of voids is dependent upon relative quantities of fine and coarse material in a unit volume, as well as gradation and shape of particles.

The following table[1] shows the range of these values for fine aggregates of natural sand and crushed stone screenings from various localities in the United States:

Fine aggregate	Range of			
	Specific gravity	Voids	Weight per cubic foot (lb.)	No. of samples
Natural sand......	2.35 to 2.72	26.8 to 46.6	82.8 to 119.9	200
Stone screenings..	2.49 to 2.90	32.7 to 49.5	80.7 to 120.0	34

Specific gravity is determined by introducing a definite weight of material into a vessel containing a liquid such as water or kerosene and noting the apparent increase in volume of the liquid. The specific gravity is the result obtained by dividing the weight of aggregate used by the volume displaced. Most sands containing silicious material do not vary greatly in specific gravity and the figure 2.65 is commonly used as an approximate average value.

Weight per cubic foot is dependent upon size and shape of aggregate particles, size and shape of container used and method of filling container. The specification tentatively adopted by the American Society for Testing Materials and given in detail in Appendix *D*, p. 728, should be followed in determining weight per cubic foot of aggregate.

Knowing the weight per cubic foot and the specific gravity, percentage of voids can be determined as follows:

$$\text{Percentage of voids} = 100 - \frac{\text{Weight per cubic foot} \times 100}{62.3 \times \text{Specific gravity}}$$

Granular Analysis.—The fineness of particles or the gradation of an aggregate is determined by screening the material through a series of sieves having different sized openings. The series of sieves known as the Tyler Standard Screen Scale is at present in general use for measuring the sizes of particles. The Tyler scale is based upon a series of openings which form a geometric series starting with 0.0029 in., the opening of the present 200-mesh standard sieve, and having the square root of two or 1.414 as the ratio For coarse aggregates, screens of heavy wire with square openings or steel plates with circular openings of the desired size are used.

For laboratory use the brass frame sieves, 8 in. in diameter and 2 in. deep are most convenient, but, for the separation of large quantities of material, wood or metal frame sieves 18 in. in diameter are used.

[1] Technologic Paper 58, U. S. Bureau of Standards.

Aggregate passing the No. 4 sieve is classed as fine material and its granular analysis is determined with the following sieves:

Sieve No.	100	48	28	14	8	4
Size opening (inches)	0.0058	0 0160	0 0232	0 0460	0.0930	0.1850

All material retained on the No. 4 sieve is classed as coarse aggregate and is usually subdivided by means of the following screens:

Screen	No. 4	⅜ in.	¾ in.	1½ in.
Size opening (inches)	0 185	0.375	0.750	1.500

The granular analysis of a group of sands ranging from fine to coarse are shown graphically in Fig. 1. This test furnishes an excellent means of measuring the variation in gradation of different lots of aggregate, but does not necessarily

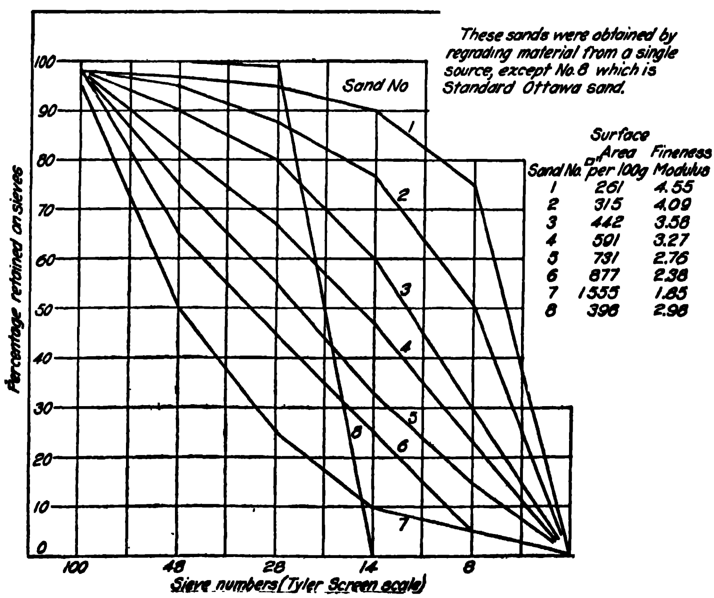


FIG. 1.—Method of graphically representing the gradation of a group of sands ranging from very coarse to very fine. Surface area and fineness modulus of each are shown in tabulation.

establish a basis for acceptance or rejectance without the results of strength tests of mortars or concretes or previous experience with the same or very similar material. From the results of the sieve tests, an approximate value for sur-

face area of the aggregate may be readily computed. The areas as computed are not the true areas and are not fully comparable with the areas of sands having different gradations or percentage of the various sizes unless the particles are similar in shape, since the assumption is made that the particles are spheres. Most aggregate particles are more or less elongated and seldom spherical in shape so that the areas as calculated are usually less than the true areas.

To illustrate the method of calculating surface areas, the area of sand No. 4 as shown in Fig. 1 is determined as follows:

Calculation of Surface Area of an Aggregate

(Sand No. 4, Fig. 1)

Sieve No.	Percentage retained	Per cent retained and passing preceding sieve (grams in 100 grams)	Sieve area factor (area in sq. in. per gram)	Total area (sq. in.)
4	0	0	½	0
8	23	23	1	23
14	47	24	2	48
28	67	20	4	80
48	82	15	8	120
100	98	16	16	256
Passing 100	2		32	64
	Total per cent, or grams....	100	Total........	596 sq. in. per 100 grams

Note.—The above value for surface area is not the true area, but is a proportional value which is sufficiently accurate for all practical purposes. The sieve area factors for the larger screens are as follows: ⅜ = ¼, ¾ = ⅛, 1½ = $\frac{1}{16}$

The values obtained in the sieve test may also be used in the calculation of the "fineness modulus," which is defined as the sum of the percentages in the sieve analysis of the aggregate divided by 100, including the following Tyler standard sieves: 100, 48, 28, 14, 8, 4, ⅜, ¾, and 1½ in.

The fineness modulus of sand No. 4 is determined as follows:

Calculation of Fineness Modulus of an Aggregate

(Sand No. 4, Fig. 1)

Sieve No..............	4	8	14	28	48	100
Percentage retained.......	0	23	47	67	82	98

Sum of percentages = 317 Fineness modulus = $^{317}/_{100}$ = 3.17

The fineness modulus of a coarse aggregate, all of which is retained on the No. 4 sieve is the sum of the percentages on the ⅜, ¾ and 1½-in. screens plus 600 (100 for each screen finer than the ⅜ in.), and the total divided by 100.

Fineness modulus increases with coarseness of aggregate. However, fineness modulus is not an absolute criterion of coarseness since a group of aggregates may be so graded as to differ as much as 600 per cent in surface area with fineness modulus constant. Variation in surface area has a very marked effect on the water requirement of an aggregate in a mortar or concrete and is a more useful criterion than fineness modulus in practice.

Of the foregoing tests of aggregates, the colormetric test is the only one which may alone definitely establish the concrete making qualities of an aggregate. The others are chiefly useful in identifying and measuring the uniformity of aggregate and are of greatest value in conjunction with compressive strength tests.

(3) *Selection of Best Available Aggregate.*—When one or more aggregates are available for use, the employment of the foregoing tests combined with the judgment of the engineer based upon previous experience, will often permit the selection of the best aggregate. While various methods have been proposed as a basis for rating the mortar or concrete making qualities of aggregates, so many exceptions can be found to such rules, that final decision, where any considerable amount of money is involved, should always be governed by the results of compressive strength tests. This method is outlined in Art. **5**.

(4) *Improvement in Quality of Aggregate.*—Mortar and concrete making qualities may often be improved at a reasonable expense by washing, screening and regrading. Washing will usually eliminate organic impurities and also reduce the quantity of silt, loam or clay. These treatments are usually applied to natural sands and gravels as separate steps in the process of excavating and loading the material for shipment. When such mechanical equipment is not available at the source of supply, washing and screening may be effected, at greater expense, by means of portable washers and rotary screens which are now on the market. The concrete mixer itself is a very efficient although a less economical washer. Aggregate prepared by crushing rock or boulders will be free from dirt if care is used in selection of the material, but excess of fine material may require removal by screening. It will seldom be found economical to separate aggregate into more than two sizes, the fine and the coarse. The advantage of the slight increase in concrete quality which may result from increasing the number of sizes will usually be overcome by confusion at the mixer, except where proportions are automatically combined.

The added cost of regrading and handling more than two aggregate sizes had better be converted into more cement which will result in a definite strength increase. The practice of using crushed stone with sand and gravel which are deficient in quality should be governed in each case by comparing the added cost of the stone with the cost of increasing the cement content to furnish concrete of the same strength. It is impossible to state definite rules which will apply to all conditions and all types and qualities of aggregates. Each problem must be considered separately with cost and quality as controlling factors.

(5) *Use of Pit Run Materials.*—Natural supplies of bank or river sand and gravel are widely distributed and probably furnish the greatest percentage of

material used for concrete aggregate. There is a marked tendency, especially in newly developed localities, where the demand is not great, to permit the use of pit run aggregate as excavated, without screening or washing. The use of pit run material is often poor economy and cannot be recommended unless compressive strength tests of concretes have demonstrated that little will be gained by screening and recombining, since there is usually an excess of fine material present which may result in a reduction in strength and permeability, so that different batches of concrete will vary greatly in quality and workability. In all work of importance, unless the value of pit run material has been indicated by strength tests, it should be screened into fine and coarse divisions by the No. 4 screen and recombined. It has become common practice in some localities to supply a combined fine and coarse aggregate ready for use. This plan no doubt has some advantages in that the ready mixed aggregate is economical to handle and requires less storage space on the job, but precautions must be taken to prevent segregation of the fine and coarse particles.

(6) *Some Characteristics of Sand in Concrete.*—Quality of concrete is affected far more by variation in quality and physical characteristics of sand than of gravel. The surface area of a sand is a function of the grading or the relative quantities of the various sized particles in a unit volume. A fine sand contains a greater number of particles per unit volume than a coarse aggregate and its surface area is greater. The "bulking effect" or swelling of sand to which water is added is dependent upon fineness, or surface area of sand. With a sand of given gradation, swelling increases with the addition of more moisture up to a percentage which is dependent upon fineness, beyond which the addition of more moisture causes a decrease from maximum volume to one less than that in the dry state. This bulking effect of sand takes place during the process of mixing concrete and the volume of concrete produced is in turn affected by the bulking tendency of the particular fine aggregate used. This characteristic of sand explains why the theory of filling voids in coarse aggregate by a carefully calculated quantity of fine aggregate fails to work out in practice.

In a similar manner the moisture content affects the actual quantity of sand in concrete when proportions are based upon volume measure. As percentage of moisture in the sand increases, up to that for maximum bulking effect, less sand will be added to the batch and cement content per unit volume of concrete is increased. If preliminary proportioning studies have been based upon dry aggregates, the relative volume of sand may be slightly increased on the job when wet sand is used. On small jobs the total cement saving over the amount which would otherwise have been used will not be great, but on large work the difference may be appreciable since it may amount to one-half bag or more per cubic yard of concrete. Since the influence of other factors, difficult to control in the field, may more than counterbalance the effect of the increased cement content it had best be considered as increasing the factor of safety. Only on work where careful supervision and inspection is employed should this volume increase of sand be permitted in proportioning.

(7) *Concrete Making Value of a Sand.*—The extensive investigation of sand mortars made some years ago by M. Feret brought out the fundamental relations between gradation of sand and strength of mortars. The later improper application of the results of these studies to the production of concrete has been

responsible for the erroneous ideas which exist with respect to the concrete making value of a fine aggregate. The arbitrary limitations placed upon gradations of fine aggregate in some specifications has resulted in the condemning of many available fine aggregates well suited to the production of concrete. Such limitations have generally been based upon tests of sands in mortars. The conclusions so drawn are not applicable to the same fine aggregate in a concrete, and tests have shown that the fine aggregate in question must be combined with the coarse aggregate in a concrete to permit of sound conclusions being drawn.

In Fig. 2 are shown the strengths and other properties of mortars made with the eight sands whose granular analyses are shown in Fig. 1. These sands were

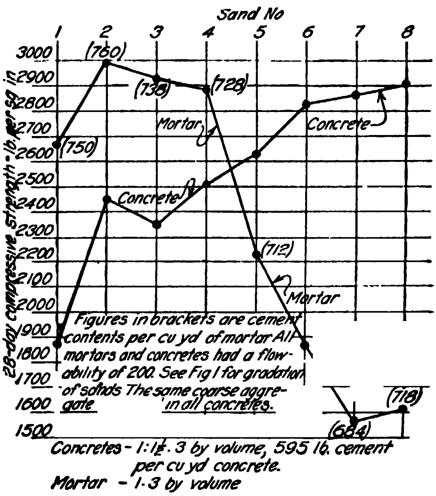


FIG. 2.—Showing the effect of fineness of sand on mortar and concrete strength. Mortar strength tests as ordinarily made often result in rejection of fine sands which would prove satisfactory in concrete. The above mortars and concretes are fully comparable since the only variable factor is the sand gradation.

obtained by screening aggregate from a single source of supply and recombining the various sizes in the required proportions. It is seen that sands Nos. 2, 3 and 4 produced mortar of the highest strength, and that sands finer than No. 2 gave strengths varying inversely as the fineness of the sand. On the basis of these mortar tests alone, sands 5, 6 and 7 would likely be considered undesirable for use in concrete. Concretes produced by combining these sands in turn with the same gradation of coarse aggregate indicate that better concretes, based on compressive strengths, are produced with the finer sands Nos. 5, 6 and 7 than with the coarse sands which gave mortars of the highest strengths. Concretes made of the finer sands are also superior in another respect which is of greater importance to the field engineer—that is, that such concretes show no tendency to segregate in the greatest flowabilities required in practice.

Tests of sand in mortars do not indicate their true concrete making qualities due to the fact that the consistencies employed in mortars are seldom comparable with those needed in concrete. In a mortar a fine sand requires a much greater percentage of mixing water than does a coarse sand with other conditions equal.

The large excess of water required for the fine sand to result in equal flowability greatly reduces strength. However, in a concrete the paste which is composed of the cement and smaller sand particles may be drier when a fine sand is used and yet furnish the same flowability. A larger percentage of the material in the concrete has the flowing property and as a result the fine sand concrete requires proportionally less water than in a mortar. This is illustrated by mortar and concrete tests of sands Nos. 2 and 7 of Figs. 1 and 2.

Sand	Flowability	Per cent mixing water	28-day strength (lb. per sq. in.)
	1:3 Mortar		
	200	13.61	2,993
	200	19.22	1,562
	1:1½:3 Concrete		
2	200	9.73	2,465
7	200	10.80	2,865

To produce mortars of the same flowability 19.22 per cent mixing water was required with sand No. 7 as compared with 13.61 for sand No. 2. In comparable concretes, only 10.80 per cent water was needed with sand No. 7 as compared with 9.73 per cent for sand No. 2. A large percentage of the finer sand particles of the former assists the cement in carrying or floating the coarse aggregate particles so that a less fluid mortar or paste is required. The small percentage of fine particles in sand No. 2 necessitates a more fluid cement paste to result in the same flowability. While the actual quantity of water used is greater in the concrete containing sand No. 7, the cement paste itself is drier, the higher surface area of the fine sand accounting for the additional water required. That this is true can be demonstrated by screening the mortars from the two concretes and testing them on the flow table which is described in Art. **3**.

Fine sands, which from the results of mortar tests appear to be unfit for use in concrete, may actually be most desirable. Realization of this fact should in many cases lower concreting costs since it will result in the use of local aggregates which would be classed as inferior under many specifications in common use. On all work of importance the expense of a small properly planned series of strength tests should definitely establish safe practice.

(8) *Density, Porosity and Permeability.*—Available test data do not permit definite conclusions as to the relations between density, porosity and low permeability. The same concrete may have a high porosity and a low permeability. Porosity is a measure of the total pore space while permeability is dependent upon arrangement, size and physical condition of the pores. A neat cement may have a high porosity and yet be impermeable for all practical purposes. A mortar may have a low pore space or high density yet be very

permeable under comparatively low heads of water. Porosity is mainly dependent upon aggregate grading and quantity of mixing water used, increasing with fineness or surface area of aggregate and with water content. *Permeability* is to a certain extent affected by aggregate grading and water content but is mainly influenced by *cement* content.

With the same aggregate grading and flowability, the variation in density or solidity ratio and porosity of a concrete may be slight for the widest range of cement contents employed in practice yet there is a marked decrease in permeability as cement content increases.

2*c*. **Admixtures.**—Inert materials in powder form which are added to cement and concrete are classified as admixtures. The most common of these are hydrated lime and powdered rocks or clays. Such materials especially in lean mixtures, may slightly increase compressive strength in case the aggregate is lacking in finer particles. More often admixtures are employed as fillers in the aim to reduce permeability. As a means of increasing compressive strength, admixtures are generally ineffective and the extra expenditure required can best be devoted to increasing the cement content of the mix, which will definitely increase strength and watertightness. More studies are needed to determine definitely the relations between compressive strength, watertightness, grading of aggregate and cement content, and when suitable tests cannot be made it should be kept in mind that more cement itself is the most efficient admixture which can be used to increase strength and reduce permeability.

2*d*. **Waterproofings and Hardeners.**—Waterproofings for mortars and concretes may be divided into two groups, integral compounds and surface coatings. Integral compounds include inert fillers, active fillers, water repelling substances, and special cements. Inert fillers are usually the admixtures of lime, pulverized rock or clay. Active fillers, containing resinates of potash or a saponifiable oil, are supposed to react with the free lime of the cement and form insoluble resinates and lime soaps. These lime soaps are neither soluble in water nor wet by it and form the basis of what are described as "water repelling" compounds. Water repelling compounds are stearates of lime, or soda and potash. It is claimed that these materials fill the voids and by their water repellant nature prevent the passage of moisture. Waterproof cements are usually ordinary cements to which water repelling materials have been added in powder form.

Coating compounds may be linseed oil, paints or varnishes, bituminous coatings, liquid hydrocarbons, soaps, and cements. Other types include the silicates of sodium and magnesium or pulverized iron particles to which a salt has been added to accelerate the corrosion of the iron when applied. These latter materials may be applied in liquid form to the concrete surface or in a mortar plaster coat. Linseed oil paints tend to soften under water and owing to the saponification of the linseed oil by the alkalies in the concrete are not permanent. Bituminous coatings include asphalts, petroleum residuum and coal tar pitches. They are sometimes used as paints but are generally applied hot with alternate layers of felt. Liquid hydrocarbons form superficial surface coatings which may be partially effective under low heads so long as the film remains uncracked. Soaps are supposed to react with the free lime in the concrete near the surface and form an insoluble lime soap.

Technologic Paper No. 3 of the U. S. Bureau of Standards sums up the value of integral compounds as follows: "Portland cement mortar and concrete can be made practically watertight or impermeable to any hydrostatic head up to 40 ft. without the use of the so-called integral waterproofing materials . . . The addition of these compounds will not compensate for lean mixtures, nor for poor materials, nor for poor workmanship in the fabrication of the concrete. None of the integral compounds tested materially reduced the absorption of mortars before they were dried by heating to 212 deg. F. Thus they have little or no practical value. If the same care is taken in making the concrete impermeable without the addition of waterproofing materials as is ordinarily taken when waterproofing materials are added, an impermeable concrete may be obtained." Recent tests in concretes of various waterproofing compounds on the market verify the foregoing conclusion and indicate that their use does not decrease permeability or absorption below that obtained for plain concrete of the same cement content and flowability.

Until further investigations have indicated the relation between aggregate grading and watertightness, the same care and precautions which are employed to produce a concrete of high strength will also be valuable in producing concrete of low permeability and any added expenditure proposed for the purpose of insuring watertightness by means of integral compounds may best be used in purchasing more cement.

To increase the hardness and resistance to abrasion of floor surfaces, boiled linseed oil, sodium silicate and magnesium fluosilicate have been found successful in practice. As one manufacturer of a concrete hardener explains, floors which are built in such a manner as to comply in every respect with the latest specifications for such work do not require the use of a hardener, yet in practice it is difficult to attain such perfection and wear may often be considerably reduced by the application of a hardener. Linseed oil may be applied by brushing it on to the surface. The silicates are usually mopped on in two or three coats, the first being a rather dilute solution of 2½ per cent followed later by solutions of 5 per cent.

The rate of hardening of concrete may be greatly accelerated by using a solution of 3 or 4 per cent calcium chloride for the mixing water. Final set of cement is attained in about one-half the normal time and the strength at 48 hours is practically the same as that normally attained at 7 days. Strengths of calcium chloride concretes at 28 days are about the same as for the untreated. Ultimate strength is not in any way affected, but the early hardening is greatly accelerated, which is of especial advantage in the finishing of floors and in permitting the earlier removal of forms in structural work. The accelerator should not be used in reinforced concrete work which will later be exposed to moisture, owing to the danger of corrosion caused by the presence of the salt. Compounds having calcium chloride as the base are sometimes advertised as integral waterproofing materials but their effectiveness in such work has not been demonstrated, neither have they merit as hardeners to resist abrasion and wear.

2e. Mixing Water.—The quality of mixing water to be used in concrete does not usually require special consideration. Any water which is satisfactory for drinking purposes and domestic use will serve for concrete. In unsettled localities where available water may be acid or alkaline or along the

coast where only sea water is available some consideration should be given to the source of supply. Just how great a quantity of dissolved salt may be present in water without injurious effect has not yet been established. Many satisfactory structures have been built in which alkali water has been used while others which have been exposed to alkali or sea water have shown signs of deterioration. It is not probable that the small quantity of salt present in the mixing water can alone account for failures which have occurred, since the reaction between the salt and cement is probably completed before initial set takes place. The presence of soluble salt in the mixing water is no doubt a minor factor so far as possibility of disintegration is concerned yet the use of fresh water is to be recommended whenever it may be obtained at a reasonable expense.

3. Consistency or Flowability.

3*a*. Importance and Effect on Quality of Concrete.—In order that concrete may be properly handled and placed, it must be plastic and flowable. Consistency required in any case is dependent upon methods of handling or transporting, shape and size of form in which it is to be placed, size and position of reinforcing steel, and accessibility for the purpose of vibrating the form or spading and tamping the mass. In any case the purpose is to completely fill the space with a homogeneous mixture, free from segregation, with all steel reinforcing thoroughly bonded, with surfaces as true and smooth as the faces of the form, at a minimum of cost. Concrete transported in steel chutes must be sufficiently flowable that it will pass at a uniform rate through the system. In modern practice large quantities of concrete must be placed during short periods of time and the amount of energy which can be expended in tamping, spading and vibrating the mass is small per unit volume. In large mass work a very stiff, slightly quaking consistency may be used provided it is transported by carts or buckets, since a small amount of tamping will cause excess water to rise and increase the wetness of the succeeding batches so that flowability may be slightly decreased as the work progresses. In structural work where forms are shallow and contain considerable steel, there is no excess water to increase the flowability or placeability of succeeding batches and concrete must come to the form in such a condition that it may be effectively placed with a minimum of labor. A very wet almost fluid consistency may in some cases be required. The use of very wet consistencies in deep forms will result in the formation of a layer of laitance having a thickness of as much as 6 in. Laitance has a very low compressive stength, is very porous and permeable, and should be removed before concreting is continued.

On any work there is some minimum flowability which must be attained to result in a homogeneous uniform mass. This minimum will depend upon the foregoing factors and is a consideration of the greatest practical importance to the field engineer.

In any given concrete, increase of flowability is gained by increasing the amount of mixing water. The addition of inert material in powder form, contrary to common opinion, does not increase flowability, but actually results in a decrease, since the dry material extracts a portion of the water from the mass. Just as strength of concrete is dependent upon cement, the only strength furnishing element in the mass, flowability is dependent upon water content. As is generally known, increase in quantity of mixing water will decrease compressive

strength. This decrease in practice may be as much as 25 to 50 per cent of the strength which might have been obtained had the flowability been restricted to the minimum usable. Therefore the question of flowability is really as important to the architect or designer as to the field engineer. Minimum usable flowability is fixed by conditions of the work, and the use of sufficient water to attain it may result in strengths far below those assumed in design. It is of prime importance to the field engineer that the concrete may be properly and economically placed, while the designer is mainly concerned with the attainment of specified quality. Increase in flowability means decrease in quality. For this reason flowability is a fundamental factor in the production of concrete which must be considered by the designer as well as the field engineer.

It is equally important that the flowability factor be kept in mind in comparing qualities of cements or aggregates. Unless flowabilities are equal, concretes are not fully comparable. Equal flowabilities and equal cement contents per unit volume are the two fundamentals which must be observed in making comparisons.

3b. Measurement of Flowability or Consistency.—Two methods are employed in practice for measurement of flowability or consistency, "the slump test" and the "flow table." In the slump test the tendency of a molded mass to distort or slump under the action of gravity when the lateral support is removed is measured while the flow table gives a measure of the tendency of a mass to spread or flow out under the influence of a definite amount of work. In the slump test, the slump or reduction in height of the molded specimen when the form is withdrawn is measured and expressed as so many inches slump. For the flow table method a mass of material is molded on a smooth surface, the form withdrawn and the surface raised vertically and dropped a given number of times from a fixed height, after which a comparison of the new diameter with the original gives a figure called the "flow figure" or "flowability."

Slump Test.—As originally employed in practice a metal mold such as the 6 × 12-in. cylinder form is used. Concrete is molded in the usual manner for the preparation of test specimens, by ramming or tamping the material in four layers and the form is then withdrawn vertically. The unsupported mass either remains standing in its original shape, it slumps or sinks down uniformly, or topples over. The difference in height between the original and the new upper surface is expressed as "slump in inches." For convenience two handles, or a loop may be attached to the form and for one-man operation projections at the bottom are of assistance in holding the form down during the process of tamping.

To avoid some of the practical difficulties encountered in the use of the cylindrical form of mold, it is now commonly made in the form of a truncated cone 12 in. high with upper and lower diameters of 4 and 8 in. respectively.

For field control and test of consistency the cylinder slump test has been found of value. It was effectively used by the Concrete Ship Section of the Emergency Fleet Corporation in the construction of concrete ships. The unusually rich and plastic concretes employed were well suited to the slump test. The slump test however is not accurate or reliable enough to be employed as a standard consistency apparatus in the laboratory. With aggregates varying in gradation or with concretes of different cement content, slumps varying considerably may be found for concretes of the same flowability. For the same batch of

concrete, slump is not proportional to flowability except for a very narrow range of water content. With harsh working concretes or concretes of low cement content, even as rich as 1:2:4 true slumps may not be obtained even with the truncated cone type of mold, as a portion of the mass often crumbles or falls away with one side remaining its full height so that decision as to the true slump figure is difficult. For field use the slump test will be found to be of considerable value when proper consideration is given to its limitations.

Flow Table.—The flow table as devised in the concrete laboratory of the Bureau of Standards and shown in Fig. 3 consists of a metal covered table top, mounted by means of a flanged coupling to the end of a short vertical shaft. This shaft carries at its lower end an adjustable bolt Below the belt is a cam

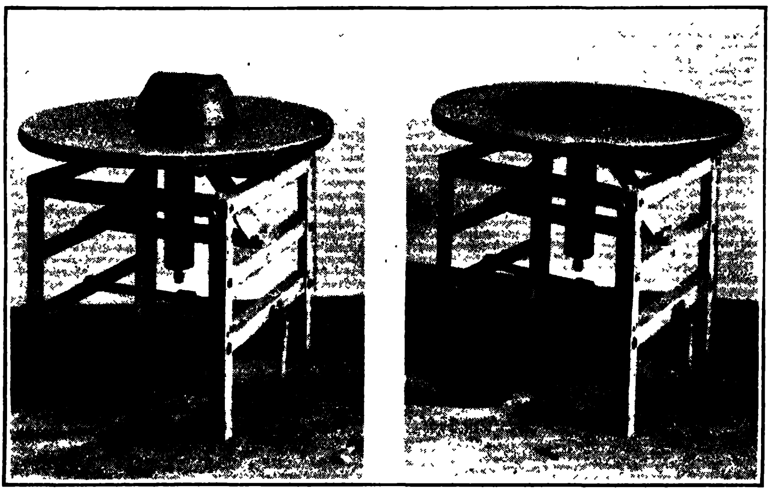


Fig. 3.—Flow table as developed at the Bureau of Standards for measuring the flowability of mortar and concrete. A representative sample is molded in the truncated sheet metal form and caused to spread by dropping the table top a given number of times through a fixed distance. The new diameter of the mass divided by the old and multiplied by 100 gives the flow figure. The 1:1½:3 concrete shown had a flowability of 160, about as dry as can be used in structural work.

having a throw of about 1½ in. attached to a horizontal shaft which can be operated by hand. The bolt is so adjusted that the table top is raised and dropped freely through ½ in. by one revolution of the cam. The mortar or concrete is molded in a sheet metal form on the center of the table, the form is removed and the top raised and dropped 15 times. The new diameter of the mass divided by the old, multiplied by 100 is designated as the flow figure. For 1½-in. maximum size aggregate a mold having the shape of a hollow frustum of a cone 6 in. high and with upper and lower diameters of 8 and 12 in. is used. For ¾-in. maximum size aggregate a mold 3 in. high with upper and lower diameters of 4 and 6 in. has been found satisfactory. A self-reading proportional caliper is used and the sum of the readings of two diameters at right angles gives the flow figure direct. If the table top is horizontal, the mass will spread concentrically and the measured diameters will be practically the same.

Tests of the flow table[1] have shown that the flow is directly proportional to change in quantity of mixing water within the necessary working limits of consistency. So far as tests have been made, flow as measured by the table has been found proportional to velocity of flow in a steel chute. The flow table will permit of a more accurate comparison of test results of different laboratories which has in the past been difficult owing to lack of a satisfactory method of measuring flowability.

4. Methods and Theories of Proportioning Concrete.—Various theories have been proposed for the use of available aggregates to produce concretes of definite quality. Since considerable time and testing equipment are needed to determine how materials can best be used by actual tests of concretes, any proportioning method which can be worked out, based upon some easily determined physical property of one or more of the materials employed, will prove of great value. Descriptions of the more prominent methods advocated, together with brief explanations of their shortcomings follow.

4*a*. Arbitrary Volume Proportions.—There are no considerations either practical or scientific which justify the use of arbitrary volume proportions. The almost universal assumption that a concrete composed of one volume cement, two volumes fine aggregate and four volumes coarse aggregate will attain a strength of 2,000 lb. per sq. in. at 28 days has been proven false in many localities yet such a proportion is often specified and owing to the general failure to test field concretes the inferior product is seldom discovered. Reinforced concrete design has been fairly well standardized and allowable stresses have been established after much study and experiment, but little attention is directed towards obtaining a concrete equal in quality to that assumed in design. The adoption of a proper system of field inspection and tests of concrete actually placed should do much to eliminate the common practice of specifying arbitrary proportions without proper knowledge of the concrete making qualities of the cement and aggregate which are to be used.

4*b*. Void Method of Proportioning Concrete.—Several variations of the void method of proportioning concrete have been proposed. In each case the idea has been to produce a concrete in which the void space will be a minimum. Knowing the volume of voids in the coarse aggregate sufficient mortar of fine aggregate and cement is specified to fill the coarse aggregate voids, often using 10 per cent excess mortar. Strength of concrete is varied by increasing or decreasing the cement content of the mortar. With a coarse aggregate high in voids this method will result in a better working concrete having less tendency to segregate than with an aggregate low in voids. The production of a concrete having zero voids is of course impossible since the mortar which is added may itself have as high as 40 per cent voids and the coarser particles of the mortar force apart the particles of coarse aggregate so that the volume of the latter in the concrete becomes greater. Concretes proportioned in this manner may or may not be of the greatest density possible for the materials employed and actual strength attained must in any case be determined by actual strength tests. An assumption sometimes made is that the strength of the concrete will be that of the mortar used. Such an assumption is erroneous since the concrete strength may be as little as 50 per cent of that of the mortar.

[1] Measurement of Flowability by Means of the Flow Table, *Engineering News-Record*, May 27, 1920.

4c. Fuller's Theory of Maximum Density for Proportioning Concrete.—The assumption is made that with fixed cement content an aggregate, containing the cement, so graded as to have maximum density, will have maximum strength. It is claimed that strength increases with increase in density. Sand and gravel are separated into a number of sizes and recombined to have a grading which will closely coincide with the "ideal" grading curve or curve of maximum density.[1] Tests[2] of aggregates from numerous sources show that an aggregate can be screened and recombined to have widely different gradings and yet produce concrete of approximately the same quality. Also with fine aggregate of the same grading, the relative proportions of fine and coarse aggregates may be so varied to range over a broad zone which results in concretes of varying densities but of approximately the same compressive strength. With the same cement content and the same supply of aggregate, the concrete of high density will usually have high strength but by changing the aggregate grading, or by modifying the relative proportions of fine and coarse aggregate of the same gradings, a considerable decrease in density of concrete may result without lowering compressive strength. An aggregate so graded so as to coincide with the "ideal" curve will often be lacking in fine particles, producing a harsh working concrete which segregates badly in the higher flowabilities.

The maximum density theory of proportioning is one application of the void theory. With equal cement contents the trend of increased strength with increased density is generally apparent, and it would seem that any theory of proportioning devised for predicting strengths should involve these properties. However, since density cannot be definitely determined without actual preparation of mortars and concretes and taking into consideration other important factors such as flowability, it is doubtful whether either of the preceding proportioning methods can be developed so as to be of great practical value.

4d. Surface Area Theory of Proportioning Concrete.—The surface area theory of proportioning concrete assumes that strength is dependent upon the ratio of weight of cement to surface area of the aggregate, that for a group of aggregates having different surface areas, and cement contents proportional to surface areas, equal strengths will be obtained. A water formula was proposed which it was claimed would result in equal consistencies for such a group of mortars or concretes. Tests have shown that this water formula furnishes such a quantity of water in each case that equal water-cement ratios will result—that is, the pastes alone will have equal consistencies with no allowance for the varying requirements of the sand which differ in the surface area or fineness. When the water formula is disregarded and consistencies are made equal, widely different strengths may be attained, indicating that strengths are not proportional to the cement-surface area ratio. Other tests[3] have shown (see also Fig. 2) that a normal sand may be regraded and so increased in fineness as to double the surface area and yet produce a concrete having the same cement content and flowability, equal in strength to the mixture having one-half the surface area. Tests do not bear out the basic claims of the surface area theory, but indicate

[1] "Plain and Reinforced Concrete," Taylor and Thompson.

[2] *Technologic Paper* **58**, U. S. Bureau of Standards.

[3] *Journal* Engineering Institute of Canada, May, 1921. *Engineering News-Record,* June 12 and August 14, 1919.

that the surface area factor is only one of the many which affect compressive strength and that it is not in itself a criterion of strength.

4e. Water-cement Ratio Theory of Proportioning Concrete.—The water-cement ratio theory[1] of proportioning concrete assumes as its basis that strength of concrete is solely dependent upon the ratio of the volume of mixing water to the volume of cement for aggregates from the same source of supply. It is further claimed that aggregates having the same fineness modulus will require the same quantity of mixing water to result in the same consistency or flowability when cement contents are equal. Quantity of mixing water to result in any desired consistency may be computed from a water formula which involves the factors of cement content, fineness modulus and absorption of aggregate and a constant water-cement ratio will result for aggregates having the same fineness modulus.

This water formula is based upon the same erroneous assumption as is the one proposed for the surface area theory of proportioning. It provides sufficient water to bring the neat cement pastes in the mixtures to the same consistency but does not make allowance for the varying water requirements of the aggregates, which with the same fineness modulus may vary as much as 600 per cent in surface area of fineness. The resulting concretes are not comparable owing to the wide variations in consistency. The addition of more water to the drier consistencies results in loss of the constant $\frac{w}{c}$ relation which is the foundation of the theory and assumed criterion for constant strength. Tests[2] have shown (1) that the proposed water formula does not result in comparable concretes which have equal consistencies as stated, and (2) disregarding the fundamental requirement that concretes must have equal flowabilities to be comparable, a constant $\frac{w}{c}$ relation does not necessarily result in equal strengths.

4f. Pit Run Method of Proportioning Concrete.—While it is recognized that the use of pit run materials should be avoided, there are often times, especially on small work, where deficiency of large aggregate particles can be most economically corrected by the use of more cement or a richer mixture. Tests generally show that, with ratio of cement to total aggregate constant, compressive strength decreases as the proportion of coarse aggregate decreases, especially when the latter falls below 50 per cent of the total. On the other hand, when cement content is increased proportional to increase in sand content, the strength also increases. The object in this method of proportioning is to specify the cement content required to result in equal strength as sand content of the aggregate increases. One method[3] proposed for proportioning pit run aggregate involves a diagram for selection of cement contents which requires the changing of usual volume proportions. Absolute volume aggregate proportions are reduced by an amount which is dependent upon the new sand content of the aggregate and this value is then re-calculated to loose volume proportions. This is made necessary because of the assumption that strength of concrete is dependent upon absolute volume of cement in the mix and the density of the mix.

[1] *Bulletin* No. 1, Structural Materials Laboratory, Lewis Institute.

[2] *Engineering News-Record*, June 12 and August 14, 1919.

[3] R. W. Crum, *Proceedings* Am. Soc. for Testing Materials, 1919.

However, it is not apparent that determination of absolute volumes and density are essential, and a study of available data indicates that a diagram showing the change in loose volume proportions and sand content will be sufficiently accurate on work where the use of pit run materials is justified. The diagram of Fig. 4 may be used to determine the reduction in volume of pit run aggregate in the mix for increases in proportion of fine material.

For example, assume that pit run material containing 40 per cent fine aggregate produces a concrete of specified quality in the proportions 1 to 5 by loose volume measure. The pit run material now contains 80 per cent sand. What change in proportion must be made to maintain the specified strength?

Increase in sand content has been $80 - 40 = 40$ per cent, and from the curve an increase in sand content of 40 per cent requires a 24 per cent reduction in volume of aggregate used. Then $5 - (0.24)(5) = 3.8$, the new aggregate volume. The proportion required is one part cement to 3.8 parts pit run aggregate.

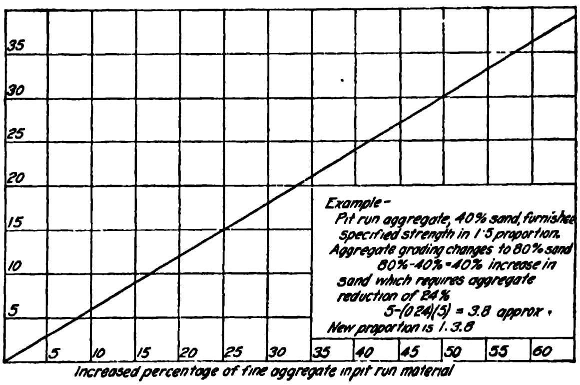


FIG. 4.—Diagram showing reduction of aggregate volume required to produce concrete of constant compressive strength as the proportion of fine material in the pit run increases. The relations shown are approximate but of sufficient accuracy where the use of pit run material is justified.

It must be kept in mind that this diagram furnishes only approximate results. The resulting concretes will be on the safe side, without the use of the large excess of cement required when cement content is made proportional to the sand content of the aggregate.

5. Method of Proportioning Concrete Based upon Preliminary Tests of Aggregates.—This method of proportioning which is described in detail and applied to aggregate used in one locality, evaluates the concrete making properties of one or more aggregates by coordination of laboratory testing methods and concreting practice in the field. Many of the individual features are not new but have been employed in whole or in part by engineers and contractors. Quality of concrete depends upon many factors such as cement, aggregate, mixing water, method of mixing and curing which are sometimes difficult to control, and more often difficult to measure. It is therefore to be expected that under the best of control conditions concrete strengths for given materials and conditions will vary within fairly definite ranges which may be determined by experiment.

In most localities concrete aggregates are obtained year after year from the same source of supply and are usually fairly constant in quality. For aggregates in common use there is no reason why their concrete making qualities with different cement contents and within the widest ranges of flowabilities employed in practice cannot be definitely established within reasonable limits for practical working conditions. Relations so established must be modified in practice, making proper allowance for differences in cement quality, minimum usable flowabilities and other variable factors. These allowances should be based upon test data aided by the results of previous experience.

In the laboratory proper consideration must be given to the conditions and factors which the engineer must provide for in the field. In comparative tests the two fundamental factors, equal cement contents and equal flowabilities must be satisfied. The range of flowabilities must be that which is required in the field for various types of work. Cement contents must vary from the leanest to the richest which may be required in practice to furnish concretes of the specified quality.

This method of proportioning may be divided into three divisions as outlined below:

5a. Step No. 1—Selection of the "Desirable" Fine-Coarse Aggregate Grading.—Divide the material from each source of supply on the No. 4 or the ¼-in. screen. Prepare concretes having the same cement contents and flowabilities, varying the relative quantities of fine and coarse aggregates in the various batches. At least three fine-coarse aggregate combinations should be employed—such as, 3 volumes fine : 7 volumes coarse, 4:6 and 5:5. If the fine aggregate is lacking in the finer particles, more than 5 parts fine may be used. If the fine aggregate contains a large percentage of fine particles, having a surface area per 100 grams of 1,000 or more square inches, the 5:5 combination may be dropped and a 2½:7½ added. Ordinarily the first three mentioned combinations will permit selection of the "desirable" combination. These tests should be fully comparable as to cement quality, cement content, flowability and other conditions. One cement content and one flowability will generally be sufficient to establish the best fine-coarse aggregate combination as well as the best source of supply. Freedom from segregation in the wetter flowabilities, as well as compressive strength should be the basis of selection. Weights per cubic foot of dry mixed aggregate should be obtained for each combination. Considering material from any one source the dry aggregate combination having the greatest unit weight will generally be found to produce the strongest concrete. For some aggregates the maximum weight combination may be harsh working and show a tendency to segregate in the wetter mixtures due to a deficiency of fine material, in which case the combination next higher in sand content should be chosen. That the "desirable" fine-coarse aggregate relation is that one having the greatest unit weight is not stated as a general law, but has been found true in working with aggregates of the same type having widely different characteristics. The aggregate combination having the greatest unit weight will of course have the highest density and a minimum of voids for aggregates of the same apparent specific gravity, while in the dry condition, and to this extent this method may be considered as a void method of proportioning in its selection of the aggregate grading. This apparent law should not be relied upon but selection should

be based entirely upon compressive strength tests of fully comparable concretes. Further study and experience may permit the omission of actual strength tests in Step No. 1 and justify the selection of the best source of supply and the "desirable" gradation on the basis of maximum unit weight or low void content, etc. This maximum density of aggregate should not be confused with maximum density of concrete since the "desirable" grading will usually result in a concrete of *lower* density or solidity ratio than a grading having a lower sand content.

5*b*. Step No. 2—Determination of Cement Content—Strength Relations for "Desirable" Gradation in Concretes of Different Flowabilities.—In Step No. 1 the "desirable" fine-coarse aggregate combination as well as the best source of supply was determined. Under Step No. 2 concretes are prepared with varying cement contents which will result in strengths throughout the ranges ordinarily specified and employed in practice for the wettest and driest consistencies. These strength results are arranged in diagrammatic form showing the relations between compressive strength, cement content per cubic yard of concrete, volumes of cement and fine and coarse aggregate. The strength values so obtained are those resulting under the accurate laboratory control employed and may be somewhat higher as well as of smaller variation than can be obtained in the field. From this diagram, knowing in advance of the beginning of work the specified strength and the probable minimum flowability which can be used, an estimate can be made of the quantity of cement and fine and coarse aggregate which will be required per cubic yard of concrete to attain the concrete strength specified. This proportioning diagram will show at a glance the quantities of materials which must be employed to result in a concrete of any specified quality for aggregates from this particular source.

5*c*. Step No. 3.—Verification and Modification in Practice under Field Conditions.—Owing to variation in brand and quality of cement, variation in physical condition of aggregate, inaccuracies in measuring aggregate volumes, differences in flowability from batch to batch, and variation in regard to curing conditions, it is not to be expected that concretes produced on the job will be as uniform in quality, except under most rigid inspection and supervision, as those of the laboratory from which the working diagram is originally devised. A systematic method of sampling and testing of field concrete under uniform inspection conditions will indicate modifications which should be made to the original diagram. After a few applications to field work, a revised diagram may be prepared which will insure production in the field of concretes equal or slightly superior to those assumed in advance. With aggregates from the same source and cement from the same mill, it is probable that the greatest variations from the laboratory test values can be accounted for by variation in cement quality, a change brought about in the concrete making qualities of the cement by age and storage conditions. A standardized compression test for the neat cement can be employed to give direct comparison between the cement which it is proposed to use on a particular job and the quality of the cement used in the tests from which the diagram is derived.

6. Practical Application of Preceding Method of Proportioning.—The method of determining the proportions of cement and aggregate which will result in concretes of the desired quality as described in the preceding article is applied below to a pit run gravel aggregate.

The gravel employed was of silicious and limestone composition with many of the larger particles covered with rough and well-bonded limestone concretions. Material from this pit usually contains a very high percentage of sand with 90 to 95 per cent passing the $\frac{3}{4}$-in. screen and 60 to 80 per cent the No. 4 screen. The aggregate was separated on the No. 4 screen, rejecting all particles over $\frac{3}{4}$ in. The colorimetric test resulted in a light yellow solution indicating that no harmful organic matter was present. Loose weights per cubic foot of the sand and gravel were determined as 98 and 93 lb. respectively. The specific gravity was found to be 2.66. Granular analysis of the sand and gravel are shown in Fig. 5 and indicate that the sand lacks a proper amount of particles retained on the 28- and 48-mesh sieves to result in a straight line gradation. Dry sand and gravel were combined in the proportions 3.5 : 6.5, 5.5 : 4.5 and 7.5 : 2.5, and loose and packed weights determined as shown in Fig. 5. Using these three combinations concretes were prepared

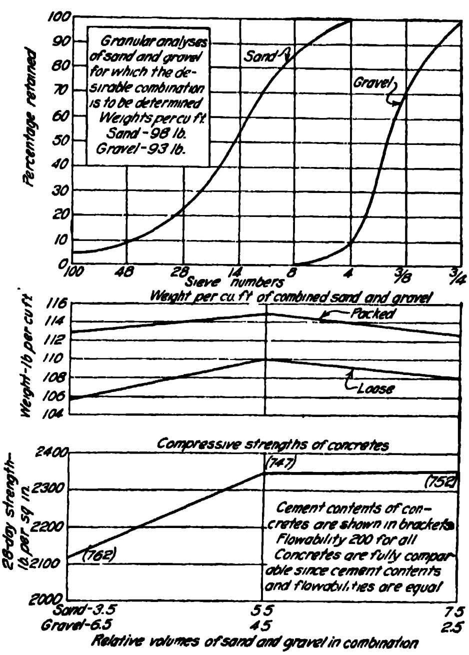


FIG. 5.—Selection of the "desirable" sand-gravel combination. Concretes having equal cement contents and flowabilities are prepared with varying ratios of sand and gravel. It has been general practice to specify the 3.5:6.5 combination, but for well graded sands a combination approaching 5.0:5.0 will usually be found desirable. The gradation or maximum size of the coarse aggregate has little effect within wide limits.

having the same flowabilities as measured by the flow table, and approximately the same cement contents. Strength tests at 28 days indicated that either combination would be suitable in concrete. The 3.5 : 6.5 sand gravel concrete was harsh working and showed a tendency to segregate, so that the 5.5 : 4.5 combination was selected as most "desirable." Ordinarily unless previous experience with the cement and aggregate can be used as a basis for selection of cement content to be used in Step No. 1, two cement contents should be employed. In this case a volume of one part cement to approximately 3.40 volumes (sum of separate volumes of sand and gravel) of aggregate resulted in a compressive strength of 2,300 lb. at 28 days with a cement content of approximately 747 lb. per cu. yd. of concrete in place.

In order to determine the range in compressive strength values for different cement contents and flowabilities, cement contents of 0.18, 0.26 and 0.34 of the sum of the aggre-

gate volumes were selected with flowabilities of 160 and 200. The 28 days compressive strength tests, together with cement content in pounds per cubic yard, and aggregate volumes per bag of cement are shown in Fig. 6, the "Proportioning Diagram." This diagram indicates at a glance the relations between compressive strength and cement content of concretes prepared with aggregates from this source of supply. The same data has been rearranged in Fig. 7. Either type of diagram may be used in practical work.

Assume that a concrete having a compressive strength of 1,500 lb. per sq. in. is required and the type of work is such that a flowability of 200 can be employed with a reasonable amount of tamping and spading. In Fig. 6 project horizontally on the 1,500-lb. line to its intersection with the strength curve for 200 flow, then project vertically to the inter-

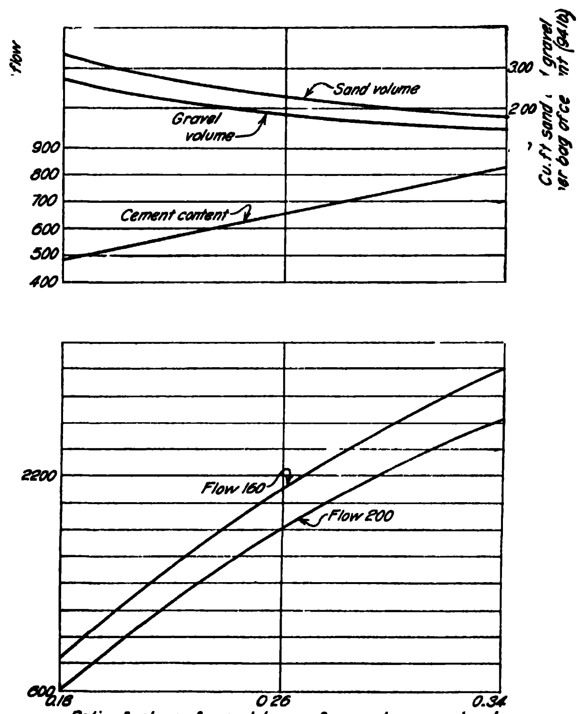


FIG. 6.—Proportioning diagram showing the compressive strengths of concretes containing the "desirable" gradation for the extremes of cement contents and flowabilities used in practice. Knowing the specified strength and the minimum flowability which can be used, the cement content required per cubic yard of concrete as well as the batch proportions of sand and gravel per bag of cement can be read from the curves.

(*Note.*—The above relation holds true for the particular cement and aggregate used in the tests.)

section with the cement content line which indicates that 600 lb. of cement are required per cubic yard of concrete in place. To determine what proportions of cement and aggregate will result in such a concrete continue upwards to the intersection with the sand and gravel curves at which the readings are 2.5 cu. ft. sand and 2.1 cu. ft. gravel per bag of cement of 94 lb. Since batches are usually measured on the basis of bags of cement, the batch quantity is shown at a glance to be 1 bag of cement to 2.5 cu. ft. sand and 2.1 cu. ft. gravel or multiples of this ratio. In Fig. 7 quantities of cement in barrels and aggregate in fractions of a cubic yard required to produce one cubic yard of concrete in place can be estimated. For each yard of concrete 1.60 barrels cement and 0.60 cu. yd. sand and 0.49 of gravel are needed.

In practice a number of conditions are met with which may alter the relations as determined in the laboratory and the final working diagram which will apply to available materials in any locality may be modified by the results of field tests and inspections. The degree of accuracy with which qualities can be predicted will be dependent upon the care employed in the field and judgment in the use of the diagram, taking into account variations in cement quality, mixing and placing methods, and curing conditions.

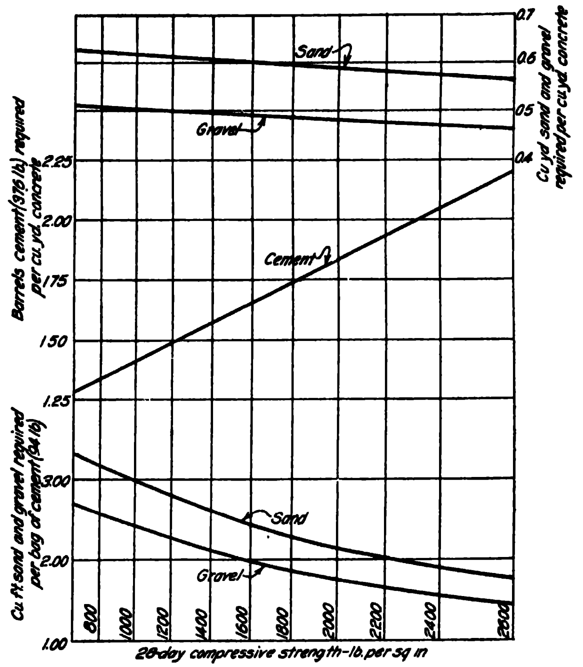


FIG. 7.—Another form of proportioning diagram derived from Fig. 6 showing the batch quantities of sand and gravel required per bag of cement; cement in barrels and aggregate in cubic yards per yard of concrete in place; for a working range of compressive strengths. (*Note.*—These relations hold as shown only for the particular cement and aggregate used in these tests.)

7. Quality of Concrete as Affected by Processes in Its Preparation.

7a. **Mixing.**—The development of machines for mixing concrete has practically eliminated the older method of hand mixing. Except on very small jobs the mechanical mixer is more economical and concrete produced is more uniform in quality. Good concrete can be produced by hand mixing but the proper procedure is too often ignored except under the strictest supervision. Continuous mixers which automatically proportion the materials should not be permitted as their product is not uniform. Continuous mixers of the pug mill type are satisfactory provided premixed cement and aggregate are fed to it. This type is most commonly used in preparing very dry consistency mortar mixtures.

Little has been done toward determining the efficiency of the various drum mixers now on the market. The few tests made have shown that type or make of mixer has little effect on quality of concrete, so long as certain necessary precautions are given consideration. The mixing of concrete is quite analogous to the mixing of neat cement in laboratory tests. Specifications state that neat

cement shall be vigorously kneaded for a period of 1 min. This length of time is required to result in a uniform plastic condition which is little changed by further work. Less work will result in a stiffer crumbling mass with the same quantity of mixing water. Variation in the time of mixing of concrete gives the same general result. With other factors and conditions the same it is found that plasticity or flowability, as well as compressive strength are increased by continuing the mixing action up to a certain point, beyond which added work produces only slight increase in these qualities. The length of time required to attain this uniform, homogeneous, plastic condition is dependent upon richness of mix, condition of aggregate and quantity of mixing water as well as type of mixer and speed of rotation of drum, but tests have generally shown that a mixing period of ¾ to 1½ min. is sufficient. The use of wet aggregate materially reduces the mixing time required. Whether or not certain machines will complete this mixing action in a shorter period of time than others can only be determined by tests which will also take account of such factors as loading and discharge speeds.

So far as quality of concrete produced is concerned there is probably little choice among batch mixers now in use. The choice of mixer is rather dependent upon plant requirements of the particular job, first cost, maintenance, and operating cost.

7b. **Placing and Curing.**—Quality of concrete is little affected by ordinary methods of transporting and placing except in so far as strength is sometimes reduced by the excessive quantity of mixing water required in flat chutes by the tower and chute system. In some cases the excess flowability is not required by the shape of form or presence of reinforcing steel and a plastic quaking consistency which might otherwise be employed would result in concrete of considerably higher strength, since the degree of fluidity is fixed by the condition of the chuting system. Flowability which must be employed is determined by size and shape of form, presence of steel and accessibility, both for the purpose of placing and tamping. In any case flowability should be restricted to the minimum required to place the concrete properly with a reasonable amount of spading. Sloppy concretes having greater flowability than is required are inferior in strength and wasteful of cement. Where fluid mixtures must be used, concretes high in sand content will be beneficial in preventing segregation. With coarse sands, the sand content may be made equal in volume to the coarse aggregate, the first being increased and the latter reduced so as to maintain the same cement content per volume of finished concrete.

Proper curing conditions are as essential as proper proportioning to obtain concrete of specified quality. Concrete will harden most uniformly under water, or in a thoroughly moist atmosphere where there will be no evaporation losses and all moisture is retained in the mass. Concrete roads are commonly cured under water by flooding and concrete floors may be protected by damp sawdust, shavings or sand. Such precautions cannot always be employed, but in all cases some attention to covering and sprinkling of horizontal surfaces and the retaining of forms in position as long as possible on vertical surfaces will be of value. Concrete as placed contains sufficient water to hydrate the cement, providing it is not removed by evaporation. Such water as is lost during the first few days will not be fully effective if re-supplied at some later period.

8. Field Inspection of Concrete.

8a. Control at the Mixer.—The quality of concrete cannot be measured or predicted for any given materials without detailed knowledge of the various processes and steps employed in its fabrication and placing, as well as of the characteristics of the materials. There can be no assurance that concrete quality may not vary as much as 50 to 75 per cent during a day's run of a mixer unless a systematic method of inspection and testing is employed. On work of importance at least one and preferably not less than two inspectors should be provided who will devote their full time to the inspection and supervision of the concreting process. On small work one man can oversee the work at the mixer as well as the placing of concrete in the forms, but when the distance between is great or one position is not easily accessible from the other a second inspector should be provided. Supervision at the mixer is of greatest importance and the inspector's more important duties consist of checking up quality and volumes of cement and aggregate, limiting flowability to the minimum usable and insuring a proper time of mixing. The mixer should be charged in such a manner that the exact volumes of cement and aggregates used can be estimated at a glance. The use of charging hopper into which the batch is placed before entering the mixer is not only economical but of assistance in controlling quantities. The use of wheelbarrows or two-wheel carts of known capacities is recommended. Materials may be charged direct from storage bins providing the charging hopper can be so divided off or marked to indicate volumes contained. The charging of a mixer by shoveling materials directly into the drum should not be permitted. The use of a continuous belt conveyor onto which both fine and coarse aggregates are loosely thrown should not be permitted unless the actual volumes of each can be definitely determined just before entry into the mixer. Careful inspection during transporting and placing of concrete will assist in bettering quality on most work, but the main effort should be centered at the mixer.

8b. Molding and Storing of Field Test Specimens.—Whenever possible, samples of concrete for test should be taken after its final deposition in the form. What is desired is the determination of quality of concrete as placed in position and this method of sampling tends to eliminate variations in individual batches.

Cylindrical test specimens are recommended, the diameter being four times the maximum aggregate size and the length two times the diameter. For ¾-in. aggregate, a 3 by 6-in. cylinder mold may be used and, for 1½-in. aggregate, the 6 by 12-in. form is required. Capacities and dimensions of available testing machines must be considered in connection with the preparation of large size test pieces since a 6 by 12-in. cylinder may exceed the capacity of a 100,000-lb. machine and an 8 by 16-in. cylinder may exceed the limit of a 200,000-lb. machine.

Molds are usually made of iron pipe or machined castings, sheet metal or paper. The cost of machined cast iron molds does not warrant their use. Pipe molds are made by splitting black or galvanized iron pipe along one element, removing a strip about ⅜ in. wide. Before molding, the mold is closed by means of a metal collar or clamp and the test specimen is later removed by loosening the clamp. It is not necessary that the inner surface be machined to size providing the true diameter of the specimen is used in computing the unit load. Sheet metal bent to cylindrical form with a suitable clamping device will be found more

satisfactory and more economical than the heavier metal types. All metal molds may be made watertight by the use of circular metal discs held in the bottom of the mold by lugs attached to the side. The paraffined cardboard mold has been found satisfactory for molding field test specimens. They are very light as compared with metal and have the added advantage that they can be split along one element and nested together for shipment. Previous to use, the seam is closed by lacing with wire staples spaced about ½ in. apart. The waterproof paper jacket, which is left in place until ready for test, protects the specimen during shipment as well as assists in retaining the moisture in the mass during the early curing period.

The following procedure should be followed in preparing field test specimens: Set up the molds, which have been previously oiled, if metal, on a plane oiled surface of metal or wood. Obtain sample of concrete from the work and thoroughly remix to a homogeneous condition if segregation takes place. Fill all molds one-third full and thoroughly spade or tamp the concrete. Continue filling and tamping until the mold is full. A very fluid consistency should be very lightly stirred or spaded. For stiffer mixtures a tamper of steel or wood having an end area of 2 or 3 sq. in. will be required. The exact methods and amounts of tamping vary greatly with the consistency. In any case the work done should be sufficient to completely fill the mold and result in a honogeneous uniform mass. After completion of tamping the mold should be heaped full and the side of the form lightly tapped. After a period of an hour or so remove excess material with a trowel to a distance of ¼ to ½ in. below the top. Refill with a neat thick cement paste, trowel smooth and cover with an oiled metal plate or smooth board. The specimen should be allowed to remain in position for approximately 24 hr., after which the mold should be removed and the specimen stored in damp sand or water. The purpose of the neat cement capping is to furnish an end which can later be ground plane on a flat steel plate with water and fine sand in case such a surface was not obtained in the process of molding. If the concrete to be molded is low in cement content, or of a dry consistency, it will be found advantageous to spread neat paste on the support before setting the form in position in case the molds are not provided with bottoms.

Since it is impossible to subject the small test specimens to the same curing condition as the large mass, storage should be standardized to result in a constant temperature and prevent loss of water by evaporation. In cold weather, specimens should be molded indoors, but they should not be subjected to abnormally high temperatures or permitted to dry out at any time during the curing period. If specimens must be shipped some distance for test, the time in storage and time in transit, during which drying will probably occur, should be made uniform during the progress of the work. In case such specimens are tested in the dry condition, proper consideration must be given this factor in comparing test results with the strength values shown in the proportioning diagram, since these field test values will probably be higher.

Owing to the limited capacities of available testing machines, it has been customary on some work, where aggregates up to 4 or 6 in. maximum size have been used, to mold small specimens with the fine material screened from the concrete. Such tests will no doubt indicate comparative qualities but may give little idea of the actual strength values obtained in the work as these

mortars may test from 50 to 100 per cent higher than the original concrete as placed.

While the cylindrical form of compressive test specimen affords the best means of checking up the quality of concrete actually placed, transverse tests of plain and reinforced concrete have been employed for this purpose. Although beam tests may fairly well indicate quality they should not be relied upon where compression tests are possible. Rather wide differences in quality as measured by compression tests may be represented in beam tests by differences not greatly in excess of the working range of error of the latter. The advantage of the beam test is the ease with which a satisfactory machine can be built. Tests of reinforced beams add to the variable factors which are difficult to control or measure, and it is doubtful whether the cost of building and operating the more powerful machine required will be justified by the results obtained. A hydraulic machine for compression tests of cylinders will prove more valuable and economical.

9. Some Clauses for the Concrete Specification.—The adoption of the method of evaluating the concrete making qualities of an aggregate as described in Art. 5, together with the employment of the flow table for measuring and controlling flowabilities will permit the adoption of a specification in which the following provision may be included.

Cement

Cement shall comply with the Standard Cement Specifications[1] and Tests for Portland Cement of the American Society of Testing Materials.

Aggregates

Aggregates shall be composed of hard and durable gravel, crushed rock, blast furnace slag or similar materials free from particles which may later disintegrate under the action of air and moisture. Slags shall not contain more than 1.5 per cent sulphide sulphur or weigh less than 70 lb. per cu. ft. For structural work no restrictions are placed upon aggregate grading except that it must be such when combined in concrete that there will be no marked segregation during the process of placing.

Concrete

Concretes of the following qualities and flowabilities shall be used as specified:

(Show a list of the various divisions of work—such as footings, retaining walls, columns and floors—with the specified maximum strength which must be attained together with the maximum allowable flowability for each grade of concrete.)

The contractor shall submit a list of the batch proportions which he proposes to employ for each quality of concrete, as obtained from the "Proportioning Diagram" and, after approval, such proportions shall be employed until modification appears desirable from the results of the field tests, or there has been an appreciable change in quality of cement or aggregate.

Fine aggregate whenever practicable shall be thoroughly wetted before being charged into the mixer.

Concrete shall be mixed in a drum type power driven machine with a loading hopper which will permit accurate measure or inspection of volumes used when materials are not first handled in carts or barrows of known capacities or by an automatic weighing device.

Mixing shall continue for a period of not less than 45 sec. after the entire batch is introduced and before discharge commences.

Ordinarily flowability will be determined from samples of concrete which have been placed in final position in the work, but determination shall also be made at the mixer if requested by the inspector.

[1] See Appendix *A*, p. 724.

10. Laboratory Methods for Use in Selecting and Determining the Concrete Making Qualities of Available Aggregates.—Laboratory methods for determining the concrete making qualities of concrete aggregates together with lists of necessary equipment are shown below. Often some of these tests may be omitted when working with materials whose properties are known from previous experience. The following details apply especially to pit run gravel aggregates and will require slight modification to apply to the other types of materials.

10*a*. Tests of Aggregates and Selection of "Desirable" Gradation.—*Important Equipment:* Scales and balance of 200 and 15-lb. capacities graduated in the metric system, cubic foot container of wood or metal; sieves Nos. 100, 48, 28, 14, 8, 4, ⅜, ¾, 1½, the latter four preferably having 18-in. diameter frames; 3 per cent solution of NaOH and 12-oz. graduated prescription bottles for colorimetric test; galvanized mixing pans not less than 24 in. square; trowels; $\frac{1}{10}$ or $\frac{1}{20}$ cu. ft. density vessel of metal, a 6×12-in. cylinder form with bottom attached will serve; a flow table; 3×6-in. and 6×12-in. cylinder molds; grinding plate for surfacing ends of test specimens; various size containers; testing machine; etc.

Order of Procedure:

(1) Visual inspection of aggregate.
(2) Weight per cubic foot of pit run.
(3) Granular analysis of pit run.
(4) Separate pit run aggregate into two lots on the No. 4 screen.
(5) Make colorimetric test for organic impurities in sand.
(6) Determine physical properties of sand, such as weight per cubic foot, specific gravity, surface area.
(7) Same for gravel.
(8) Determine loose weight per cubic foot of various sand-gravel combinations—such as 3:7, 4:6, 5:5—by volume.
(9) Employing the aggregate combinations of No. 8, prepare batches of concrete so proportioned that all will have equal cement contents. Since bulking effect of high sanded combinations is great, batches of high sand proportions should contain a greater quantity of cement. Assuming that one volume of cement is to be used with five volumes (sum of individual) of aggregate, the volumes required will be as follows:

Volumes (Loose)

Cement (Approx.)	Sand	Gravel
1	$(0.3 \times 5) = 1.50$	$(0.7 \times 5) = 3.5$
1.02	$(0.4 \times 5) = 2.00$	$(0.6 \times 5) = 3.0$
1.04	$(0.5 \times 5) = 2.5$	$(0.5 \times 5) = 2.5$

Make all flowabilities equal by means of the flow table. Calculation of quantity of cement in each batch will serve as a check on the volumes of cement assumed. Variation in cement content of wet concrete should not be greater than 3 per cent.

Selection of aggregate grading for use should be determined on the basis of strength and freedom from harshness and segregation. Generally the combination lowest in sand content which shows no tendency to segregate should be chosen.

For sand having a surface area between 400 and 600 sq. in. per 100 grams, the 5:5 combination will ordinarily prove best. When surface area exceeds 1,000 sq. in. per 100 grams "over-sanding" will not usually be beneficial and the more common sand gravel relation, approaching 3:7 will likely prove best. In general the coarser the sand, the greater the relative quantity which may be employed. With one aggregate available, the selection of the "desirable" grading requires at least three batches of concrete with tests to be made at the 28-day period. When aggregate from more than one source of supply is available, the number of batches will be multiplied accordingly. For accuracy, volumes should be transposed to weight measure. The amount of time and labor required for this step may be greatly reduced by rejecting all coarse material retained on the ¾-in. screen, thereby permitting the use of 3 × 6-in. cylinder molds, for which the small batches can be readily mixed by hand. Discarding the coarse particles will usually have little effect on strength.

10b. Strength of the "Desirable" Gradation in Concrete.—Not less than two flowabilities and three cement contents should be employed. Previous experience with the same or similar aggregates will be of assistance in choosing the cement contents. Ordinarily three cement contents of 1:3, 1:4½, and 1:6 by volume, the aggregate values being the sum of the individual volumes, will cover the range of strength values assumed in design. Flowabilities of 160 and 200 will usually furnish sufficient information as to effect of variation of water content on strength. A flow of 160 is plastic and quaking suited only to large volumes or deep forms where the mass may be spaded and vibrated. A flow of 200 will prove sufficient for most structural work except in special cases.

Assuming that the 5:5 combination has been selected, batch proportions may be selected as follows:

Volume proportions	Volumes		
	Cement	Sand	Gravel
1:3 = 0.330	1	1.5	1.5
1:4½ = 0.220	0.67	1.5	1.5
1:6 = 0.166	0.50	1.5	1.5

For the two flowabilities 6 batches will be required for test at 28 days. Aggregate of ¾-in. maximum size with 3 × 6-in. test pieces will reduce the cost of the work. Quantity of water required will be governed by the flowability requirement but should be accurately weighed in every case. Determination of unit weight of wet concrete will permit the calculation of such values as quantity of cement per cubic yard, density and surface area of aggregate per unit weight of cement in the mix.

Strength values obtained when worked up in diagrammatic form furnish the "Proportioning Diagram" which must be considered as an approximate relation until the strength values may be checked against the results of tests of field concretes which have been prepared under a systematic method of inspection.

10c. Calculation of Test Data.—Assume that a 1:2:4 by volume concrete has been prepared with materials having the following physical characteristics:

1 volume cement — weight 94 lb. per cu. ft. — specific gravity = 3.15
2 volumes sand — weight 110 lb. per cu. ft. — specific gravity = 2.65
4 volumes gravel — weight 100 lb. per cu. ft. — specific gravity = 2.65

Batch of concrete is made up of:

1 volume cement = 4.270 kilograms (9.4 lb.)
2 volumes sand = 9.980 kilograms (22 lb.)
4 volumes gravel = 18.140 kilograms (40 lb.)
Water = 2.850 kilograms (6.3 lb.)

35.240 kilograms, total batch weight

Net weight of ⅙ cu. ft. of concrete = 13.15 kilograms (in density vessel)
Ratio of ⅙ cu. ft. to volume of batch = 0.373

Pounds Cement per Cubic Yard of Concrete in Place:

$0.373 \times 4.27 = 1.59$ kilograms cement
1.59×2.98 (constant for ⅙ cu. ft. measure) = 474 lb. cement per cu. yd.

Density or Solidity Ratio:

The ⅙ cu. ft. vessel has a volume of 5,660 c.c. approx.

Absolute volume of cement $= (0.373)\left(\frac{4.27}{3.15}\right)(1{,}000) = 505$ c.c.

Absolute volume of aggregate $= (9.980 + 18.140)\left(\frac{3.73}{2.65}\right)(1{,}000) =$ 3,960 c.c.

Total solids in 5,660 c.c. = 4,465 c.c.

Density or solidity ratio $= \frac{4{,}465}{5{,}660} = 0.790$

Surface Area of Aggregates per Grams of Cement:

From the results of the granular analysis the areas of sand and gravel have been computed as 580 and 48 sq. in. per 100 grams, respectively.

Total area of aggregate in batch:

Sand (99.8) (580) = 57,900 sq. in.
Gravel (181.4) (48) = 8,700 sq. in.

66,600 sq. in.

Grams cement in batch = 4,270

Surface area per gram of cement $= \frac{66{,}600}{4{,}270} = 15.6$ sq. in.

Water Cement Ratio:

$$\frac{w}{c} = \frac{\text{Volume mixing water}}{\text{Volume cement}}$$

Volume of cement $= \frac{4.270}{1.5} = 2{,}850$ c.c. (approx.)

Volume of water = 2,850 grams c.c.

then $\frac{w}{c} = \frac{2{,}850}{2{,}850} = 1.000$

Of the above calculations, that for cement content per cubic yard should always be made. There are two fundamental requirements which must always be provided for in comparing concrete lengths. That of equal consistencies is complied with when flowabilities are made equal. Unless cement content is intended to be a variable, it is equally essential that cement contents be approximately equal. Even when strength comparisons are not to be made, the cement content is necessary to accurately compute costs. In checking up the proportions used in the field it will often be difficult to obtain a correct value for the quantity of water contained in the batch, a value which is essential for computing the cement content per cubic yard. This may be fairly accurately determined by mixing a small batch of concrete by hand, employing the same relative quantities of cement and aggregate and adding mixing water until the flowability as measured on the flow table is the same as that of the field concrete. The comparative weights of the two wet concretes in the same density vessel will serve to indicate how closely the field concrete has been duplicated.

TRANSPORTING AND PLACING CONCRETE

By James Cowin

The transportation of the mixed concrete from the mixer to its position in the forms is an operation which will repay careful study, as every job is a new problem and the method which will deliver the concrete in the best condition and at the least cost on one piece of work may not be the most advisable to use on a very similar structure under different working conditions or at a different season of the year.

The object of every transportation system is to deliver the concrete to the forms efficiently and economically. The concrete must not have an opportunity to take its initial set during the operation. Also it must be carried in such a way that segregation will not take place and the deposition must be continuous and the system water tight. The method to use is the one which will fulfill these specifications and will do it at the least expense, considering both the first cost and the subsequent cost of operation.

Speed of operation has been, perhaps, over-emphasized in considering the merits of the various methods of transportation for it is the centering and forming which consumes the time in ordinary building construction and not the placing of the concrete after the forms are ready to receive it. The ideal system would be one in which the forming, the laying of the reinforcing steel, and the concreting proceeded at the same rate, having a trained concreting crew who would be working continuously at the same operation from the pouring of the footings to the completion of the concrete skeleton. This is not entirely possible, due to the numerous construction joints which might result, although the danger of obtaining a poor bond between new and partially set concrete is also overstressed if the connection is properly made. But the best modern practice is, nevertheless, tending towards small continuously operating units.

11. Methods of Transporting Concrete.

11a. Barrows.—Ordinary barrows are used on small jobs which will warrant only the most primitive equipment and have no place on work involving any considerable yardage except under special conditions. These

special conditions will include small operating space, difficult trestle centering, scattered work, where the making of runways is an important item, and in general, where facility of movement is the controlling factor.

11*b*. Carts.—Two wheeled carts of 6 cu. ft. capacity are next in point of cost above the barrow and for the average job it would be hard to find a more pliant, satisfactory and economical method of transportation. A man can handle a cubic yard of concrete in one-half the time required by a man with a wheelbarrow and with considerably less physical exertion The carts will require two lines of runways instead of the one required for the barrow but as these runways will be partially salvaged this additional expense is not of much importance in comparing costs. Using either of these methods the concrete can be mixed at ground level, poured into the cart or barrow, and hoisted to the floor required, an empty being returned to the mixer on the counterweight platform of the hoist at the same time. Carts or barrows can also be kept to the floor on which they are being used by elevating the mixed concrete to a hopper with skips or buckets and drawing from this bin. This gives more even operation as pouring is not interrupted by slight delays at the mixing plant or the hoist. This hopper should hold at least a batch and a half and should be emptied from time to time to prevent settling of the heavier aggregate and to keep the sides free from setting concrete.

The hoisting tower will preferably be built of wood as it is more economical and can be easily extended as the work progresses. The steel tower of the spouting plant is not required as the building follows it up and gives the support which the greater height of the spouting tower does not get. The bucket is hung from a heavy bail which runs in wooden guides and the contents are dumped by omitting the front guide and placing a block to catch and overturn the bucket at the proper place.

On floors above the basement, the concrete is transported to its destination over forming on which the reinforcing has already been laid. To avoid disturbing the steel and more especially to give the carts an even roadbed, the runways should be carried on wooden trestles or horses, 8 to 10 in. high, which can be withdrawn from the concrete when the slab has been poured up to them. Turnouts should be provided at short intervals so that the empty and full carts are not kept waiting.

The cart or buggy is generally the last step in distribution, even if spouting is used, as it is practically impossible to design a spouting system flexible enough to pour directly into the forms.

The importance of transporting with carts cannot well be exaggerated and this method should always be carefully studied in planning a plant layout before passing on to the consideration of other forms of transportation.

11*c*. Cars.—Cars are used where the conditions are such as to justify their greater cost and where a large quantity of concrete is to be transported some distance. Cars are built in a number of sizes, ranging in capacity from 14 to 54 cu. ft. of wet concrete and run on tracks of 16- to 20-lb. rails, built ordinarily in 15-ft. portable sections. They can be obtained either with a bottom dump or with a swing or side dump. Under special conditions, such as where the building to be constructed is some distance from the point at which the concrete materials are received, it is sometimes more economical to establish

the mixing plant at the point of receipt and transport the mixed concrete to the building. Dump cars are invaluable for this work and also for road and tunnel work where the pouring is confined to long narrow sections. The capacity and inflexibility of this system will suggest other classes of work to which it is fitted.

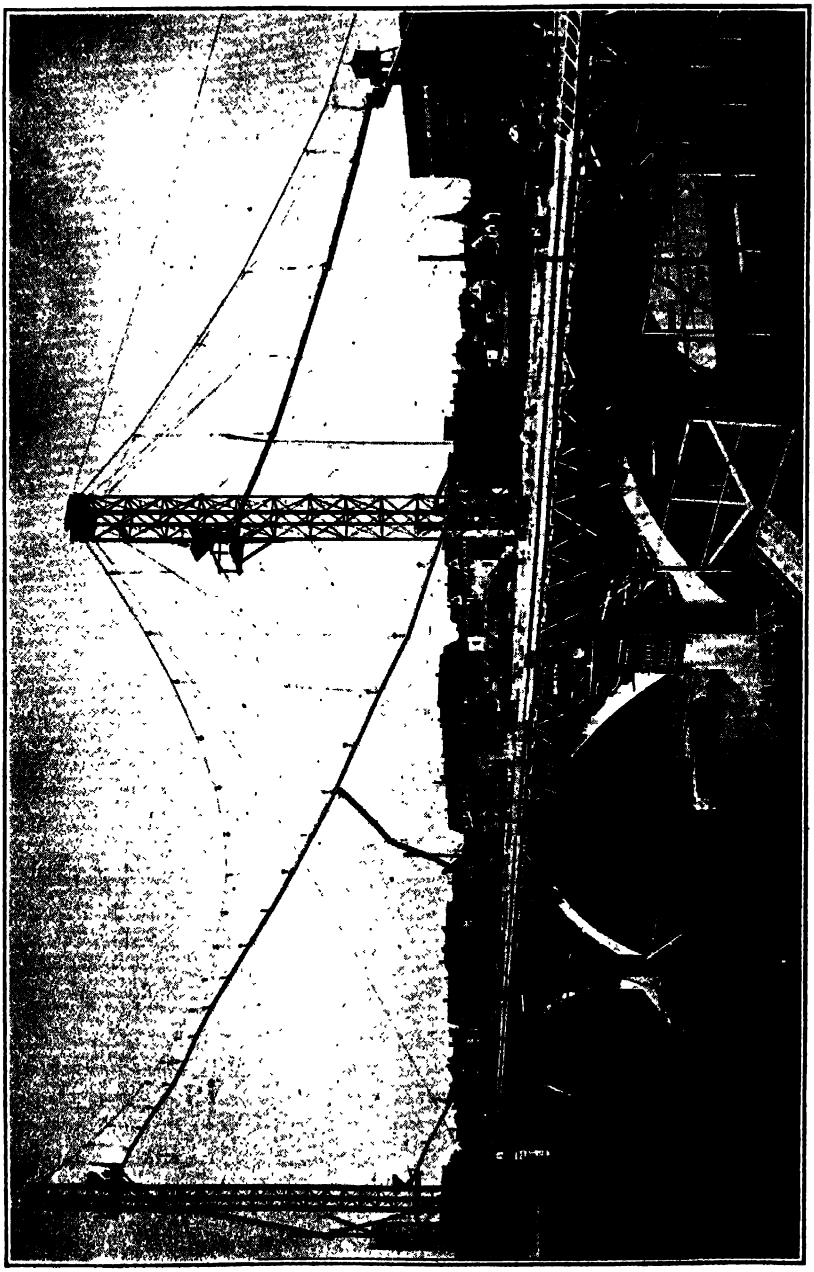
11*d*. Spouting.—Transporting concrete by gravity is claimed by its exponents to have many advantages over the two methods just described. It largely eliminates the human element from this portion of the work and it is that item which can not be accurately figured in making an estimate. It increases the mixer capacity as the output is automatically removed and there are none of the delays which seem unavoidable in the cart or car systems. It saves time as the concrete can be distributed as fast as the mixer can prepare it and the forms be made ready to receive it. The formwork can go ahead at two or more separate points as the stream of concrete can be diverted from one to the other without the necessity of providing a continuous line of supports for the runways.

Notwithstanding its manifest advantages in some ways, a spouting plant is not advisble for many kinds of work and the first cost, cost of erection and depreciation should be well considered. A gravity plant will not ordinarily pay on scattered work, on a many storied building with a comparatively small amount of concrete to the floor, or in general, to work involving less than 2,000 cu. yd. of concrete. This limit is being reduced, as the manufacturers of spouting systems are constantly bringing out new improvements and economics which in time will make a gravity system much more adaptable to general construction than it is at present.

Concrete to flow successfully through a line of chutes should not be too wet but should be of the consistency which would be selected for making the best concrete regardless of the method by which it was transported to the forms. A sloppy concrete will separate and the heavy aggregate will settle and clog the chutes. The addition of a small amount of lime to the mix will tend to make it flow smoother. The slope of the chute should be for ordinary conditions, 1 in 3, although for very long lines or where the pitch is not uniform this will have to be increased in some cases to as much as 1 in 2. The condition of the mix will also influence the pitch of the chutes—as, for example, a washed gravel concrete will flow more rapidly than a crushed slag concrete. The object is to have a slope which will deliver the concrete slowly but continuously in a well mixed mass to the forms and this can soon be determined after the operation has started. There are three general methods for supporting the chutes: (1) Suspending from a cable, (2) using a boom, and (3) carrying on tripods.

The suspension cable system is especially adapted to cases where the concrete is to be conveyed over 200 ft. and is limited only by the height of the tower. In this system the chutes are hung from an overhead cable, one end of which is attached to the hoisting tower. The average height of a tower is 200 ft. and if the distance is too great to be covered from one tower the concrete can be relayed to a second tower where it is again hoisted and distributed. The principal field for this system is for dam and bridge or similar work as the distribution is confined to a short distance on either side of the line of the suspension cable.

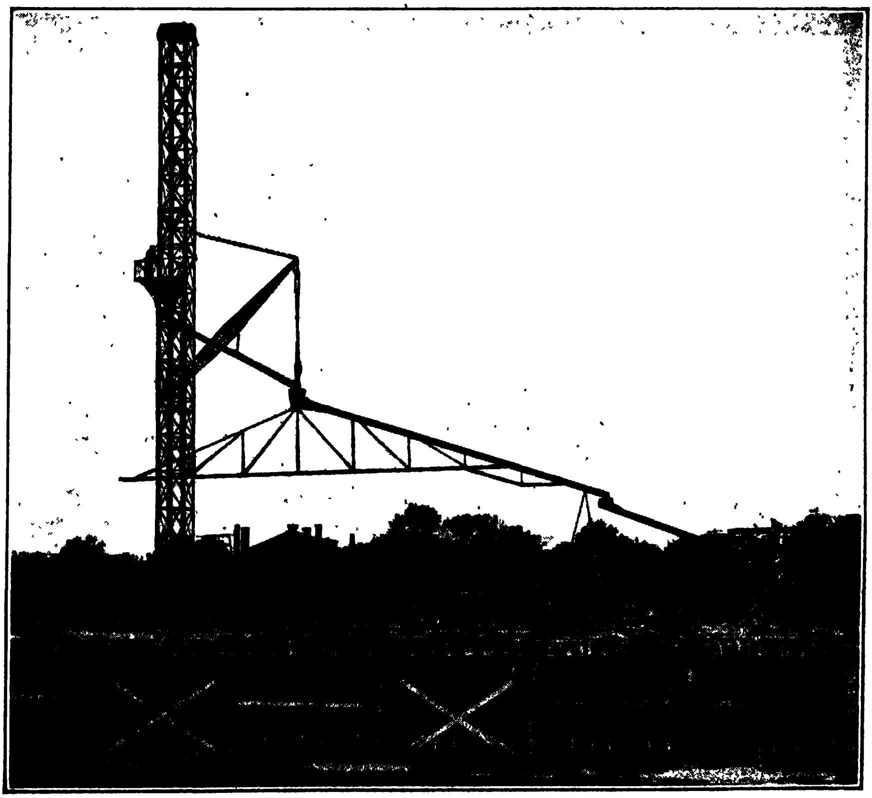
The boom plant can be used to advantage where the concrete does not have to be chuted to distances much in excess of 200 ft. The first section of chuting is



(*Courtesy Insley Mfg. Co.*)

FIG. 8.—Typical continuous line system.

supported by a boom fastened to the tower. This is followed by a counterweight chute and then by such sections of swivel head chuting as may be necessary, or the counterweight chute may be omitted and the boom chute followed directly by swivel head sections. Floor supports, such as tripods or gin poles, carry the chuting beyond the radius of the boom. This is the ideal plant for distances within 200 ft. of the tower as it is flexible both vertically and horizontally, covering the entire areas with very little time lost in shifting supports.



(Courtesy Insley Mfg. Co.)

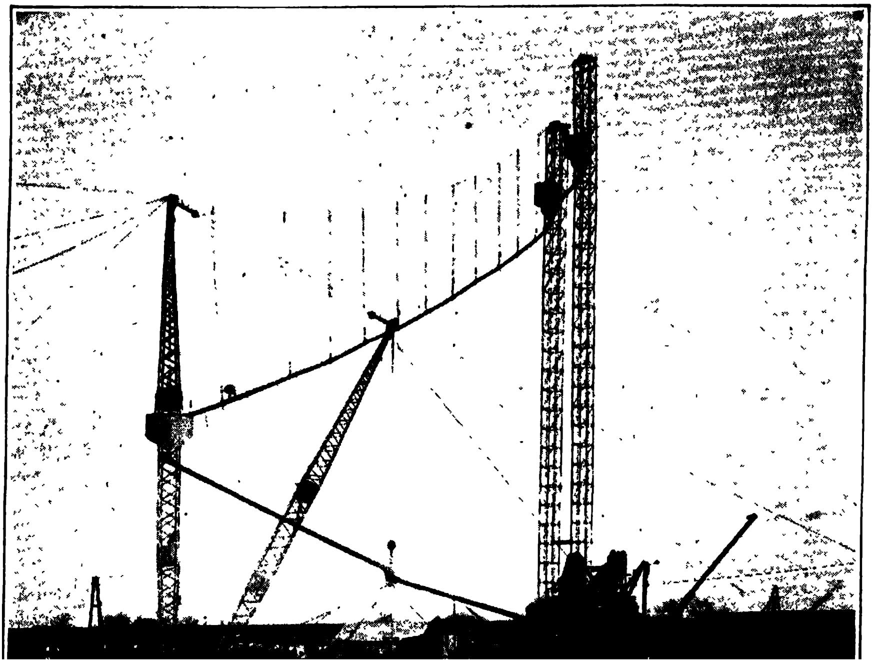
FIG. 9.—Typical boom plant installation.

The tripod system is very limited in the distance it will cover and its principal use is in extending the length of the cable and boom chute lines. Tripods or similar supports carry the chutes from the hopper to the forms and it is a cumbersome and unwieldy method involving considerable labor for every change in the point of deposit.

These three methods can be combined in a number of different ways. A cable line may carry the concrete to a second hoisting tower where the concrete is again elevated and distributed by a boom plant, or the two systems may be fed from a single tower, the boom plant handling the yardage near by and the cable line taking care of the more distant distribution. In selecting a system or combination of

systems, future use of the equipment should be kept in mind as the expense of a gravity plant is too great to be absorbed in one ordinary contract.

Up to the point where the concrete enters the chute line the equipment is the same for all systems and consists of a mixing plant on the ground, a tower, a hoist bucket and equipment for hoisting, and a receiving hopper.



(*Courtesy Insley Mfg. Co.*)

FIG. 10.—Combination of continuous line and derrick supported counterweight chute equipment.

The tower is generally built of steel on account of the height required and will be securely guyed in order to hold one end of the cable line or the weight of the boom. The average height is 200 ft. although towers as high as 375 ft. have been erected.

The hoist buckets are made in three sizes: 18-cu. ft. capacity for the output of a ½-yd. mixer, 27-cu. ft. capacity for a ¾-yd. mixer, and 36-cu. ft. capacity for

the use of a 1-yd. mixer. The receiving hopper should have a capacity of 50 per cent in excess of the capacity of the hoist bucket to provide for contingencies. In some plant designs the hopper and dumping guides as well as the boom and boom supported chuting are carried on a sliding frame which makes changes in the elevation of the receiving end of the line comparatively easy.

Chutes are made of No. 14 gage steel and the 12-in. oval chute has been found the best for the ordinary range of work. The life of a chute varies from 10,000 to 20,000 yd. depending on the position in the line and the character of the aggregate used. Where the direction of the line is fixed, rigid connections are made between the sections by bolting the flanges but where lateral play is desired, a swivel connection should be used.

11e. Pneumatic and Miscellaneous Transportation.—Using air under pressure to convey concrete to its position is sometimes necessary due to the inaccessibility of the point of placement or in cases where a thin coating is to be applied. Tunnel linings are generally placed pneumatically as it is practically impossible to pour them but, in distinction from a coating, forms and centering are used to support the green concrete.

The gun used for tunnel linings consists roughly of a large cylinder into which the mixture is dumped and a piston which pushes the concrete from the cylinder into a pipe line through which it is forced by compressed air to its position in the forms. A 6-in. pipe has proven very satisfactory if the size of the coarse aggregate is confined to 1 in. or under. The coatings are applied either for ornamental purposes or to fireproof and protect existing work. An example of the latter is in tank construction where it is necessary to tighten the bands after the concrete has set. To protect these bands from rusting a coating sufficiently thick to cover the steel is applied pneumatically.

Special conditions have brought out a number of ingenous ways for getting concrete to its destination, ranging from bucket elevators to a belt conveyor which one contractor used very economically in pouring a sewer. The object of every method of transportation is to get the concrete to the forms in perfect condition, quickly, efficiently, and economically, and every job is a new problem to be solved by the builder with this end in view either from his own experience or from the experience of others.

12. Depositing in Forms.—The final stage of transportation, the depositing of concrete in the forms, is the most imporatnt step in the process. Any separation in the mix at this point cannot be corrected and directly affects the strength of the building. For this reason the concrete should not be allowed to fall any considerable distance as the momentum of the heavier particles will carry them to the bottom of the member being poured.

Footings will in some cases be spouted if the mixing plant is located at ground level and the same precautions against too steep an inclination of the spout line should be observed here as in other parts of the work. The mixture should be spread in horizontal layers. There will be considerable laitance on the top of the footing on account of the quantity of concrete poured in one mass and this should be chipped off for the area of the column to be superimposed as the full compressive value of the concrete will be required at this point.

Columns should be poured in the center to eliminate the possibility of the coarse aggregate jamming in the 1½ or 2 in. between the hooping and the forms

and causing pockets which require patching after the forms are removed. A smoother finish is also obtained as the concrete is sieved through the spiral wire and only the more liquid portion reaches the outside faces. Columns should be poured up to the bottom of the beam boxes, or of the depressed head, and allowed to take the initial set. Six to eight hours is sufficient time. This precaution is to avoid the plane of weakness which will develop at this point if the beams, slabs and columns are poured together. The compressive strains of the setting concrete are dissipated at the free end of the columns.

Beams and slabs should be poured from the column outward, filling the form to the required depth at each cart load and keeping the juncture planes as nearly vertical as possible. The reinforcing bars should be hooked up from time to time to aid the concrete in flowing under them.

Walls should be poured in horizontal layers similar to footings and any construction joint should be in as nearly a horizontal line as possible.

13. Continuous and Even Deposition in Forms.—Continuous pouring of a unit should be maintained if it does not entail too much sacrifice in economy or smooth operation. Construction joints, if properly handled, are not especially objectionable and sometimes the haste to complete a slab in one pouring results in poorer work than in the case where there is more careful, slower mixing and handling with the work interrupted at the right place. Construction joints should be avoided at points and in planes of shearing stress. For this reason beams and slabs should be stopped in the middle of the span where there is no vertical shear, and the joint should be a vertical one in order to cut across the lines of the horizontal shearing forces.

14. Bonding Set and New Concrete.—The common and also the best method for obtaining a good joint between old and new concrete is to chip back the face of the set concrete until a fresh surface is exposed, clearing away all free and loose particles, wetting thoroughly with water and smearing on a thick grout. This should be allowed to set slightly when it is ready for the addition of the fresh concrete.

There are patented chemical compounds on the market which are guaranteed by the companies producing them to make a perfect joint.

15. Spading, Puddling and Tamping.—Air pockets are liable to occur, especially in long columns, and can largely be avoided by poling with a ⅝-in. rod as the form is being filled. Tapping the forms either with a hand or an air hammer will result in denser concrete. The amount of spading, puddling and tamping which a given concrete can stand will depend on the amount of water used in mixing. Very much disturbance of a wet concrete will result in segregation of the coarse aggregate while the strength of a dry mixture will be increased by a moderate amount of working, especially tamping. The concrete should be agitated enough so that it will completely fill out the forms, eliminate air pockets, and cover the reinforcing steel. As the concrete will start to set almost immediately after it is poured, any spading or puddling should be done while the pouring is going on as any subsequent working will break up the crystals which are forming and very decidedly weaken the concrete.

16. Depositing Concrete Through Water.—Casting concrete under water is best accomplished by pouring through a pipe or tremie which reaches to the point of deposition or by lowering in a closed bucket with a trap door which can

be opened and the concrete released when it has reached the proper position. The discharge end of the tremie should be kept buried in the cast concrete and the pouring should be continuous—otherwise, the charge will be lost and the pipe become filled with water which will wash out part of the cement of the next batch fed into it. A certain amount of the cement will be leached out of the concrete before it has an opportunity to set, even with the most careful placing, but the curing conditions are so much superior to any that can be obtained in air that the net result will compare very favorably with that of concrete poured in the open.

17. Remixed Concrete.—The re-use of concrete which has been spilled in mixing or transporting is advised against by every writer on concrete and while the danger is, perhaps, exaggerated, this partially set concrete at best, replaces only so much aggregate and at worst may cause a weak spot in the building, due to the lack of cohesion within the lumps themselves on account of disturbance of the partially completed chemical action. Dr. Michaelis has shown that if set and hardened cement is reground it can be used again and considerable strength developed. In this case, however, the action of mixing does very little towards breaking down the coating which has already formed around the particles of cement so that practically no new strength is added and the pieces of re-used concrete, like so much stone, must depend on cement from without to bond them into the new mixture. In other words, the cement has lost its value and the safest course is to not use this partially hardened concrete if the set has proceeded far enough to be noticed.

18. Time of Set and Removal of Forms.—Cement is required by most specifications to have attained its initial set in not less than 45 min. and its final set in not more than 10 hr. The rate of hardening from the time of final set is so very largely dependent on a number of different conditions that no general rule can be made, but ordinarily concrete will have attained approximately one-half of its ultimate strength in 28 days. The time required for the concrete to become hard enough to permit removal of the forms will vary with its richness, amount of water used in mixing and more especially with the temperature at the time of setting. Under normal summer weather conditions, the column forms can be removed the second day and the forming from the sides of beams and girders can be removed in the same length of time. The bottom supports, of course, remain in place to carry the weight of the beam or girder for at least two weeks. The centering for heavy beams should remain in place longer than for light, narrow beams on account of the greater weight and slower set. Special attention should be paid to slab centering as deflected slabs are of very common occurrence and whether they are due to deflection in the forms themselves or to too early removal of the forms and centering, the impression given to the general public is one of insecurity. Removal of the forms in a week or ten days, keeping up a number of supports set at close intervals to take the load for another two weeks is a good and safe procedure as the removal of the forms gives an opportunity to wet down the underside of the slab which is quite as important as wetting the top and also permits any necessary patching while the concrete is still in condition to make a satisfactory bond.

19. Concreting in Cold Weather.—The season in which concrete can be poured under favorable temperature in the Northern States is so short that it is abso-

lutely necessary that cold weather building practice be so perfected that work can be carried on throughout practically the entire year. Under present conditions, although more building is being done each succeeding winter, the contractor must not only make his profit for the year in seven months but he must also earn his overhead for the other five. One result is a shortage of skilled labor and a consequent bidding up of wages during the rush period. Another is that good men are going into lines offering steadier employment, the building industry losing its best and most ambitious element.

The actual additional expense of cold weather construction has been estimated by contractors to be between 5 and 10 per cent—depending, of course, upon the character of the work—but the additional cost to the owner, will be less than these percentages for the reasons already implied. The general contractor is satisfied with less profit, his sub-contractors are glad to get work, materials are at their seasonal low point and the pick of the workmen can be secured at standard rates. The advantage to the owner in getting his building several months earlier will very often outweigh the slight difference in cost which may still remain after these deductions have been made.

Concreting can be successfully done in cold weather by heating the water, sand and gravel, and by enclosing the skeleton and keeping the cast concrete warm until it has set. The addition of salt and chemical anti-freeze mixtures should be mentioned to make the list complete.

19*a*. Heating the Aggregate and Water.—When the temperature gets down to 40 deg. F., active measures should be started to add artificial heat. Even at 50 deg. F. the action of setting has slowed down to less than one-half of what it was at 70 deg. but, if this is recognized, the forms can be left up the additional time necessary for the concrete to harden. At 40 deg. F., however, the action has become so slow that the concrete lies practically inert. At this temperature, if the chemical action is started by warming up the concrete, enough heat will be generated in the process of setting to complete the hardening. Even in actual freezing weather, concrete in relatively large masses can be handled in this way on account of the large amount of heat developed and the insulation afforded by the concrete itself. Thermometer readings taken from the interior of concrete cast in large masses have shown temperatures as high as 90 deg. F. a year after the concrete was poured. Where any attempt at all has been made to heat the concrete, it is a very rare thing to see a column that has been structurally damaged by freezing. The fireproofing may scale off, but the core is generally sound. The sand and gravel or crushed rock should above all be heated as they make up the bulk of the concrete and the only object of heating the ingredients is to obtain a uniformly heated mix by introducing thermal units in the most convenient way. If the water can conveniently be warmed, additional heat can be added in that form but under no conditions should boiling water be added directly to cement. The aggregate should preferably be heated over coils of steam pipes rather than by any direct application of fire unless it is very carefully watched and kept from getting too hot. A piece of rock which has been heated to 500 or 600 deg. will retain its heat for some time and in the mixture will, by evaporation of the water, cover itself with a thin film of dry cement which renders any bond impossible.

A large bank of steam coils which will heat enough material at one operation so that plenty of time can be taken to heat the entire mass slowly and uniformly is the best practical method. An ideal way would be to have a slowly moving endless belt similar to that of an ore roasting furnace running over the steam coils, the cold sand and gravel being thrown on one end and the heated aggregate released to the mixer at the other.

On work the size of which will not warrant the expense of fitting up an old boiler and steam pipes, the standard method of banking the gravel over a metal culvert pipe in which a fire is built will be satisfactory, provided the precautions are taken against overheating part of the gravel and leaving another part in frozen lumps.

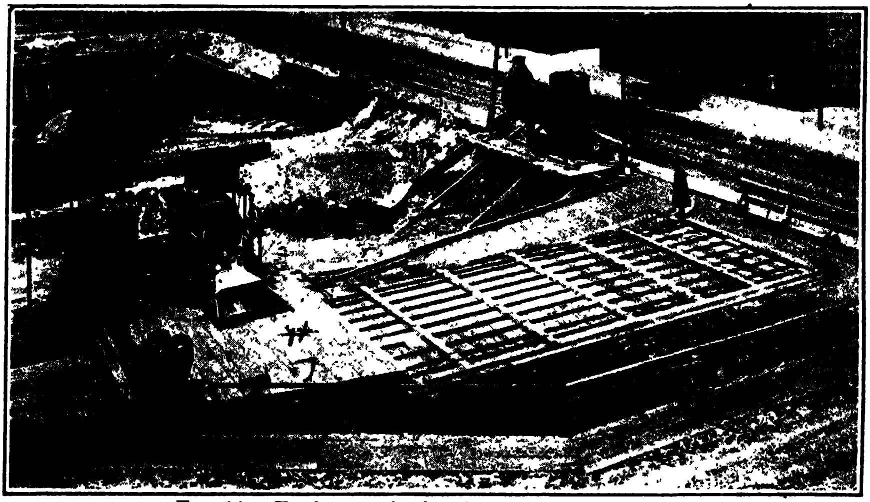


Fig. 11.—Equipment for heating concrete aggregates.

As already stated warm water may be used but the cement should not be heated. The bad possible effects will more than offset the small benefits of introducing a little additional heat in this way.

19b. Enclosure and Heating Forms.—In severe weather the work should be enclosed and heated, especially for the first 48 hr. after being poured. Any concrete given a proper start and carefully cared for during the first two days is almost safe. Enclosing and heating a building or portion of a building is mainly for the benefit of the thin members, like slabs and beams, or for protecting the faces of the larger members, as experiments have shown that concrete 2 in. from a face will not register daily changes in temperature and the seasonal changes do not penetrate over 6 in. If the building is of skeleton construction, a canvas curtain can be carried around the story to be poured and if the boiler used in heating the aggregate is not of sufficient capacity to carry a steam line to the story which will raise the temperature well above the freezing, salamanders can be used. The number of salamanders required will vary with the outside temperature and it will require constant attention to keep the room properly heated in this way.

19c. Protection against Frost.—As soon as possible after pouring, the top of the slab should be protected by covering with tar paper and straw. A fall of snow should be left undisturbed as it makes the best of protective coverings.

19d. Frozen Concrete.—Concrete that is frozen can be saved by enclosing and heating. After thawing the concrete will resume its setting from the point it had reached at the time that it was frozen. A great danger is to mistake frozen for set concrete and to remove the forms before measures have been taken to thaw out and properly cure the concrete. Putting a small sample near the stove or radiator will soon show what has happened by the sweating

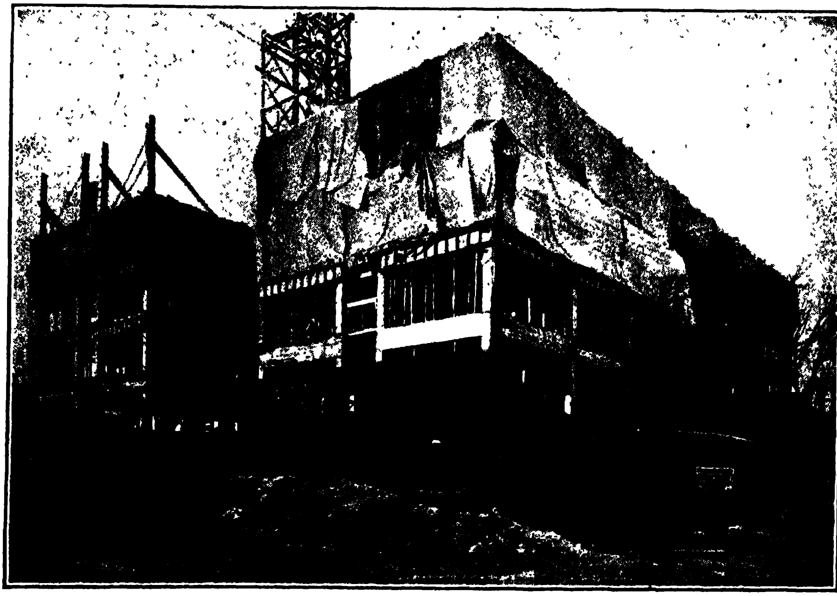


Fig. 12.—Canvas walls as used in winter covering.

and softening which will occur if it is frozen. Concrete which has frozen after setting has started is damaged to a certain extent due to the mechanically expansive action of the ice crystals and the amount of damage will depend on the degree of setting which has preceded the freezing. It is this action of the ice that makes a twice frozen concrete crumbly and worthless. The ordinary case of frozen concrete, however, is one which has been poured cold and then has been caught by an unseasonal spell of cold weather. As a general thing, this concrete can be saved as the cement was so inert when the concrete was poured that practically no action had taken place prior to the freezing.

19e. Use of Anti-freeze Mixtures.—Common salt has been used since the early days of cold weather concreting and it has been shown by inference to be of no value as the only claim made for it is that it lowers the freezing point of the mixture a few degrees. It is not the actual freezing of concrete which is to be guarded against but the lowering of the temperature to a point at

which chemical action will not take place. Furthermore, salt retards the setting, corrodes the reinforcing and introduces soft salt crystals into the body of the concrete.

Calcium chloride and chemical hardeners based on calcium chloride do accelerate the setting and for that reason may be of certain problematical value by decreasing the time necessary to keep the concrete heated.

Alcohol like salt, reduces the freezing point and for that reason has no more value than common salt.

20. Protection against Heat.—The protection of concrete against heat is not provided for because of any bad effect of the heat itself, but because of the action of the heat in evaporating water which the concrete requires in setting. Therefore, any concrete which can be given plenty of water is helped by heat,

FIG. 13.—Canvas cover for curing.

as it accelerates the setting. Thin concrete members or ornamental exposed faces should be protected from the direct rays of the sun. Moist earth is one of the best protective coverings for slabs and floor finishes. Tank walls or other thin walls should be reinforced against the daily change of summer temperature which may amount to 50 deg. in a few hours as no concrete can stand the strain regardless of the methods used in protecting it during curing.

CONCRETE CONSTRUCTION IN ALKALI SOILS AND SEA WATER

BY G. M. WILLIAMS

21. Constituents and Distribution of Soluble Salts in Alkali Ground Water and Sea Water.—The presence of various soluble salts in sea water and in the ground water of the arid and semi-arid districts of the west requires special attention to the production of concrete which is to be exposed to such conditions.

Waters carrying in solution the salts of calcium and magnesium as well as sodium and potassium are commonly known as alkali waters. All ground waters contain appreciable quantities of one or more of these salts, but it is usually in the

arid regions of the west that concentrations are sufficiently high to result in such waters being classed as alkali waters. The weathering and breaking down of rocks by long exposure to the elements has resulted in the formation of the soil, and these salts which were once among the constituents of the rock are now found distributed throughout the soil, usually in greatest quantities in the clay and shale. Water percolating through the soil dissolves a portion of the salts with which it comes in contact and by this means they are slowly carried away through natural drainage channels to the sea or later deposited on the surface of the ground by evaporation of the water. In the humid regions years of heavy rainfall have resulted in the leaching of the greater portion of the soluble material from the soil, so that the ground water is now low in salt content. In the arid and semi-arid regions relatively small quantities of soluble salts have been removed except where by irrigation or other means, large quantities of water have been available together with good natural drainage conditions. In irrigated districts heavy crusts of white salts may often be seen on the surface of low lying land and on the shores of small lakes and ponds, resulting from the evaporation of water which has passed some distance through the soil. The salt content of sea water has resulted from the constant inflow of streams carrying salts in solution.

What is known as "white alkali" is usually a mixture of the sulphates and chlorides of sodium, calcium and magnesium, while "black alkali" so called because of its tendency to dissolve vegetation or organic matter and stain the surface of the soil a brown or black color, contains a large percentage of sodium carbonate, together with smaller amounts of the white alkali. The white alkali is the most widely distributed type, and experience has shown that it is more injurious to concrete.

In sea water chloride and sodium (common salt) predominate with appreciable quantities of sulphate, magnesium, calcium and potassium. Due to the changes brought about after the salt carrying waters of the soil reach the sea, the constitution of sea water is not the same as that of the waters entering. Sea water may be considered as an alkali water of the chloride type of high and quite uniform concentration while soil waters in the arid region are of the sulphate, carbonate or chloride types, of which the sulphate is by far the most common. Whereas the concentration of sea water is fairly constant, the salt content of soil water may vary several hundred per cent in distances only a few feet apart, depending upon available supplies of salt in the soil, quantity of soil water and drainage conditions. Sea water has a quite definite constitution and concentration while soil water although varying greatly in concentration is usually quite constant as to various constituents present in any locality.

In the following table some typical analyses[1] of alkali soil waters found in the western states are shown. These well show the wide ranges in concentrations and constituents which are encountered.

[1] All analyses except No. 4 are taken from Technologic Paper 95, U. S. Bureau of Standards. No. 4 is taken from *Bull.* 616, U. S. Geological Survey.

ANALYSIS OF ALKALI WATERS

Number	Per cent soluble solids in water	Percentage reacting values							
		Na	K	Ca	Mg	Cl	SO_4	CO_3	NO_3
1	0.11	23.5	...	16.3	10.2	15.4	12.9	21.7	
2	0.46	23 9	...	19.4	6.7	27.1	20.0	2.9	
3	1.67	32 0	...	4.7	13.3	2.0	46.3	1.6	
4	1.85	48 3	1.5	0	0	6.1	7.8	36.3	
5	2.31	42 7	...	1.7	5.6	46.4	2 5	1.1	
6	3.31	27 5	...	1.8	20.6	0.4	47.8	1.8	
7	4.80	36.5	...	1.5	12.0	4.0	43.2	1.1	1.7

Waters Nos. 3, 6 and 7 are of the sulphate type, most commonly encountered in the west, the concentration of the latter being unusually high. Water No. 5 is of the chloride type and is typical of the soil water near Great Salt Lake, Utah. Water No. 4 is an example of a soil water high in carbonate or "black alkali."

The constitution of sea water is quite constant and its concentration is high and fairly uniform, being lowered considerably near to the outlets of fresh water streams. In the following tabulation are shown the averages of 77 analyses[1] of sea waters, which may be considered as representative for comparative purposes.

TYPICAL ANALYSIS OF SEA WATER

Approximate concentration (per cent)	Percentage reacting values						
	Na	K	Ca	Mg	Cl	SO_4	CO_3
3.5	38 0	1.8	1.7	9.5	45.1	4.7	0.2

This average analysis of sea water is quite similar to Water No. 5 in the preceding table as to relative quantities of constituents, but somewhat higher in concentration.

22. The Effect of Alkali Solutions on Concrete.—The durability of sea water structures of concrete has been a problem of interest to engineers for many years and the study of the combined effect of sea water and various exposure conditions has been in progress ever since Portland cement began to be employed in such work. The possibility of disintegration of concrete by soluble salts present in ground became markedly apparent with the great development of irrigation in the western United States. As in sea water work, concrete has been generally adopted as the most useful material in the construction of various irrigation structures as well as for foundations, sewers and other works. The continued use of large quantities of water for irrigation has resulted in a gradual rise in the water table, or soil water level in many localities, and this water has become alkaline due

[1] Recalculated from data in *Bull.* 616, U. S. Geological Survey.

to dissolving quantities of soluble salts in the soil. These waters, coming into contact with underground and surface structures of concrete have in many cases caused disintegration in comparatively short periods of time. This disintegration is generally first noted, and is most marked, at and just above the ground line and will vary with soil water conditions. Where the alkali water level is constantly below the limits of soil capillarity, disintegration will usually be confined to that portion of the structure which is immersed for a large portion of the time, with perhaps some deterioration at the ground surface due to the occasional presence of surface water. Disintegration at or near the surface is no doubt accelerated by frost action and alternate wetting and drying, but these effects are only contributory.

Deterioration of mass concrete in sea water is generally found between the limits of the high and low tide, where the concrete is alternately wet and dry. The action is greatly accelerated by freezing and thawing, by the mechanical effect of the waves and abrasion caused by floating bodies in the water.

The appearance of concrete affected by alkali salts varies with the quality of the concrete, the concentration of the solution and duration and condition of exposure. With good quality concrete, the first stage of the action is marked by the flaking or shelling off of the surface skin of neat cement or rich mortar, exposing sand grains and aggregate particles. As the action continues larger aggregate particles are exposed and the new surface has the appearance of being abraded with white deposits of salts apparent in the pores.

In the last stage the material loses its identity as concrete and appears to be a mixture of aggregate particles distributed throughout a white, soft, putty-like, lime paste. This change may be accompanied by a considerable increase in volume. If soil water conditions change so that the mass becomes dry soon after disintegration has been partially or fully completed, there will be an appreciable hardening but the strength regained is very slight as compared with that of the original concrete. Distintegration of concrete of low permeability is essentially a surface action which gradually progresses into the mass. In some cases the outer skin appears to be immune to attack but it does not entirely prevent the penetration of alkali water, which in reacting with the inner portion causes swelling which shatters the surface skin. Rapidity of penetration is dependent upon quality of concrete and concentration of salt in the water, but usually chipping away the affected portion will reveal concrete which is apparently unharmed.

A somewhat different and more rapid action occurs in the case of more permeable concrete or in mortar mixtures of the type commonly used in hand tamped drain tile and sewer pipe. The comparative ease of penetration of the alkali water through the wall exposes a large volume of cement to the action of the salts and a rapid swelling and increase in volume results, the whole mass finally breaking down to form the typical white putty-like paste.

23. The Theory of the Disintegration of Concrete by Salt Solutions.—When deterioration of concrete in alkali soils and sea water was first studied, the action was generally ascribed to the use of poor aggregates or improper methods of mixing and placing concrete and it was believed that disintegration was brought about merely by the crystallization of alkali salts in the pores of the concrete. Since the salts in crystal form occupy a greater volume than in solution, it was believed that the mechanical forces exerted were sufficient to disrupt the mass.

This being true disintegration could be avoided by producing concrete of low permeability which would prevent appreciable quantities of salt from entering.

Laboratory investigation has shown that disintegration is not due to disruptive forces exerted by salt in crystallizing, but rather to the chemical reaction between salts in solution and constituents of the cement. Disintegration is primarily due to chemical action and any disruption which may occur under certain conditions as the result of crystallization is secondary. The constituents of the cement attacked by the salts are lime, silica and alumina. During the process of hardening of concrete calcium hydrate is formed, and it is this material which is most readily attacked. The sulphates of sodium and magnesium in alkali waters react with the calcium hydrate to form calcium sulphate and sodium and magnesium hydrates. The magnesium sulphate in sea water likewise reacts with lime in the cement to form insoluble magnesium hydrate and calcium sulphate. The part played in the process of disintegration by these newly formed compounds is not well understood and probably varies with the permeability of the concrete and quantity and movement of the alkali water but the final effect cannot be disputed since the changes produced in the laboratory under controlled conditions have been verified by the inspection of good quality concrete exposed to similar conditions in the field.

Burke and Pinckney in *Bull.* No. 81, Montana Agricultural Experiment Station, state the following conclusions:

(1) The disintegration of cement by alkali salts is principally due to reaction between these salts and the calcium hydroxide necessarily present in set cement. As a result of these reactions relatively insoluble new compounds are formed in the body of the cement structure . . . these new compounds have greater weight and require greater space than the $Ca(OH)_2$ replaced . . . New compounds force apart the particles of cement, thus weakening or breaking the binding material.

(2) A certain *weakening*, not a disruption of the cement is due to the loss of a portion of the binding material, crystallized calcium hydroxide, which is merely dissolved and moved in solution.

(3) In order for destructive action to become marked, the alkali solutions must percolate through the cement, or at least must penetrate beyond the surface.

(4) Any measures that hinder the penetration of the alkali solutions into the interior of the mass will delay the destructive action.

Steik in *Bull.* No. 122, University of Wyoming Experiment Station has drawn the following conclusions from laboratory studies:

(1) A solution of magnesium chloride had the greatest disintegrating effect, due to the action of hydrochloric acid produced by the hydrolysis of the salt.

(2) The presence of sodium carbonate in solutions of the other salts retards the disintegrating effect.

(3) The ultimate cause of the disintegration of cement is by the alkalies forming compounds with the elements of the cement which subsequently are removed from the cement by solution.

Bates, Phillips and Wig in *Technologic Paper* No. 12, U. S. Bureau of Standards, after an investigation which included exposure tests in sea water as well as laboratory experiments, outlined the following conclusions:

(1) Portland cement mortar or concrete, if porous, can be disintegrated by the mechanical forces exerted by the crystallization of almost any salt in its pores, if a sufficient amount of it is permitted to accumulate and a rapid formation of crystals is brought about by drying. Porous stone, brick and other standard materials are disintegrated in the same manner.

(2) In the presence of sea water and similar sulphate-chloride solutions:
(a) The most soluble element in the cement is the lime. If the lime of the cement is carbonized it is practically insoluble.
(b) The quantity of lime, alumina or silica present in the cement does not affect its solubility.
(c) The magnesium present in the cement is practically inert.
(d) The quantity of SO_3 present in the cement up to 1.75 per cent does not affect its solubility.

(3) The change which takes place in sea water when brought into intimate contact with the cement is as follows:
(a) The magnesia is precipitated from the sea water in direct proportion to the solubility of the lime in the cement.
(b) The sulphates are the most active constituents of the sea water and are taken up by the cement. This action is accelerated in the presence of chlorides. No definite sulphate compound was established.
(c) The quantity of sodium and chloride taken up by the cement is so small that no statement can be made as to the existence of any definite chloride or sodium compound formed with the cement.

The U. S. Bureau of Standards in cooperation with the U. S. Reclamation Service, the Drainage Investigations of the Department of Agriculture, and the Portland Cement Association have been carrying on an extensive field investigation[1] since 1913 to determine the durability of cement drain tile in alkali soils and waters. Exposure tests of large size concrete specimens have been made in twelve western states so that a wide variety of exposure conditions have been encountered. Their conclusions to date may be summarized as follows:

(1) Most serious disintegration is found in waters of the sulphate type.

(2) Disintegration is primarily due to chemical action between the salts in solution and the constituents in the cement.

(3) Rapidity of disintegration is dependent upon concentration of salts in solution.

(4) Resistance to disintegration varies with cement content or richness of mix.

(5) Alkali salts are not uniformly distributed throughout the soil or body of soil water so that it is impossible to determine in advance the maximum concentration to which a structure may later be exposed.

(6) Seepage water and alkali soil conditions may be encountered which will disintegrate concrete of the best quality.

(7) The outer surface skin of concrete in which the lime has been carbonized may be either very slowly attacked or immune to attack, but the carbonized coating is not in itself waterproof or impermeable so that disintegration may begin just within the shell.

In the foregoing it is seen that all of the investigations have led to the conclusion that disintegration is brought about by chemical action between salts in solution and the constituents of the cement. Results of field tests and investigations are in accord with and amplify conclusions drawn from laboratory studies.

A field investigation which involved inspection of practically all concrete structures exposed to sea water along the coast of the United States, besides a great number in Canada, Cuba and Panama was made by Wig and Ferguson and reported in *Engineering News-Record* in September, 1917. Their study showed that structures exposed to sea water could well be divided into two groups, plain concrete and reinforced concrete. Where protected from mechanical abrasion due to wave action and floating objects, they concluded that permanency of structure of plain concrete of good quality is assured against disintegration.

[1] *Technologic Paper* 214, U. S. Bureau of Standards.

Structures protected by wooden casings have shown no disintegration after 25 yr., while unprotected concrete of the same quality and exposed to the same conditions rapidly deteriorated. The problem of obtaining safe structures reinforced with steel is more difficult. Their investigation showed that the majority of all reinforced concrete structures on the American coasts subjected to the action of sea water or sea air are now showing evidences of deterioration due to the corrosion of the embedded reinforcement above the water line. They conclude that "steel embedded in first class concrete to a depth of 1 or 2 in. in accordance with existing theory and practice is not perfectly protected against corrosion by sea water action." Failure is evidenced usually by cracks which follow along the lines of the reinforcing steel, *starting above the high water line.* Cracking is caused by the corrosion of the steel, the corroded metal occupying a larger volume than the original, and during the process of rust formation stresses are set up which exceed the strength of the concrete. They state "undoubtedly the real cause of the trouble is the accumulation of salts in the pores of the concrete above the water line by capillarity and evaporation, and the absorption by the concrete of air carrying very minute particles of sea water."

The condition of reinforced concrete structures subjected to sea water, as found in the foregoing inspection, casts doubt on the permanency of such structures in which steel is used and built according to present good practice. No doubt the use of richer, less permeable, concretes with deeper embedment of steel will eliminate much of the trouble now encountered.

24. Precautions to Minimize or Prevent Deterioration of Concrete Exposed to Alkali Solutions.—All investigations have led to the conclusion that concretes of low permeability are more resistant to alkali and sea water action than concretes which more easily permit water to penetrate. The production of concrete of such quality is dependent upon a rich mixture with well-graded aggregates together with proper methods of mixing, placing and curing. The greatest factor influencing permeability is cement content. Regrading and washing of aggregate may be of value but increase of cement content is far more effective. The use of very dry or very wet consistencies should be avoided. The best consistency is one which is plastic and quaking and requires little tamping or spading to place properly.

In this connection it is often specified that concrete should be non-porous and of maximum density. All concrete is porous and absorptive and tests indicate that there is little relation between density, absorption and permeability. A concrete of fairly high porosity may have a low permeability and consequently greater resistance to disintegration. The quality desired is low permeability. While such concretes are most resistant to alkali action, all are apparently subject to disintegration if concentrations are sufficiently high. It is not possible to determine what maximum concentrations may be encountered at some later period even though investigations at the time work is under way indicate concentrations to be low. Therefore the ultimate safety of concrete structures subjected to alkali soils and waters can only be assured by employment of such precautions as will prevent ground waters from coming in contact with the concrete. Such precautions are manifestly impossible in some types of construction. Ground waters may often be intercepted and carried away by means of vitrified tile or gravel drains. Integral waterproofing compounds have not demonstrated their

effectiveness in resisting alkali action and cannot be relied upon to give the desired protection. Tests have shown that tar and bituminous coatings are not permanent in resisting the penetration of moisture, although deterioration is retarded for a time. The membrane system of waterproofing should furnish protection as long as the coating remains unbroken. It is difficult to apply this material to all surfaces in practice and any method of waterproofing employed should be considered merely as supplemental to a complete system of drainage

For structures placed in alkali soils and waters it may be stated that permanence can be guaranteed only so long as ground waters are effectively drained away from the concrete. Such protection is sometimes impractical and in any case the plans will be dependent upon the particular conditions at hand. On all work where there is possibility of alkali conditions developing, protection from action should be considered in the preparation of the plans, as effective drainage can usually be most economically provided for at this time.

In sea water the protection of mass concrete from the action of salts can be made effective by protecting surfaces from wave action and abrasion by floating objects. Timber jackets have proven of value but will require renewal after a period of years. A facing of granite blocks will be more expensive but permanent. While deterioration in sea water has usually been found above low water level, there have been cases of disintegration of concrete constantly immersed, so that every precaution should be taken to insure a uniformly good quality of concrete throughout.

Best results will be obtained by unwatering the sections to be concreted so that the concrete can be more uniformly placed and furnished protection from the action of sea water during the early period of hardening, but satisfactory results have been obtained in depositing concrete below the water surface by means of the tremie.

To sum up our present knowledge of the durability of concrete in alkali soils it may be stated that the best of Portland cement concrete will be affected when exposed to sulphate waters and soils of high concentrations. Good quality of concrete when exposed to waters of the carbonate and chloride types is very slowly, if at all, attacked. Average quality of concrete exposed to these latter waters in the Western States for a number of years shows no apparent signs of deterioration. Where soils and ground waters contain large amounts of sulphates, no assurance can be given of the permanency of Portland cement concrete unless by drainage and other means such waters can be prevented from coming in contact with the concrete. In any case concretes of high cement content and low permeability will be found most durable.

A rich concrete of low permeability can safely be used in unreinforced structures exposed to sea water if precautions are taken to prevent mechanical abrasion of the surface. Long life of reinforced structures exposed to sea water is less certain owing to the danger of corrosion. Present practice requires special attention to obtaining a rich concrete of low permeability together with a minimum covering of the steel of not less than 3 or 4 in. The adequacy of these requirements can best be determined by observing the history of structures now being built.

SECTION 2

FORMS FOR CONCRETE

By A. B. McDaniel

1. General Conditions.—An important feature of concrete construction is the formwork. The design of a reinforced concrete building will be governed to some extent by the design, use, and re-use of the forms. For example, uniform story heights and column sizes are adopted as far as practicable to secure economy in the use of form panels and column molds. In order to have full size structural members and true, uniform surfaces, it is necessary to build all formwork true to line and grade, strong, durable, and rigid, and braced sufficiently to withstand all bulging, twisting, or sagging.

Form surfaces adjacent to the concrete should be dressed smooth and true, and be free from joints, cracks, and imperfections, which would cause leakage and mar the appearance of the finished work.

As the cost of the formwork of a concrete structure varies from 10 per cent in a simple structure to 30 per cent in a complex one, economy in the use of forms is of the greatest importance. The use of beam, girder and column sizes to utilize standard widths of lumber will save mill expense. As far as practicable, the various units of a building such as beams, girders, and columns should be spaced and arranged so that the form panels, or units can be re-used a maximum number of times. It is often true that the use of a slight excess of concrete to secure uniformity or symmetry of construction will ensure a considerable saving in the final cost through simplification of formwork.

2. Material for Forms.—Forms for concrete construction are generally built of two materials; lumber and steel. The former is the natural and original material, which is in general use at the present time. Steel has come into popular favor for special features of construction, such as round columns, circular bins, tanks, tunnels, etc. Steel forms will be discussed fully in a later article.

Lumber for forms should be partially seasoned, as kiln dried lumber will absorb water from the fluid concrete and swell and bulge, while green lumber will dry out and shrink. In using ordinary, marketable lumber, care should be taken to frame the formwork to allow for some swelling without distortion.

The kind of lumber to be used for the various parts of the formwork depends to a great extent on the cost and availability of material. Spruce, fir and Southern pine are generally used for studs, posts and joists. Yellow pine, Norway pine and fir are adaptable for the sections in contact with the concrete such as the sheathing and flooring. White pine is often used for the sides and bottoms of beams and girders, but is ordinarily too expensive for extensive use.

All form lumber should be free from knots, twists, shakes, decay and other imperfections which would affect its strength and the finished surface of the concrete.

The size and thickness of lumber to be used in the various parts of formwork depends upon the design, the load to be supported, the kind of material to be

used, etc. However, it has become current practice to use certain common sizes of lumber for the various elements of formwork, and arrange these members to carry the required load. For example, for sheathing and flooring 1-in. material is used when the form panels are to be utilized only a few times, while 1½-in. stock will be used when the panels are to be employed many times. Posts and shores are 4 × 4-in. or 6 × 6-in., while 1 × 6-in. or 1½ × 6-in. material is generally used for ledger boards, cross-ties and bracing.

Face lumber should be dressed all four sides, sheathing dressed two edges and one face, while braces, supports and ties may be of undressed material. Floor and wall forms should be of tongue-and-grooved or beveled-edge material. The tongue-and-grooved material makes a smooth surface, but is hard to re-use and is apt to bulge from swelling. Beam, girder and column sections are generally made of pieces dressed true to edge, forming simple square or butt joints.

3. Design of Forms.

3*a*. Principles.—The fundamental principles of form design are economy, simplicity, strength and symmetry. The first two principles are closely related and obvious. Strength should only be sufficient (with a low factor of safety) to provide for the dead load and live load, during construction. Very often deflection rather than strength will govern the design of a section of the formwork, such as the floor joists.

In the past, form design has been left largely to the carpenter foreman, who has prepared rough sketches for the building of the forms as occasion required. However, on recent work of any magnitude, it has become the practice to prepare detail plans for formwork in the drafting room, under the supervision of the chief or designing engineer. This method is strongly recommended for economy and accuracy of design.

Forms should be designed on the basis of the greatest practicable amount of re-use. Hence the design should consider the use of uniform story heights with the higher stories at the bottom; the proper allowance for and location of construction joints, the kind of windows and floor finish to be used, etc. To secure the most efficient and economical form design some extraneous factors such as the sizes and lengths of lumber readily obtainable in the local market, the location of lumber yard and mill with relation to the structure at the site and the size, character and location of the concrete plant, must be considered.

3*b*. Values to Use.—In the design of forms, the loads to be applied, during and subsequent to pouring the concrete, and the physical qualities of the materials to be used, are important and fundamental factors which must be determined previous to the actual preparation of the design.

The loads on the forms consist of the construction loads as the weight of runways, workmen, loaded barrows or cars, etc. The impact of the falling concrete should be considered in column and high wall design when the concrete is to be discharged from barrows or cars. The final load is simply the weight of the concrete and its reinforcement if any. The use of sections of a concrete structure for the storage of materials during construction should not be allowed.

The pressure of wet concrete against forms depends on the rate of filling, the method of placing, the type of structure and the temperature. The tests[1]

[1] From *Eng. Rec.*, Jan. 15, 1910.

of Major Francis R. Shunk, Corps of Engineers, U. S. Army, were made in 1908, and indicated that the pressure of concrete follows the linear law

$$p = wh$$

with w equal to 152 lb. per sq. ft. until a time T has elapsed after pouring begins, when the concrete assumes initial strength

$$T = C + \frac{150}{R}$$

where C is a constant depending on temperature of the concrete in the forms, as follows:

Temperature (deg.)	80	70	60	55	50	40
Values of C.	20	25	35	42	50	70

and R is the rate of filling in vertical feet per hour.

Tests made by Hector St. George Robinson to determine the pressure of wet concrete in columns of small dimensions, thin walls and similar light concrete construction indicated that the lateral pressure for average conditions was equivalent to that of a fluid weighing 85 lb. per cu. ft.

Subsequent tests on full size columns made by E. B. Germain of the Aberthaw Construction Co., Boston, in 1913, and by the writer in Illinois in 1913, 1914, and 1915, indicate that an equivalent fluid pressure value of 145 lb. per sq. ft. per foot height is a rational and safe value.

The following values are recommended:

Dead weight of concrete, 150 lb. per cu. ft.
Construction live load on floor forms, 75 lb. per sq. ft.
Pressure of wet concrete on columns and walls per foot of depth, 145 lb. per sq. ft.
Coefficient of elasticity for spruce and equal, 1,200,000 lb. per sq. in.
Extreme fiber stress in spruce or equal for timbers, 1,200 lb. per sq. in. for column yokes, 1,800 lb. per sq. in.
Horizontal shear for spruce or equal, 200 lb. per sq. in.

The moment formula which should be used in the design of members subjected to flexural stress, such as floor stringers and joists, depends upon conditions of loading and support. Generally $M = \frac{wl^2}{10}$ is used, except in the case of single spans, where the construction load may be concrete cars moved over portable tracks. In the latter case, the positive moment at the center of the span would undoubtedly be much greater than the negative moment at the supports, due to the dead load. For a bending moment of $M = \frac{wl^2}{10}$, the corresponding deflection should be ascertained by the formula $D = \frac{3}{384} \cdot \frac{w''(l'')^4}{EI}$ (see "Notation," p. 59). The coefficient $\frac{3}{384}$ is a mean between $\frac{1}{384}$ for beams with fixed ends and $\frac{5}{384}$ for beams with ends simply supported.

3c. Preparation of Drawings.—The detailed plans for the complete formwork for a concrete structure of large magnitude are generally prepared in the drafting room of the main office of the contractor under the supervision

of the designing engineer after the construction drawings have been adopted. The complete form design is similar to that which would be drawn for the framework of a steel skeleton building, special attention being given to the re-use of the forms and the reduction to a minimum of the amount of lumber required for the job. The following schedule is the order of procedure in the drafting room work:

(*a*) The preparation of a general assembly plan showing the arrangement of the various parts of typical section or elements of the formwork (see Fig. 1).

(*b*) A location or key plan is drawn to show the location of the various parts of the structure; columns, girders, beams, etc. From an inspection of Fig. 2, it will be noted that the general location of the building is given, and the various structural elements and their form sections are designated by symbols.

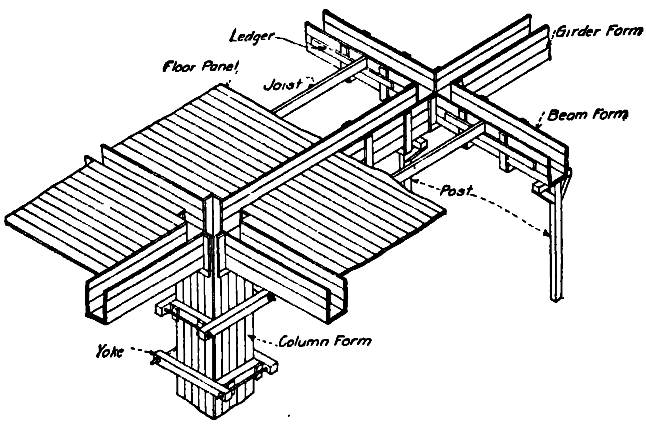


FIG. 1.—General assembly plan.

(*c*) The detail plans are made, beginning with the footing form and proceeding up through the building in about the following order: Footings, basement columns, basement walls, first story columns, girders, beams, and slabs, second story columns, girders, beams and slabs, etc.

The type of form construction to be used on any job depends on the structure. It is often governed to a great extent by the judgment and experience of the superintendent and his carpenter foreman and the standard methods in use by the company. It is always advisable to adopt and maintain standard methods of form construction with which the designing office and field force are familiar. Such standards will result in simplification and economy of the drafting, make-up, erection and removal of the forms.

The following points in the design of formwork are used by the Aberthaw Construction Company of Boston, Mass:[1]

(1) Joists and girts should be in as few lengths as possible to save time in sorting on the job.

(2) Use stock sizes and lengths of lumber.

(3) Keep number of panels and pieces to a minimum.

[1] From "Forms for Concrete Work" by R. A. SHERWIN, *Concrete*, April, 1916.

(4) Provide easy stripping.

(5) Allow clearance enough for slight inaccuracies in making up and erecting, swelling of panels, etc.

(6) Panels should be a whole number of boards in width, if possible, for ease in making up.

(7) Units to be as big as can be handled and joists used as panel cleats where possible.

(8) Provide for re-use of panels.

(9) Beams to be handled as trough units when the job is regular and units can be re-used.

(10) Consider use of floor domes or inverted boxes when beams are close together. In this way beam sides and slab are erected, stripped and moved as a unit. When either of the last two systems is used the beam sides should be given a slope to prevent hard stripping.

(11) Provide for re-shoring if necessary.

(12) Have bracing above the men's heads.

(13) When four beam haunches occur at a column consider making haunches as a unit similar to a column head in flat-slab construction.

(14) Consideration of steel forms.

The assembly plan generally serves as the basis for a study of costs, as several combinations of structural units can be made and their relative costs ascertained before a final design is determined upon.

The use of a minimum amount of material is predicated upon the greatest possible simplification of the forms, the duplication of parts and the re-use of the form sections. The determination and location of the various sections are facilitated by the use of a location, erection or key plan.

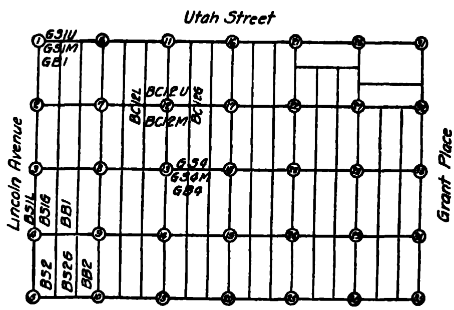


Fig. 2.—Location or key plan.

A very useful feature of the key-plan is the system of marks or symbols used. Figure 2 illustrates the adaptation of mnemonic symbols (*i.e.*, memory symbols) for this purpose. The initial letter or figure in the column notation indicates the story location of the form section; the second letter is *C* for column side; the third symbol is one or more figures designating the number and hence the plan location of the column, and the last symbol is the initial letter of the name of the street which the section faces. In the case of beams and girders, the same forms may be used in several floors and hence the story location is not stated. The first symbol for beams and girders is composed of two letters which designate the section as either a side or a bottom of a beam or a girder; the second symbol is the beam or girder number (all beams and girders are numbered consecutively); the third symbol is the initial letter of the name of the street which the beam or girder side faces. In many cases the third symbol is placed on only one side; the omission from the other side thus indicating its position. As an illustration of the use of this code, reference will be made to some symbols in Fig. 2. At column 12 are four sets of symbols, each one designating a side of the column form. Thus *BC*12*U* signifies that the section on which this mark is placed is located in the basement, is a side of column 12, and faces Utah Street. The three symbols *GS*4, *GS*4*M*, and *GB*4 represent respectively, the side form of

girder 4, facing Utah St., the side form of girder 4 facing Montana St., and the bottom form of girder 4.

The system of marking described above has been simplified in some cases by the omission of the letters *S* or *B* for the sides, and the last letter such as *U* or *M* for designating the position, thus giving the mark *B*4 or *B*9, and having only one number for any one panel. The advantages of this system are simplicity of marking both on key-plan and on form sections and the elimination of confusion in the making up and handling of forms.

The detail plans of the form sections for the footings, slabs, beams, girders, and columns are made in the order in which they will probably be needed at the building site. However, it is desirable to prepare these plans complete before the actual construction work begins, so that there will be no delay in the ordering of the material from the lumber mill. The beam, girder and column sections or panels are detailed on sheets of tracing cloth or paper, about 25 × 33 in., sub-

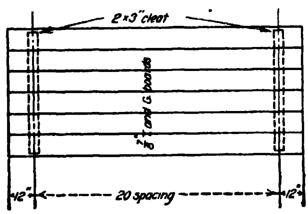


Fig. 3.—Detail of floor panel.

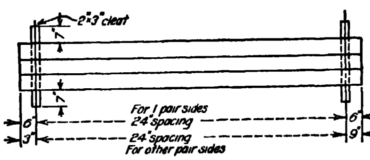


Fig. 4.—Detail of column side.

divided so as to have two rows of five spaces each. When the job orders go to the field office, the prints are cut up into 5 × 6-in. units, with a detail on each unit and are used by the mill man for cutting up the material and by the carpenters for making up the panels at the benches. Figure 3 shows a section of a sheet of floor details, and Fig. 4 a section of interior column details. It should be noted that each drawing of a form panel is made complete, giving sizes, dimensions, and bill of material for all the lumber necessary to make up the panel. The lines of the cleats are shown dotted as these pieces are placed first on the make-up bench and the form boards are laid on top and nailed to them. The cutting lines for the re-use of the form sections on upper stories are clearly shown and marked. End elevations or sections are shown when the piece is complicated.

As each beam and girder section is made up of several boards held together by cleats about 2 ft. apart and each column section is likewise made up of several boards held together by the yokes, which in this case serve as cleats also, each board is marked with a symbol consisting of the letter *M* and a number. In like manner, the cleats are given a number preceded by *K*, and the yokes are marked with a number preceded by *Y*. Each *M* number represents a board of a certain thickness, width and length, regardless of the location of the board in a beam, girder or column. The draftsman in making the detail drawings should use as many boards of such regular widths as 5¾ in., 6¾ in., and 7¾ in., as possible. These widths are the exact finished dimensions of 6-, 7- and 8-in. stock.

The proper symbols are marked with lumber crayons on the pieces as soon as they are cut out at the mill. The bench carpenters are thus able to select with-

out delay the pieces necessary to make up any required form panel of which they have the drawing.

After the completion of the drawings a material list is prepared, the various kinds of pieces (M's, C's Y's, etc.) being listed independently. The lumber is ordered in advance from this material schedule, a copy of which is sent to the job to serve as a guide for the checking up upon the receipt of the material.

4. Tables and Diagrams for Designing Forms.[1]

4a. **Notation.**—The following notation is used in the tables and diagrams:—

b = breadth of member in inches.
d = depth of member in inches.
l = span of member in feet.
l'' = span of member in inches.
w = uniform load per linear foot.
w'' = uniform load per linear inch.
w' = total load on floor in pounds per square foot = dead weight of slab per square foot plus 75 lb. per sq. ft. live load.
h = head in feet.
D = deflection in inches.
E = modulus of elasticity in pounds per square inch.
I = moment of inertia in inches^4.
f = maximum fiber stress in pounds per square inch.
M = bending moment in foot-pounds.
M_r = resisting moment in inch-pounds.
s = spacing in inches.
n = number of spaces.
n' = number of spaces between column yokes.
k = largest dimension of column in inches.
P = concentrated load in pounds.
V = total maximum vertical shear in pounds.
v = maximum unit horizontal shear (pounds per square inch) = $\frac{3}{2}\left(\frac{V}{bd}\right)$.

4b. **Allowable Fiber Stresses.**

Maximum fiber stress in spruce or equal:
for timbers . . . . 1,200 lb. per sq. in.
for column yokes . . . . 1,800 lb. per sq. in.
Horizontal shear for spruce or equal, 200 lb. per sq. in.
Crushing perpendicular to grain for spruce
or equal . . . . 400 lb. per sq. in.

4c. **Formulas.**—The spacing of joists was determined in Table I by the formulas:

For flexure

$$s = 2{,}000 \frac{bd^2}{w'l^2}$$

[1] From "Concrete Engineers' Handbook" by HOOL and JOHNSON, except as noted.

Table I.—Spacing of Joists

Based on $M = \frac{wl^2}{10}$ Deflection unlimited

For supported ends $\left(M = \frac{wl^2}{8}\right)$, deduct 20 per cent. from values given—except for the few values determined by shear which should be multiplied by $\frac{d}{2l}$.
Deflection for any spacing not governed by shear:

For $M = \frac{wl^2}{10}$, $D = 0.0225\frac{l^2}{d}$ For $M = \frac{wl^2}{8}$, $D = 0.030\frac{l^2}{d}$

Slab thickness	Size of joist (inches)	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
3 in.	2X4	31	24	20	16	14*	12*	10*						
	2X6			47	39	32	28	24	21	18	16	14*	13*	12*
	2X8						50	43	38	33	29	26	23	21
	2X10									52	47	41	37	34
	3X4	47	37	30	25	21*	18*	15*	13*	11*	10*			
	3X6					49	41	36	31	27	24	21*	19*	17*
	3X8								57	50	44	39	35	32
	3X10												56	50
	4X6						55	48	41	36	33	30	26	23
	4X8											52	47	42
	4X10													
4 in.	2X4	28	22	18	15	12*	10*							
	2X6		52	42	35	29	25	21	19	16	14	13*	11*	10*
	2X8					53	45	39	34	30	26	23	21	19
	2X10								54	47	42	37	33	30
	3X4	42	33	27	22	18*	16*	13*	12*	10*				
	3X6				52	44	37	32	28	24	22	19*	17*	16*
	3X8								51	45	40	35	32	29
	3X10											56	50	45
	4X6						50	43	37	33	30	27	23	21
	4X8										53	47	42	38
	4X10													
5 in.	2X4	25	20	16	13	11*								
	2X6		47	38	32	26	23	19	17	15	13	12*	10*	
	2X8					48	41	33	31	27	24	21	19	17
	2X10							56	49	43	38	34	30	27
	3X4	38	30	24	20	17*	14*	12*	11*					
	3X6			57	47	40	34	29	25	22	20	17*	16*	14*
	3X8							53	46	41	36	32	29	26
	3X10											51	46	41
	4X6					53	45	39	34	30	27	24	21	19
	4X8									54	48	43	38	35
	4X10													55
6 in.	2X4	23	18	15	12	10*								
	2X6	55	43	35	29	24	21	18	15	14	12	11*	10*	
	2X8				53	44	38	32	28	25	22	20	17	16
	2X10							51	45	39	35	31	28	25
	3X4	35	27	22	18	15*	13*	11*	10*					
	3X6			53	43	36	31	27	23	20	18	16*	14*	13*
	3X8						56	49	42	37	33	29	26	24
	3X10										52	47	42	38
	4X6					49	41	36	31	27	25	22	19	17
	4X8								56	50	44	39	35	32
	4X10												56	50
7 in.	2X4	21	17	14	11									
	2X6	51	40	32	27	22	19	16	14	12	11	10*		
	2X8				49	41	35	30	26	23	20	18	16	15
	2X10						55	47	41	36	32	29	26	23
	3X4	32	25	20	17	14*	12*	10*						
	3X6			48	40	34	29	25	21	19	17	15*	13*	12*
	3X8						52	43	39	34	30	27	24	22
	3X10									55	48	43	39	35
	4X6				53	45	38	33	29	25	23	21	18	16
	4X8								52	46	41	36	32	29
	4X10												57	46
8 in.	2X4	20	16	13	10									
	2X6	47	37	30	25	21	18	15	13	12	10			
	2X8			55	45	38	32	28	24	21	19	17	15	13
	2X10						51	44	38	34	30	27	24	21
	3X4	30	24	19	16	13*	11*	10*						
	3X6		55	45	37	31	27	23	20	17	15	14*	12*	11*
	3X8						48	42	36	32	28	25	23	20
	3X10									51	45	40	36	32
	4X6				50	42	35	31	27	23	21	19	16	15
	4X8							56	48	42	38	34	30	27
	4X10											53	48	43
9 in.	2X4	18	15	12	10									
	2X6	44	35	28	23	19	16	14	12	11				
	2X8			51	42	35	30	26	23	20	17	16	14	13
	2X10					55●	48	41	36	31	28	25	22	20
	3X4	28	22	18	15	12*	10*							
	3X6		52	42	35	29	25	21	18	16	14	13*	11*	10*
	3X8					53	45	39	34	30	26	23	21	19
	3X10								54	47	42	37	33	30
	4X6			56	46	39	33	28	25	22	20	18	15	14
	4X8							52	45	40	35	31	28	25
	4X10										56	50	45	40
10 in.	2X4	17	14	11										
	2X6	41	32	26	22	18	15	13	11	10				
	2X8			48	40	33	28	24	21	18	16	15	13	12
	2X10					52●	45	39	34	29	26	23	21	19
	3X4	26	21	17	14	11*	10*							
	3X6		48	39	32	27	23	20	17	15	13	12*	11*	10*
	3X8					50	42	38	32	28	25	22	20	18
	3X10								50	44	39	35	31	28
	4X6			53	43	36	31	27	23	20	19	17	14	13
	4X8							49	42	37	33	29	26	24
	4X10										52	47	42	38
11 in.	2X4	16	13	10										
	2X6	39	30	25	20	17	14	12	11					
	2X8			45	37	31	27	23	20	17	15	14	12	11
	2X10					49●	42	36	32	28	24	22	20	18
	3X4	25	19	16	13	11*								
	3X6		46	37	31	26	22	19	16	14	13	11*	10*	
	3X8					47	40	34	30	26	23	21	18	17
	3X10							55	48	42	37	33	29	27
	4X6			49	41	34	29	25	22	19	18	16	13	12
	4X8						53	46	40	35	31	28	25	22
	4X10									56	49	44	39	35
12 in.	2X4	15	12	10										
	2X6	36	29	23	19	16	14	12	10					
	2X8		49●	42	35	29	25	22	19	16	15	13	12	10
	2X10				50●	46●	40	34	30	26	23	20	18	17
	3X4	23	18	15	12	10*								
	3X6	55	43	35	29	24	21	18	15	13	12	11*		
	3X8				53	44	38	32	28	25	22	19	17	16
	3X10							52	45	39	35	31	28	25
	4X6			47	39	32	28	24	21	18	17	15	13	11
	4X8						50	43	38	33	29	26	23	21
	4X10									52	46	41	37	33

To left of solid zigzag lines, deflection is less than $\frac{1}{8}$ in, using formula $\frac{3}{384}\frac{w(l)^4}{EI}$
To right of dotted zigzag lines deflection is greater than $\frac{1}{4}$ in, using formula $\frac{3}{384}\frac{w(l)^4}{EI}$

* indicates deflection is greater than $\frac{1}{360}$ of the span.
● indicates spacing is determined by horizontal shear.

For deflection

$$D = 0.0225\,\frac{l^2}{d}$$

$$\text{When } D = \frac{1}{8}\text{ in.}\quad l = \frac{\sqrt{800d}}{12}$$

$$\text{When } D = \frac{1}{4}\text{ in.}\quad l = \frac{\sqrt{1,600d}}{12}$$

$$\text{When } D \quad \frac{l''}{360} \qquad \frac{40}{27}d.$$

For horizontal shear

$$s = 3,200\,\frac{bd}{w'l}$$

(Depths for joists are taken ¼ in. less than nominal sizes.)

In Table II, Part A:

For flexure

$$s = 2,000\,\frac{bd^2}{w'l^2}$$

For $D = \frac{1}{8}$ in.

$$s = 11,100\,\frac{bd^3}{w'l^4}$$

For horizontal shear

$$s = 3,200\,\frac{bd}{w'l}$$

In Table II, Part B:

For flexure

$$s = 1,600\,\frac{bd^2}{w'l^2}$$

For $D = \frac{l''}{360}$

$$s = 1,780\,\frac{bd^3}{w'l^3}$$

For horizontal shear

$$s = 3,200\,\frac{bd}{w'l}$$

Table III:

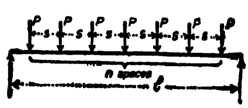
Deflection of ⅛ in. governs.

$$l'' = \sqrt[4]{1,590,000(145)d^3}$$

In Table IV the concentrated loads from the joists were considered on a simple span in calculating the bending moment. The worst case was assumed—that is, when one joist comes at mid-span. Since a girt is usually continuous for at least two spans and the full live load never reaches it, the moment of resistance of the timbers was multiplied by 1.2. Nominal sizes of the girts were used in the computations.

For flexure

$$l = \frac{240bd^2 + \frac{s}{8}P(n^2 + 2n)}{3P(n + 1)}$$



Horizontal shear is to be considered separately. From the above considerations it would seem that an allowable horizontal shear of (200) (1.2) = 240 lb. per sq. in. may safely be used.

Table II.—Spacing of Joists

Deflection limited

Distance center to center of joists in inches

Part A.—Based on $M = \frac{wl^2}{10}$ with deflection limited to $\frac{1}{8}$ in.

Slab thickness	Size of joist (inches)	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
3 in.	2X4	31	24	16	11									
	2X6			47	39	29	21	15	12					
	2X8						50	38	29	22	17	14	11	
	2X10								57	44	35	28	22	18
	3X4	47	37	25	17	12								
	3X6					43	31	23	18	14	11			
	3X8							57	43	33	26	21	17	14
	3X10										52	41	34	27
	4X6					58	42	31	23	18	14	11		
	4X8								58	45	35	28	22	18
	4X10											55	45	36
6 in.	2X4	23	18	12										
	2X6	55	43	35	29	21	15	11						
	2X8				53	44	38	28	21	17	13	10		
	2X10							51	43	33	26	21	17	13
	3X4	35	27	18	12									
	3X6			53	43	32	23	17	13	10				
	3X8						56	43	32	25	20	15	12	10
	3X10									50	39	31	25	20
	4X6					43	31	23	18	13	11			
	4X8							57	43	33	26	21	17	14
	4X10										52	41	34	27
9 in.	2X4	18	15	10										
	2X6	44	35	28	23	17	12							
	2X8			51	42	35	30	23	17	13	10			
	2X10					55	48	41	34	27	21	16	13	11
	3X4	28	22	15	10									
	3X6		52	42	35	26	19	14	10					
	3X8					53	45	34	26	20	16	12	10	
	3X10								52	40	31	25	20	16
	4X6			56	46	34	25	18	14	11				
	4X8							46	35	27	21	16	13	11
	4X10									53	42	33	27	22
12 in.	2X4	15	12											
	2X6	36	29	23	19	14	10							
	2X8		49	42	35	29	25	19	14	11				
	2X10				50	49	40	34	29	22	17	14	11	
	3X4	23	18	12										
	3X6	55	43	35	29	21	15	11						
	3X8				53	44	38	28	22	17	13	10		
	3X10							52	43	33	26	21	17	13
	4X6			47	39	29	21	15	12					
	4X8						50	38	29	22	17	14	11	
	4X10								57	44	35	28	22	18

Strength governs to left of zigzag lines.

Part B.—Based on $M = \frac{wl^2}{8}$ with deflection limited to $\frac{1}{360}$ of the span.

Slab thickness	Size of joist (inches)	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
3 in.	2X4	25	18	13	10									
	2X6	58	46	37	31	26	22	17	14	11				
	2X8				56	47	40	35	30	26	23	20	17	14
	2X10							55	48	42	37	33	30	27
	3X4	37	27	20	15	11								
	3X6			56	46	39	32	26	21	17	14	12	10	
	3X8							52	45	40	35	30	25	22
	3X10										56	50	44	40
	4X6					52	43	35	28	23	19	16	14	12
	4X8									53	47	40	34	29
	4X10													54
6 in.	2X4	18	13	10										
	2X6	44	34	28	23	19	16	13	10					
	2X8			51	42	35	30	26	22	20	17	15	13	11
	2X10						48	41	36	31	28	25	22	20
	3X4	28	20	15	11									
	3X6		52	42	35	29	24	19	16	13	11			
	3X8					53	45	39	34	30	26	22	19	16
	3X10								54	47	42	37	33	30
	4X6			56	46	39	32	26	21	17	14	12	10	
	4X8							52	45	40	35	30	25	22
	4X10										56	50	45	40
9 in.	2X4	15	11											
	2X6	35	27	22	18	15	13	10						
	2X8		50	41	34	28	24	21	18	16	14	12	10	
	2X10				53	45	38	33	29	25	22	20	18	16
	3X4	22	16	12										
	3X6	52	41	33	28	23	19	15	13	10				
	3X8				50	42	36	31	27	24	21	18	15	13
	3X10						57	49	43	38	33	30	27	24
	4X6		55	45	37	31	26	21	17	14	11	10		
	4X8					57	48	41	36	32	28	24	20	17
	4X10								57	50	45	40	36	32
12 in.	2X4	12												
	2X6	29	23	18	15	13	11							
	2X8	53	42	34	28	23	20	17	15	13	12	10		
	2X10			54	44	37	32	27	24	21	18	16	15	13
	3X4	18	13	10										
	3X6	44	34	28	23	19	16	13	10					
	3X8			51	42	35	30	26	22	20	17	15	13	11
	3X10						48	41	36	31	28	25	22	20
	4X6	58	46	37	31	26	22	17	14	11				
	4X8				56	47	40	34	30	26	23	20	17	14
	4X10							55	48	42	37	33	30	28

Deflection governs to right of zigzag lines

Diagram I:

For flexure

$$s = \sqrt{\frac{336,000d^2}{145h}}$$

For deflection

$$s = \sqrt[4]{\frac{1,590,000d^3}{h}}$$

Diagram II:

For flexure

$$s = 2,380 \frac{bd^2}{h(l'')^2}$$

For deflection ($D = \frac{1}{8}$ in.)

$$l'' = \sqrt{\frac{95,400d}{238}}$$

When $d = 2$ in.	When $d = 4$ in.	When $d = 6$ in.
$l'' = 28.3$ in.	$l'' = 40.0$ in.	$l'' = 49.0$ in.

These results are based on a value of $k = l''$ and the values given may at least be increased to 30, 42, and 50 respectively.

For shear ($v = 200$ lb. per sq. in.)

$$l'' = 9d - (l'' - k)$$

Table V:

$$n' = \frac{(hl'')^2}{19,060}$$

$$h = \frac{1}{l''}\sqrt{19,060n'}$$

Formulas to use with table:

$$\text{Diameter of bolt} = \sqrt{\frac{0.095bd}{2l'' - }}$$

$$\text{Net area of washer} = \frac{3bd^2}{2l'' - k}$$

Illustrative Problems.—1. Required the proper spacing of 2 × 8-in. joists having a span of 7.5 ft. to support forms for a 5-in. slab in beam-and-girder construction, assuming 1-in. sheathing.

Table I shows that 2 × 8-in. joists spaced 31 in. on centers will give sufficient strength if the bending moment is assumed equal to $\frac{wl^2}{10}$. For this spacing the table indicates that the deflection is somewhat over $\frac{1}{8}$ in. but less than $\frac{1}{4}$ in. and much less than $\frac{1}{360}$ span. Accurately, $D = 0.0225 \frac{(7.5)^2}{7.75} = 0.16$ in. From Table III we find that the spacing cannot be greater than 30 in. without the deflection of the sheathing exceeding $\frac{1}{8}$ in., which is not advisable.

For $M = \frac{wl^2}{8}$, the spacing would be (0.8)(31) = 25 in. from Table I and the deflection $D = 0.030\frac{(7.5)^2}{7.75} = 0.22$ in. From Table II, the spacing would be $22 + \frac{1}{8}(8) = 24.7$ in.

For $M = \frac{wl^2}{10}$ and the deflection limited to $\frac{1}{8}$ in., Table II shows the proper spacing to be $21 + \frac{1}{8}(8) = 23.7$ in.

2. Assume joists in the preceding problem to be supported midway between beams. Determine their economical size and proper spacing, assuming $M = \frac{wl^2}{10}$ and deflection limited to $\frac{1}{8}$ in.

It will be sufficiently accurate to assume the span as 4 ft. From either Table I or II, we find that 2 × 4-in. joists may be employed spaced 25 in. on centers.

3. Determine size and span length of girt in the preceding problem to support the joists midway between beams.

Load coming from each joist is $(3.75)(2\frac{5}{12})(137.5) = 1{,}075$, or say 1,000 lb., accurately enough. From Table IV we find that a 3 × 4-in. girt with posts spaced 3.8 ft. c. to c. could be used, or a 3 × 6-in. girt with posts 5.6 ft. c. to c., or a 4 × 6-in. girt with posts 6.6 ft. c. to c.

Horizontal shear must be considered separately considering a joist to occur close to one support. Assuming a 4 × 6-in. girt with posts 6.6 ft. on centers, $V = 2{,}260$ lb. and $v = \frac{3}{2} \cdot \frac{2{,}260}{(4)(6)} = 142$ lb. per sq. in., which is less than the allowable value.

A 3 × 4-in. post could sustain $(3)(4)(400) = 4{,}800$ lb. without injuring the fibers of the girt. It would only be required to support 3,450 lb., consequently this size of post is suitable.

4. Assuming the cross-section of beam below slab as 14 × 18 in. (23 in. total depth) determine the safe span for the beam bottom to be made of 2-in. plank.

Live plus dead load on beam bottom is $75 + 2\frac{3}{12}(150) = 3{,}625$ lb. per sq. ft. Diagram I shows the maximum span to be 43 in.

5. Find the proper spacing of 3 × 4-in. posts to support the forms for 8 × 16-in. beams (cross-section given below slab) spaced 6 ft. on centers with a 4-in. floor slab. Assume that no girt is placed at midspan of joists.

Total load on beam per linear foot is $(125)(6) + \frac{(8)(16)(150)}{144} = 883$ lb. not considering the weight of the forms, which may be neglected. Safe bearing of post on fibers of cap = $(3)(4)(400) = 4{,}800$ lb. Safe spacing of posts $= \frac{4{,}800}{883} = 5.4$ ft.

6. Determine the size and spacing of joists, girts, and posts to support an 11-in. flat slab floor.

Assuming 1-in. sheathing we find from Table III that the joists cannot be placed more than 27 in. on centers. Table I shows that for 2 × 8-in. joists the spacing of girts may be made 6.5 ft. The load coming from each joist is $(6.5)(2\frac{7}{12})(212.5) = 3{,}110$ lb., or accurately enough 3,000 lb. From Table IV we find that for 4 × 6-in. girts the posts may be placed 3.8 ft. on centers. Horizontal shear on girts must be considered separately. $v = \frac{3}{2}\frac{4{,}380}{(4)(6)} = 273$ lb. which is somewhat greater than the allowable value and a 6-ft. spacing of the girts is necessary.

TABLE III.—SAFE SPAN FOR FLOOR SHEATHING (INCHES)

Based on $M = \frac{wl^2}{10}$ with deflection limited to $\frac{1}{8}$ in.

Slab thickness (in.)	Weight in pounds per square foot (live plus dead)	1-in. stock	1¼-in. stock	1½-in. stock	2-in. stock	Slab thickness (in.)	Weight in pounds per square foot (live plus dead)	1-in. stock	1¼-in. stock	1½-in. stock	2-in. stock
3	112.5	32	39	46	57	8	175.0	29	35	41	51
4	125.0	31	38	45	56	9	187.5	28	35	41	50
5	137.5	30	37	44	54	10	200.0	28	34	40	50
6	150.0	30	37	43	53	11	212.5	27	33	39	49
7	162.5	29	36	42	52	12	225.0	27	33	39	48

TABLE SPAN OF GIRT

Based on $M_T = \frac{1}{6} fbd^2$ (1.2)

center to ter of posts in

Spacing of | Size of gir | ing

5.9 7.8 9.8 | 6. 3.2 4.4 5.6

4X4 4X6 4X8 4X10 | 4.0 4.9

8. 11.

4. 6.

4. 4X6 4X8 4X10

2.4 3.8 4.

6X6 6X8 6X10 | 8. 10. | 8. 11.

sing form

Spacing of joists | Size of gir. (inches)

4 6 8 | 3 5 8

4. 6.8 9.5 11

3 3 3 3 | 4 4 4 4 | 4 4

8 10 | 4 6.0 | 2 5 6.6 | 4 4.4 6.3 | 8 6.0

1.8 3 5

8 6 | 8 9.

3 3 3 | 3.8 8. 10. 12 | 2.4 4.4 8

4 4 4 4 | 3 7. 9.9 4 | 5 8. 10. | 4.6 8.0 | 3.8 6.8 8.9 | 1.3 5.8 8.3

6.9 9.4 11.7 | 4. 8 6

st in | pou

4 4

7. What spacing of vertical studs is required for a wall form with $1\frac{1}{2}$-in. sheathing and a height of 12 ft. Diagram I shows the spacing to be 18 in.

8. Assuming a 24 × 24-in. column with $l'' = 37$ in., determine the spacing of the column yokes. For 2 × 4-in. yokes placed on edge, the value of l'' to be used in Table V should be

$$(37)(1.23)\sqrt{\frac{(2)(37)(24) - (24)^2}{(37)^2}} = 42.5 \text{ in., say } 42 \text{ in.}$$

For 4 × 4-in. yokes, the value of l'' to be used should be $(37)(0.87)(0.935) = 30$ in

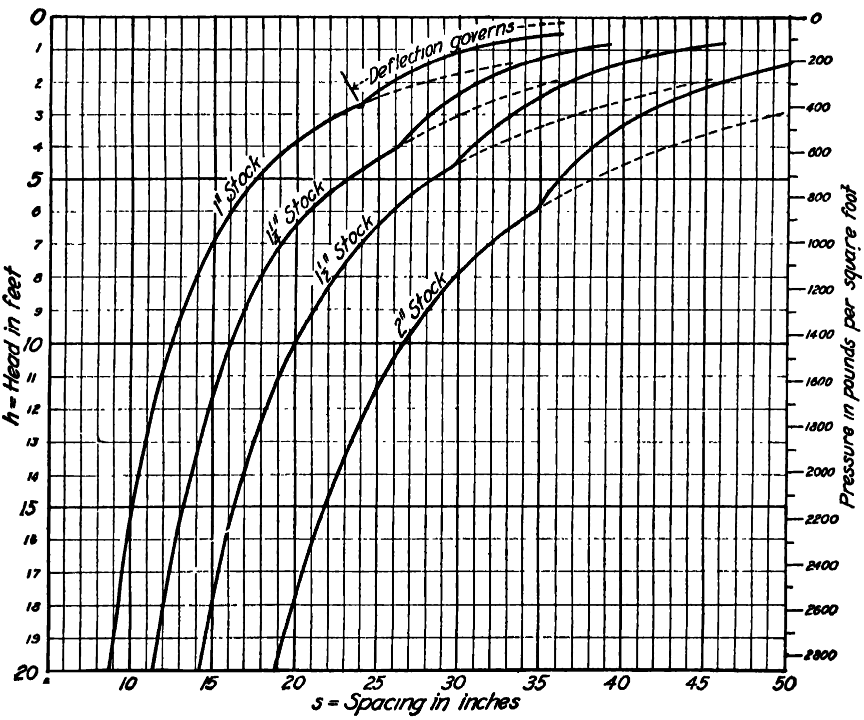
For shear the actual value of l'' must not be taken less than $9d - (l'' - k) = (9)(4) - (37 - 24) = 23$ in. for either the 2 × 4-in. or the 4 × 4-in. yokes. Evidently the shear in the yokes will be less than the allowable.

DIAGRAM I

SPACING OF VERTICAL AND HORIZONTAL STUDS (OR COLUMN YOKES) AT ANY GIVEN DEPTH BELOW SURFACE OF CONCRETE

(Based on strength and deflection of sheathing)

$M = \frac{wl^2}{10}$ Deflection limited to $\frac{1}{8}$ in



Other things being equal, Diagram II shows that 1-in. sheathing would be more economical than $1\frac{1}{2}$-in. when 2 × 4-in. yokes are used. The table shows that the same number of yokes would be used in the two cases.

The spacing center to center for the 2 × 4-in. yokes for a 10-ft. column with 1-in. sheathing would be as follows in inches starting at the top: 30–20–14–11–10–9–8–7.

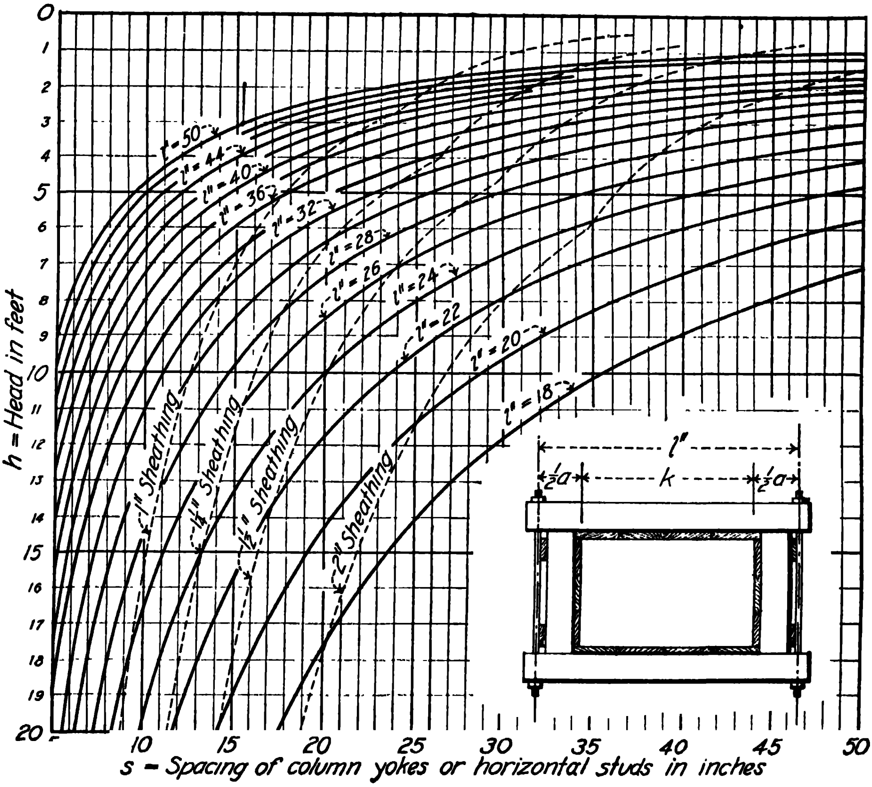
Where the strength of yokes governs their spacing, the bolts must have a diameter of $\sqrt{\frac{0.095bd^2}{2l'' - k}}$ using actual values of k and l''. Thus for the 2 × 4-in. yokes, diameter of the bolts must be at least $\sqrt{\frac{(0.095)(2)(16)}{50}} = 0.25$ in. The net area of washer should be $\frac{3bd^2}{2l'' - k} = \frac{(3)(2)(16)}{50}$ 1.92 sq. in.

Diagram II

Spacing of Column Yokes or Horizontal Studs at Any Given Depth Below Surface of Concrete

(Based on strength and deflection of the yokes or studs)

Based on $M = wl^2$ Deflection less than ⅛ in.



Directions for Using Diagram II and Table V

For 2 × 4-in. yokes or studs (flat) multiply actual l'' by 1.73 before using diagram or table.
For 2 × 4-in. yokes or studs (on edge) multiply actual l'' by 1.23 before using diagram or table.
For 3 × 4-in. yokes or studs (on edge) multiply actual l'' by 1.00 before using diagram or table.
For 4 × 4-in. yokes or studs (on edge) multiply actual l'' by 0.87 before using diagram or table.
For 3 × 6-in. yokes or studs (on edge) multiply actual l'' by 0.67 before using diagram or table.
For 4 × 6-in. yokes or studs (on edge) multiply actual l'' by 0.58 before using diagram or table.
For 6 × 6-in. yokes or studs (on edge) multiply actual l'' by 0.47 before using diagram or table.

For $b \times d$-in. yokes or studs (on edge) multiply actual l'' by $\sqrt{\frac{18}{bd^2}}$ before using diagram or table.

For columns the value of l'' to be used in diagram or table should be the value of l'' as found above multiplied by $\sqrt{\frac{2l''k - k^2}{(l'')^2}}$, in which expression the actual values of l'' and k are to be substituted.

In determining spacings from the diagram or table for $\left\{\begin{matrix} 3 \times 4\text{-in. and } 4 \times 4\text{-in.} \\ 3 \times 6\text{-in., } 4 \times 6\text{-in., and } 6 \times 6\text{-in.}\end{matrix}\right\}$ yokes, actual values of l'' greater than $\left\{\begin{matrix} 42.0 \\ 50.0 \end{matrix}\right\}$ in. will give a deflection of yokes greater than ⅛ in., and actual values of l'' greater than $\left\{\begin{matrix} 60.0 \\ 72.0 \end{matrix}\right\}$ in., will give a deflection greater than ¼ in.

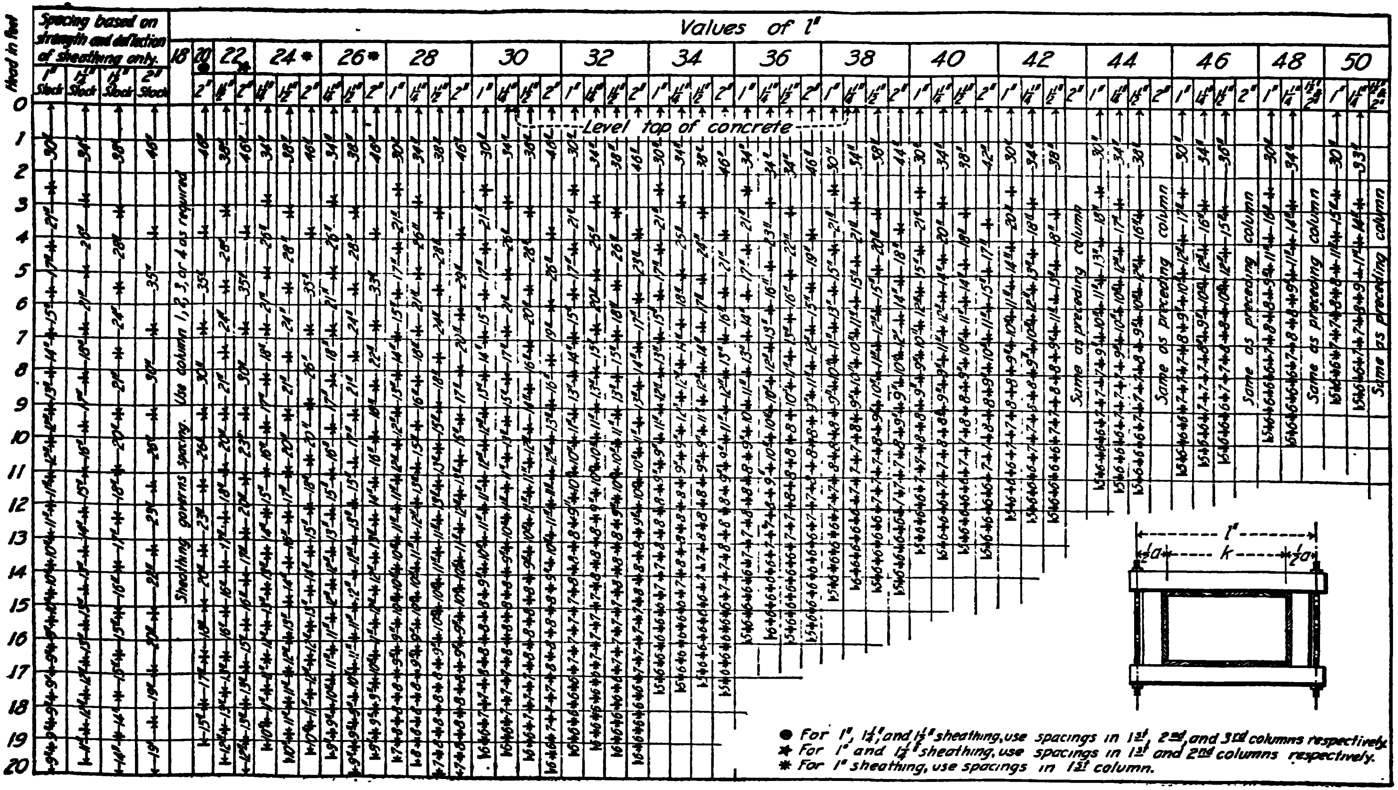
In determining spacings from the diagram or table, *actual* values of l'' must not be considered as less than determined by the formula

$$l'' = 9d\text{-}(l'' - k)$$

otherwise horizontal shear will be greater than 200 lb per sq. in. The corresponding *actual* value of k (which will be called k') should be determined by subtracting the value of a (see sketch) from the value of l'' found by the above formula. The value of l'' to use in diagram or table should then be found as explained above and finally multiplied by $\sqrt{\frac{k}{k'}}$.

TABLE V.—SPACING OF COLUMN YOKES OR HORIZONTAL STUDS

Based on $M = \frac{wl^2}{8}$ Deflection less than 1/8 in.



9. Assuming a 12 × 12-in. column with l'' = 22 in., determine the spacing of 2 × 4-in. yokes placed on edge.

The limiting value of l'' for shear on yoke is (9)(4) − (10) = 26 in. Thus actual values of l'' and k must be considered as 26 in. and (26 − 10) = 16 in. respectively, which gives a value of l'' to be used in Table V of $(26)(1.23)\sqrt{\frac{(2)(26)(16)-(16)^2}{(26)^2}}\sqrt{\frac{12}{16}} = 25.6$ in., say 26 in.

Jack Spacing.[1]—When 3 × 4-in. spruce jacks are used under beams, their spacing should be about as follows:

Beam Depth	Jack Spacing
Up to 17 in. inclusive	4 ft. 6 in.
18 in. to 23 in. inclusive	4 ft. 0 in.
24 in. to 33 in. inclusive	3 ft. 6 in.
34 in. to 47 in. inclusive	3 ft. 0 in.
48 in. and over inclusive	2 ft. 6 in.

Table VI.—Sizes of Cleats on Beam Sides

t/d in.	0 in.	3 in.	6 in.	9 in.	12 in.
32	2 × 4	2 × 3 (22) 2 × 4	2 × 3	2 × 3	2 × 3
34	2 × 4	2 × 4	2 × 3 (20) 2 × 4	2 × 3 (22) 2 × 4	2 × 3
36	2 × 4	2 ×4	2 × 4	2 × 3 (20) 2 × 4	2 × 3 (20) 2 × 4
38	3 × 4	2 × 4	2 × 4	2 × 4	2 × 4
40	3 × 4	2 × 4 (20) 3 × 4	2 × 4 (22) 3 × 4	2 × 4	2 × 4
42	3 × 4	3 × 4	3 × 4	2 × 4 (20) 3 × 4	2 × 4 (22) 3 × 4
44	2 × 6	3 × 4 (22) 2 × 6	3 × 4	3 × 4	3 × 4
46	2 × 6	2 × 6	3 × 4 (20) 2 × 6	3 × 4 (22) 2 × 6	3 × 4
48	2 × 6	2 × 6	2 × 6	2 × 6	2 × 6
50	2 × 6	2 × 6	2 × 6	2 × 6	2 × 6

Spacing other than 24 in. shown thus (20).
d = depth of beam in inches.
t = thickness of slab in inches.

[1] **Aberthaw Construction Company, Boston, Mass.**

When the jacks support beam forms which carry a slab load, it is necessary to provide for the design of the jack as a column (see Figs. 41 and 42).

Cleat Spacing.[1]—Cleats on beam sides are generally spaced 24 in. on centers and are dressed two edges. For beams up to and including 30 in. in depth, 2 × 3's are used. For beams of greater depth use the size and spacing noted in Table VI:

5. Make-up of Forms.

5a. Plant Design.—The lay-out of the plant or construction work is generally left to the job superintendent, who arranges his machinery and

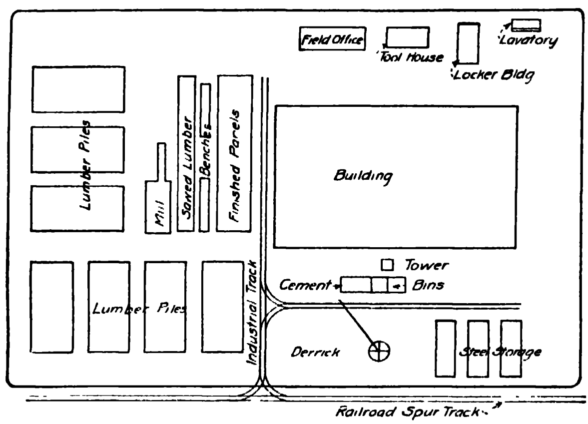


FIG. 5.—Typical plant layout.

equipment in accordance with his judgment. However, on work of any magnitude, it is advisable for the designing office to make a field study of the building site and then prepare a plant lay-out and schedule of operations. In some companies, a production or efficiency engineer has direct charge of the planning and routing of the construction work. The routing clerk is under the supervision of the production engineer, and has direct charge of the routing of the material for the job he is connected with. The production engineer and the routing clerk should visit the site and in company with the job superintendent, locate the various parts of the plant so as to secure the greatest efficiency in the unloading and handling of material. This inspection (together with an accurate map of the site, transportation facilities, etc.) furnishes the designing office with the necessary information for the preparation of the plant lay-out.

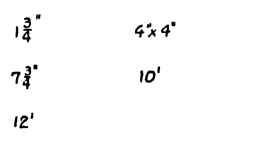


FIG. 6.—Markers for lumber piles.

A typical plant lay-out is shown in Fig. 5 and indicates the location of the mixing plant, the cement shed, the lumber piles, the reinforcement storage, the mill, the benches, the routing office, etc. From this general plan, detailed drawings of the storage places for lumber, reinforcement, etc. can be made.

[1] Aberthaw Construction Company, Boston, Mass.

The lumber storage yard should be located as near the sawmill as is practicable and should be readily accessible to a railroad siding, wharf, or road. The available area should be divided into spaces so that as far as possible, all boards of the same width, thickness and length may be stacked in the same pile. When the lumber is unloaded from the wagon, car, or barge, it will be carried to its proper pile, where it can readily be found when wanted. Each pile should have a marker or labeled stake in front, as shown in Fig. 6.

An inspection of Fig. 5 shows the location of the saw-mill, make-up benches and routing office with relation to the lumber yard, the source of raw material, and with relation to the building site, the point of disposition of the completed form sections. The progress of the raw material should be continuous from the unloading of the lumber to the erection of the forms in the structure.

5b. Job Office Methods.—The office of the routing clerk should be located near and face the mill and benches. Along the front wall should be placed a slightly inclined shelf, upon which the plans and drawings may be placed. Underneath are shelves for drawings, order slips, etc., not in immediate use. In the center of the front wall is placed a small sliding window, which serves as a ticket window through which the order slips are passed. Upon the rear of the office is placed a cabinet or order-slip board. This board has projecting from its face three horizontal rows of steel hooks, on which the order slips are hung. There should be hooks for vertical rows of slips for the labor foreman, the mill man, and two places for each making-up bench. A lay-out of an order-slip board for a job with one mill and two benches is shown in Fig. 7.

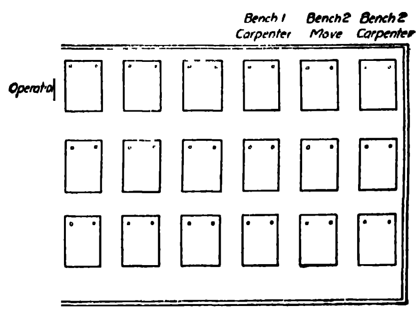


FIG. 7.—Order slip board.

The order slips are 4 × 5 in., and each slip is divided into spaces, which are filled in by the routing clerk or his assistant with the information necessary to instruct the men receiving the slips as to what is required. The slips are of different colors, so that they may be distinguished at a glance. There are six kinds of slips used: Stock order, white; mill order, yellow; mill order duplicate, blue; move order, white; bench order, orange; bench order duplicate, pink. Each slip has two small holes near the top and these provide for the hanging of slip on the cabinet or board.

The slips should be filled out in triplicate, by placing them in groups of three in the following order: Stock order, mill order, and duplicate mill order. The same arrangement can be applied to the other three slips. A single holder for each set of slips can be made by driving two 20d nails up through a board. The bottom ends of the slips may be held in place by a heavy rubber band. The slips for stock order, move order, mill order and bench order are shown in Fig. 8.

The operations in the make-up of forms will now be explained by the consideration of the procedure in the construction of the section for the bottom of a typical beam.

When everything is ready for making up the forms, the routing clerk will place on the middle row of hooks of the order-slip board the order slips for stock

to be moved from the stock piles to the mill. On the lower row of hooks he will place the corresponding mill order for the cutting of this stock. The labor foreman or move man goes to the routing office window, where he is handed a white slip labeled "Stock Order." The routing clerk stamps the date and time in the space labeled "Issued" just before he hands out the slip. The move man goes with the slip to the stock yard and selects the material from the piles, marks or tags the boards with their proper numbers and has his laborers carry it to the mill. The move man signs the "Stock Order" slip and returns it to the routing office, where the clerk stamps it again in the space headed "Returned" and places it on the upper row of hooks of the order-slip board. The routing clerk then gives to the move man the next order slip from the middle row of hooks, and the latter carries out the order as before. As soon as a stock order has been filled and the order slip moved to the upper row of hooks on the order-slip board, the clerk moves the corresponding mill order and its duplicate from the lower to the middle row of hooks. The mill order is now issued to the mill man and the duplicate order slip placed on the top row of hooks. The mill man takes the stock which he finds at the cut-off saw and cuts as required by the order. When the order is finished he marks the pieces with the M, C, Y, or other number as stated on the slip, signs the slip, which he then returns to the routing office, where he receives another order slip. The returned mill order slip is placed on file. The routing clerk now issues the move order to the move man, who moves the stock listed thereon from the mill to the bench. The move order, when returned and stamped, is placed on the top row of hooks, and the bench order and duplicate, which calls for the use of this stock are placed on the middle row of hooks. The bench order is now issued to the carpenters and is accompanied by a blue print sketch of the plan of the panel which is to be made up. This sketch would be cut out of the detail form drawing. The duplicate bench order is placed on the top row of hooks, upon the issuance of the bench order. An inspection of the order-slip board at any time will show the work which the move gang, the mill and the benches are doing. The order on any vertical row of hooks should be kept in the same relative position with respect to its corresponding orders on other rows; thus if the mill order to cut the stock for beam bottom, *BB*2, is issued just before that for column side, 1*C*18*G*, the remaining orders for these sections should follow in the same order.[1]

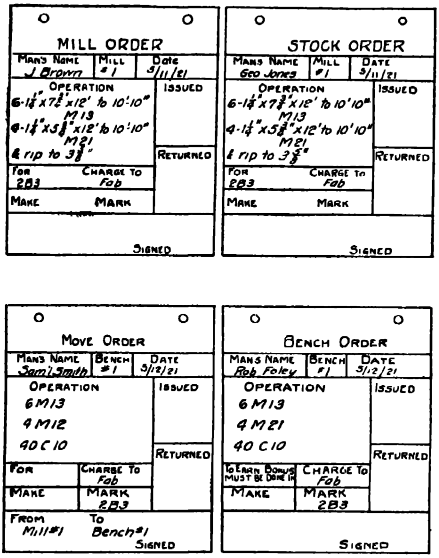


FIG. 8.—Order slips used in form construction.

[1] From *Concrete*, Aug., 1918. Description of a system initiated by Thompson & Lichtner, Consulting Engineers for Aberthaw Construction Company, Boston, Mass.

All order slips should be turned in to the routing clerk at the end of the day, or whenever the work called for on the slip is stopped, whether it is completed or not.

The routing clerk should provide for the cutting of all pieces of the same dimension at one time, so as to economize time in the setting of the saw-bench stops at the mill. Each bench order should call for the make-up of all like sections at the same time, so that the carpenters utilize one bench set-up for as long a time as possible. It is advisable to cut all the cleats and pile them in labeled sections near the benches before the regular work on the boards is begun. The cleats may be piled under the benches by the move gang when called for on the proper move order.

5c. Sawmill.—The sawmill should be located so that the lumber will go directly to the cut-off saw, then to the rip saw, and then to the bench, without moving backward or crossing its path. A mill should be of the portable type with a swing cut-off saw and emery wheel, grindstone, etc. The labor crew necessary to operate such a mill would be a mill man and a helper.

5d. Bench Work.—The benches should be located near the mill, but at a sufficient distance to allow piling of milled stock which is ready to be made up.

(Courtesy of Aberthaw Construction Co.)

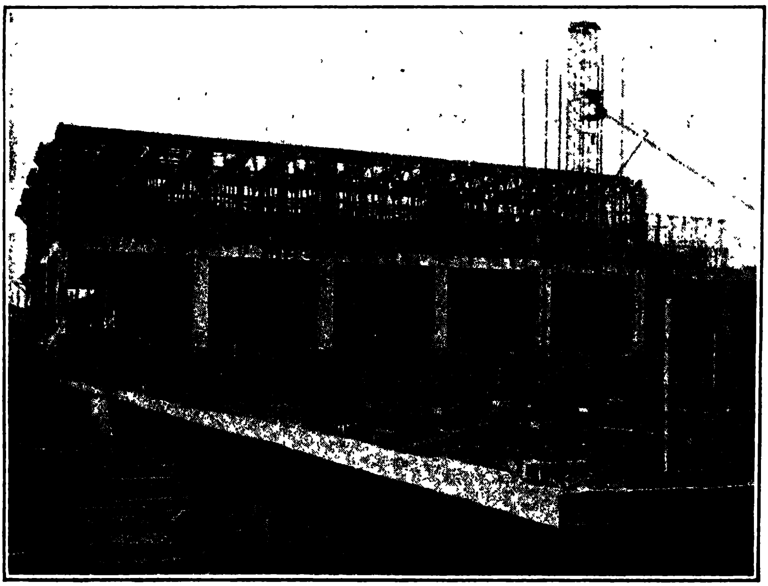
Fig. 9.—View of carpenters' benches.

The number of benches required depends upon the size of the job. On a large job it is desirable to use one bench for making up column sides, another for beam sides, etc. This plan provides for the maximum use of the same bench lay-out. A detailed view of a bench is shown in Fig. 9. Form clamps are generally used, but where there are a number of panels of equal width, or nearly equal widths, a couple of hardwood eccentrics, bolted to turn on the table, can be used. The cleat strips should be spaced in accordance with the standard cleat spacing, about 2 ft. A thin steel plate is fastened on the top of the bench between each pair of strips and serves to turn and clinch the nails as they are driven through the cleats and boards. There should be one skilled carpenter and one assistant for each bench.

6. Erection of Forms.—As soon as a form section is completed it is placed in a pile of similar form sections near the benches. The surfaces of the forms, which are to be in contact with the concrete, are given a coat of light lubricating oil.

Several hours before the forms are to be used they should be moved on to the building site and stacked near the place where they are to be erected. The movement of the forms from the finished piles should be made by the move gang according to directions given by the routing clerk.

The erecting of forms is done by carpenters under one or more move gang foremen. These foremen receive their instructions from the routing clerk on written tickets and use them for the direction of their men. The stripping of forms is done by a stripping gang and foreman, and the moving by the move gang



(*Courtesy of Aberthaw Construction Co.*)

Fig. 10.—Forms for second story of building.

and foreman. A construction and inspecting or a carpenter foreman has supervision of these three classes of gangs, and inspects, instructs or disciplines them as occasion requires. The erection, move and stripping foremen receive written instructions similar to the tickets used in making up of the forms.

As far as possible all form sections of a kind should be erected at one time, that is, the column sections of one story be assembled, erected and braced before the erection of the remainder of the formwork is begun (Fig. 10). In a building covering a large area, it will probably be advisable to erect the forms for only one section of each story at a time.

The method of framing and erection of the studding or wall framing of a frame house can be used to advantage in the erection of the supports for a flat slab floor. All the posts in a transverse line should be laid flat on the floor in their proper position, and the girt or heading piece nailed to their outer ends. The scabs and bracing should then be nailed on and the bent well tied together. The bent can

be easily raised into position by several laborers and held in place temporarily by longitudinal and diagonal floor braces. This method can be efficiently used for the assembly and erection of the centering or supports for floor panels, beams and girders.

The construction and inspection foreman should inspect the erection work sufficiently to insure that the forms are in their proper location, as shown by the Key Plan (see Fig. 2). The forms should be in proper alignment, well nailed preferably with double-headed nails and all bevel tenon strips, etc., should be in place. These strips should be put on the forms by the bench carpenters and not by the erection gang. High priced carpenters should not be employed in raising and holding form sections in place during erection. This class of work should be done entirely by laborers. Column and beam forms should be well braced so that the pouring of the concrete will not cause their distortion, or deflection. Distorted or displaced forms can be remedied during the pouring of the concrete, but under no conditions should the forms be moved after the concrete has begun to harden.

7. Removal and Re-use of Forms.—The removal of forms should be done by ordinary laborers under the direction of an experienced labor foreman. Care should be taken to preserve the sections intact and not injure the corners or surfaces of the concrete. Wrecking bars should be used. All form sections should either be removed to the place where they are to be used again or taken to the mill for re-making. On large jobs it may be advisable to use a portable saw operated by a small electric motor, and the whole self-contained on a frame-work, that can be raised from floor to floor as the work progresses. The forms to be re-made can then be raised directly to the next floor and be cut without the expenditure of the extra time for lowering them to the ground and raising them again.

The time of removal of the forms depends on the character of the concrete, the location of the form, the temperature and moisture conditions, etc. The column sides can be removed first in from 2 to 4 days, care being taken to provide sufficient shoring for the slabs, beams and girders. The slab forms and beam sides may next be removed in from 7 to 14 days from time of pouring. The beam and girder bottoms, with their supports, should be left in place from 10 days to 3 weeks after pouring. The girder sides are removed after the beam sides, and the girder bottoms should be left in place after the removal of the beam bottoms, especially when the beam sides and bottoms are removed as units and under light loading and favorable temperature conditions. The wall forms are usually built independent of other forms and can be removed as soon as the concrete has become sufficiently hard.

The forms should be thoroughly cleaned and re-oiled before stacking for erection in a new location. All rough surfaces should be smoothed and repairs made to put the sections in first-class condition.

8. Forms for Buildings.—In the preceding articles concerning the design, make-up, erection and removal of forms, some reference has been made to the use of forms in building construction. It is the purpose of this article to discuss more fully some of the details of forms for the various elements of a reinforced concrete building, namely; walls, columns, beam and girder floors, flat slab floors and miscellaneous structures, such as stairways, cornices, etc.

8a. Walls.—Wall forms may be of two types, continuous and panel. The former type is best adapted to low walls where the breaks are uniform and

similar in character, as in a one- or two-story building or the basement wall of a higher structure. The continuous type of wall form for a low wall such as the cellar or basement wall of a building consists of 1 in. boarding nailed to vertical studs and held in place by horizontal and diagonal braces. Figure 11 shows a cellar wall. Where the outside earth is hard and firm no formwork is necessary below the ground surface. Figure 12 shows the formwork for a low wall where there is no cellar or basement below grade. Note the method of supporting the forms on the ground over the foundation wall.

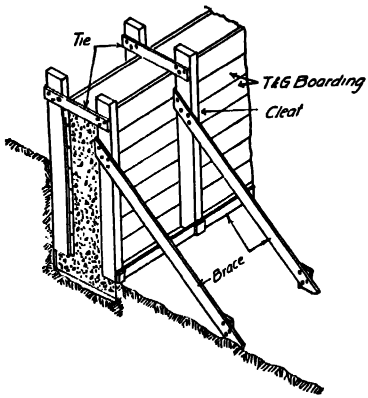


FIG. 11.—Basement wall form.

Panel or sectional forms are generally more efficient and economical for walls of appreciable height. The size of the form depends on the panel length of the building and the weight that can be handled. The essential parts of a panel form are the sheeting or planking, the studs and the wales. The sheeting is nailed to the vertical studs, which are held in place by horizontal wales or waling pieces. The latter are single timbers, or in some cases, two smaller timbers set edgewise against the studs and spaced apart to allow the tie-bolts to be carried through.

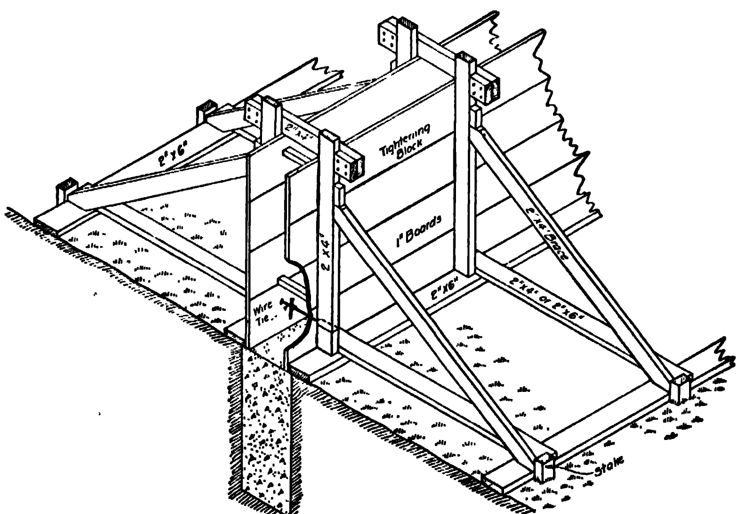


FIG. 12.—Self supporting wall form.

Wire is often used for tieing the opposite wall panels together and is tightened by wedges or twisting the wire. Spreaders of wood or metal pipe hold the adjacent sections the proper distance apart, and are removed as the concrete level rises.

Figure 13 shows a typical wall panel construction used by the Turner Construction Company of New York. In this case the wales or rangers are single

timbers and the studs are double timbers separated by block spacers. Bolts are used as ties and the spreaders are iron pipes, through which the bolts pass. Several methods are used for removing the bolts upon the removal of the form panels. The simplest method is to place a wood washer at each end of the pipe adjacent to the form and these can be readily cut out of the wall after the bolts are withdrawn and the panels removed. The block holes can be easily pointed up. Another method consists in having the bolt made in three pieces, with the middle piece connected to the end pieces by ordinary unions. The end sections are removed by a few turns of their heads with a wrench, and the holes are plugged with cement mortar after the removal of the forms.

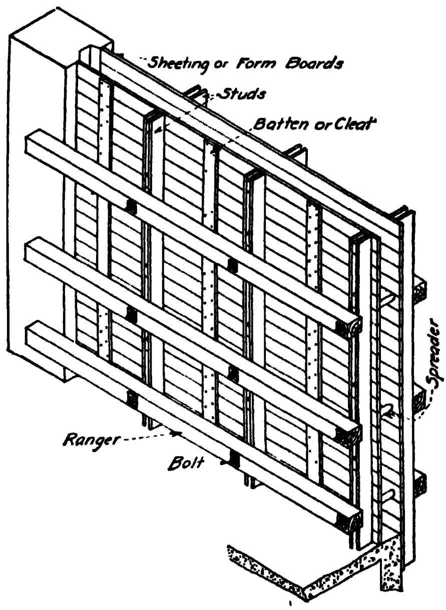


FIG. 13.—Form for curtain wall of building.

Several special patented devices are used to tie wall form sections together and are based on the principle of removable end sections provided with means of tightening the tie to draw the panels into line or to the required spacing. Figure 14 shows the three section bolt used in battered wall work, with "Universal Cone Nuts," as made by the Universal Form Clamp Company, Chicago, Ill. Figure 15 shows a unique combination of a central wire section and end bolts, called

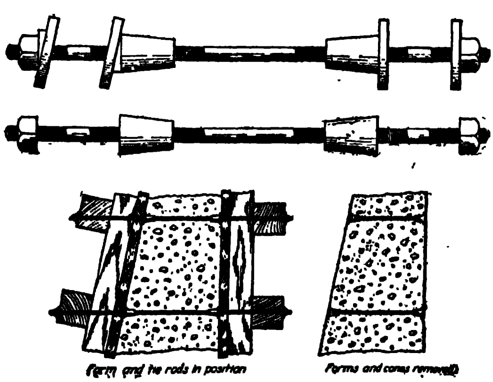


FIG. 14.

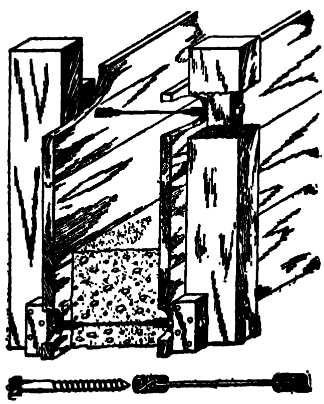


FIG. 15.

"Tysern," marketed by the Unit-Wall Construction Company, New York. Figure 16 shows a combination rod and wire tightening or twisting brace, which is made by the Universal Form Clamp Company, Chicago, Ill.

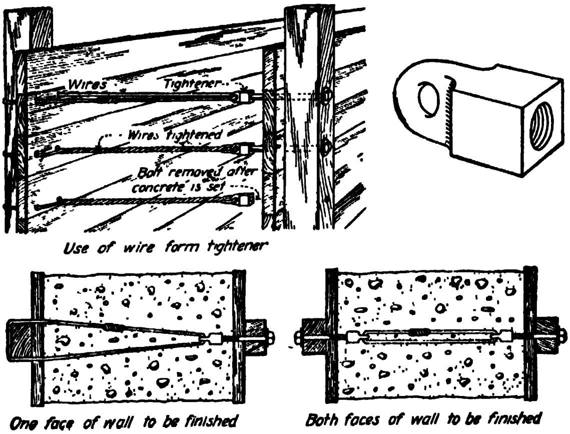


FIG. 16.

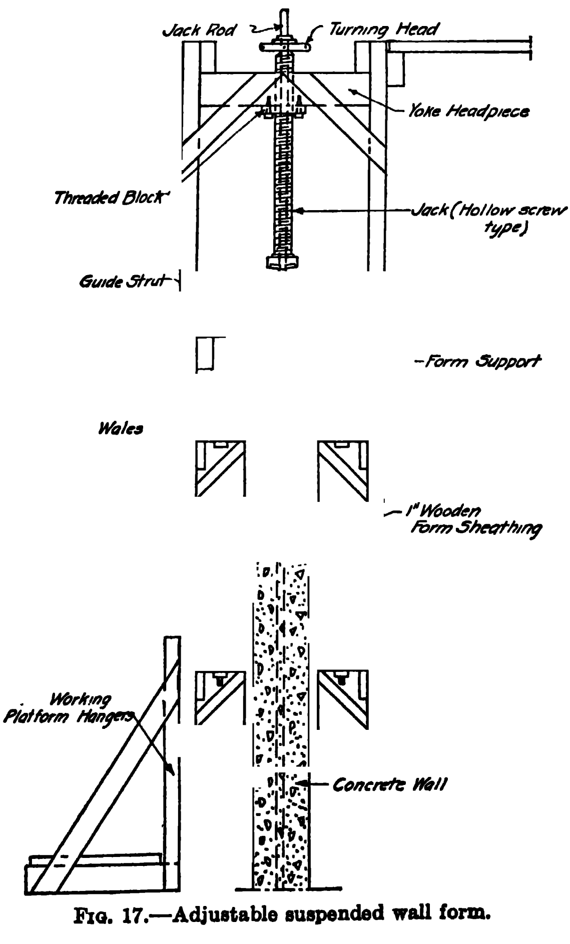


FIG. 17.—Adjustable suspended wall form.

For high walls, the form panels may be made large and raised by jacks supported on a gallows frame (see Fig. 17).

The formwork for the coping of a brick or concrete wall is clearly shown in Figs. 18[1] and 19.[1] A simple form for a window sill is shown in Fig. 20.[1]

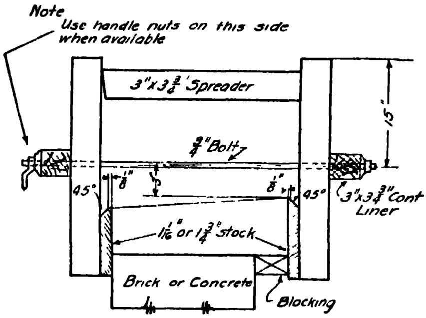


FIG. 18.—Sectional view of coping forms.

8b. Columns.—Columns of reinforced concrete buildings are generally square, rectangular or circular in section. Occasionally the corners of the square and rectangular columns are chamfered. The square and rectangular columns are built with wood forms, while the circular column is generally made in a metal form or mould.

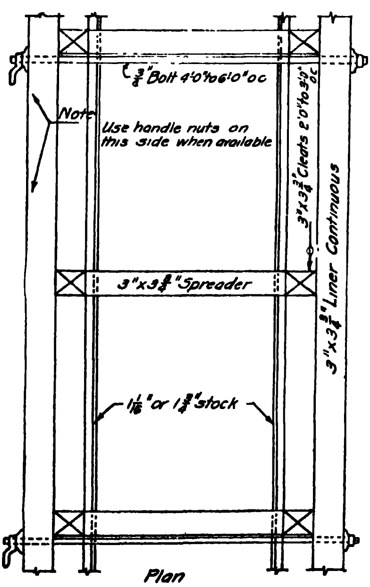


FIG. 19.—Plan of coping forms.

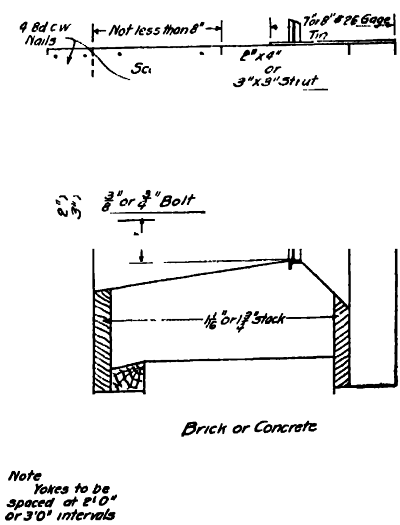


FIG. 20.—Detail of form for window sill.

Column forms are made in sections, each section or panel forming a side which is made up, erected and removed as a unit. Wood panel units consist of the sheathing or lagging which is held together by the yokes which act as cleats

[1] Aberthaw Construction Company, Boston, Mass.

and also serve as clamps to hold the panels in place. The forms are built and erected complete for a full story in height. An opening is left at the bottom as a "clean-out" for the removal of sawdust, shavings and dirt after the form is erected and before the concrete is poured.

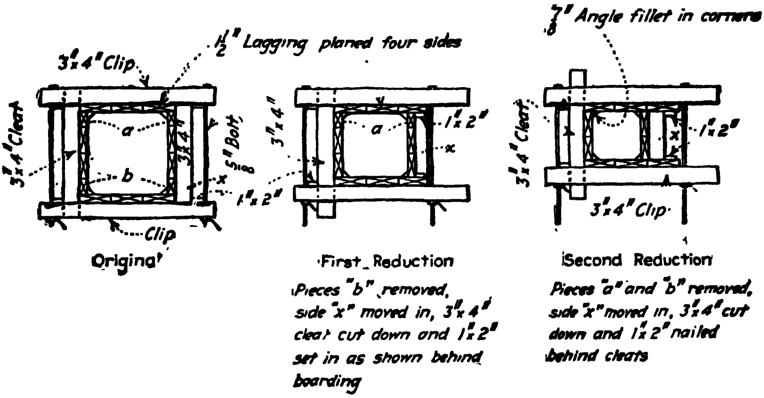


FIG. 21.

The upper section of a typical form for a square column is shown in Fig. 1. Note the simple method of holding two opposite sides of the form together by the long bolts, and the other pair of sides by the hardwood wedges.

Column form panels are often made up of narrow sheathing strips in order that a reduction in the size of the column may be made in the upper stories.

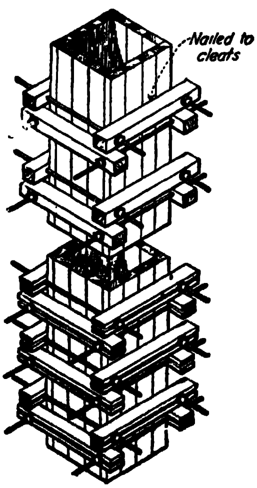


FIG. 22.

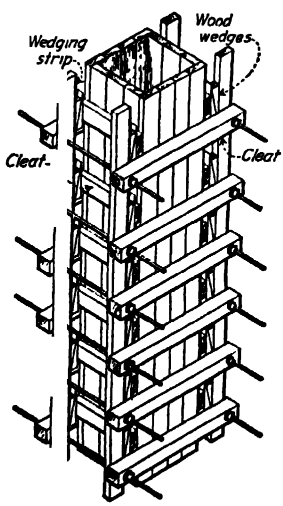


FIG. 23.

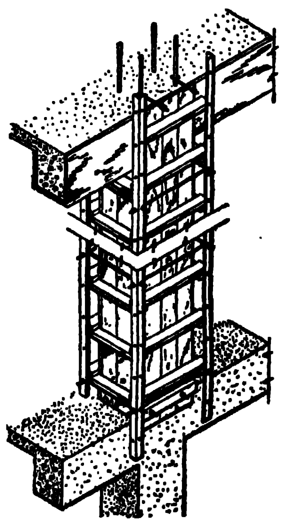


FIG. 24.—New England column clamp.

This method applies especially to interior columns, as exterior columns are generally made of the same width throughout the height of the building. The method of reduction used by the Aberthaw Construction Company, Boston, Mass., is shown in Fig. 21.

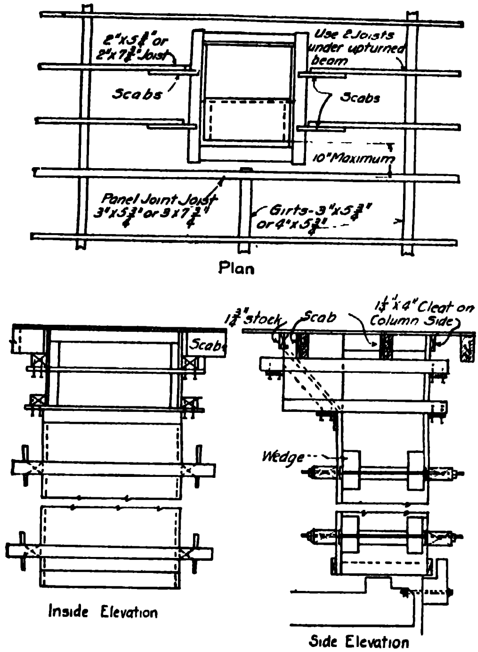


FIG. 25.—Formwork for typical wall column.

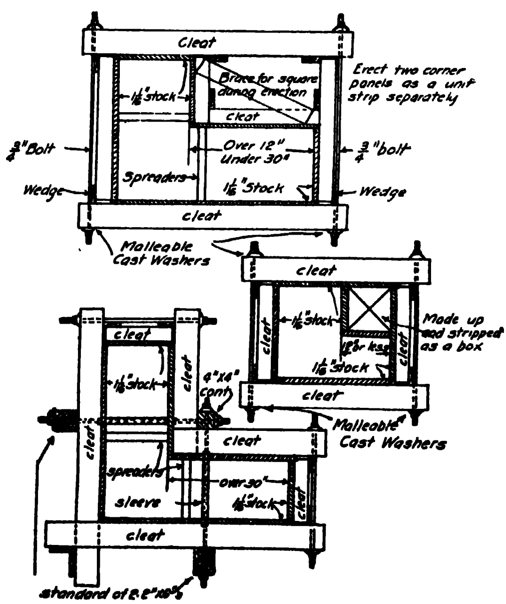


FIG. 26.—Details of corner column forms.

A form with yokes used in sets of alternate yoke and bolt is shown in Fig. 22. At the lower section of the figure, the yokes are shown as made up of two timbers

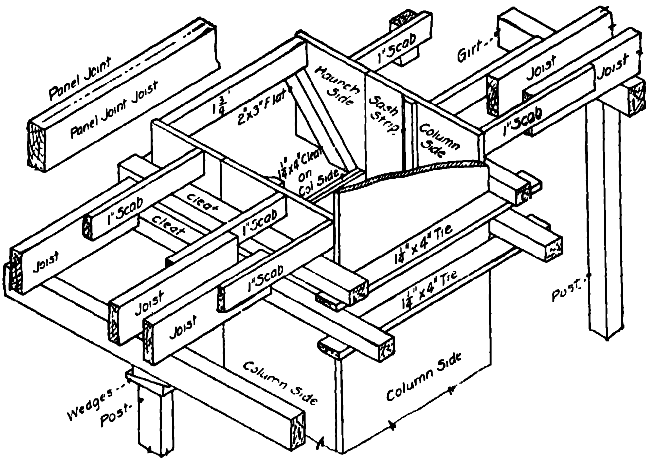


FIG. 27.—Form construction of wall column with haunch.

each and separated by a space equal to the bolt thickness. In Fig. 23 is shown a column form, the yokes of which are lined up and tied together with vertical or

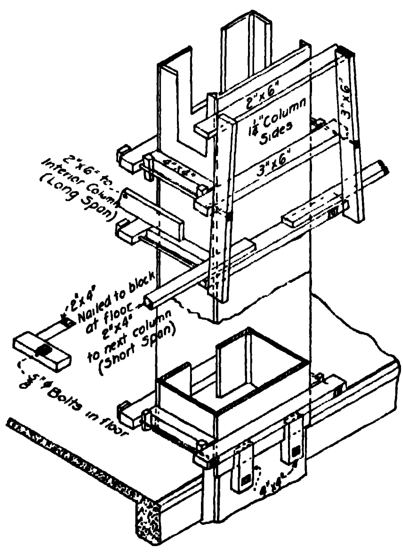


FIG. 28.—Bush system of exterior column forms.

wedging strips. The form shown in Fig. 24 uses vertical angle irons at each corner to brace and tie together the cleats. The bracing is done by bolts run through the legs of the angles at frequent intervals.

A detail view of the formwork for a typical exterior column is shown in Fig. 25.[1] Details of the forms for corner columns are shown in Fig. 26.[1] The arrangement of the formwork for a wall column with haunch is given in Fig. 27.[1] The Bush type of form for an exterior column is shown in Fig. 28 and is used by the Turner Construction Company, New York, N. Y.

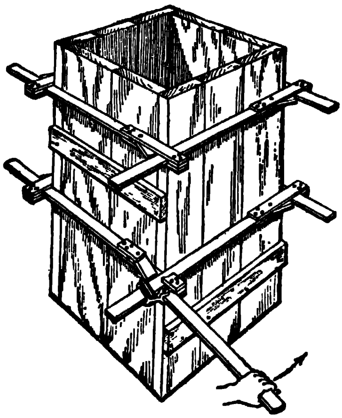


FIG. 29.—Gemco square column clamp.

FIG. 30.—Gemco round-column clamp.

Various types of special or patented clamps are on the market, and require no wood framing to hold the column sections or panels together. Figures 29 and 30 show the Gemco clamps for square and round columns. The Sterling clamp is shown in Fig. 10 and the Universal, for round columns, in Fig. 31. A simple wedge clamp is the K. & W. clamp composed of steel bars (see Fig. 32). The construction and method of operation of these clamps is easily understood from an inspection of these illustrations.

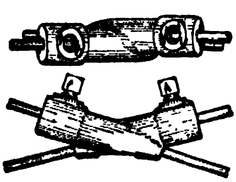


FIG. 31.—Universal round-column clamp.

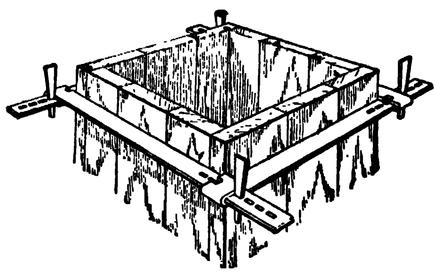


FIG. 32.—K. and W. clamp.

Interior columns of reinforced concrete buildings of the flat slab floor type are generally circular with flaring column heads. Sheet steel has proved to be the most adaptable material for such columns, as it is light, adjustable, and can be utilized economically.

Metal column forms are described in detail in Art. **13**. The method of joining up the column heads to the formwork of a drop panel of a flat slab floor is shown in Figs. 38 and 39.

[1] Aberthaw Construction Company, Boston, Mass.

8c. Floors.—The floors of reinforced concrete buildings are of three types: Flat slab or girderless, beam and girder (panel), and slab.

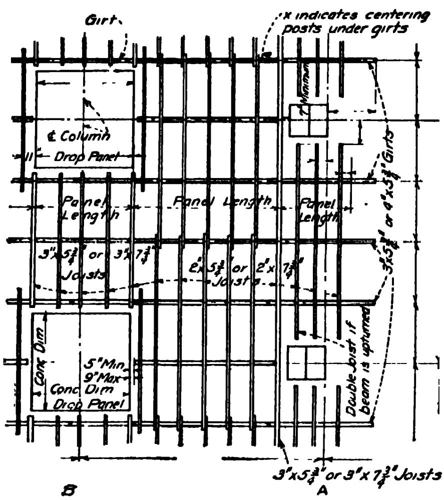


FIG. 33.—Assembly plan for exterior bay—flat slab floor.

The flat slab or girderless floor consists of a slab supported at regular intervals (generally the corners of square areas) by columns. There may or may not be a plinth or depressed portion of the slab over the column head. Due to regular

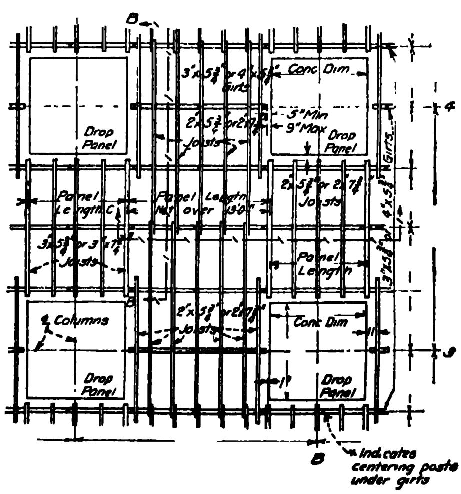


FIG. 34.—Assembly plan for interior bay—flat slab floor.

sized bays and simplicity of construction, the flat slab floor involves simple and inexpensive formwork. The details of the forms for the drop panel, flat slab floor, used as a standard by the Aberthaw Construction Company, Boston, Mass.,

are shown in Figs. 33, 34, 35, 36, 37, 38 and 39. It will be noted that the essential feature of the formwork is a simple floor construction of girts and joists, supported at regular intervals on posts. On the tops of the joists are laid the floor panels of 7/8-in. sheathing, plain steel or corrugated steel. The

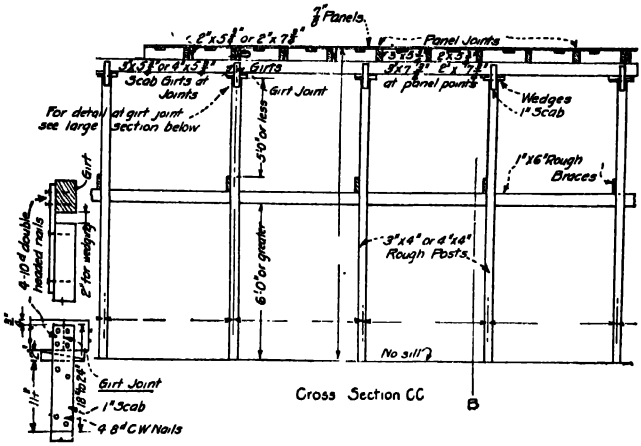


FIG. 35.—Assembly plan for interior bay—flat slab floor.

leveling up of the flooring is done by wedges at the tops of the posts and under the girts. Girt joints are always made over posts, and panel joints over joists At the drop panels, a special construction is used; a series of plinth joists suspended from girts and supporting the panel (see Figs. 38 and 39).

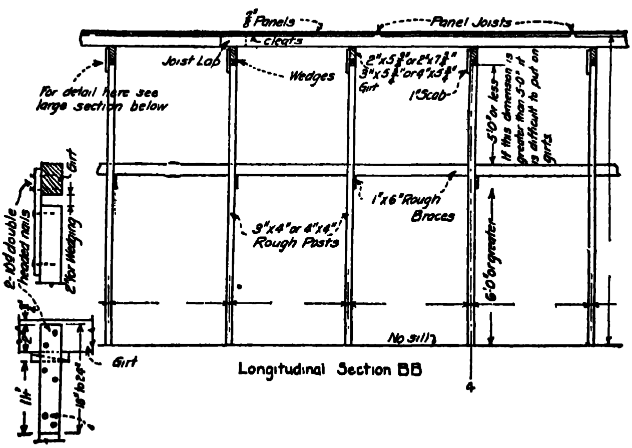


FIG. 36.—Assembly plan for interior bay—flat slab floor.

The beam and girder (panel) type of floor consists of a system of girders supported on columns at regular intervals, and in their turn supporting beams which carry the floor slab. The square or rectangle included within the four adjacent columns is called a floor panel, the structural members of which are generally two girders, two or more beams and the slab. The formwork for this type of floor is considerably more complex and expensive than for the flat slab

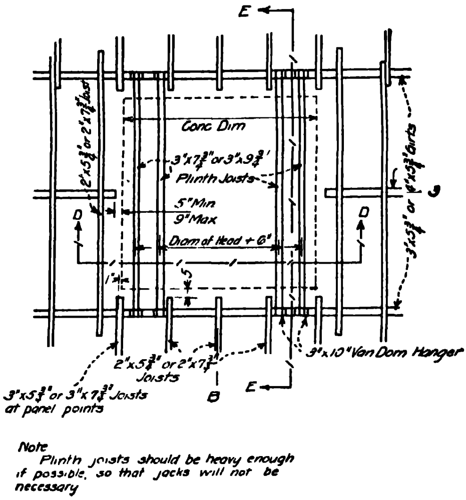


FIG. 37.—Assembly plan—drop panel—flat slab floor.

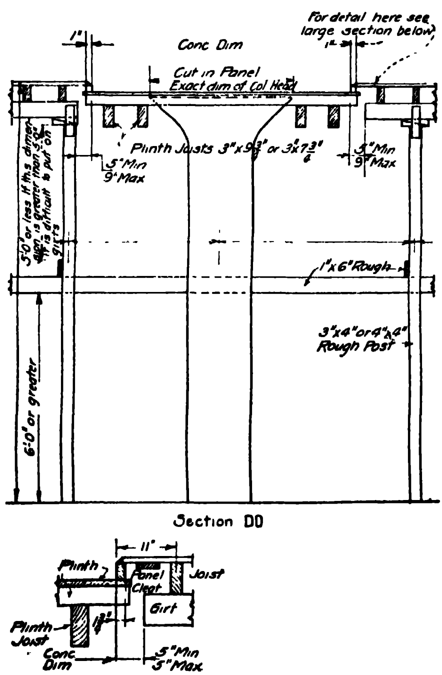


FIG. 38.—Assembly plan for drop panel.

type, as may be seen from an inspection of Figs. 1 and 40. It will be noted that, in Fig. 40,[1] the girder sides are made continuous, and that the bottom edges of

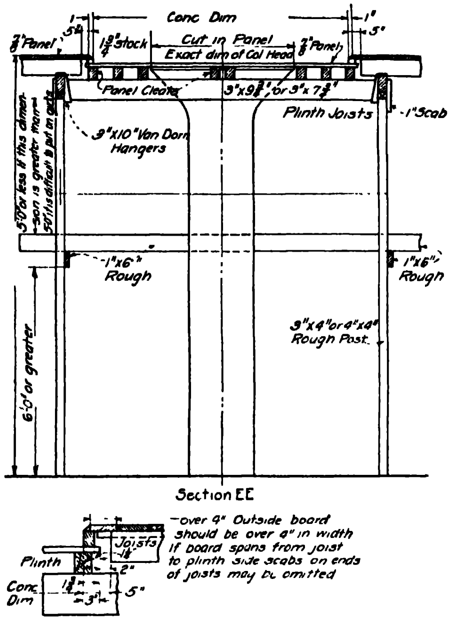


FIG. 39.—Assembly plan for drop panel.

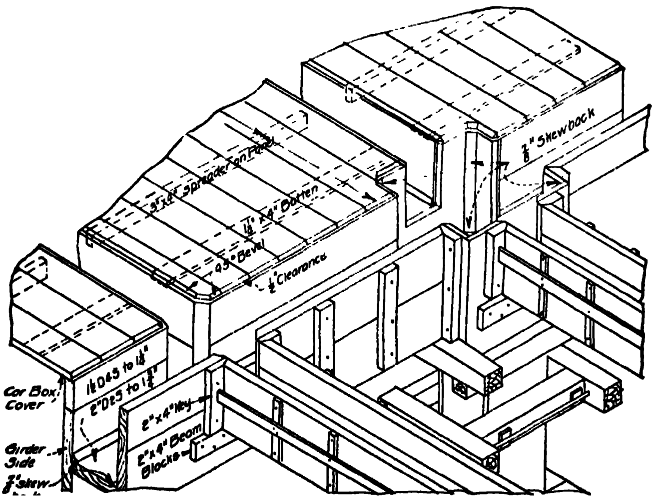


FIG. 40.—Beam and girder floor system.

beam and girders and their intersections are beveled by the use of skewbacks in the forms. The form units, beam and girder sides and bottoms and slab

[1] Turner Construction Company, New York, N. Y.

panels are made up separately (see Art. 5) and set up in place after the column forms are erected. The relationship and sizes of the various parts of a system

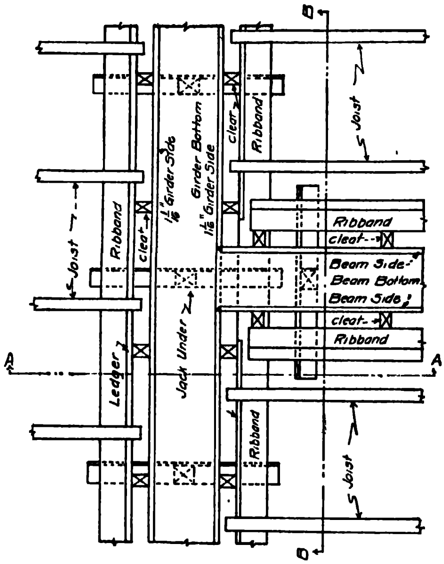


FIG. 41.—Plan of beam and girder floor formwork.

used by a well-known construction company[1] are shown in Figs. 41 and 42. Note the use of ribbands to clamp the beam and girder sides to prevent their

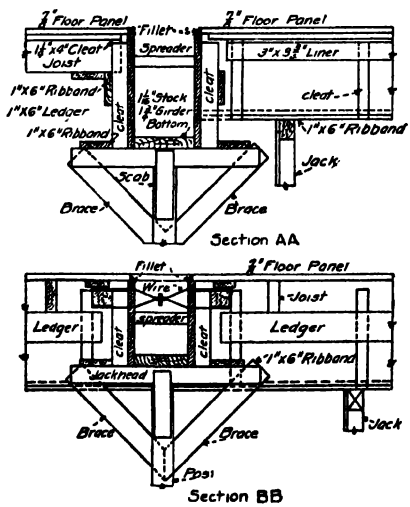


FIG. 42.—Details of beam and girder floor formwork.

spreading due to the pressure of wet concrete. The spacing of the jacks is given in Art. 4c. The beam sides are made of $1\frac{1}{16}$-in. white pine S4S and the bottoms

[1] Aberthaw Construction Company, Boston, Mass.

of 1¾-in. white pine S4S. The spacing of the joists depends on the weight of the floor slab to be supported. The ends of the joists are supported on joist bearers or ledgers which are fastened directly to the cleats on the beam or girder sides.

The slab type of floor consists of panels in which the slab acts as both beam and slab and is supported by girders which are carried by columns at the corners of the floor panels. This form of floor is constructed by the use of cores of wood or metal set in lines parallel to one direction or side of the panel. The spaces between the adjacent lines of cores form the beam stems, while the slab lies above the tops of the cores (see Figs. 90 and 91).

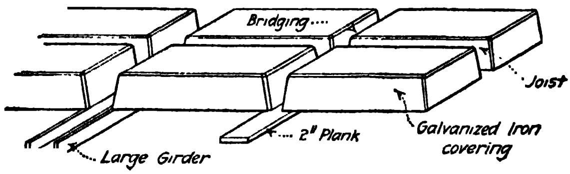


FIG. 43.

A simple type of core form was used at the University of Wisconsin, (see Fig. 43), and consisted of metal covered boxes supported in two directions on 2-in. planks. A novel form of collapsible core box is shown in Fig. 44. The molds are made in two equal sections with a hinged joint through the longitudinal center near the upper surface, and held apart by transverse struts between the lower edges. The beam is formed in the space between adjacent molds and a plank resting on strips nailed to the sides. The girder is molded in the space between the ends of the molds in one panel and the corresponding molds in the adjacent panel.

The use of metal cores for slab form construction is described in Art. 13.

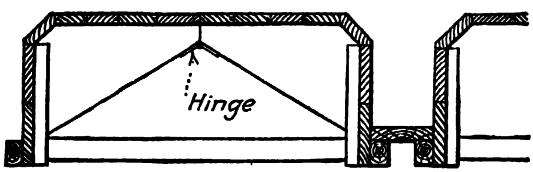


FIG. 44.

8d. Miscellaneous.—There are some sections of a reinforced concrete building which are so irregular in design and shape that it is necessary to construct special forms of a rather complicated type for them. Two typical examples of this special construction are the stairway and the cornice.

As a reinforced concrete stairway is generally an inclined slab with projections to form the steps, the formwork must provide for the support of the wet concrete on a slope and give final shape to the stairway. The system of forms used by the Turner Construction Company, for a typical two-flight stairs with intermediate landing, is shown in Fig. 45. Note that the forms are supported by posts called "hangers," which are bolted to the side walls of the stairwell.

The cornice of a building is generally molded independent of the main framework, especially if the cornice is large and a special feature of the architectural

design. As the cornice is a projecting member, a special framework must be built to support the forms and placed concrete until set. Figure 46 clearly illustrates

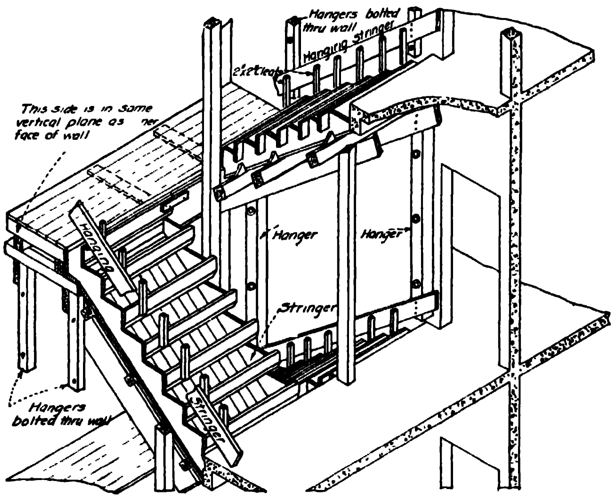


FIG. 45.—Details of stairway formwork.

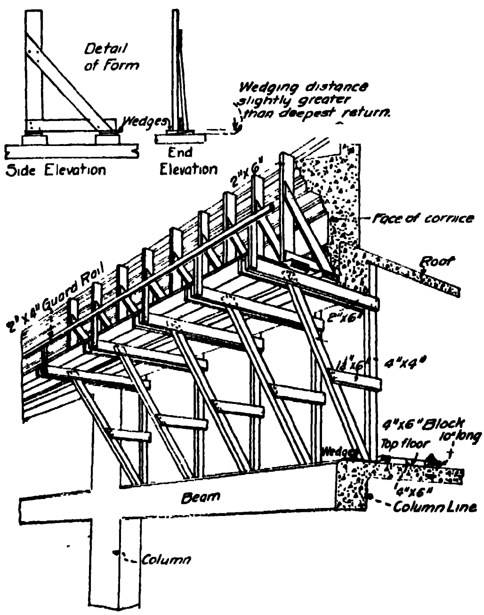


FIG. 46.—Supports and forms for cornice.

the formwork used by the Turner Construction Company. The projection of the cornice beyond the face of the building is provided for by a whole system of brackets which can be adjusted independently of each other by wedges.

9. Forms for Walls, Bins and Tanks.—Forms for walls with plain surfaces are described in Art. *8a*, so that this section will be devoted to a discussion of curved walls, which are largely used in circular bins and tanks.

Forms for curved walls may be of wood or sheet metal, generally steel. The forms are generally built in rings or lifts of from 2 to 4 ft. in height, each ring or lift being made up of several segments. These segments are bolted together through flanges at their abutting ends, and are also braced and held in position by horizontal ribs, which are generally part of the sections. The various parts of a simple form for a small circular bin or silo are shown in Fig. 47.[1] Note that the sections are held together by 2 × 6-in. connection strips 2 ft. 6 in. long bolted top and bottom to the ends of the section ribs. Wedges are used to line up the forms to the correct position, and also provide for their easy removal. The outer form is made of two sections of galvanized sheet steel fitted with lugs for tightening and hooks for handling. The following table[1] gives the dimensions of the forms for various sizes of silos:

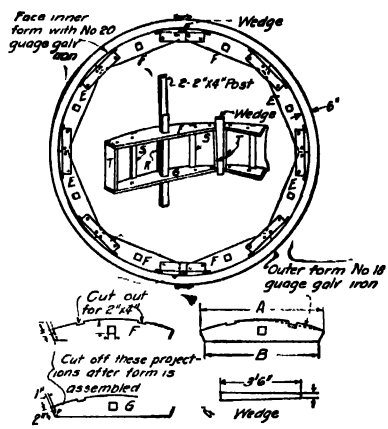


Fig. 47.—Form for small circular bin or silo.

DIMENSIONS OF INNER AND OUTER FORM SECTIONS

	Inner form				Outer form
Inside diameter of silo (ft.)	Number of sections in inner form	Length *A*	Length *B*	20-gage galv. iron 36 in. wide, length of each piece	18-gage galv. iron 36 in. wide, 2 pcs., length of each piece, *C*
10	6	5 ft. 0 in.	4 ft. 7½ in.	5 ft. 2¾ in.	18 ft. 3 in.
12	8	4 ft. 6¾ in.	4 ft. 1½ in.	4 ft. 8½ in.	21 ft. 5 in.
14	8	5 ft. 4 in.	4 ft. 11½ in.	5 ft. 6 in.	24 ft. 7 in.
16	8	6 ft. 1 in.	5 ft. 9½ in.	6 ft. 3 in.	27 ft. 9 in.
18	8	6 ft. 10½ in.	6 ft. 7½ in.	7 ft. 0¾ in.	30 ft. 10½ in.
20	10	6 ft. 2 in.	5 ft. 10 in.	6 ft. 3 in.	34 ft. 0 in.

Material for 14-ft. silo form:

- 5 pieces 2 × 12 × 16 ft., for ribs
- 1 piece 2 × 12 × 6 ft., for ribs
- 4 pieces 2 × 6 × 12 ft , for studding
- 6 pieces 2 × 4 × 12 ft., for studding
- 4 pieces 2 × 6 × 10 ft., for connections
- 3 pieces 2 × 6 × 8 ft., for continuous door form
- 2 pieces 2 × 2 × 8 ft., for continuous door form
- 64 pieces ½ × 4½-in. carriage bolts
- 2 pieces 18 gage galvanized iron, 3 ft. wide, 24 ft. 7 in. long
- 8 pieces 20 gage galvanized iron, 3 ft. wide, 5 ft. 6 in. long
- Nails, rivets, lugs, hooks, wedges, etc.

[1] Concrete Silos, Portland Cement Association.

An example of the use of wooden forms for a circular bin or tank is shown in Fig. 48. This form design[1] was used in the construction of a concrete water tower at Victoria, B. C. The structure consisted of a hollow cylinder of plain concrete, 109 ft. high, an inside diameter of 22 ft., walls 10 in. thick for the first 70 ft. and 6 in. thick for the upper 39 ft., and ornamented with six pilasters 3 ft. wide and 7 in. thick. Collapsible forms were used throughout for the interior of the tower. These forms were made in six sections and a key, 6 ft. high and used in three tiers bolted end to end horizontally and to each other vertically. The outside forms were also made collapsible for the main body of the tower and consisted of six sections for the wall between pilasters and six pilaster sections. All the sections were bolted together circumferentially and made as shown in Fig. 48.

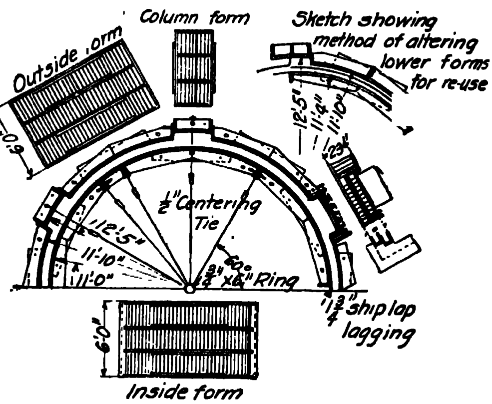


FIG. 48.—Forms for concrete water tower.

The form sections were used in three tiers and were operated as follows. After the concrete had been poured and set in the top tier, the horizontal bolts in the lower inside and outside forms were removed, as was also the small key section on the inside. The lower forms were then hoisted by hand, using four-part tackles, and placed in position on the top tier. The adjacent flanges were carefully aligned and bolted together. The column sections in the outside and the wedge and key sections on the inside, were set last. The forms were aligned from a plumb line suspended from the inner scaffold and centered over a point set in the rock at the base of the tower. The upper right-hand corner of Fig. 48 shows the method of splitting outside form sections longitudinally to form the forms for the wall sections and small pilasters of the top part of the tower.

The concrete grain bins of the Canadian Pacific Railway at Port Ontario, Canada, were built with movable cylindrical forms 4 ft. high. The curved surfaces were made of 2-in. vertical planks spiked to inside and outside circular horizontal chords. The latter were made like ordinary arch centers with four thicknesses of 2 × 8-in. scarfed planks bolted together to break joints and make complete circles inside the tanks and circular segments of 270 deg. or less on the outside of the tanks. The forms were faced with No. 28 galvanized steel, and were maintained in concentric positions with a fixed distance between them

[1] "A Concrete Water Tower," by A. KEMPKEY, JR., *Trans.* A. S. C. E., Vol. LXX, 1910.

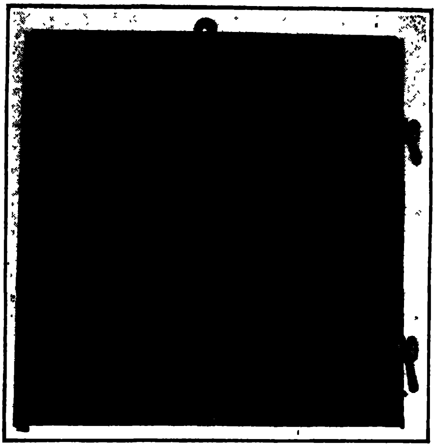
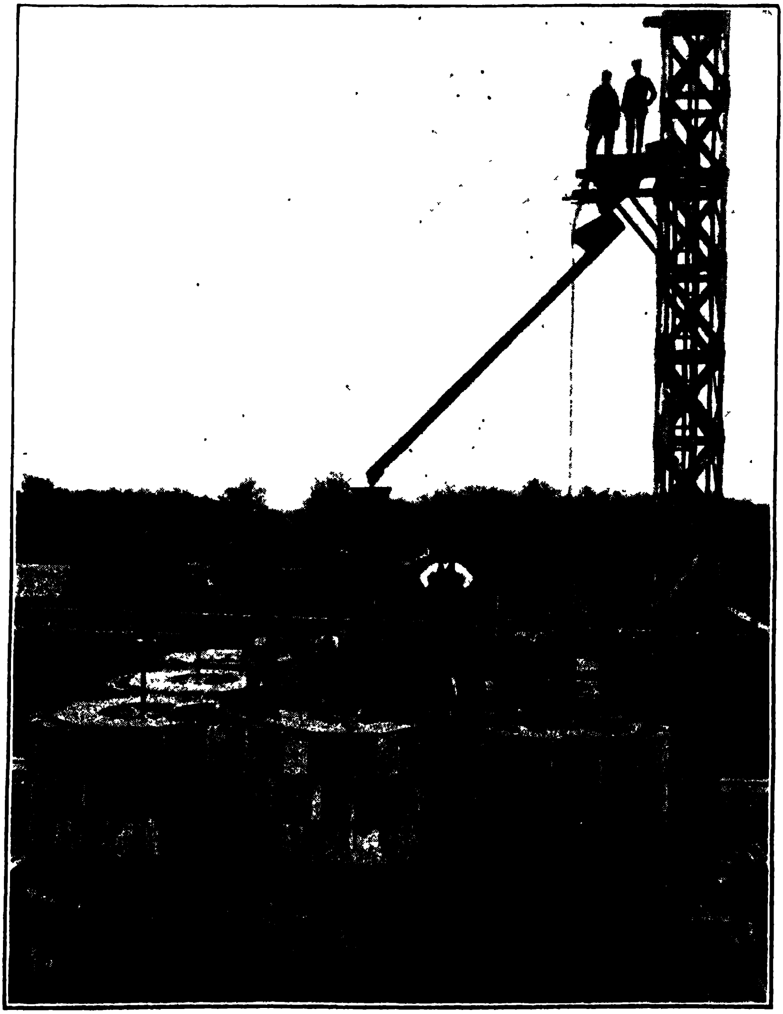


FIG. 49.—Metal form unit for circular wall.

(*Courtesy of Metal Forms Corporation*)

FIG. 50.—Use of metal form units in large circular tank.

by means of 8 U-shaped steel yokes in radial planes. Each yoke consisted of an inside and an outside vertical post with a radial web and flanges engaging the inner and outer faces of the circular chords. The posts projected about 2 ft. above the tops of the forms and were rigidly connected there by means of heavy braces and an adjustable tension rod. These yokes were bolted to the inner and outer



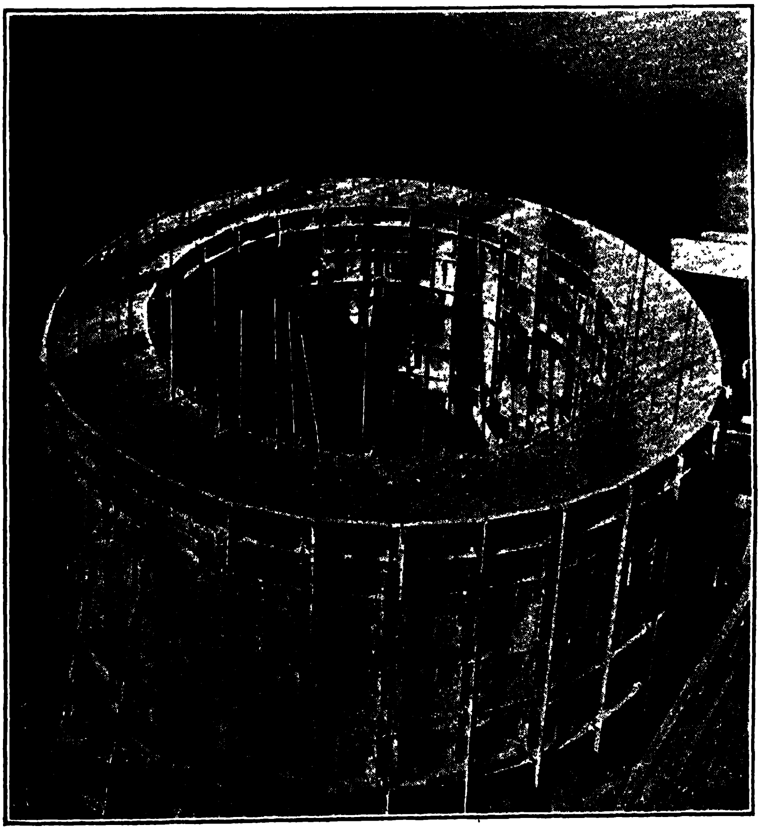
(Courtesy of Metal Forms Corporation)

FIG. 51.—Use of metal form unit in nine-bin elevator.

chords of the forms and virtually united them in a single structure. The lower ends of the vertical yoke posts were seated on jackscrews, which were supported on falsework built up inside the tanks as the walls progressed. The inside surface of the mould was a complete cylinder, but the outer surface was made in two,

three or four sections to allow for the connections between the concrete walls at the points of tangency of the different tanks.

Metal forms are in common use for the moulding of circular walls for bins, tanks and towers These forms are often made in standard size units which are easy to handle and assemble. A typical unit is made of No. 10 gage sheet steel stiffened vertically by 1 × 1-in. angle irons riveted to the edges, and horizontally by 1-in. band iron similarly riveted to the top and bottom edges. Two



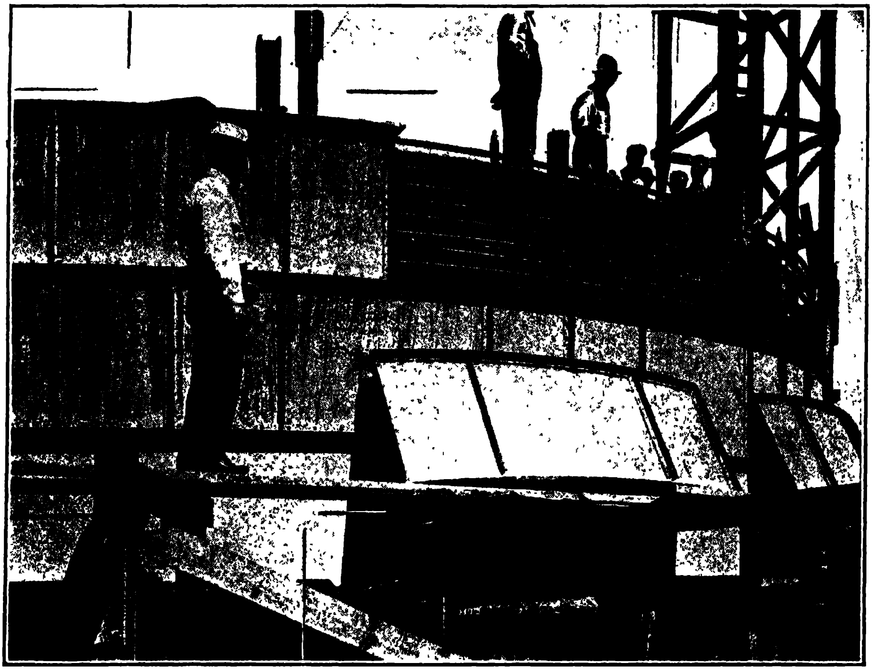
(Courtesy of Blaw-Knox Co.)

Fig. 52.—Metal forms for heavy circular wall.

sets of plates, one concave and the other convex, compose the inner and outer shells of the form. Different curvatures are secured by the use of fractional plates of various widths. The height of these plates is generally 2 ft. and three tiers or rings are used, the lower course being removed and placed on the upper course successively as the work progresses. Figure 49 shows a unit made by the Metal Forms Corporation, Milwaukee, Wis. This plate can be bent to any practicable radius. Note the side clamps to lock the side flanges of the plates together. The hook clamp at the center of the top edge engages a raised rivet on the bottom edge of the plate above, making a smooth, tight joint.

The use of the metal form unit in the construction of a concrete slime settling tank 44 ft. in diameter is shown in Fig. 50. The construction of a nine-bin grain elevator with the metal form units is shown in Fig. 51. These bins were built integrally and each bin had an internal diameter of 10 ft.

Heavy circular walls are sometimes built in long sections of forms which are well supported, inside and outside, by circular ribs fastened to vertical posts spaced at regular intervals about the circumference. Figure 52 shows this type of formwork.



(Courtesy of Aberthaw Construction Co.)

Fig. 53.—Metal forms for large standpipe.

Steel forms for the Waverly, R. I. standpipe were made of 1⅛-in. boiler plate reinforced on sides and vertically with 3 × 3 × ¼-in. angles (see Fig. 53). The inside form was made 6 ft. high and had a key section in which the plates lapped about 6 in. On either side of the joint, angles were riveted to the plates and connected by short turnbuckles, so that the whole form could be sprung in and reduced in diameter and be raised. The outside forms were made in seven segments and 3 ft. high. Two complete sections were used. When one 3-ft. section had been erected and the concrete placed, the next section was placed on top and another layer of concrete placed. On the outside forms all rivets were countersunk and the faces of angles making joints were machined to secure a smooth fit and thereby a smooth outside surface to the concrete.

A movable steel staging was located inside the tower and comprised four 5-in. channels in the form of a cross joined at the center with a standard connec-

tion. Around these channels were bent two channels in concentric circles of 14- and 19-ft. radius, and braced with 2 × 2 × ¼-in. angles. The floor was plank, giving a platform 5 ft. wide around the inside of the tank. This platform was raised as the work progressed and held in place by 4 × 4-in. guide posts spaced 45 deg. apart.

10. Retaining Walls and Dams.—Formwork for retaining walls is very similar to that for the walls of buildings, especially the basement walls below grade.

The essential parts of the formwork are the wall sections and the bracing (see Figs. 11 and 12). The forms may be made continuous in place or built in sections or panels and set in position as required. The latter method is the more economical where a considerable length of wall of the same cross-section is to be built. The principal elements of the wall forms are the sheeting, boarding or lagging, the joists or studs, the rangers for a high wall, tie rods or wire ties and spreaders. The pressure of the wet concrete is carried from the lagging to the studs and thence to the rangers and bracing to the ground.

The design of the forms is like that for walls, and the tables and diagrams given in Art. 4 may be adapted for this purpose. It should be noted that the deflection of the form panels should not exceed ⅛ in., and that the lagging should be designed as simple beams. In order that the deflection may not exceed ⅛ in. on the basis of a simple span, the distance between adjacent joists, in feet, should be less than 25 times the square root of the thickness of the lagging, in inches, $L < 25\sqrt{h}$.

The tie rod is designed as a tension member, and its diameter will depend on the location and the area of adjacent panel supported. The unit stress in the steel is generally taken at 16,000 lb. per sq. in. The following table gives the loads which tie rods of different diameters at various unit stresses will carry.

Loads on Tie Rods
(Pounds)

Rod diameter (in.)	Allowable unit stresses			
	12,000	16,000	20,000	24,000
⅛	150	200	250	295
¼	590	790	980	1,190
⅜	1,320	1,750	2,200	2,650
½	2,350	3,150	3,900	4,700
⅝	3,700	4,900	6,100	7,400
¾	5,300	7,000	8,800	10,300
⅞	7,200	9,600	12,000	14,300
1	9,400	12,600	15,700	19,000
1¼	14,800	19,700	24,600	29,500

The formwork for concrete retaining walls used by the C. B. and Q. Ry. Co. gives a good example of a combination of continuous and sectional forms; the sectional part consisting of the studs, coping and bottom forms for the face, and

the sectional forms or panels for the back of the wall. Sectional forms were found to be impracticable for the face of the wall, due to warping and the impossibility of securing close joints between adjacent sections. Hence, continuous forms were used in order to secure a smooth and clean wall surface (see Fig. 54).

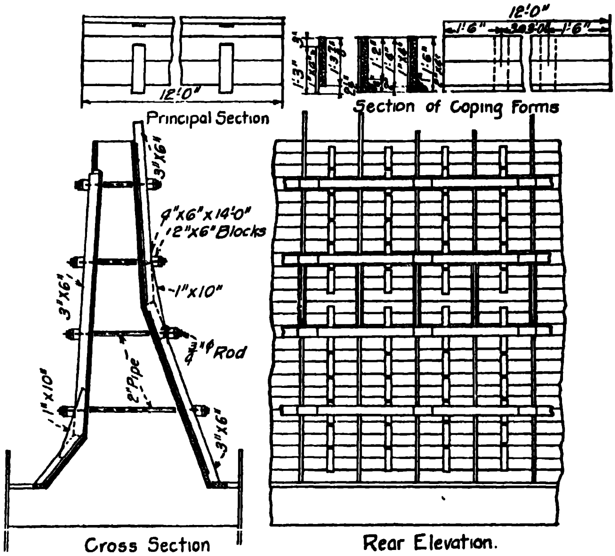


FIG. 54.—Sectional forms for retaining wall—C. B. & Q. Ry.

It was found necessary to tie down the forms by 3 × 8-in. braces extending from the vertical studs to 2 × 2-in. stakes set out several feet beyond the face of the wall. The braces were placed from 6 to 10 ft. apart, depending on the height of the wall.

The concrete retaining walls of the yards of the Grand Central Terminal were from 11 to 26 ft. in height with battered faces and rear offsets 3 ft. in height. The forms were made in sections 52 ft. long and formed of 2-in. pine planking, ship-lapped ½ in. and laid ⅛ in. apart to allow for swelling Five lines of 6 × 6-in. wales were held together at the corners by U-straps of 2 × ½-in. iron bolted to the longitudinal pieces and projecting to engage the chamfered ends of the transverse wales. The transverse bolts or ties were pairs of horizontal ½-in. rods spaced 2¹⁵⁄₁₆ in. apart, engaging cast-iron sockets, which were secured to short bolts passing through the horizontal walls and secured by washers and nuts (see Fig. 55). After the concrete had set, the nuts were unscrewed and the bolts removed, leaving the rods and castings in the wall. The form panels weighed from 2 to 3 tons and were handled by the 45-ft. boom of a locomotive

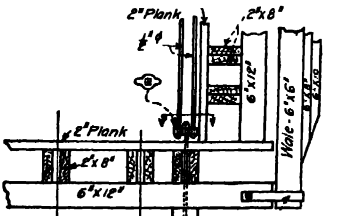
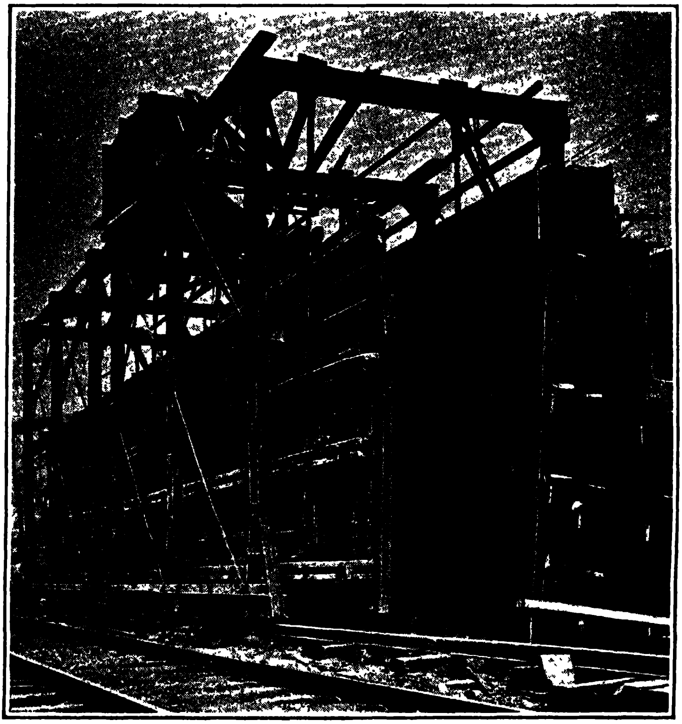


FIG. 55.—Detail of retaining wall forms—Grand Central Terminal, New York.

crane. The first section was assembled with both ends closed by sheeting. Later two sections were used and sections were built on both ends alternately, allowing the concrete in one section to set while another section was being built.

The forms for heavy walls are sometimes made in sections and mounted on travelers, which allow of their easy transfer from one place to another as the wall is built, section after section. The form panels may be built of wood or steel, but the traveler is generally a structural steel framework which moves along a track. When the wall is so placed that the track can be laid on one side of the wall only,



(*Courtesy of Blaw-Knox Co.*)

FIG. 56.—Gantry traveler for heavy wall forms.

the cantilever type of traveler is used. This consists of a structural steel frame, mounted on a truck, and projecting up and over the wall so as to engage the forms by means of hangers and jacks. The forms are adjusted by means of these hangers and jacks to their proper elevation and alignment. Thus the forms for the face and back of the wall are moved and set in one operation. When it is possible to have the track on both sides of the wall, the gantry type of traveler is used. As shown in Fig. 56, one rail on each side of the wall is required and the structural frame straddles the wall. The forms for both faces of the wall are attached by hangers and jacks to the frame.

The concrete pressure is distributed from the sheeting or plates to the studs or reinforcing angles, then to the wales and thence to the trusses of the traveler. Generally, one row of tie rods is used near the base to tie the trusses together to

take the pressure of the wet concrete. Occasionally it may be possible to secure sufficient bracing at the foot of the wall to take this pressure, in which case all tie rods can be eliminated and the thrust taken by the traveler. This requires a heavy traveler, well trussed and braced by a system of wedges and jacks between the wales and the traveler frame.

Forms for dams are generally built in sections or panels and moved up in rows or tiers as the wall is built, as is described in Art. 9 for curved walls of bins and tanks. The sections are attached to the completed wall by bolts or rods, the outer fastenings of which are easily removed when it is desired to move the sections up to a new position. This type of form provides for the enclosure of a vertical or inclined surface of concrete without outside bracing and independent of any other part of the structure. The strength, weight and surface of the last completed course is utilized to hold the forms in place, preserve their alignment and resist the thrust of the wet concrete.

[1] In the building of the massive walls of the dam, power house and ship locks of the Tennessee River power development at Hales Bar, near Chattanooga, Tenn., adjustable, movable, cantilever forms were used. The forms were made in sections or panels 15 ft. long, each of which had a single adjustable shutter of the same length and 5 ft. high to correspond with the depth of the course, and was made with horizontal 2-in. planed planks ship-lapped and nailed to transverse 2 × 4-in. outside cleats 5 ft. apart. The horizontal lower edge of the panel rested on a single corresponding plank, which for most of its width, engaged the upper edge of the last completed course of concrete, protecting the latter and locating the plane. The principal supports comprised 6 × 8-in. cantilevers 12 ft. long and 3 ft. apart. They were anchored to the concrete with ¾-in. bolts, 3 to 4 ft. long and 3 ft. apart, set in the concrete. The vertical studs were aligned with three horizontal 4 × 6-in. wales and wedged against them (see Fig. 57).

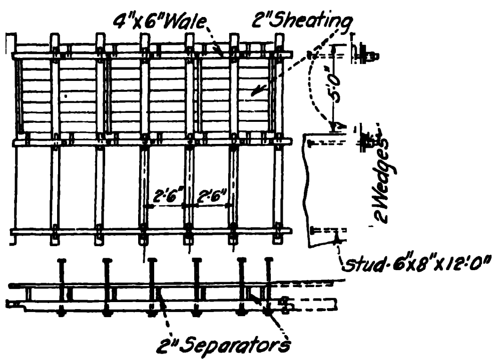


Fig. 57.—Adjustable form for heavy battered wall.

In the construction of the great reservoir dams of the U. S. Reclamation Service, the movable, adjustable type of form panel supported entirely by the concrete of the dam, was used with great success. Figure 58 gives the details of the simple wooden panel supported by bolts anchored back into the concrete for the construction of the Elephant Butte Dam. The panels were 18 ft. long, 4 ft. high, and for the downstream face were lined with No. 24 gage sheet metal to give this surface a more finished appearance.

The use of panels on a dam of curved alignment or so-called arch type, is shown in Fig. 59. Note the method of anchoring to the concrete and the cantilever frame to resist the thrust of the wet concrete. On the downstream face where the batter and curvature introduced a little complication, the V-shaped

[1] *Engineering Record*, April 24, 1909.

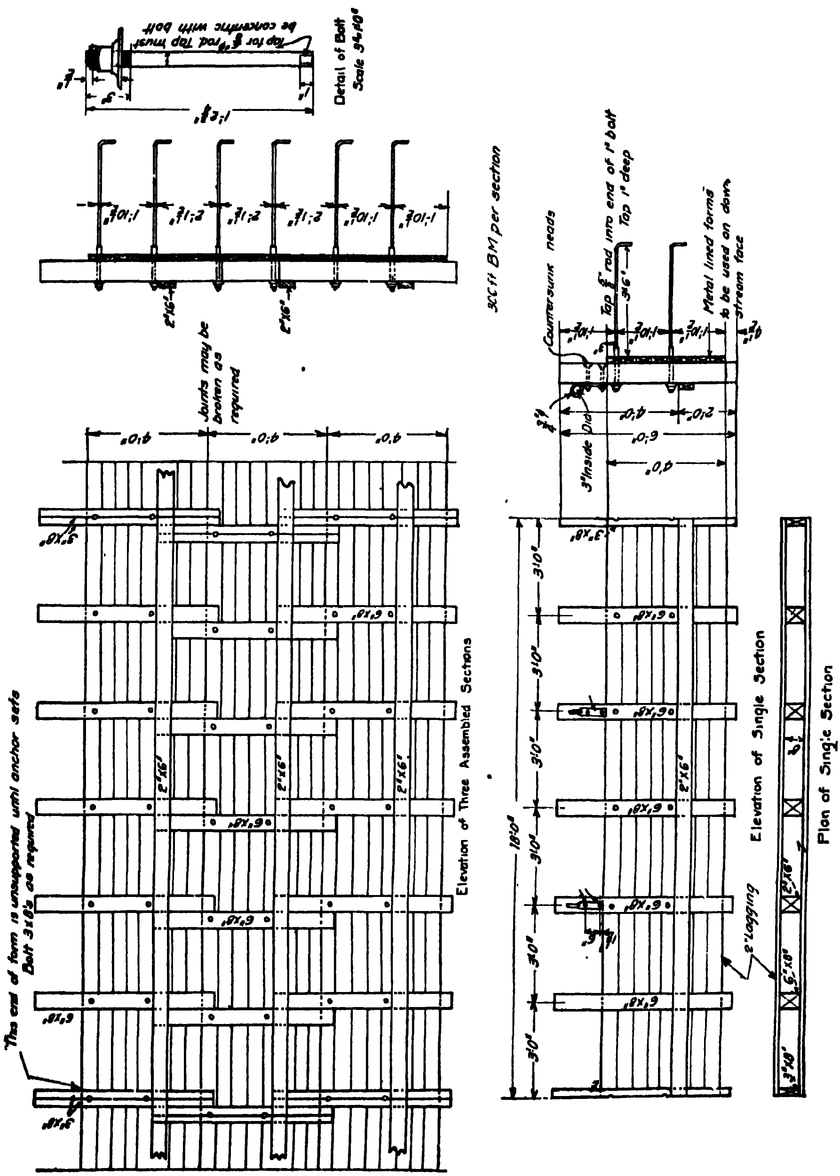


FIG. 58.—Movable wood forms for Elephant Butte Dam—U. S. Reclamation Service.

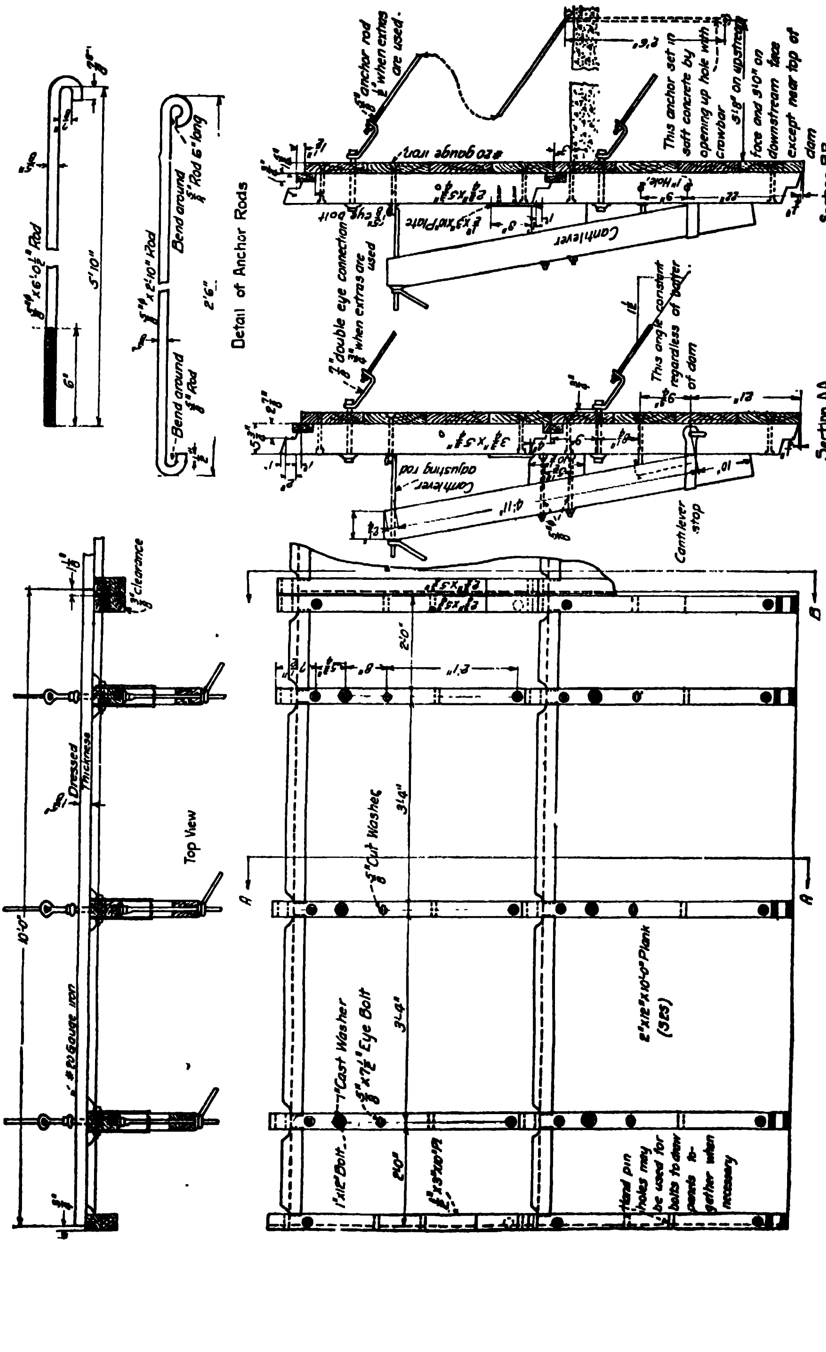


FIG. 59.—Face form panel for Arrowrock Dam—U. S. Reclamation Service.

opening between the ends of the panels was closed with a piece of light metal; and every ten panels in height, when the width of opening amounted to 5 in., the forms were set in close contact again, moving them towards the center between contraction joints. As the radius of curvature changed for every row of forms, on the downstream face, it was necessary to have them set each time by the field party.

11. Forms for Open Channels, Culverts and Bridges.—This article will discuss formwork for open channels or waterways, box culverts and for slab and girder bridges, leaving to Art. **12**, a description of forms for channels of curvelinear section, such as pipe, siphons, arch culverts, tunnels and closed conduits.

During recent years, the lining of open earth and rock channels of irrigation systems in the arid West, has necessitated the use of concrete forms. Figure 60

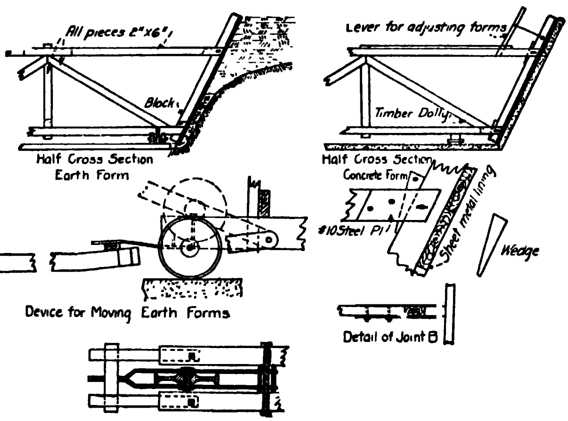


FIG. 60.—Form for lining channel with concrete.

shows a unique form which was used by the U. S. Reclamation Service at El Paso, Texas, to line an old channel with concrete. The unique device for moving the form section for the earthwork is shown in detail.

Open channels above the ground surface, or flumes, have been built of reinforced concrete on several large irrigation projects in recent years. These flumes are of two general types; the flume with bell-and-spigot joints, and the flume without joints. Figure 61 shows the formwork used by the U. S. Reclamation Service at King Hill, Idaho, for a flume of the bell-and-spigot type. The formwork for flumes of the continuous or "without joints" type are very similar, consisting of panels or sections supported by braced frames or bents, which also carried the runway over which the concrete was transported. The forms are generally made in sections of about 16 ft. in length and were set up as follows. The outside panels together with the posts and templets were set up and held together by top cross-braces. The steel reinforcement was placed, and the inside panels and posts were suspended from the top cross-beams. Then the inside templets were placed on the brackets and all securely braced by the struts and diagonal braces. Bulkheads were used in the forms at the end of each day's work.

Box culverts have been extensively used in replacement work in railroad construction. The formwork is plain, simple and requires little space, especially

as compared with that required for an arch culvert. As the loads on the foundation of a box culvert are generally light and well distributed, inexpensive foundations can be used. Figure 62 shows the essential features of the forms for a 10 × 10-ft. box culvert used on the C. B. & Q. Ry. Note that the formwork

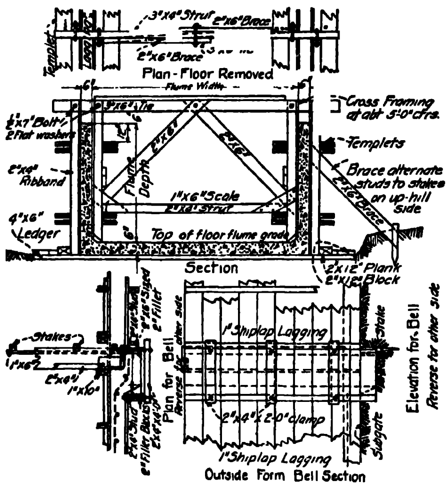


Fig. 61.—Forms for bell and spigot type of concrete flume—U. S. Reclamation Service

consists of the inner box and the outer panels, which are braced back to the earth or embankment on each side. Forms of this type are used for culverts ranging in size from single 4 × 4 ft. to double 20 × 20 ft. and triple 16 × 20 ft.

The formwork of a reinforced concrete box culvert used in highway construction is shown in Fig. 63, which is taken from a model of the U. S. Bureau of

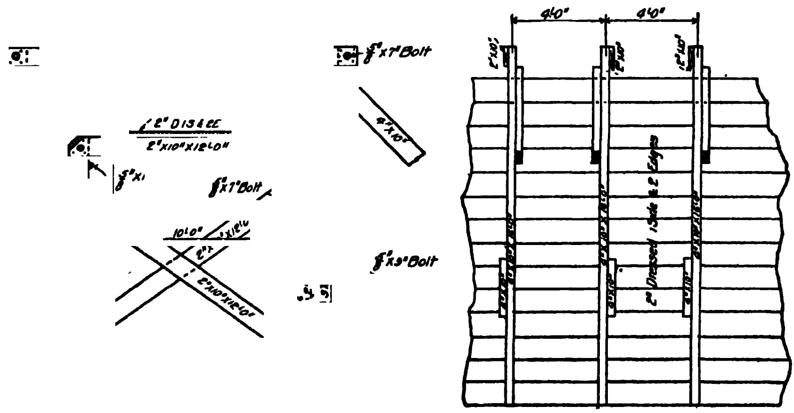


Fig. 62.—Forms for 10 × 10 ft. reinforced concrete box culvert—C. B. & Q. Ry.

Public Roads. The wing wall forms, are at *E*, the side wall panels at *B* and the reinforcing steel at *C*. Figure 64 illustrates the complete forms for a small box culvert and head wall.

The formwork for a slab and girder bridge consists of the substructure or falsework and the superstructure of slab and girder panels and their supports.

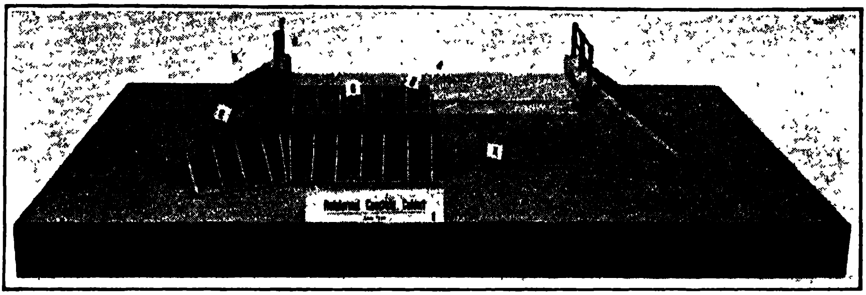


FIG. 63.—Forms for highway box culvert.

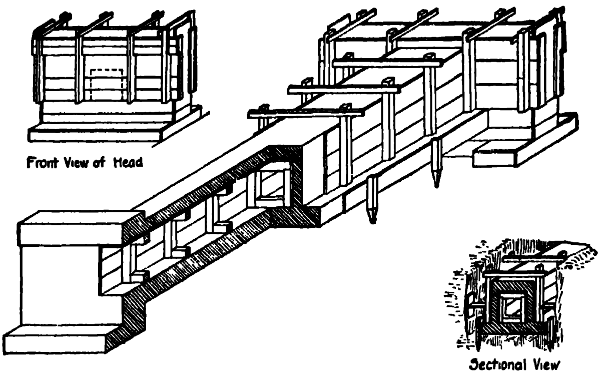


FIG. 64.—Forms for small box culvert—Mass. Highway Commission.

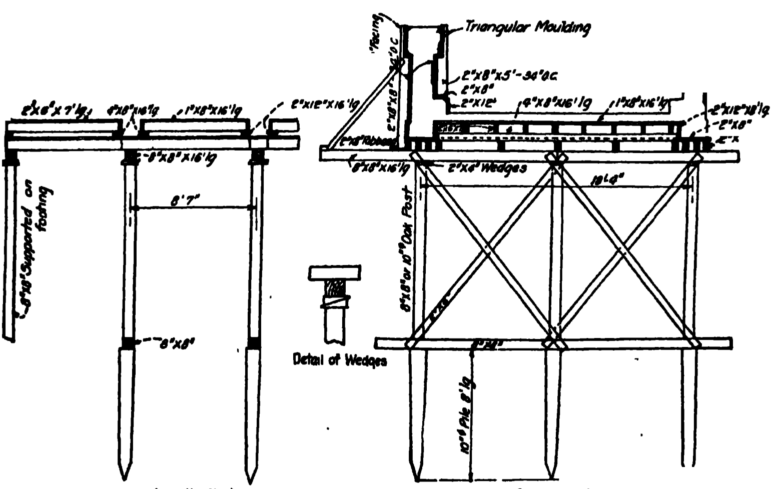
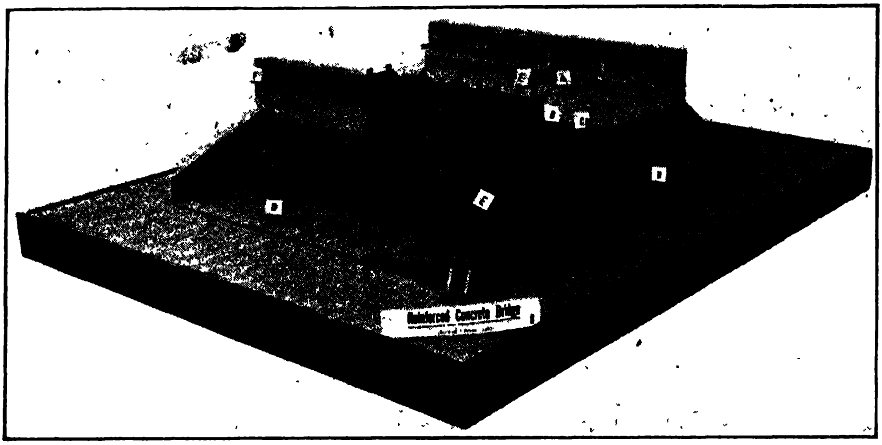


FIG. 65.—Formwork for through concrete bridge—60-ft. span—16-ft. roadway.

Figure 65 shows a design prepared several years ago by the Illinois Highway Commission to furnish contractors with an approved method of building formwork for a standard through slab and girder type of reinforced concrete highway



(*Courtesy of U. S. Bureau of Public Roads*)

Fig. 66.—Forms for typical highway bridge of encased I-beam type.

bridge. The wedges on the top of the posts of the falsework furnish an easy method of removing the lower form panels after the side panels are taken off.

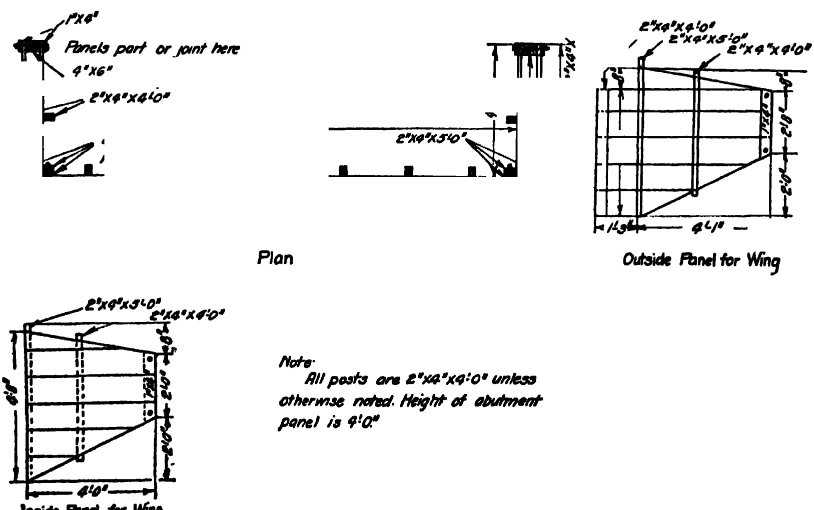
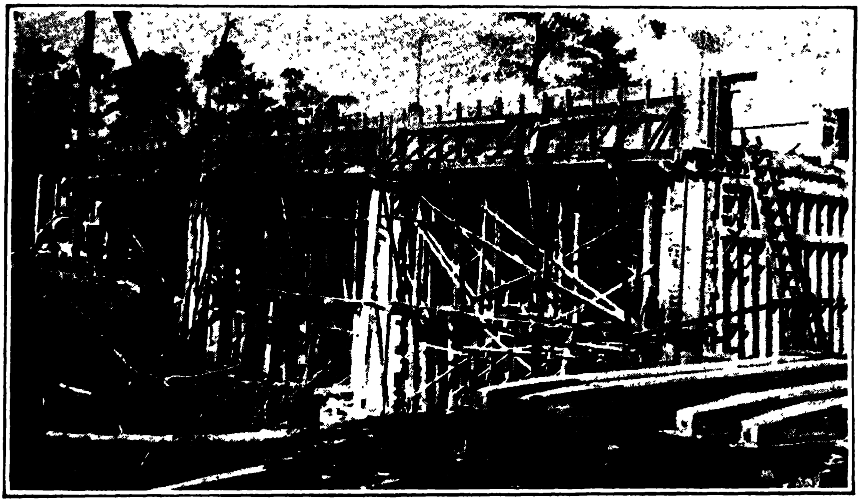


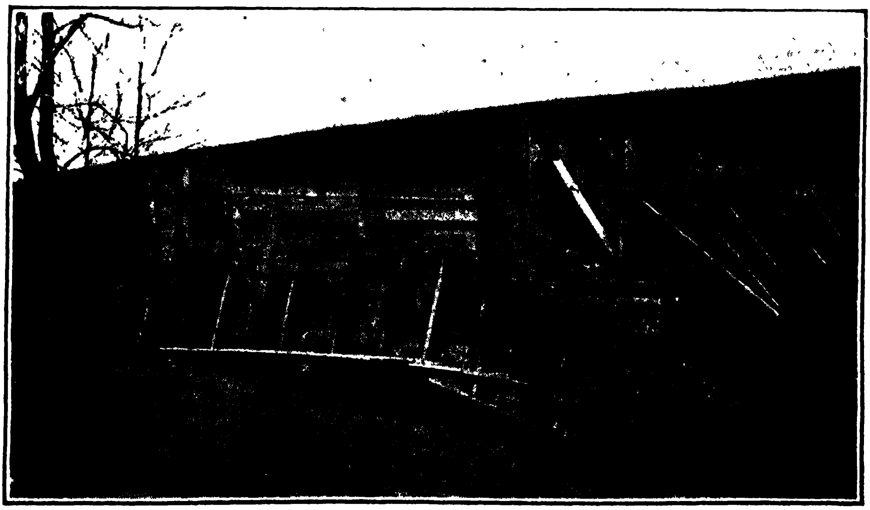
Fig. 67.—Panel forms for bridge abutments—U. S. Reclamation Service.

The essential parts of the forms for a small highway bridge are given in Fig. 66, which is a view of a model of the U. S. Bureau of Public Roads. Note the wing wall forms at *E*, and the balustrade forms at *F*.

The details of the panels for small concrete bridge abutments of the U. S. Reclamation Service are given in Fig. 67. The wings in this case are at right angles to the face wall.



(*Courtesy of U. S. Bureau of Public Roads*)
Fig. 68.—Formwork for multiple span through girder bridge.



(*Courtesy of U. S. Bureau of Public Roads*)
Fig. 69.—Formwork for small highway bridge.

The falsework and forms for a multiple span through, slab and girder bridge are clearly shown in Fig. 68. Note the simple panel construction of the pier forms and the thorough bracing of the side panels of the girder.

In Fig. 69, the side form panels of the girder are braced back to the ground and the coping form is held in place by vertical wedges.

The details of the formwork for a deck girder in Indiana are shown in Fig. 70. The falsework comprises several parallel bents of heavy timbers supported on mud sills or leveling blocks. The whole superstructure is carried on parallel lines of 3 × 12-in. stringers and deck plank of the same size timber laid on side and transversely. The railings are 6 × 8-in. precast concrete blocks.

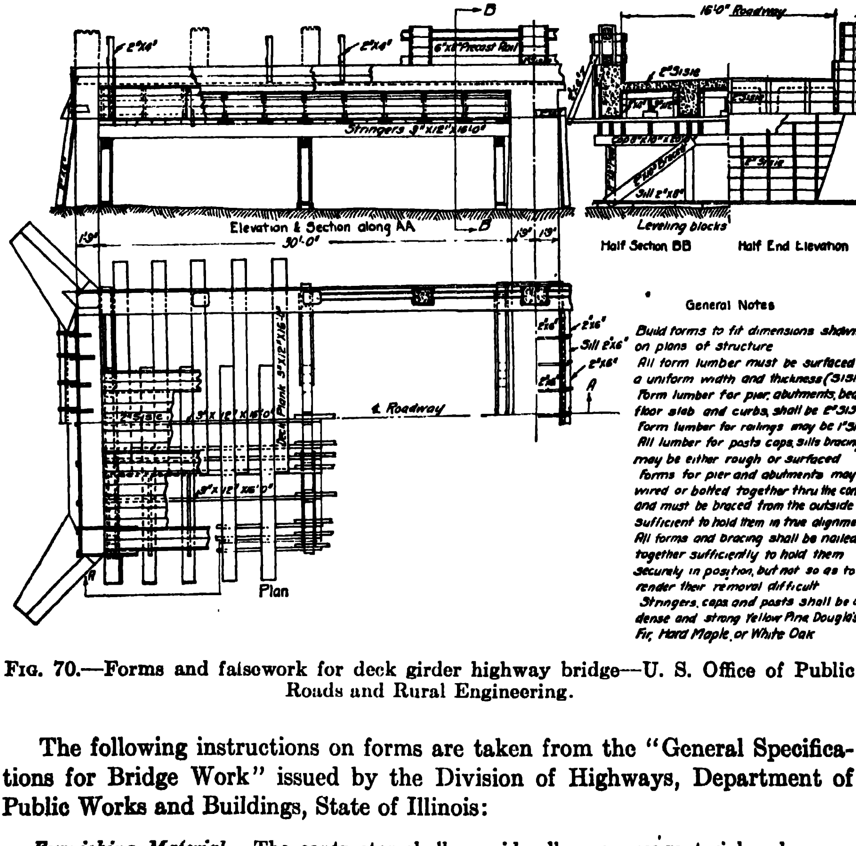


Fig. 70.—Forms and falsework for deck girder highway bridge—U. S. Office of Public Roads and Rural Engineering.

The following instructions on forms are taken from the "General Specifications for Bridge Work" issued by the Division of Highways, Department of Public Works and Buildings, State of Illinois:

Furnishing Material.—The contractor shall provide all necessary material and means for building the forms for all concrete masonry.

Construction.—All forms shall be so constructed as to be held rigidly in place, line and elevation. If at any point of the work, after the concrete has been placed, the forms show signs of bulging or sagging, that portion of the concrete shall be immediately removed on notice by the inspector, and the forms shall be properly supported.

All forms are to remain in place until, in the opinion of the engineer, it is safe to remove them.

Moulding.—Moulding having ¾ or 1-in. sides shall be used on all exposed corners. The surface of such moulding in contact with the concrete may be rounded to a uniform radius or may be flat. In the latter case the resulting angles between adjacent surfaces in the finished concrete shall be equal.

Beveled Faces.—Forms shall be given a bevel of 1 in. to 1 ft. wherever projections of the concrete, such as copings, floor beams, etc., would otherwise cause binding upon the removal

of the forms. The pitch shall be so arranged as to increase the thickness of copings, floor beams, etc., at the base, and the narrowest parts of such projections shall have dimensions not less than as shown on the plans.

Removal of Forms.—The forms covering what will be the exposed face of the concrete masonry shall be removed as soon as the engineer decides that it is safe to do so, and all crevices neatly filled with a stiff 1 to 2 cement mortar thoroughly rammed into place.

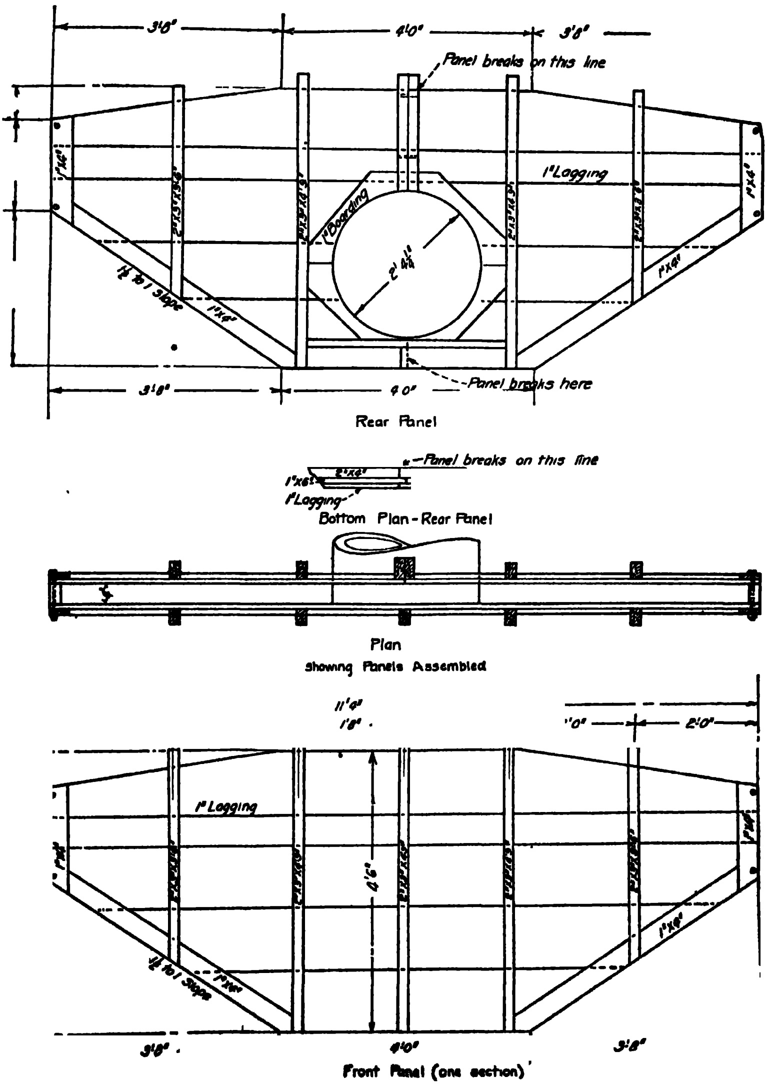


FIG. 71.—Panel forms for headwalls of pipe bridge—U. S. Reclamation Service.

12. Forms for Conduits and Tunnels.—This section will discuss the formwork for closed channels such as culverts, aqueducts and sewers of curvelinear section, pipes, siphons and tunnels. No mention will be made of conduits or pipe laid

up with molded concrete blocks or of sections of molded pipe. The former are made with little or no formwork, while the latter is made of pipe sections generally precast in patent forms or molds and later laid in place. This article will deal with conduits and pipe molded in place as a monolithic structure.

Formwork for conduits and pipe must be built so as to be rigid, able to take alternate wetting and drying and be portable. The surface of the form must be so made or lined as to give a smooth surface to the interior of the conduit. As conduit forms are expensive, they should be made in sections that can be

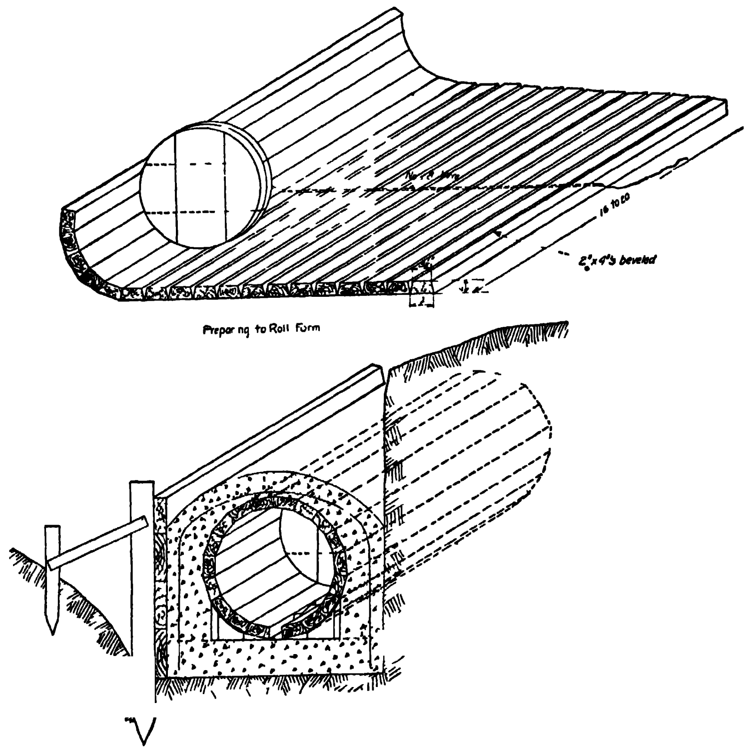


FIG 72 Collapsible form for small culverts.

re-used a large number of times. For economy of use, the formwork should be made in portable sections, which can be readily and quickly erected, taken down, moved and re-erected. To accomplish this purpose traveling forms have been devised.

Conduit forms may be built of wood or metal. The contact surfaces of the former must generally be lined with metal to give a smooth and true finish to the interior surface of the conduit. Metal forms are often advantageously used on account of their durability, freedom from warping and low weight for required strength.

The concrete culvert of circular section used in highway and railway construction is often molded about a galvanized corrugated steel or iron pipe as a form. In the cheaper types of culvert, the pipe may be laid through the embankment with the ends only embedded in concrete walls, as shown in the design of

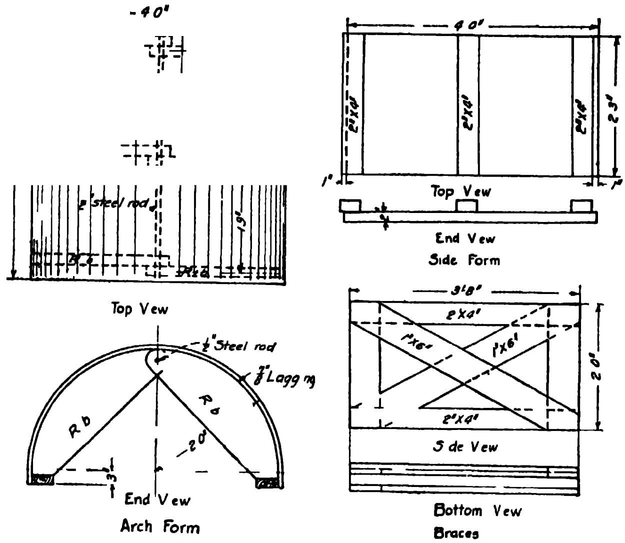


FIG 73 —Forms for large arch sewer, Washington, D C.

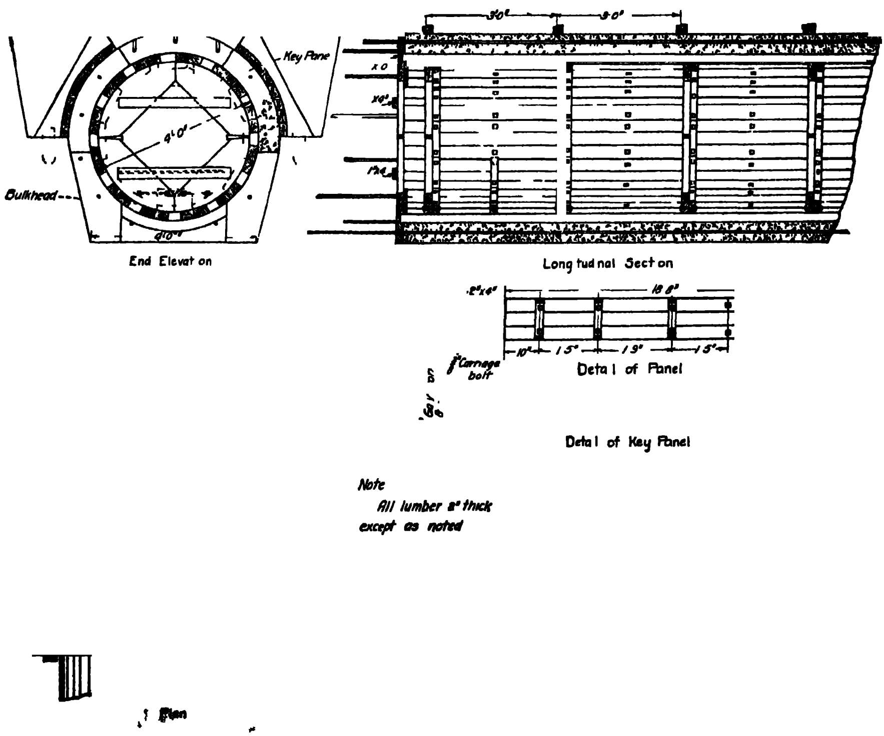


FIG. 74.—Forms for pressure pipe—U. S. Reclamation Service.

the U. S. Reclamation Service for headwall panels for a pipe conduit (Fig. 71). Generally for more permanent work, the entire pipe is embedded in concrete, in addition to suitable headwalls.

A simple type[1] of collapsible form for the barrel of concrete culverts 18 in. to 4 ft. in diameter consists of a series of staves made of beveled 2 × 4's held together with malleable iron wire (Fig. 72). The staves act very much like the top of a roll top desk, and are rolled around a circular head which has been cut to the required size of the culvert. The staves are held in place by wire bands wrapped around their outer surfaces. Wedges are driven in to hold the staves firmly. The form is removed by knocking out the wedges and knocking in the head.

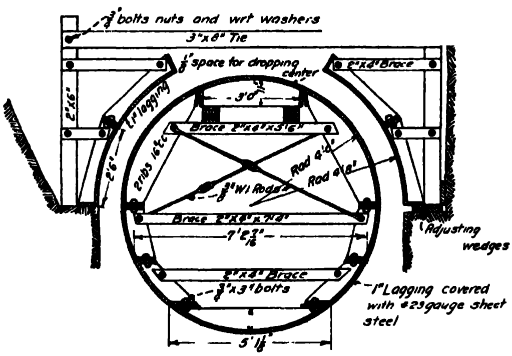


FIG. 75.—Forms for 8-ft. circular pipe—U. S. Reclamation Service.

Forms for circular culverts and pipe are made in sections to provide for the pouring of the structure in parts; first the invert, then the side walls to the springing planes and finally the arch. An important feature in the design of such forms is the provision for ready removal. There are two methods in common use; using a double hinged section for the arch form, and using wedges between sections near the springing planes. The former type is clearly shown in Fig. 73, which gives a view of the culvert arch forms used in the construction of the Connecticut Avenue Trunk Sewer, Washington, D. C. Note the ½-in. steel rod at the crown, which holds the sections together and is driven out for their removal. The latter type is described in the following account of pipe forms used by the U. S. Reclamation Service.

Forms for continuous monolithic concrete pipe construction are made in sections of lengths readily handled, usually from 10 to 18 ft. There are usually 5 to 8 sections; an invert section, 4 or more side sections and a key section. Each section consists of a set of ribs over the curved edges of which are fastened the sheeting or lagging. The sections are bolted together at their ends transversely and along their sides longitudinally with the axis of the conduit. Wooden or steel rod cross-braces and wedges are used to give the formwork final shape and position.

The setting of the forms for a conduit ordinarily begins with the invert section, which is placed on the invert concrete roughly poured as a bed or on blocks of

[1] From Second Annual Report of Illinois Highway Commission.

concrete which are embedded in the invert. The side forms are placed in corresponding pairs as the sides of the conduit are poured, care being taken to carry up the work uniformly and simultaneously on both sides. Typical examples of concrete pipe form construction are taken from the recent work of the U. S. Reclamation Service. Figure 74 illustrates the formwork for heavy pressure pipe construction partly in excavation. Note the bulkheads which are placed at the end of each day's work. Figure 75 gives the details of the forms for an 8-ft. diameter concrete pipe. Note the exterior panels and bracing for the upper exposed section of the conduit. Siphon forms are shown in Figs. 76 and 77.

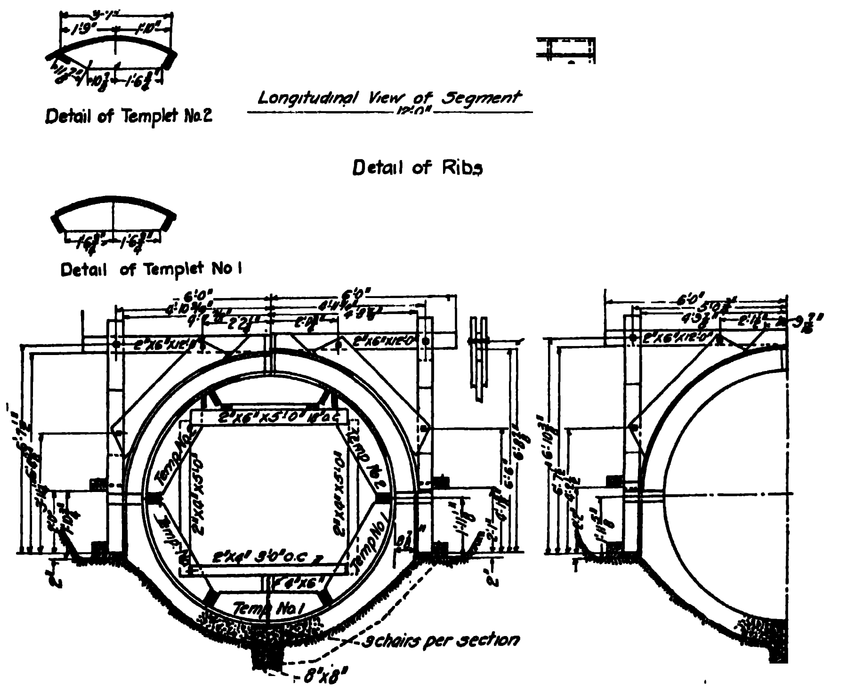


FIG. 76.—Details of barrel siphon forms—U. S. Reclamation Service.

Forms for conduits of other than circular cross-section are made similar to those described for those of circular cross-section. The forms for a 5-ft. egg-shaped sewer built in Washington, D. C., are shown in Fig. 78.[1] The forms consist of two parts; an invert form and an arch center, made in sections 16 ft. long. Each part was built up of lagging nailed to plank ribs spaced 2 ft. on centers.

The invert form was set in place, the wedge timbers A and B laid in their grooves, and then the arch center set and locked to the lower form by the latches L on the outside of the end ribs.

The hole C provided for a square gudgeon timber to pass lengthwise through the centering. This timber projected at each end of the section beyond the

[1] *Engineering News*, February 18, 1904.

ribs and was rounded so that the whole section could be revolved and wrapped with a continuous spiral steel strip 6 in. wide and $\frac{1}{24}$ in. thick.

When the concrete had been placed, the centering was removed, leaving the spiral steel in place to support the concrete. When the latter had set, the strip of steel was removed by pulling on one end. The center after its removal, was rewound with another steel strip and used in the next section. This form was devised by A. C. Chenoweth of Brooklyn, N. Y.

Metal forms for circular conduits are made in rings of from 5 to 10 ft. in length and in diameter from 3 to 12 ft. Each complete ring is composed of four or more pieces, with longitudinal joints. At one of these joints a wedge-shaped piece is inserted to allow of the easy removal of the forms. These longitudinal joints are made non-continuous in adjacent 5-ft. rings. The form

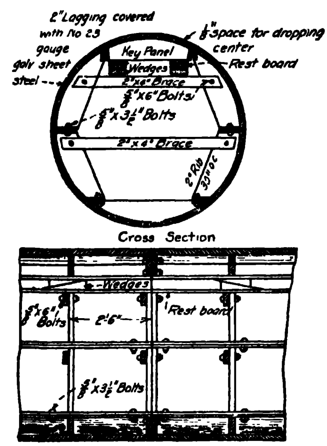


FIG. 77.—Inner form for siphon—U. S. Reclamation Service.

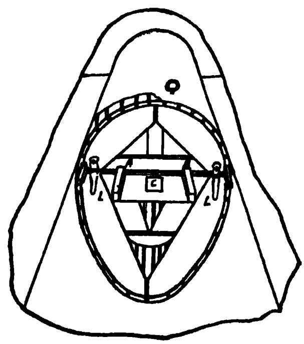


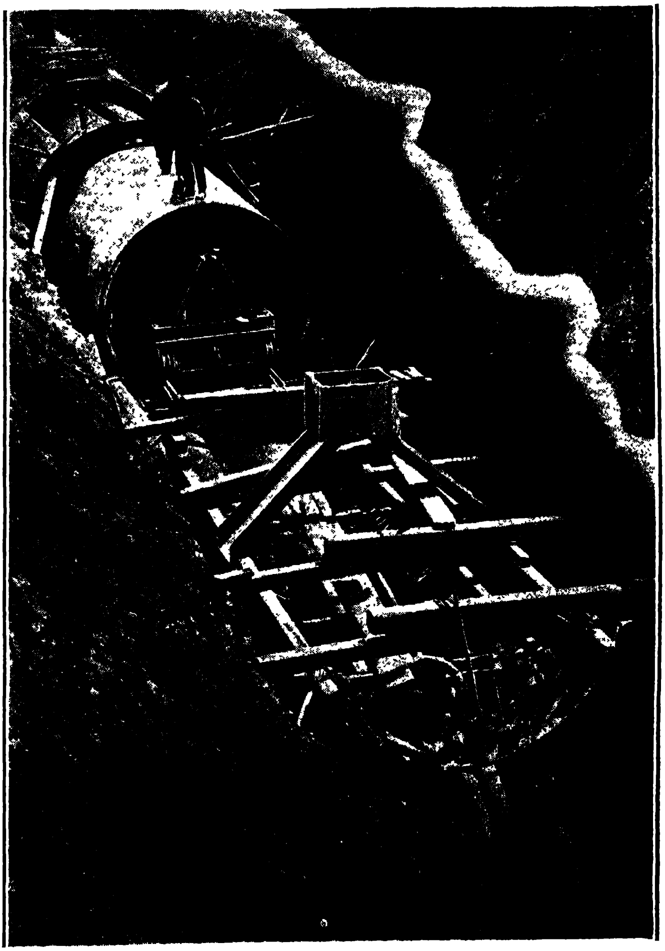
FIG. 78.—Forms for egg-shaped sewer.

for the invert section of the conduit is set first and the concrete poured, thus forming a working support for the erection of the rest of the conduit (see Fig. 79).[1] A track may be framed up in the larger size conduits and a small car operated for the transfer of the form rings of the upper part of the conduit (see Fig. 80).[1] Thus by the use of several rings, concreting may be carried on uninterruptedly, using the car to move a form ring from the rear completed part to the front part. All joints are bolted together with machine bolts.

A large variety of formwork has been devised for the lining of tunnels with concrete, the details of the forms depending on size and shape of the tunnel, the nature of the material penetrated, and as to whether the lining is to be done while the tunnel is in service. Important elements in form construction for tunnel work are strength, simplicity and portability. Strength and rigidity are very important, especially in metal forms, where deflections or bending of the parts may render their re-use difficult and expensive. All formwork should be simple in design and construction so that it may be easily erected, removed and re-erected and thus provide for portability.

[1] Blaw-Knox Telescopic Forms, Concrete Sewers, Detroit, Mich.

Forms for tunnels may be made of wood or metal or combinations of the two kinds of material. For long sections of tunnel of the same cross-section, traveling forms may be used to advantage. The formwork is generally built in sections which are readily and easily handled. The following are given as a few typical examples of the use of forms in concrete tunnel construction.



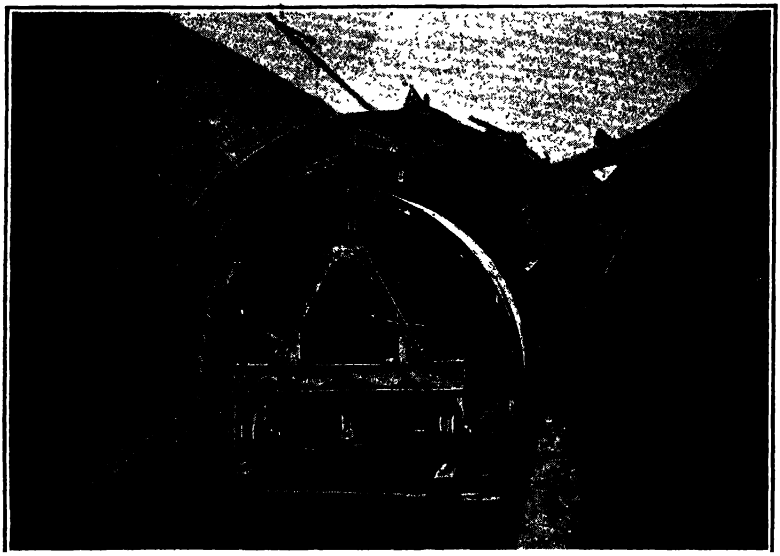
(*Courtesy of Blaw-Knox Co.*)

FIG. 79.—Invert forms for circular sewer.

The Selby Hill Street Railway Tunnel,[1] St. Paul, Minn., was built entirely in open cut through a coarse glacial sand. The open cut was heavily shored and braced and the concrete invert placed. Timber bents were erected on the invert to form a center for the forms of the walls and the arch of the tunnel. The bents were 12 ft. wide, 11 ft. high and spaced 6 ft. on centers. Each bent consisted of a 6 × 8-in. sill, two 6 × 8-in. plumb posts and an 8 × 12-in. plate (see Fig. 81).

[1] *Engineering Record*, Sept. 21, 1907.

When the bents had been erected as far as the side walls were to be built, the lowest row of temporary sheeting cross-braces between them was removed, transferring the pressure to the 6 × 8-in. posts of the bent. The inside forms for the walls up to the springing lines of the arch, 5 ft. above the invert, were



(Courtesy of Blaw-Knox Co.)

Fig. 80.—*Use of car for transfer of form sections.*

then set and braced against the top and bottom of the bent. The sheeting of the trench formed the outside forms. After the side walls were poured and set, the cross-braces of the second row of walls were removed and the load transferred to the posts of the bents. The arch forms were then erected on the bents up to a height of 11 ft., and the walls built up to this elevation. When this concrete

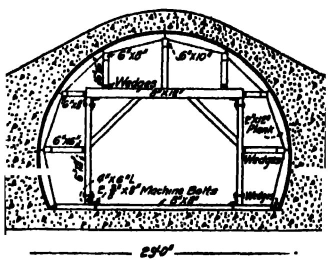


Fig. 81.—*Centering and forms for street railway tunnel.*

had set, the third row of temporary cross-braces and the top braces between the tops of the posts were removed and the balance of the arch completed.

The forms had ribs of 2 × 12-in. plank cut to a template to conform with the curve of the arch. These ribs were each in 6 sections, a section on each side extending up to the 5 ft. level of the walls, a second section extending up to

the 11 ft. height and the two top sections in the closing part of the arch. They were spaced 2 ft. on centers and were lagged with 2 × 4's.

The construction of a tunnel through loose, shaly rock, as a short section of one of the main distribution canals of the Belle Fourche Project of the U. S. Reclamation Service, involved the use of some unique forms for the placing of the concrete lining. Figure 82 clearly shows the essential parts of the formwork and method of construction. After the building of the continuous concrete footings along each side of the tunnel excavation, the steel ribs were set up 8 ft. on centers and resting on 4 × 8-in. timber sills set on the concrete. Wedges were placed between these sills and the shoes of the rib. A series of timber sections

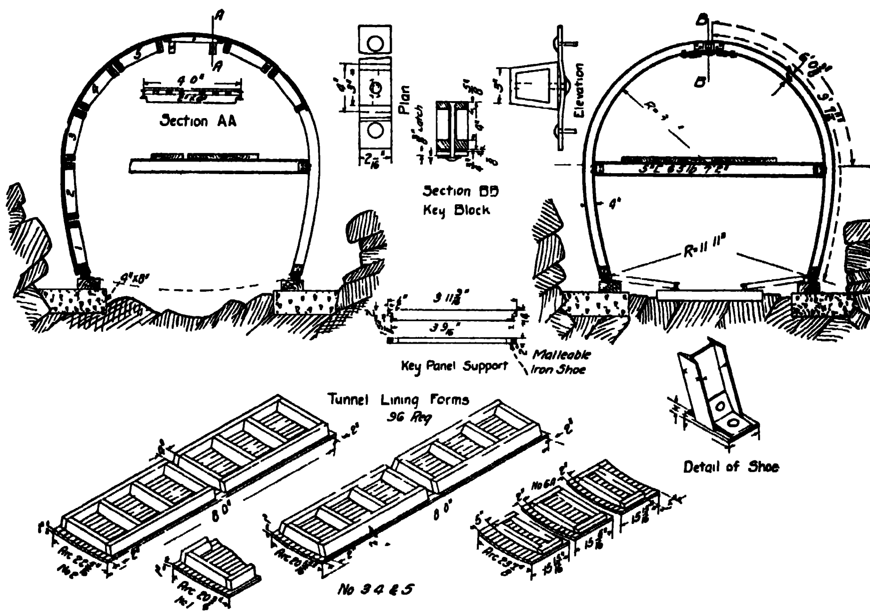
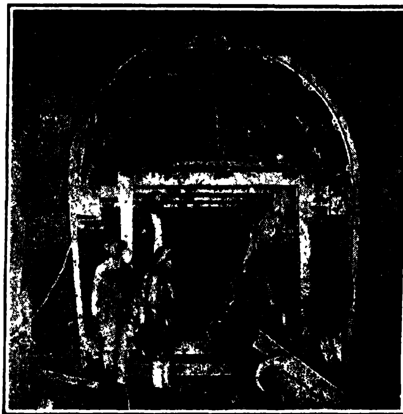


FIG. 82.—Details of tunnel lining forms—U. S. Reclamation Service.

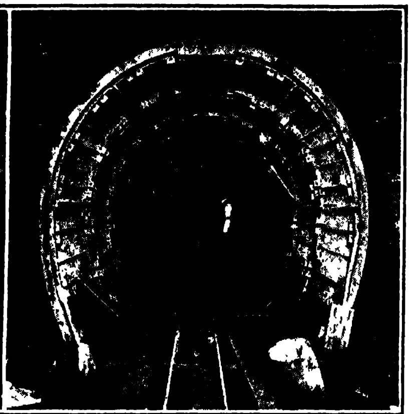
were set on the ribs, five sections on each side and a key section at the top. Note especially the details of the key block and method of supporting key section. As in most tunnel formwork an intermediate support is provided to carry a runway for the transportation of concrete.

Metal forms for tunnel construction or lining are made in various forms and sizes, but practically all are made up of sections of from 5 to 16 ft. in length and consisting of braced ribs covered with steel plates. The ribs are usually of some common form of structural shape such as I-beam, channel or angle, bent to the required form. The outstanding flanges of the ribs are provided with holes for bolting adjacent sections together. The larger sizes are ordinarily braced with cross-braces placed near the springing lines of the arch and also serve to support runways or platforms for the transportation of material. These cross-braces commonly have some method of adjustment such as turnbuckles or sleeve nuts. Note the turnbuckles near the gusset plates of the cross-bracing in Fig. 83. In

the smaller sections of horseshoe type where the arch action is sufficient to take the pressure of the concrete, no bracing is used (see Fig. 84). This form requires

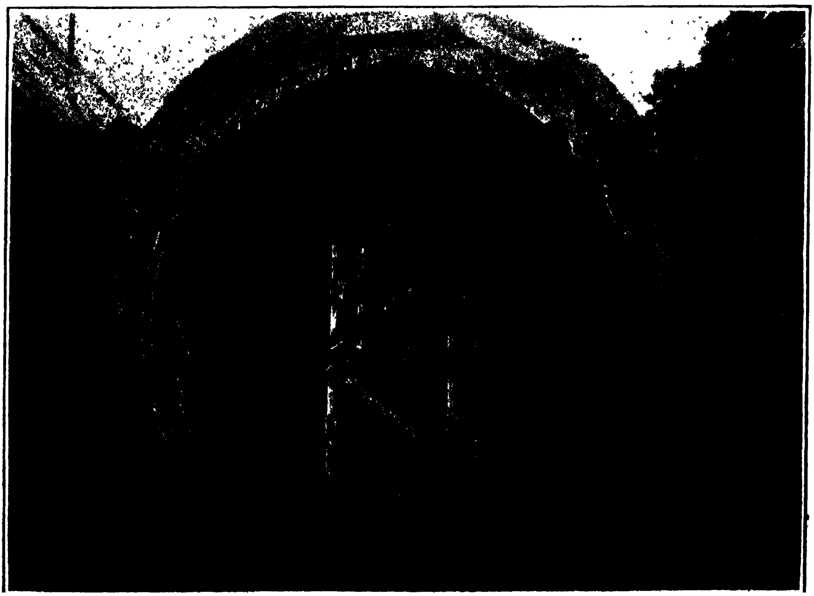


(*Courtesy of Blaw-Knox Co*)
Fig. 83.—Collapsible tunnel forms.



(*Courtesy of Blaw-Knox Co*)
Fig. 84.—Forms for horse-shoe tunnel.

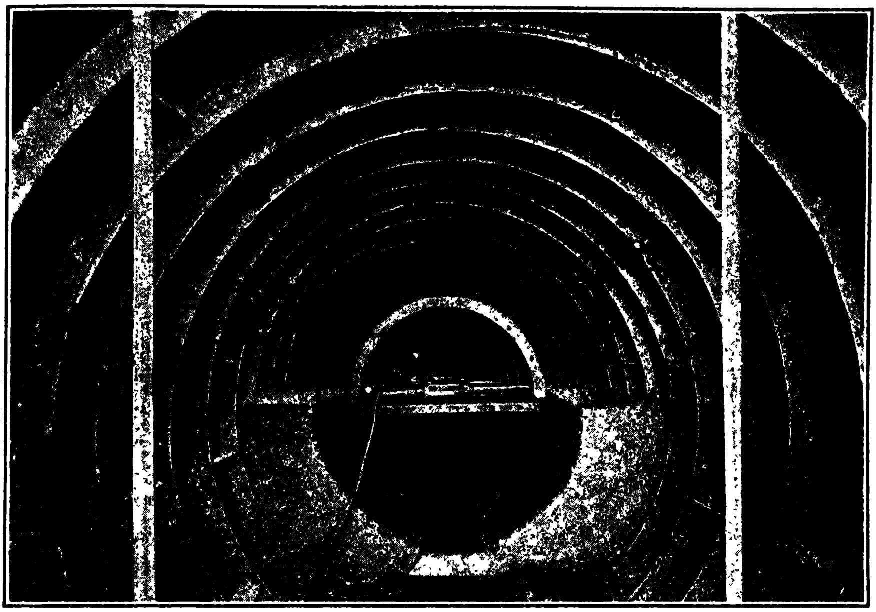
ribs and plates of great strength in order to take the great liquid pressure of rapidly poured concrete. For example, in aqueduct tunnels, built in Chicago



(*Courtesy of Blaw-Knox Co.*)
Fig. 85.—Hinged forms for large circular conduit.

and the Catskill Aqueduct, New York State, 6-in. channel ribs spaced 5 ft. apart were covered with $\frac{3}{16}$-in. steel plates.

The erection of metal form sections is made on concrete footing courses built accurately to line and grade at the bottom of each side of the tunnel exca-



(*Courtesy of Blaw-Knox Co.*)
FIG. 86.—Steel liner plates for concrete sewer.

vation. On these footings are laid timber sills upon which are placed the feet of the ribs. Wedges are placed between the shoes of the ribs and the sills to facili-

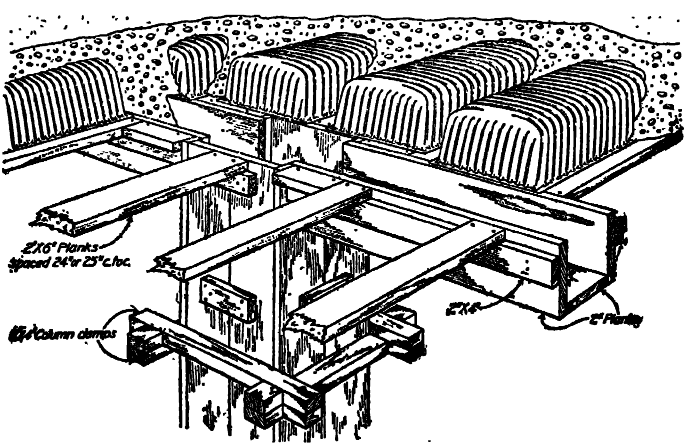


FIG. 87.—G. F. steel-tile.

tate the removal of the form sections. Generally three sets of form sections were used at one time; placing concrete in one set, the concrete setting in the second,

and the removal and set-up of the third set. Each set or unit has been used in lengths of from 20 to 40 ft.

A circular tunnel under construction with steel form sections is shown in Fig. 85. Note that each section is made in three parts which are hinged so as to form the two side-wall and the arch sections. The invert is poured and the form sections set on timber blocks. Note the narrow gage track and framed steel

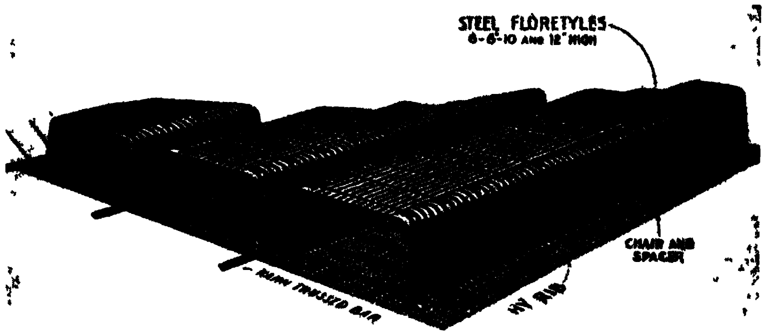


Fig. 88.—Steel Floretyles—for reinforced concrete floors.

traveler for the transportation of form sections and material. In large tunnel sections, interior braces are necessary to resist the lateral thrust of the green concrete. These braces are often made in the form of light steel framed trusses, attached to the ribs and providing space between for the traveler.

Arch rings of steel plates or cast-iron sections are used as a temporary tunnel

Fig. 89.—Steel Floredomes—for reinforced concrete floors.

lining in soft soil. Concrete is placed inside of this lining to form the permanent lining of the required cross-section. Figure 86 shows steel liner plates used in the construction of a concrete sewer at Milwaukee, Wisconsin.

13. Metal Forms.—The increasing scarcity and cost of suitable lumber for forms during the past decade has led to the more general use of metal forms in building construction. The preceding articles have referred to the use of metal

Fig. 90.—Meyer removable steel floor forms.

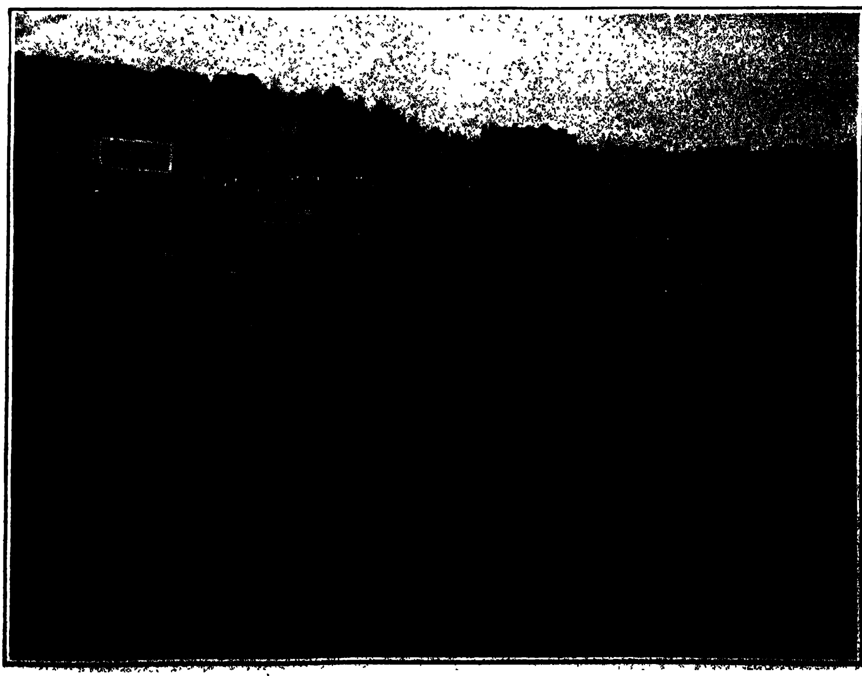
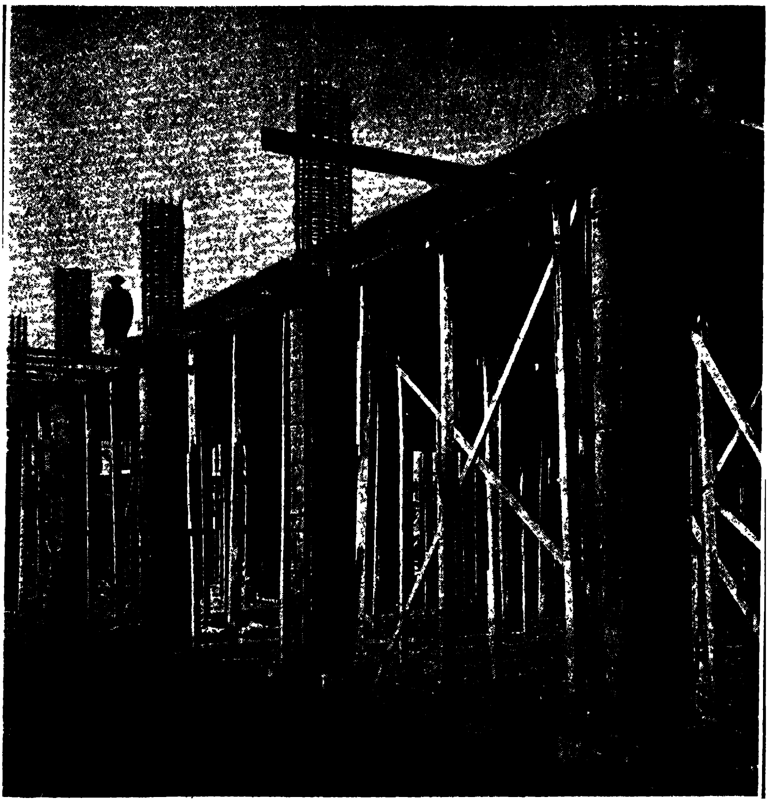


Fig. 91.—Berloy steel floor forms (10 in.).

forms in the construction of concrete sewers, tunnels, tanks and retaining walls. Hence this article will discuss the use of metal forms in wall and floor construction of buildings.

Metal forms may generally be considered in two classes; the first in which the form unit is left in place and becomes a part of the structure; and the second, where the form unit is removed and utilized many times. In the first class are the steel floor domes and cores, special types of corrugated plates, and the various



(Courtesy of Blaw-Knox Co.)

Fig. 92.—Steel circular column forms.

forms of ribbed metal lath which serve as a form during concreting and later serve as reinforcement. The second class comprises largely column, wall and floor forms.

In certain types of buildings, notably school houses where the floor loads are light and well distributed, the steel form type of concrete floor construction (similar in principle to the terra cotta hollow tile floor), has come into usage. The various well-known makes are similar in character, and consist of metal domes or arch-shaped forms which are often ribbed or corrugated to secure greater stiffness. These forms are made in depths of 6, 8, 10, 12 and 14 in. A typical example of steel form floor construction is shown in Fig. 87, where G. F.

Steel Tile[1] is used. Steel "floretyles"[2] and "floredomes"[2] are shown in Figs. 88 and 89, respectively. Steel floor forms which may be removed and re-used in the successive floors of a building are the Meyer Steelforms[3] (Fig. 90) and the Berloy Floor Cores[4] (Fig. 91).

The forms for circular columns and for flaring column heads are universally made of metal. The adjustment in the height of columns is secured by telescoping the ends. The sections of the column are steel panels held rigidly in place by stiff steel bands or rings (see Fig. 92). Forms and flaring heads are made in various standard sizes; 12 to 26-in. diameter columns with 40 to 57-in. heads, 26 to 36-in. diameter columns with 48 to 84-in. diameter heads, and 38

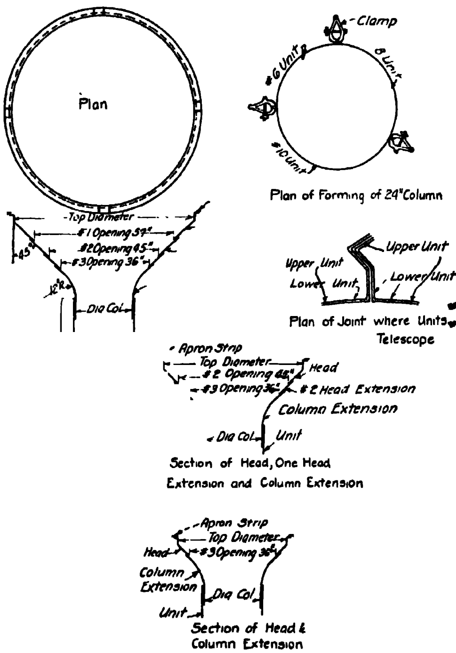
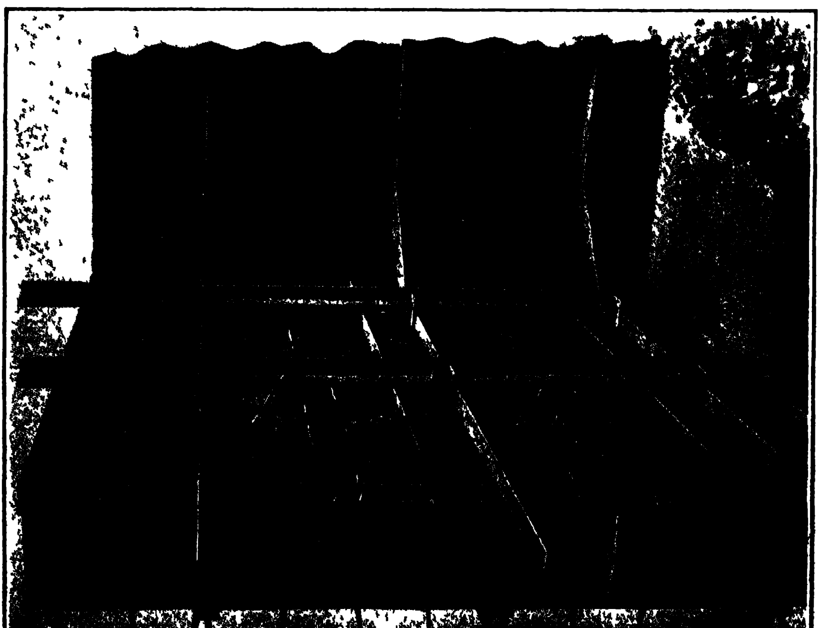


FIG. 93.—Steel column forms—Hydraulic Pressed Steel Co.

to 54-in. columns with 60 to 84-in. heads (Fig. 93). The variation in diameter is adjusted to fit form panels of standard widths, where metal panels are used. The steel column forms are not used to support any part of the floor, as is generally the case in wooden forms, so that the floor forms are ordinarily set up before the column molds.

Wall forms of metal are generally made in panels which are held together by some special form of clamp. Three well-known examples of this type of form are the Blaw, the Hydraulic Steelcraft, and the Metaforms. The Blaw forms[5] consist of standard metal panels 2 ft. square, reinforced on all four sides, with small steel angles. Fractional panels, lap and corner panels are used for adapting

[1] General Fireproofing Company, Youngstown, Ohio.
[2] Trussed Steel Company, Youngstown, Ohio.
[3] Concrete Engineering Company, Omaha, Nebr.
[4] Carlem Engineering Company, Pittsburgh, Pa.
[5] Blaw-Knox Company, Pittsburgh, Pa.



(*Courtesy of Blaw-Knox Co.*)
FIG. 94.—Steel wall forms.

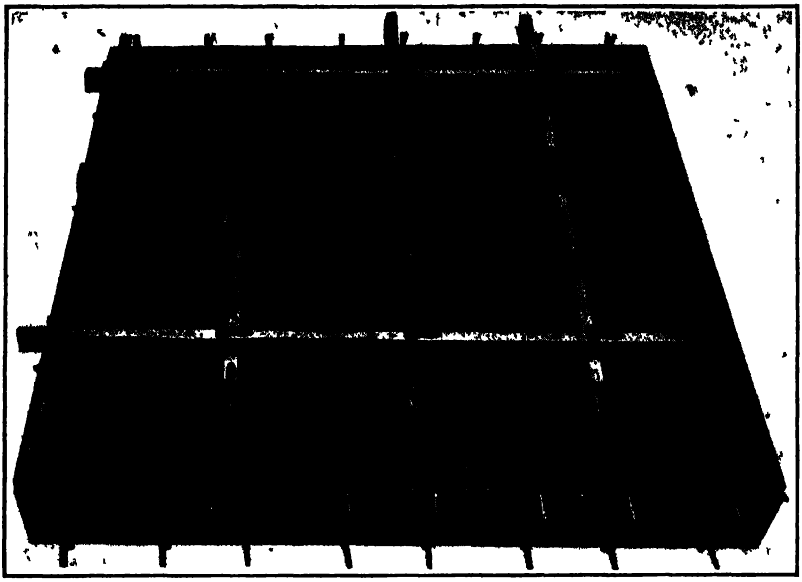


FIG. 95.—Metaforms metal wall forms.

the forms to various sizes and shapes of walls. The forms are secured to each other by wedge keys that are interchangeable and slotted. The wedges are inserted in holes in the flanges of the forms and a second wedge is driven into the slot of the first wedge, thus locking the forms securely. When the forms are to be

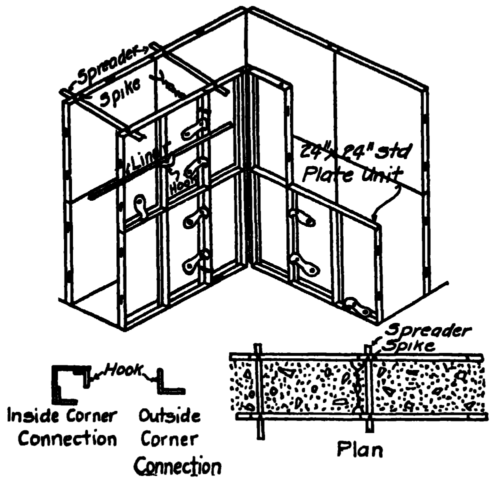


Fig. 96.—Reichert metal wall forms.

shifted by hand, the panels are assembled in courses 2 ft. high and containing not more than 24 sq. ft. of surface. These units are then fastened to liners about 11 ft. long. The horizontal liners are attached to the forms by means of small

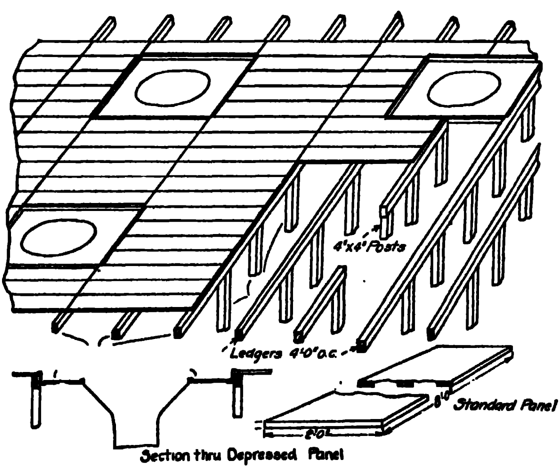


Fig. 97.—Metal panel forms for flat slab floor.

plates slotted to take the leg of the steel angle liner. These plates have an integral latchpiece which is inserted in corresponding flange slots in the form panels. When the horizontal angle liner is placed in the slot in the plates, the latch piece is drawn up tightly and the liner is made secure with the wedge key. The forms are now securely fastened and aligned (see Fig. 94).

The Hydraulic Steelcraft forms[1] consist of light pressed steel U-shaped vertical liners and horizontal ribs supporting steel form plates backed with wood. The edges of the wood backing are clad with light steel plates. Key hole slots in the back of the vertical liners enable the horizontal ribs to be clamped on by means of a key wedge and U-clamp. In erection, the framework of ribs and liners is erected and aligned. The plates may be left off while reinforcing steel is being placed, or they may be removed at any point to give access to the forms for cleaning or other purposes. Floors and roof slabs are constructed by using the same equipment as the walls.

"Metaforms"[2] are based on a 24-in. square light metal unit. The latter consists of a blue annealed sheet with a 1 × 1-in. angle riveted to all four sides and in the middle as a stiffener. Spikes or pins are placed in corresponding holes in the outstanding legs of the angles to secure alignment. The units are secured tightly together by malleable iron clamps pinned to the side angles. The units are wired similar to wood forms. The plates are held apart by stay rods, made adjustable to the wall thickness (see Fig. 95).

The Reichert system[3] is similar in design and use to the Metaforms and is shown in Fig. 96.

Metal forms for constructing concrete flat slab floors have recently come into use. A well-known make, the Deslauriers system,[4] use 2 × 8-ft. standard steel panels which are supported on lines of parallel ledgers 4 ft. on centers and supported by posts 4 ft. on center (see Fig. 97).

[1] Hydraulic Steelcraft Company, Cleveland, Ohio.
[2] Metal Forms Corporation, Milwaukee, Wis
[3] Reichert Metal Concrete Forms Company, Milwaukee, Wis.
[4] Deslauriers Column Mould Company, New York.

SECTION 3

BENDING AND PLACING CONCRETE REINFORCEMENT

By James Cowin

1. Ordering.—Reinforcing steel is ordered either directly from the mill or from one of the bar companies. Companies handling steel bars carry stocks in their warehouses and take over the responsibility of seeing that the steel is delivered at the proper time. In general they hold the same relative position between the mill and the consumer as do jobbers in other lines.

In ordering from the mill it should be borne in mind that no mill can make definite promises of delivery and the steel should be ordered far enough in advance of when it is needed to allow for delays due to strikes, breaking of machinery, or to any of the other mishaps which seem unavoidable in a large manufacturing plant.

The quality of the steel required should be stated on the order, as should also the approximate shipping date, and the routing if there is a decided advantage in getting delivery over some special road. Otherwise, the routing can be safely left to the traffic men of the steel company who are often in a better position to determine the best and quickest route to use, on account of their better knowledge of the seasonal freight movements.

The order should also specify that the bars are to be tied in 50 to 100-lb. bundles as this will make the unloading and handling much easier and will keep the steel in better condition during storage.

Mill test reports on each shipment will be very desirable in case any question arises as to the quality of the steel furnished and will serve as a check on the field inspection tests.

By dividing the specification into smaller orders it is possible to get shipments about as needed without having to handle and store the entire order at one time. This is especially advantageous if the storage space is limited.

Except under special conditions, it will be impossible to place orders directly with the mill and get the shipment required, as under normal conditions the mills are scheduled some time ahead in order to assure continuous operation. The usual course, therefore, will be to place the order with one of the jobbing companies specializing in reinforcement and leave to them the responsibility of having the steel on the ground when needed. In ordering from the bar company the same information regarding the grade of steel, bundling, and especially a schedule giving dates when the steel for the different floors will be required, should be clearly specified on the order and made a part of the contract. If the bar company has a warehouse in the vicinity of the building under construction, it will save storage space and the labor of sorting and storing if it is requested to supply the steel as needed. This can be done by giving the bar company a copy of the construction progress schedule and notifying it promptly if any changes are made which will advance or retard the work.

2. Shipping.—Considerable time and money can be saved by proper attention to shipping instructions and to the making up of bar lists in lengths and quantities to secure the lowest freight rates. A minimum single car load for reinforcing bars for interstate shipments is 36,000 lb. and 45,000 lb. minimum double load. For intrastate shipments 60,000 lb. is the minimum double load.

The maximum length for single load shipments is 40 ft. while a double car would carry lengths up to 82 ft. if it were possible to get bars of that length rolled by the mills. The maximum lengths carried in stock are 60 ft. and the mills will, under special conditions, roll bars up to 65 or even 70 ft.

Double cars are securely chained together and the coupling is blocked with wedges to prevent end motion. The steel is blocked up on a number of ties which are bolted to the floor of the cars. A plate is spiked on top of each tie and thickly smeared with grease. This permits lateral play in the load of steel when the train is going around curves and prevents derailment of one or both the cars of the double header.

Unless there is a commodity rate between the point of origin and the destination, fifth class rates will apply to car lots and fourth class to less than car lots. Commodity rates on steel are less than fifth class rates and apply only to car lots. Commodity rates will often be found where least expected.

On less than car lots the lengths should be confined to 22 ft. unless the quantity is sufficient to make up an amount of freight at the regular rates to equal 1,000 lb. at the first class rate, which is the minimum for a shipment of this kind.

A warehouse company can often save a contractor money by allowing him a fabrication in transit rate. Under this arrangement the bar company can receive the bars in standard stock lengths from the mill, cut and bend them and forward them to their destination at the same freight rate as if they went directly from the mill to their destination.

3. Checking, Assorting and Storing.—The steel should be checked as it is unloaded from the car or truck and any shortages should be immediately reported. In the case of a railroad shipment, if a claim is immediately reported to the local office of the railroad, it can be verified by having one of the railroad employees make a recheck and thus avoid the delay and annoyance of preparing affidavits. However, if the bill of lading has been made out properly by the shipper, showing the number of bundles or number of bars on the shipment, there will be no question about getting a refund from the railroad eventually even if the material is unloaded in such a way that a re-check is impractical—that is, provided the man responsible for the unloading will make out and send to the claim department of the railroad an affidavit stating the facts. In case of a truck shipment the driver should not be given a receipt until the steel shown on his loading tally has all been checked off the load.

The steel should be sorted in a convenient way for further handling as it is unloaded. The different sizes and lengths should be piled in neat, easily accessible piles, the different floors being kept separate. If the steel has not been bought already bent, the portion which is to be bent should be placed apart where it will be convenient to the bender without further handling. If not already tagged, the different piles should be labeled so that there will be no confusion or delay in finding the proper bars when needed.

If the steel is to be stored any length of time before being used, it should be blocked up from the ground and piled compactly. Plain rounds will rust down only two or three layers, even if exposed to the weather for a long time, and deformed bars do not rust badly, but square twisted bars should be protected as the shape permits water to run through the entire pile and the operation of twisting breaks the outer skin and makes them very susceptible to rusting. A thin film of rust, of course, is not objectionable but if the bars become scaly they should be cleaned with a sand blast or wire brushes. It is very seldom that the bars get in such shape as to make this necessary, as it will take at least a full season of exposure to render a bar so scaley that cleaning is required.

4. Testing.—It is well to order some extra bars of each size for testing purposes, as regardless of the reliability of the firm supplying the material, the satisfaction of knowing exactly the quality of steel being used is worth the small extra expense.

5. Bending of Reinforcement.—Bending is divided roughly into heavy bending, light bending and spiral winding.

Heavy bending includes the beam rods, column verticals which need to be off-set to carry into a smaller column above, some footing steel and any miscellaneous items such as stair hangers for which sizes larger than ½ in. are used. Light bending is understood generally to mean stirrups and column ties. Spiral winding consists in rolling Bessemer wire or hard grade hot rolled rods to a definite diameter and pitch.

Light and heavy bending can be done successfully either by hand or by machinery but spirals cannot be rolled true to diameter and pitch by hand. In the early days of reinforced concrete, spirals were rolled through a tire bender and a passable spiral was produced provided great care and patience were used. Spirals rolled with a tire bender or around a drum should not be figured to the stress allowed for machine made spirals.

Heavy bending should be done with a slow steady pressure and the pin around which the bending is done should be of at least twice the diameter of the rod to be bent if structural or intermediate grade steel is being used, and should have a diameter of three times the rod if the steel is of hard grade. The pin should have a movable collar fitted around it as this will relieve the bar of considerable strain and will make bending much easier. Some difficulty will be experienced in keeping the bends in the same plane but careful attention to the way the bar is held will obviate that.

Stirrup bending is more a matter of speed and skill than of strength, and a fast man can turn out from 75 to 100 stirrups an hour. Stirrups should be made with sharp corners—otherwise, the beam rods are prevented from taking their proper position in the forms.

6. Types of Benders.—All hand benders work on the same principle, which is simply that of holding part of the bar stationary while the balance is moved through an arc of a circle. There are a number of hand benders on the market and any good blacksmith can make one in a few hours. A hand stirrup bender is a small edition of the beam bar bender, constructed as light as possible and with a short handle, as speed rather than strength is desired in a stirrup and tie bender.

Power benders for heavy bending are of two classes, one in which the bending is done in a vertical plane and the other in a horizontal plane. The Kardong

bender is the best example of those bending in a vertical plane and is a well-designed and constructed piece of machinery run either by a gasoline engine or an electric motor. It is mounted on wheels so that it can be hauled to the job. The great objection to the vertical bender is the difficulty in holding the heavy bar when the bend is some distance from the end but as very few bars require this special bend the objection is not serious. The horizontal bender works on the idea of a turntable, the bar being laid on a table the center of which revolves. A projection in the revolving part catches the end of the bar and carries it to the angle desired.

Power stirrup benders are small bulldozers and stirrups can be bent economically on an ordinary bulldozer if there is a large number of the same dimensions. They are not economical for the ordinary user but can be used to advantage by a large fabricating shop.

Spiral benders equipped to do column spiral bending are rare and the contractor will do well to buy spirals made up rather than to attempt to make them. The wire comes in coils about 3 ft. in diameter and weighing approximately 200 lb. The coil is placed on a drum and is run through horizontal and vertical straightening rollers before recoiling is attempted. After leaving the straightening rolls the wire goes between two live rollers which force it against a third roller set at an angle which regulates the diameter of the finished spiral. This roller can be moved up or down and thus give any diameter wanted. The pitch is governed by a roller which pushes against the wire at right angles to the plane of the three diameter rollers. The spirals are made up after they are rolled by attaching two or more spacing bars. Practically every form of spacing bar in general use is patented but all have the same function—that of holding the turns of wire at the proper distance apart to give the pitch. These will permit the spiral to be collapsed for convenient handling and shipping.

7. Spacers.—Slab spacers not only give the proper spacing to the bars but also support them the proper distance from the forming. They are made of flat bands or of wire, the bands being punched or folded down to form the supporting legs and up to form the lips which are bent over the bars. Wire spacers are bent to make the supports and bar holders. As the spacing and the sizes of slab bars used vary greatly these are not ordinarily carried in stock but are made to order.

Beam and joist spacers are similar to slab spacers but are made of heavier material.

8. Supports and Ties.—In some cases, supports are required where spacers are not needed. There are a number of different kinds of chairs manufactured for this purpose made of sheet metal, flats or wire. These are carried in stock and can be had in any standard length.

Ties are clips of wire which fasten two intersecting bars and save considerable time over tying with wire. There are also ties which combine the functions of ties and supports. Spacers, supports and ties are made and sold by practically all reinforcing steel dealers. The kind used is more or less a matter of personal preference as they all serve the same purpose and the cost is approximately the same.

9. Assembling.—Reinforcing steel is assembled to a more or less extent before it is placed in the forms. Spirals will be opened out and the vertical

column rods wired in position. In some cases the beam reinforcing will be wired to the stirrups and handled as one unit. The slab rods should be in bundles as the handling is much simplified and considerable time is saved in placing.

10. Placing.—As already stated, the column reinforcing will be assembled on the ground and will be a complete unit when dropped into the forms. Light columns can be placed by hand but heavy or long columns can be better handled with a gin pole. Care should be taken to avoid twisting the columns as this will throw the verticals out of line. The hand holes which have been left at the bottom of the column forms to clean out shavings and other debris can be used to swing the column cage around so that the verticals line up with the stubs projecting from the column below.

If the beams are assembled before placing, they can be handled similarly to the columns but, if not, the stirrups should, of course, be placed first in their proper position and should be tacked down to avoid displacement. Beam saddles, if used, can then be placed in the bottom of the beam boxes and the beam rods fastened in place. If two layers of beam rods are used, short pieces of 1⅛ or 1¼-in. rods will be found to make excellent separators. If, as will most probably be the case, slab rods are to cross or reach the beam, a ½ or ⅝-in. round rod run the length of the beam and wired to the tops of the stirrups will elevate them to the proper distance below the top of the slab. With the stirrups tied to the horizontal beam rods the beam is ready for concreting.

Slab rods should be wired or clipped at intersections if in a multiple-way slab, or should be fastened with spacers. The rods in one-way reinforced slabs should be held in place with spacers. Two lines of spacers will be sufficient for the span of ordinary length. Three lines in a panel will hold the four belts of a flat slab if one is placed at the intersection of the diagonal bands and the intersecting belts are wired together. No. 16 soft annealed wire is used for wiring. Where it is necessary to raise part or all of the slab rods over a beam, it is most easily done by laying and tying them in position without bending and then bending them up with a "hicky." A hicky is a piece of steel 5 or 6 ft. long with an opening in the side near the bottom which slips around the rod. By pulling back on the top of the hicky the slab rod is bent up.

After the steel is all in place it should be carefully inspected and any loose bars which might become displaced during pouring, should be wired. Careful placing is more important than is generally thought as a slab rod can very easily lose 10 per cent of its value by being ½ in. too high.

SECTION 4

FINISHING CONCRETE SURFACES AND WATERPROOFING

FINISHING CONCRETE SURFACES

By James Cowin

The treatment of a concrete surface is determined by the effect the exposed concrete is to have on the architectural appearance of the building. Generally the removal of the moulded look from concrete is the primary consideration, but where the character of the building warrants the additional expense, the con-

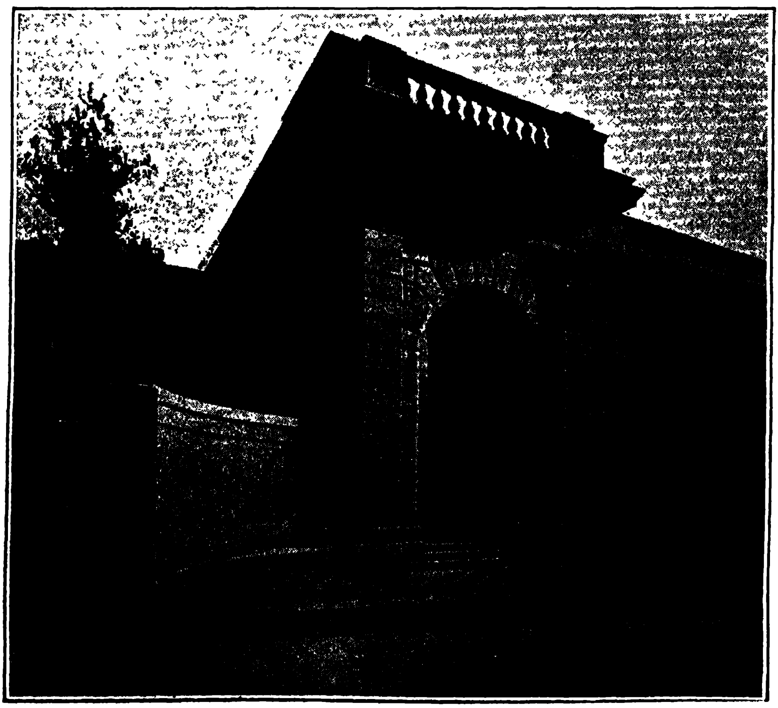


Fig. 1.—**Sixteenth Street entrance to Meridian Hill Park, Washington, D. C. Massive construction in which ornamental detail is well executed in concrete.**

crete surface may be finished so as to bring out and accentuate the texture of the concrete and the color of the aggregate.

The objection to concrete from the esthetic standpoint can be obviated by removing the outside film of cement and sand, mechanically or with acids, breaking up the surface to introduce points of light and shade, bringing out the

texture by emphasizing the aggregate and by adding color through proper selection of the aggregate, or by the introduction of coloring matter into the concrete mixture.

Regardless of whether any surface treatment is intended, special attention should be paid to the formwork on any surface which will be exposed. A smooth surface with the irregularities of the form markings reduced to a minimum by careful construction will present a neat, clean appearance which, in many cases, is quite satisfactory without further work being done upon it. The mixture should be a little wetter than normal and should be especially well puddled and spaded back from the forms to avoid air pockets. In the case of columns and walls the concrete should be poured to the center and puddled out against the forms as, otherwise, a stone or a piece of crushed rock may become lodged between the reinforcing and the forming causing a pocket which will have to be patched

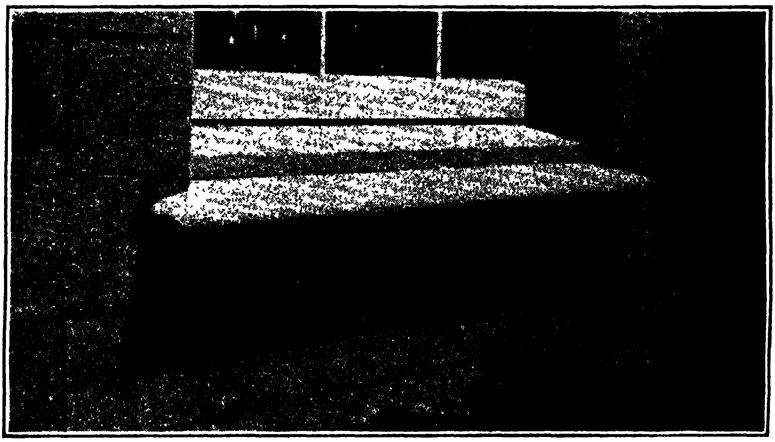


Fig. 2.—Acid etched surface.

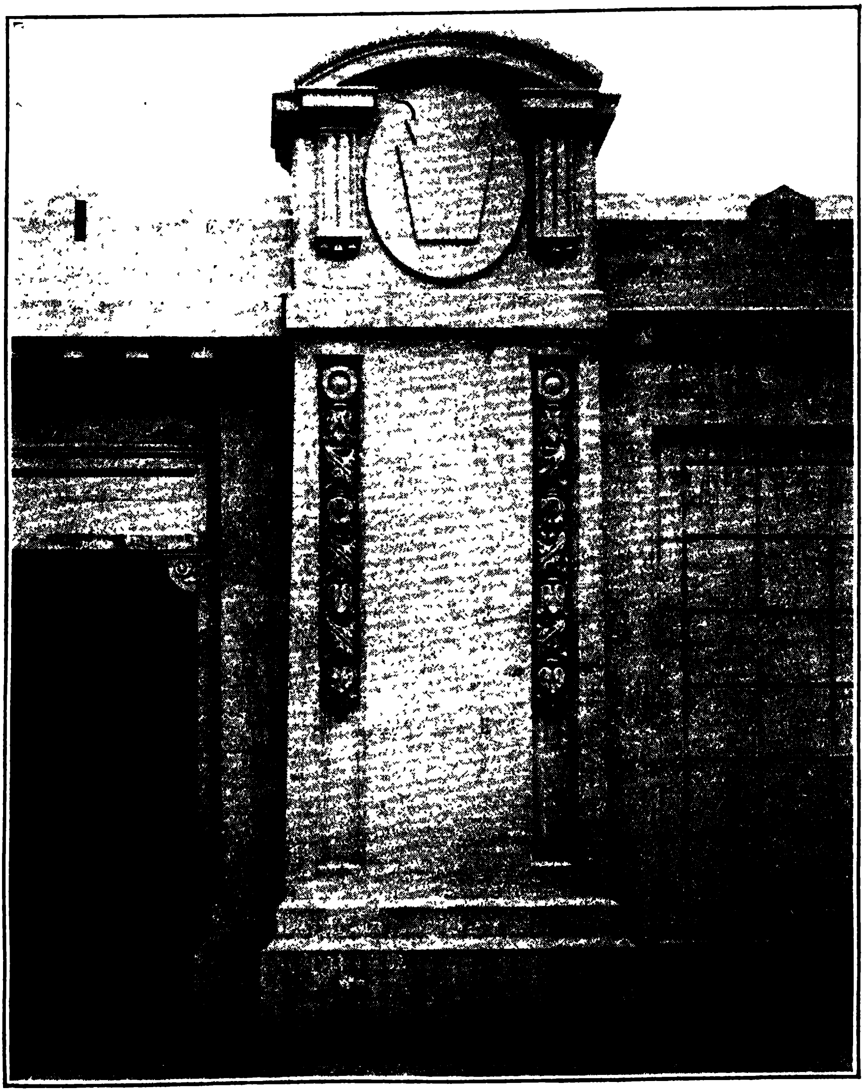
after the forms are removed. On the other hand, a sloppy mix and excessive spading should be avoided or there will be segregation and the surface concrete will be composed of fine sand and cement.

The different methods for treating the surface of monolithic concrete in order of simplicity are washing, brushing, acid treatment, rubbing, tooling, bush hammering and sand blasting.

1. Washing and Brushing.—Washing, as its name implies, simply means the removal of the outer film of cement and sand with a stream of water before the concrete has an opportunity to set. The force of the water will be determined by the amount of set which has taken place and as the success of this method presupposes the removal of the forming before the concrete has fully attained even its initial strength, it is doubtful if the effect produced is worth the danger of leaving green concrete unsupported at such an early age.

Brushing is also performed while the concrete is still green but while washing to be effective should take place in from 6 to 8 hr. after the concrete is poured, brushing should not be started until the mix is hard enough to hold the particles of aggregate firmly against the friction of the brush—otherwise, some will be

removed, resulting in a pitted surface. The time required for sufficient hardening will vary from 24 hr. in the summer to several days in cold weather and can only be determined by experimenting with the particular surface to be treated. An ordinary scrubbing brush with stiff fiber bristles will be quite satisfactory if

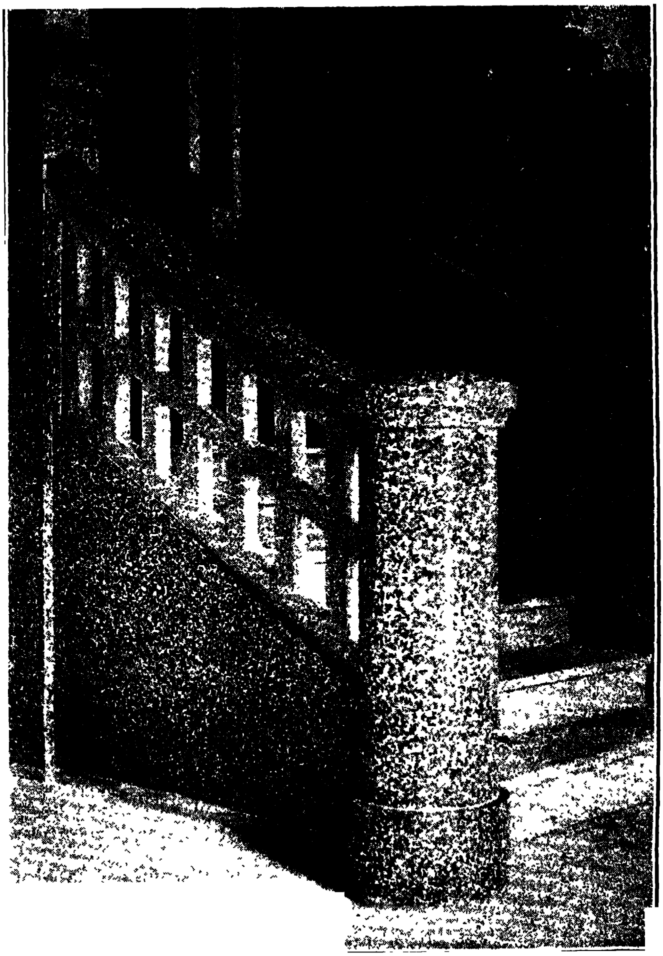


(Courtesy of the Portland Cement Association).

Fig. 3.—Column with a rubbed finish.

the concrete is still green but a wire or metal brush will be required if the concrete has been allowed to become firmly set. An even pressure with careful attention to the corners should be observed.

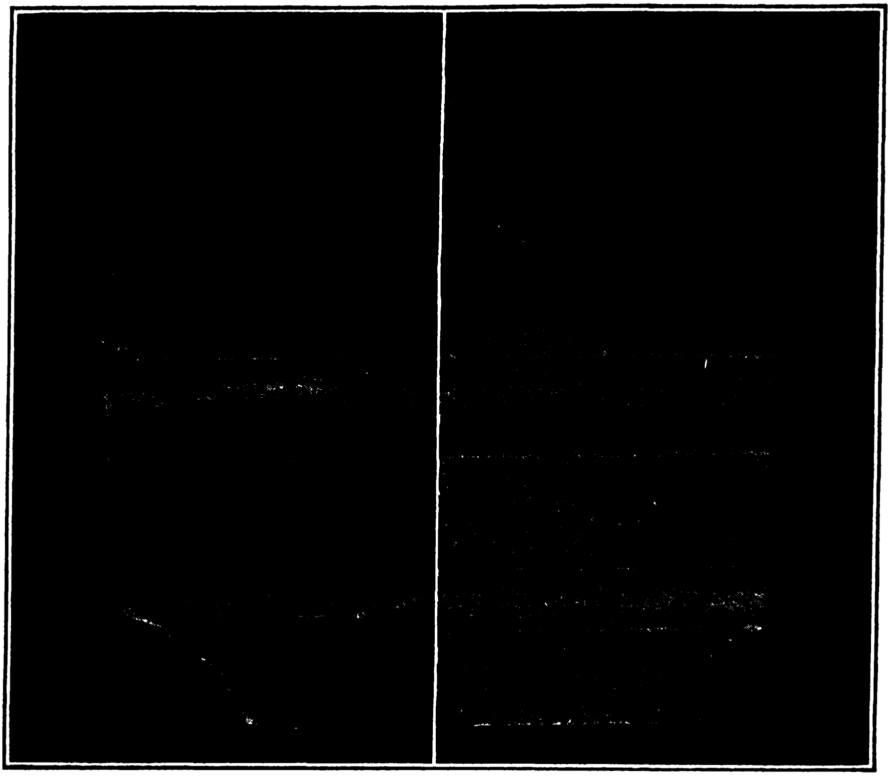
2. Acid Washes.—Acid washes are often combined with a brush treatment either following the brush treatment in order to sharpen the effect of the brushes or wetting down the surface while it is being scrubbed. Acids must be thoroughly and quickly removed with a stream of water or they will discolor the surface and leave the concrete shaly.



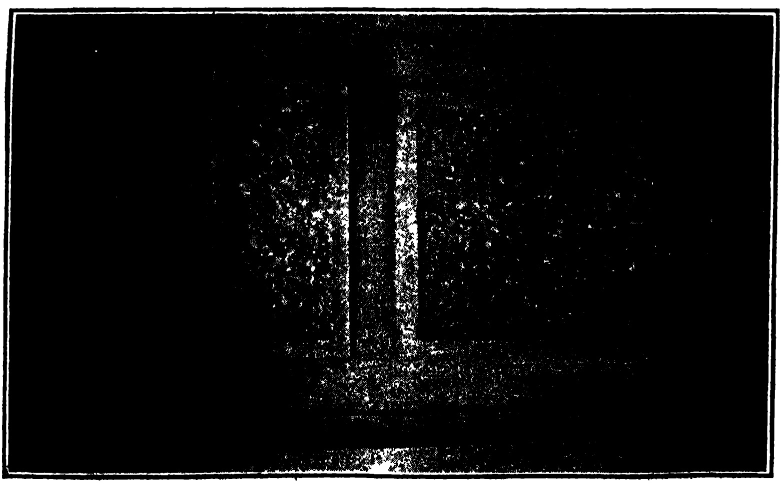
(Courtesy of the Portland Cement Association)

Fig. 4.—Special surface finish. Concrete rubbed and polished.

The action of the acid is to eat away the cement and leave the aggregate exposed, and experimenting will show the proper solution to use. One part of hydrochloric (muriatic) acid to six or eight parts of water is strong enough for very green concrete while one part in four will require the addition of vigorous scrubbing to affect concrete which is firmly set. Sulphuric acid of the same strength should be substituted for the muriatic acid if white Portland cement has been used in the mix.



(*Courtesy of the Portland Cement Association*)
FIG. 5.—Sample showing concrete before and after tooling.



(*Courtesy of the Portland Cement Association*)
FIG. 6.—Ornamental design brought out by bush hammering.

Good effects have been obtained by painting the surface with full strength acid and then thoroughly washing down. If the object is to particularly emphasize the aggregate, the surface may even be scrubbed with the wet acid provided the acid is at once entirely removed by further scrubbing with water.

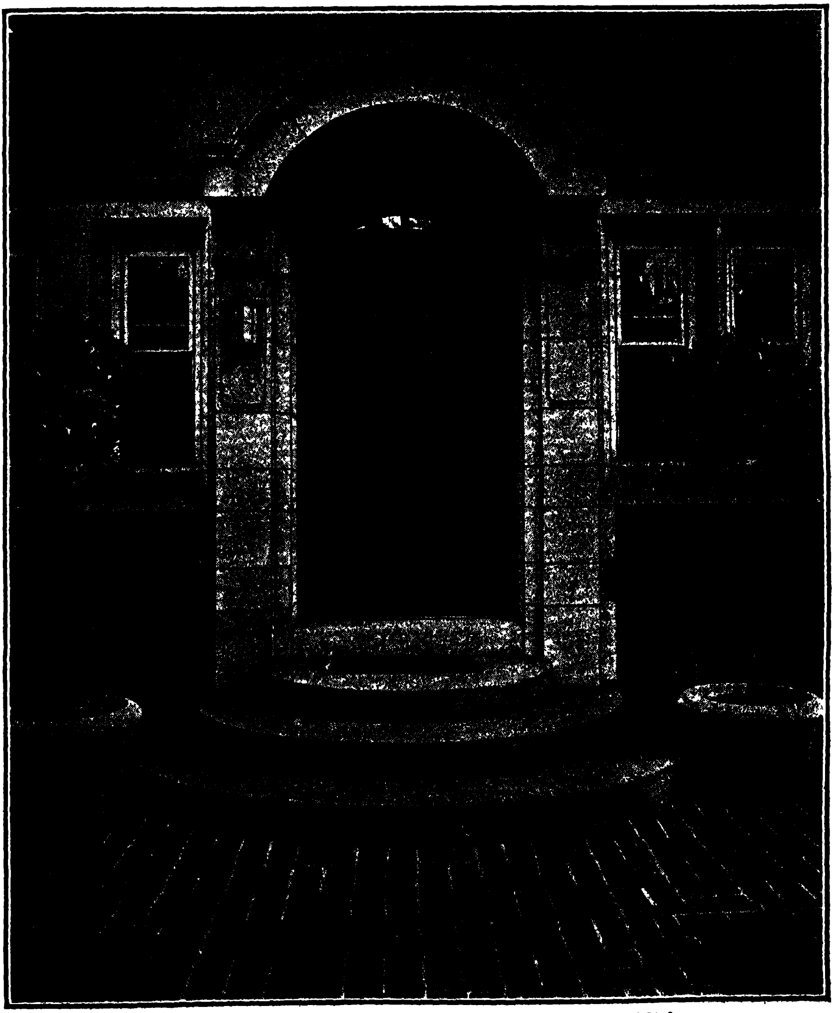
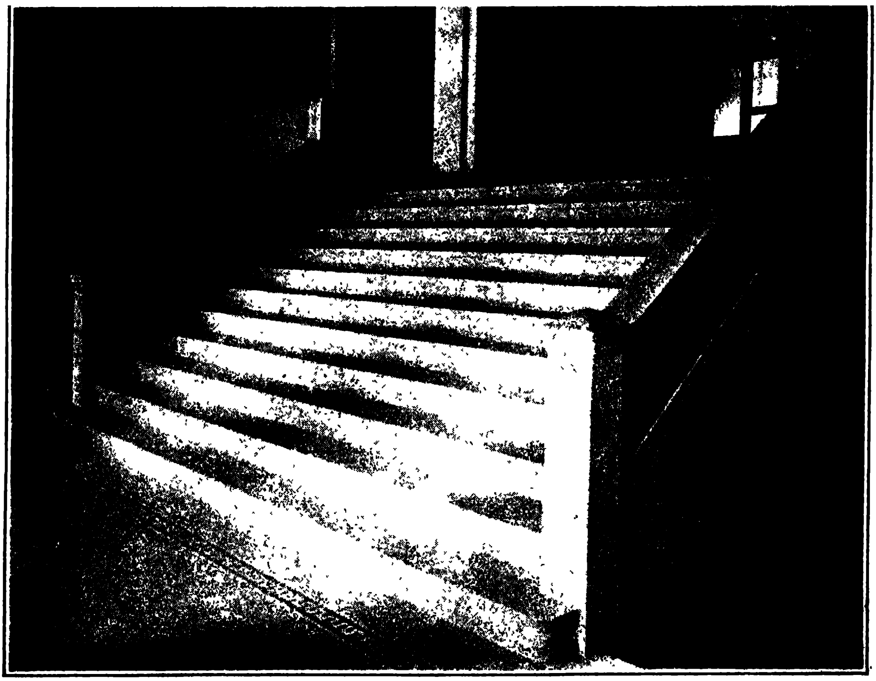


Fig. 7.—Entrance, Albert Kahn's residence, Detroit, Mich.

3. Rubbing.—Rubbing the concrete with bricks of fine material, soft stone or a commercial abrasive, such as carborundum or emery, will give a good smooth surface free from hair cracks and form marks. When a rubbed surface is contemplated, the concrete should be well spaded back from the forms in order to keep the coarse aggregate away from the surface where it will interfere with the

rubbing. The forms should be removed while the concrete is still green. This can be done in 24 hr. for compression members such as columns and walls and the sides of beams can be exposed in two days if the bottom shoring is undisturbed. In case it is impractical to remove the forming so early, the rubbing can be performed when the concrete is older, although the process is more difficult. The forming should be carefully removed to prevent spalling. In case any spalling does occur or air holes are found the following procedure should be followed: Brush off any loose particles, mix up a mortar which will correspond in composition to that of the surface to be repaired, allow to set from 2 to 5 hr. depending on weather conditions and apply after wetting the surface with water.



(Courtesy of the Portland Cement Association)

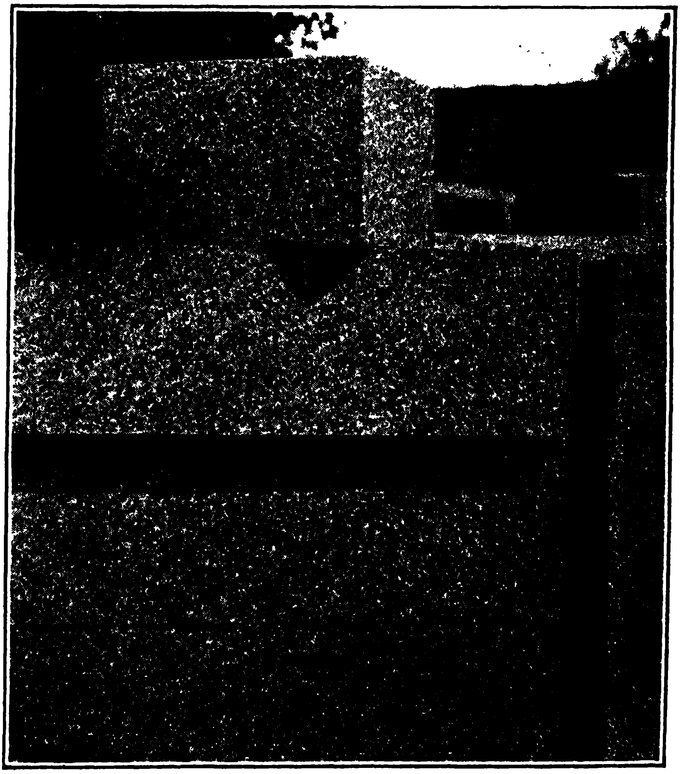
FIG. 8.—Concrete stairs, stair railing and floor finished by using colored aggregate and rubbing.

Rubbing consists in removing the outer surface of the concrete by abrasion. The amount of rubbing to be done will depend on the finish desired. Rubbing in a thin grout of cement and sand with a No. 16 carborundum stone, washing with water and finishing with a No. 30 stone will give a good surface well sealed.

4. Tooling and Bushing.—After the concrete has become thoroughly hard it can be tooled or bush hammered the same as cut stone. This should not be attempted until the concrete is at least two months old as the particles of aggregate in green concrete will be picked out leaving unsightly cavities in the surface. Bush hammering can be done by hand or by machinery and is essentially stone mason's work. The best results are obtained with 16-point hammers

although as low as 4-point or as high as 36-point may be used satisfactorily. Tooling is also done by power tooling machines which operate several carborundum wheels revolving parallel to each other and which give an embossed stripe effect which is very pleasing. Concrete which is to be tooled or bush hammered should be prepared as for rubbing, keeping the agregate well spaded back from the surface.

5. Sand Blasting.—The surfaces to which sand blasting are applicable are comparatively rare as this method is especially suitable to large unbroken areas. Furthermore, only a large amount of work will justify the expense of setting up



(Courtesy of the Portland Cement Association)
Fig. 9.—Exposed aggregate.

a sand blasting outfit. The effect, however, is very pleasing as the character of the concrete is brought out as in no other way. The concrete should be thoroughly hardened, preferably, at least three months old, and any air pockets or depressions which are to be patched should be repaired some time in advance of the operation—otherwise, the blast will eat out the green fillings before the harder surface is affected.

All ridges and pronounced form marks should first be removed by tooling as the sand blast will effect the surface uniformly and they will be apparent on the finished work. Corners and angles should be protected with boards to preserve

their sharp outlines. A quarter inch nozzle, using clean sharp silica sand which will pass a No. 8 screen, will be found the most satisfactory under ordinary conditions and a nozzle pressure of from 50 to 80 lb. will be required.

6. Colored Aggregates.—One of the main objects of all the foregoing methods of treatment is to bring out the character of the aggregate and the finished effect can be considerably heightened by careful attention to color, shape and texture in selection of the aggregate. As the local market will generally limit the choice, it will be well to make up a number of samples using different proportions of the aggregates available before making a final selection.

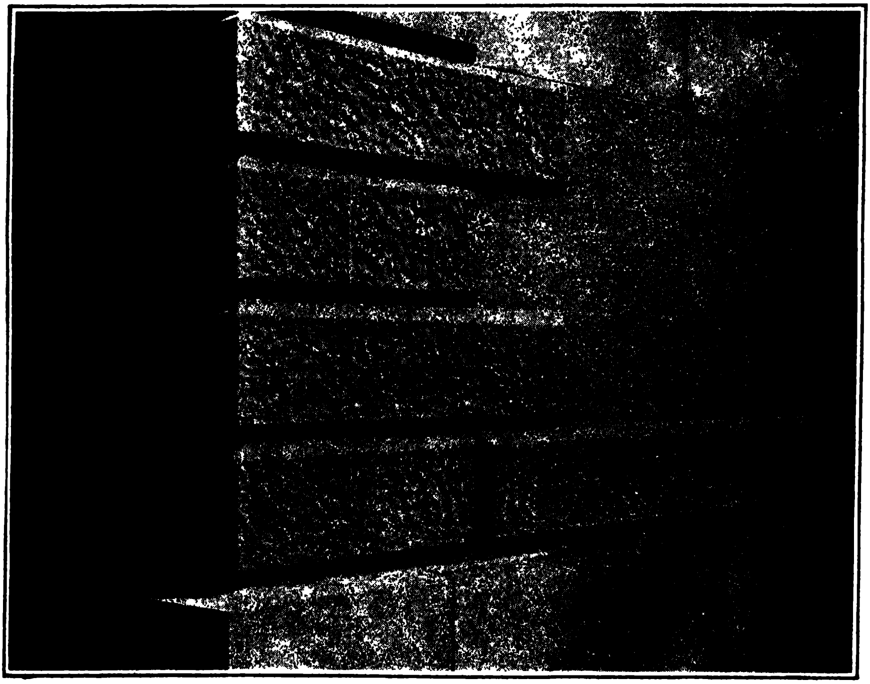


FIG. 10.—Synthetic stone as used in Catskill Aqueduct Building.

The color or combination of colors which is most pleasing to the eye when the surface treatment is completed will be the main consideration. If the desired aggregate is too expensive to use throughout the work, a veneer of at least 1 in. can be put on the exposed face. This can be done by spading the facing up against the form and forcing the backing behind, or by putting some loose boards against the face or the form which can be removed after the backing has obtained a slight set; the facing can then be poured in the resultant opening. This method is confined to very special cases and small units. The best method is to divide the form with iron plates and pour both facing and backing together, gradually raising the plates so that they interset. Aside from the wide variety of aggregates suitable for facings obtainable in every locality there are a number of companies with wide distribution specializing in special aggregates for this purpose.

7. Addition of Colors.—While the most satisfactory way to obtain color in concrete surfaces is in the selection of the aggregate, it is possible to color the matrix as well by adding suitable pigments. Any coloring material added will, at the best, be so much inert material and at the worst, will act injuriously on the cement and be itself acted on by the cement. For that reason only mineral colors should be used and then, in general, in amounts not greater than 6 per cent of the cement by weight. The yellows, browns, reds, and blacks are permanent colors, being ground hematite ores (iron oxides) but the blues and greens will have a tendency to fade. Chromium oxide is an exception to this general statement for, while too expensive for very general use, it will give a green which will not fade. The correct amount of color to be added to obtain the desired shade should be determined by experiment as the sand and gravel used will have a decided influence on the result.

Concrete surfaces can be painted with a number of reliable brands of paints which have been put on the market for that purpose. If the surface is given a priming coat with a solution of magnesium zinc fluosilicate or zinc sulphate in water or with one of the commercial floor hardeners, the free lime on the surface will be neutralized and there will be considerably less danger of the paint peeling off.

A third method of adding color to a concrete surface is by the insertion of clay tiles or mosaics. These can be glued to the form or can be inserted afterward in openings left for them by nailing blocks of wood of the requisite size and shape to the inside of the forms.

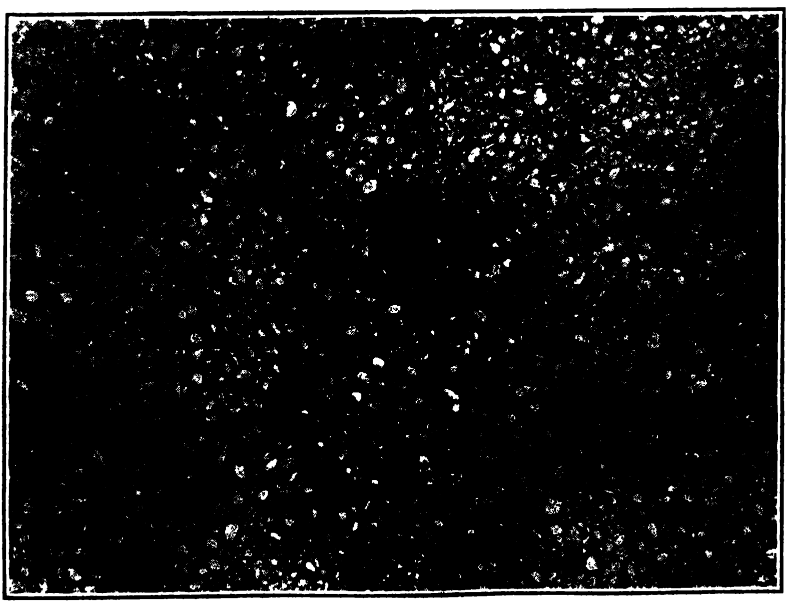
8. Plaster Finishes.—Plaster finishes are a makeshift at best and serve no purpose which cannot be better served by treating the surface of the concrete itself. If a plaster finish is to be used, the surface of the concrete should be treated the same as in bonding new and old concrete. It should be brushed clean and then thoroughly soaked with water. The facing mortar should consist of one part cement to three parts sand or screenings and applied as stiff as practicable.

9. Surfacing Concrete Floors.—Concrete floors call for a different quality in concrete from any that have previously been considered. Concrete is essentially a compression material and it is for that quality that it finds its greatest use, but a successful wearing surface must resist abrasion without disintegration and this requires that the concrete be given different treatment from that given it as a load bearing material. The value now lies in the outer skin which all treatments for bringing out the decorative side of concrete seek to remove and which in structural members is considered waste material, useful only as a fire-proofing. More dissatisfaction with reinforced concrete buildings has arisen from the inability of concrete to stand up under the wear and tear of iron truck wheels than from its failure as a structural material. As there is no penalty attached to the placing of poor concrete floors other than the annoyance and expense of patching them when they fail as against the disasters which can occur with poor design or execution in other phases of concrete, there has developed no standard universally accepted practice for laying concrete floors.

A wearing surface can be finished on the load bearing slab or a separate layer of concrete can be added to the slab for the sole purpose of taking the wear. Finishing the load bearing slab is not to be recommended as the mixture is too

(Courtesy of the Portland Cement Association)

Fig. 11.—Method of laying the finish coat.

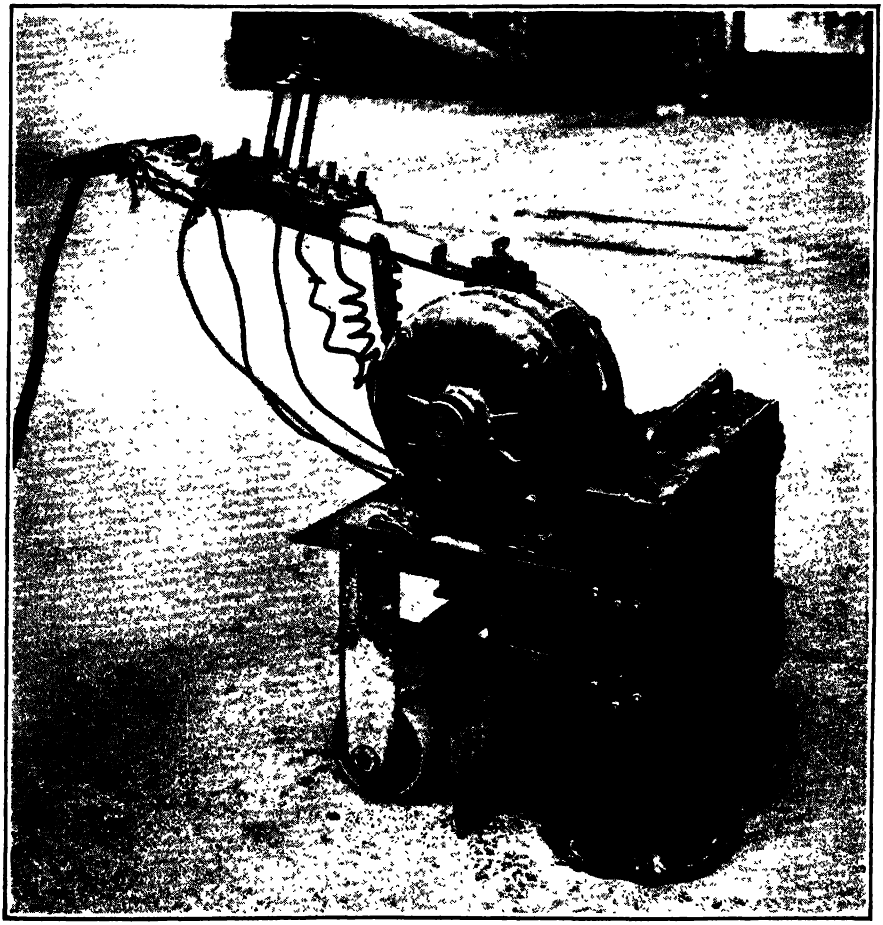


(Courtesy of the Portland Cement Association)

Fig. 12.—Close up photograph of terrazzo floor after finishing with the rotary surfacer, waxing and polishing.

lean to make a good wearing surface and, more important, a mixture which is wet enough to pour will leave laitance on the surface in setting.

A better method is to cast the wearing surface in a separate operation but before the base course has a chance to set. This should be done as soon as possible but in no case should the interval between the two operations be more



(*Courtesy of the Portland Cement Association*)

Fig. 13.—Rotary surfacing machine used in giving a concrete floor a terrazzo effect.

than 45 min. The mixture for this top coat should consist of one sack of cement to not more than two parts of clean silicous sand, or equally durable material, uniformly graded from ¼ in. down to that passing a 100-mesh screen. These should be mixed thoroughly with a minimum of water to the consistency of a stiff batter. The minimum thickness should be 1 in. although some specifications permit ¾ in. However, with the most careful attention to leveling the base course, there are necessarily elevations and depressions which bring the slab

very close to the top of the wearing surface even when 1 in. is used so that less than that amount of covering should not be considered.

The procedure is to level off the mixture for the finish coat with a straight edge run over parallel screens set to grade taking care that plenty of material is kept ahead of the straight edge to avoid pockets in the surface. After stiffening has commenced, the work should be finished with a steel trowel to a smooth surface avoiding excessive troweling which breaks up the newly formed crystals. It should be troweled again with a heavy trowel just before the mixture gets too hard to be workable and covered in 24 hr. with burlap or sawdust which should

Fig. 14.—Finished concrete floor with tile inserts.

be kept moistened for two weeks. Damp earth can also be used and one contractor obtained very good results by building up water tight side boards and keeping the floor covered with 2 in. of water.

If the finish coat is not to be added until some time after the slab is cast, the base course should be gone over with a wire brush within the first 24 hr. after it is poured, in order to roughen the surface. If the concrete has attained its final set, it should be roughened by picking or hammering. After wetting down, a thick grout should be applied and then the top coat added at once. This coat should have a minimum thickness of 1½ in. and should preferably be 2 in.

A granolithic finish can be obtained by substituting for the mixture given above 1 part cement to ¾ parts coarse sand to 1¾ parts crushed granite or trap screened through a ¾ screen and caught on ⅜₆-in. screen. After a few days the

floor can be ground down with terrazzo floor grinder. By care in selection of the aggregate as to size and color a very good effect can be obtained.

If a nailing surface is desired a mixture of 1 part cement to 1 part sand to ¾ parts sawdust has been found very satisfactory.

9*a*. **Mechanical Hardeners.**—There are a number of substances on the market to be used as the aggregate, or part of the aggregate, to increase the hardness. Their value is doubtful, unless the aggregate obtainable is extremely soft. Some of the prepared rock aggregates are valuable for their color effects.

9*b*. **Chemical Hardeners.**—The chemical hardeners are sold under a number of different trade names and are based on the reaction between magnesium or zinc fluosilicates, zinc, lead or aluminum sulphates or sodium silicate and the cement in the top surface. The free lime is neutralized and insoluble silicates, sulphates and florides are formed which increase the density and hence the hardness of the finish coat. They can be applied when the concrete is new or after it has become thoroughly hardened and set. The method of application is to flush on with pails or a sprinkling can and spread with a long handled brush for 3 min. After 24 hr. a second solution is applied of four times the strength of the first. The cost is about 1 ct. per sq. ft. and the value of the treatment seems well established.

The Bureau of Standards in a bulletin "Report of Service Tests on Concrete Floor Treatments" gives the results of experiments on seventeen floor sections treated with as many different proprietary hardeners and five sections treated with what they call home treatments, such as sodium silicate, aluminum sulphate, etc.

The conclusion arrived at, based on about two years of observation of the effect of ordinary foot traffic on the different sections was that hardeners of the magnesium fluosilicate and zinc sulphate classes gave excellent results. Of the home treatments, aluminum sulphate did not as effectively harden the surface but did keep down the dust while sodium silicate was very highly recommended as a hardener. As the sodium silicate treatment is simple and economical, the operation is here given in detail:

Take 1 gal. of commercial sodium silicate, a 30 to 40 per cent solution, and dilute with 4 gal. of water. This will cover about 1,000 sq. ft. with one coat. The floor surface to be treated should be cleaned free from grease, dirt, etc., and then scrubbed with clear water. After it has become thoroughly dry the solution can be applied. This should be brushed over the surface for several minutes with a mop or broom to secure even penetration. After an interval of 24 hr. the surface should again be scrubbed with clear water. After it has dried it is ready for a second application. Three or even four applications may be necessary for thorough saturation of the floor surface but the result, when properly applied, is a very hard wearing surface, bright and uniform in appearance, somewhat lighter in color than before being treated.

9*c*. **Accelerators.**—The Bureau of Standards has also conducted a series of experiments to find out the effect of calcium chloride when added to a concrete mix and have conclusively proved that the strength of the concrete is about doubled in its early stages by the addition of 5 per cent calcium chloride or commercial preparations having it as their base. This shortens the time necessary to wait before the finish can take its final troweling and in that way

is oftentimes an economy. It has no effect on the final strength of the concrete according to specimens which have been tested at the end of 3 yr.

9*d*. Linseed Oil and Soap.—Linseed oil with or without the addition of coloring is successfully used to keep down dust where the wear is light, but has little value under heavy traffic. The effect is very much the same as when a wooden floor is painted. Until the paint is worn off, the base beneath is protected.

China-wood oil is the basis of a number of concrete paints and while more effective than linseed oil, is subject to the limitations of any coating. After the paint is worn off the dusting is resumed as concrete will not absorb oils to any great extent.

An oiled surface should be re-treated every year, in which case, if the traffic is light, the floor will be quite satisfactory.

Soap treatments are without very much value and the scrubbing with soap and water which a floor gets in the ordinary course of events has about the same effect as an extended series of soap treatments.

9*e*. Colored Floors.—Making a colored floor surface is preferably accomplished by using a colored aggregate, and fine effects can be obtained by using red granite chips, feldspars, or even colored marbles, although it is better to use only the harder igneous rocks on account of their wearing qualities.

Mineral colors can be added to the mix, using in general about 6 lb. of pigment to the bag of cement, or the surface can be painted with one of the commercial floor paints manufactured for that purpose. As stated in the discussion of the benefits of painting with oils to prevent dusting, a floor paint will be quite satisfactory if the wear to which the surface is subjected is not too heavy. It is stated that using a priming coat of zinc sulphate solution (8 lb. of zinc sulphate to the gallon of water) will prevent the paint coat from peeling or scaling.

WATERPROOFING

By Philip George Lang, Jr.

10. General Considerations.—In numerous instances, concrete structures owe the early impairment of their utility, and occasionally their complete and untimely destruction, to the action of water. With the broadening use of concrete as a material of construction, the subject of waterproofing becomes one of increasing importance.

While it cannot be stated with absolute assurance that the early deterioration, due to the action of water, of every concrete structure on which waterproofing protection has been omitted will invariably occur, neglect of such precaution greatly increases the hazard to the investment represented, and it can, with reasonable certainty, be presumed that the prospective duration of the life and utility of such a structure is materially reduced.

The introduction of water within the body of concrete composing a structure is attributable to a variety of causes. A brief consideration of the origin and character of these agencies may well be considered worthy of notice as a prelude to a discussion of those means which contemporary practice indicates as most effective in the prevention of such conditions.

Prominent among these is capillary attraction, which, due to the porous nature of the material under consideration, acts in a marked degree to facilitate the absorption of water which may be brought into contact with the surface, and to accumulate within the concrete mass the agency through which its destruction is frequently wrought.

Gravity also plays an important role, and this phase of the question is especially prominent in the case of concrete bridge floors. It is with peculiar reference to this element that concrete protection should be designed. It is probable

Fig. 15.—Effect of lack of waterproofing on intrados of masonry arch.

that, in the larger portion of failures due to water penetration, the ingress of the destructive agency was occasioned by the operation of this natural law.

The destructive action of water, after its introduction within the mass of concrete, is accomplished, in most cases, by physical action along the lines peculiar to its familiar characteristics. In some instances, chemical action may also occur, but this appears improbable unless the water contains some foreign or unusual ingredient.

The passage of water through concrete may be attended by some degree of solvent or erosive action, which has a tendency to weaken the concrete, and, to a varying extent, to remove some portion of its essential part. This fact has, in numerous instances, been proved by examination of water which had, for experimental purposes, been caused to pass through concrete, and which, after its passage, displays, in every case, a pronounced alkaline reaction.

The absorption of water into concrete, especially when this occurs through chance or accident, is by no means uniform. The consequent uneven permeation of the mass causes disproportionate expansion, producing stresses within the body of the concrete, which sometimes prove the means of internal fracture and consequent deterioration.

The freezing of water with which concrete has become impregnated is a very fruitful source of deterioration, as the solidification and consequent increase in volume of the contained water produces fractures within the body of the concrete.

Fig. 16.—Effect of lack of waterproofing on intrados of masonry arch.

The vicinity of a railroad, and the consequent presence in the atmosphere of locomotive fumes and cinders frequently causes the adjacent water to acquire a perceptible sulphuric acid content, and this characteristic is, in a milder degree, frequently imparted to rain in its descent through such gas-laden air. When water of this character is absorbed into concrete, its acid content is a source of slow but sure destructive chemical action.

The destructive effects of water are well exemplified in the case of the masonry arch viaduct, views of which appear as Figs. 15 and 16. The bridge in question is situated near Columbus, Ohio, a locality which reflects the general climatic environment of the Middle West, and may be considered typical of American conditions.

This arch was constructed in 1905, but no waterproofing protection was applied. As the application of waterproofing protection to the extrados of

these arches subsequent to the discovery of the conditions shown would have occasioned very heavy expense, and possible interference with traffic, remedial measures were sought by the boring of additional weep holes. This alleviated, but failed to entirely relieve the conditions.

It can with reasonable certainty be stated that, unless proper waterproofing is applied, the destructive effect of water will eventually, in some degree, become apparent in any concrete or masonry structure. The application of such waterproofing increases the structure's life, and consequently serves as a comparatively inexpensive and effective means of insuring the safety of the investment represented. Waterproofing should, therefore, especially in view of its negligible expense compared with the total cost of the structure, receive due consideration in the estimate and design.

11. Methods of Waterproofing.

11*a*. Integral Method.—The *integral* method consists of the inclusion among the components of the concrete mixture of a compound which, by the operation of its inherent chemical properties or physical characteristics, has a tendency to render the mass more or less impervious to water. The efficacy of the integral waterproofing is dependent upon the preservation of the absolute solidity of the mass which it is designed to protect, and it becomes largely useless upon the development of fissures or cracks, by means of which the ingress of water is effected. Integral waterproofing is, for this reason, not adaptable to use in connection with bridge floors or other forms of construction subject to the passage of heavy rolling loads, and since, as previously stated, it depends not only upon the original density of the mass, but also upon freedom from rupture due to any cause whatsoever, integral waterproofing will not be further considered.

11*b*. Surface Method.—The *surface* method of waterproofing seeks the exclusion of water from a concrete mass by means of a protective coating applied to the surface. Waterproofing is effected by means of chemical or mechanical action, or a combination of the two. In general, the chemical action is such that the material applied, or this material in conjunction with the concrete, will form a surface which, in itself, becomes, in a sense, water-repellant. The mechanical action is such that the material tends to fill the surface pores, and, in some cases, after application, the particles expand, sealing the pores, and forming a protective agent which is waterproof in the sense that water cannot enter, due to the absence of pores and surface cracks.

Satisfactory results may be obtained from such methods, so long as the waterproof surface remains unbroken, but it cannot be satisfactory if cracks or other defects develop. Surface waterproofing is, for this reason, not recommended for use on structures subject to vibration or settlement, which may cause the waterproof surface to crack.

11*c*. Membrane Method.—The *membrane* method consists in applying to the surface of the structure, or material to be protected, a waterproofing membrane or coating which will adhere to that surface and prevent the penetration of water. A membrane or coating of this nature can be so constructed as to retain its adhesion to the structure under any ordinary conditions of distortion due to vibration, traffic or other causes, and its utility is not impaired by the development of cracks or the necessity of providing for construction or

expansion joints. One of the most severe conditions under which waterproofing is used is that of protection on railroad bridge floors.

The membrane method, when suitable waterproofing materials are applied to a well-designed structure by efficient workmen and under proper conditions, has been found to render satisfactory protection under practically all conditions of vibration, and its use is recommended on all such structures to be waterproofed. It is desired to again emphasize the fact that the cost of waterproofing protection is relatively small.

12. Essentials of Successful Waterproofing.— Successful waterproofing is dependent upon three essential factors, considered in the order set forth below:

(*a*) Design of the structure to be waterproofed.

(*b*) Workmanship in the application of waterproofing.

(*c*) Waterproofing materials.

12*a*. Design of the Structure to Be Waterproofed.—Every structure, whether the ultimate application of waterproofing is intended or not, should be so designed as to provide all possible facilities for the shedding of water which may be deposited. In designing structures to be waterproofed the general fact must constantly be borne in mind that, under no conditions, must any facility be afforded for the permanent accumulation of water on the surface to be waterproofed.

Care must be exercised to avoid the formation of any depressions or pockets in which water can collect, and, where the requirements of construction demand the placing of an indentation of any nature, adequate means for its drainage must be provided. Where any element of doubt exists as to the proper gravitational discharge of water from a section of the surface to be waterproofed, drainage channels, pipes or downspouts should always be placed.

Unless the general design is such that the unwaterproofed structure constitutes a natural watershed, it is doubtful whether special means subsequently adopted for waterproofing can be made absolutely effective, and certainly difficulty in waterproofing will be encountered. The more or less permanent accumulation of water, even though it be slight, upon the surface to be waterproofed, notwithstanding the fact that this surface is thoroughly protected by membrane waterproofing, may, either by solvent action, decay or chemical activity, destroy the utility of the protecting membrane.

The efficacy of membrane waterproofing applied to any structure is dependent upon the preservation of an unbroken surface. If this waterproofing surface is fractured, even in a slight degree, the utility of the membrane protection over a considerable adjacent area is impaired. Surface water enters and gradually diffuses itself over the concrete surface in a widening circle, throughout which area it will have a tendency to loosen the bond between the concrete surface and its protective coating. Deprived of contact with the air, such water can only remain as a saturant or be absorbed into the material which it was intended to waterproof.

The possibility of such a condition should be recognized, and can easily be removed by due cognizance on the part of the designer of the special needs created by the placing of waterproofing membrane.

Any expansion joints introduced into the structure should be such as to provide ample space for ordinary and even exceptional changes due to variations in

temperature. Such joints should be held to the absolute minimum, and proper steps taken to see that the waterproofing membrane will not subsequently be broken at these joints.

The structure should be designed so as to avoid excessive deflection under load, in order that the protective membrane may not be broken. The question of flashing—that is, sealing the ends of the waterproofing membrane to the structure to be waterproofed—must be given careful consideration, and proper means for preventing the entrance of water at such points provided. Wherever possible, satisfactory metallic flashing should be used, and, in addition, where required, conduits and drains should be installed to carry away excess water. Great care should be exercised to prevent the ingress of water where flashing is used and where the waterproofing membrane is broken to provide for drains.

12b. Workmanship in the Application of Waterproofing.—In providing effective means for the exclusion of water from a concrete structure, the character of workmanship and conditions under which the work is prosecuted are matters whose importance is secondary only to the soundness of principle embodied in the general design. Faultless design combined with the highest quality of waterproofing materials will fail to produce the desired results unless the actual application is performed in a proper manner by skilled and experienced workmen, under intelligent supervision, suitable weather conditions, and without undue interruption in the progress of the work, occasioned by operating necessity or other causes.

The application of waterproofing materials to any structure should be the subject of an especial and carefully written specification, and such work should, during its progress, at all times be under the constant and rigid scrutiny of an inspector, whose duty it should be to thoroughly familiarize himself with the methods of procedure adopted for this particular branch of the work under his jurisdiction.

The forces employed in this application should, in part, at least, consist of men acquainted by previous experience with work of this character, under the charge of a competent and responsible foreman. The waterproofing materials should be applied, preferably only on clear days, and under no conditions should this work be attempted during falling weather of any nature, or at a time when the general temperature is below the freezing point.

Under frost conditions, especially in a moist atmosphere, the surface to be waterproofed is frequently covered with minute particles of water, and, in some cases, with a thin film of ice. The attempt to apply waterproofing in such cases will at least prevent the securing of proper seal between the waterproofing materials and the structure to be waterproofed.

Any surface to which waterproofing is to be applied should be scrupulously cleaned, and all dust, dirt, and other foreign or loose material of any nature whatever removed. Every possible precaution should be taken to insure for this purpose a clean, dry surface, and avoid the presence of any substance which might impair the continuity of the seal, and consequently become the means of forming a leak in the waterproofing protection.

The importance of uninterrupted prosecution of the work as a factor in successful waterproofing renders this feature a matter worthy of special consideration in devising a field schedule. This is particularly true in view of the

fact that it is essential to hold the number of construction joints in the waterproofing membrane to a minimum, since such joints are a potential source of weakness in the membrane. The foregoing consideration should be particularly borne in mind in connection with the waterproofing of a bridge structure carrying railroad traffic, and, in general, it may be stated that, under no conditions except those which are absolutely unavoidable, should a railroad bridge structure be waterproofed under traffic. In such cases, experience has shown that it is well nigh impossible to secure a watertight protection..

The permanent utility of waterproofing is dependent upon the preservation of an unbroken membrane, impervious to water, containing no holes, fractures or seams by means of which the penetration of water can be effected. In addition to the possible causes of such orifices already enumerated, an element of danger lies in the pressure of any superimposed material—in the case of railroad bridges, the ballast; in the case of arch viaducts, the filling material. The operations of workmen engaged in tamping or the performance of other maintenance tasks on railroad structures imposes a hazard which should be guarded against.

In the case of arch viaducts or other structures which are waterproofed, and upon the waterproofed surface of which fill is deposited, proper steps must be taken to prevent the injury of the waterproofing membrane during the depositing of fill. It is consequently essential that, after a waterproofing membrane is applied, some form of protection be used with a view to eliminating, so far as possible, the liability of contact with any destructive agency. This can be accomplished in several manners.

A layer of concrete, preferably not less than 2 in. nor more than 3 in. in thickness may be deposited over the membrane. This concrete may be reinforced with suitable metallic mesh, in order that it may be properly held together. The waterproofed surface may, if preferred, be protected with brick, properly placed. In any event, the protection used for the waterproofing membrane should be carefully moulded to the contour of the structure, and its surfaces so constructed that they will, in themselves, provide suitable drainage.

In the case of highway bridges on which paving is used, it would appear that the use of a protection course of some sort for the waterproofing membrane should, at least, be considered. The necessity for the occasional renewal of the pavement, with attendant possibility of fracture of the waterproofing membrane, should be borne in mind. With the waterproofing membrane properly protected, a definite and recognized plane of demarcation between the waterproofing and the paving is established.

Single or multiple arch bridges, whether of concrete or masonry, on which the application of membrane waterproofing to the extrados prior to filling is contemplated, constitute a feature of construction work which merits special consideration, so far as the waterproofing detail is concerned. It can, with reasonable certainty, be assumed that the filling material will contain stones or other bulky objects which, during the filling, unless the work is handled with unusual care, become potential agencies of destruction, causing punctures to the waterproof coating. The use of suitable protection for the waterproofing on structures of this class should, at least, be carefully considered.

12c. Waterproofing Materials.—The waterproofing materials in common use are the output of several firms by whom the American market

is supplied, and all of which products can, with proper application, be relied upon to give efficient protection. Such protection consists, in general, of two essential parts, namely the *mastic* or *binder* and the *fabric*, to which, in some cases is added a third component, the *primer*.

Mastics or binders are, in general, derived from two sources, namely coal tar pitch—that is, the inert residuum of coal tar from which the volatile components have been expelled—and asphalt, whose adaption to the needs of this service is the object of several proprietary treatments. The mastic is the agency by means of which the waterproofing is actually accomplished. Experience indicates that its adaptability to this purpose is vastly increased by the use of layers of fabric, from the introduction of which elasticity and strength are imparted to the membrane.

The primer, when used, is a binding agency, somewhat more fluid in its consistency than the mastic, and is applied to the bare surface to be waterproofed for the purpose of forming an anchorage for the subsequent waterproofing coat.

The art of waterproofing is yet in a formative stage, and the determination of the materials to be used on any specific structure should be regulated by the judgment of the engineer, supplemented by such data as he can obtain concerning the utility of the materials under consideration.

The mastic or binder used must be elastic within the ordinary range of temperatures. It must adhere to the surface to be waterproofed, and to the fabric used. It must be of such composition that it will not be soluble. At atmospheric low temperatures it must not become brittle, and at atmospheric high temperatures it must not flow. It must further be of such nature that it can be readily applied without heating to an excessive degree in order to facilitate its proper application.

The fabric used in the waterproofing membrane must be of material which will not deteriorate, rot out nor enter into chemical action with the waterproofing mastic or binder used. It should consist of properly woven material, especially adapted to the needs in question. Its quality and durability are essential elements in determining the success and life of the waterproofing. The fabric should have a tearing resistance of from 40 to 50 lb. per in. width for both warp and woof, and a stretch in either direction, without fracture of from 10 to 12½ per cent.

In the selection of fabric, the primary object of its use should be borne in mind—namely, that of holding the waterproofing material together, giving it elasticity, preventing it from being cracked due to deflection, expansion and contraction, and of giving it body, so that the proper application will permit the use of methods which will provide for some give and take in the waterproofing membrane. It is essential to this end that the fabric permanently retain its distinct and separate identity. Its homogenization with the mastic or binder will ultimately impair the quality of the waterproofing.

13. Specifications.—Specifications for guidance in the application of waterproofing may be varied to suit particular brands of material or special working conditions. It is believed that the form given below is readily adaptable to any special or local need, and can, with minor changes, be used as a rule of procedure on any ordinary form of construction work:

Inspection and Tests

All materials used in the work shall be inspected and tested in accordance with the latest methods recommended by the American Society for Testing Materials. Materials which are condemned shall be promptly removed from the work.

No work shall be started until all materials have been tested and found to meet the specification requirements. If there is reason to believe that the materials used are not in accordance with the materials tested, additional tests should be arranged for.

The work shall be performed under the supervision of an inspector.

Number of Tests

At least two 1-lb. samples of waterproofing asphalt or plying cement, taken from separate drums, shall be tested on all work requiring 2,000 lb. of material or less. For work requiring more than 2,000 lb. of asphalt or plying cement, the number of tests shall be increased by 1 for each 2,000 lb. of material or fraction thereof.

The number of rolls of fabric tested shall equal the cube root of the number of rolls to be used—for example, if 8 to 27 rolls are to be used, 3 rolls shall be tested; if 28 to 64 rolls, 4 rolls shall be tested. Samples for test purposes shall be cut from the ends of the rolls. They shall be 12 in. wide, full width.

Tests of elastic cement shall be made as required.

Delivery and Storage of Materials

All materials shall be delivered in their original packages, clearly marked with the name of the manufacturer and brand, at least three weeks prior to use. Unless so marked materials will not be accepted.

All material shall be promptly unloaded upon arrival and protected from injury. Fabric rolls shall be placed in suitable weatherproof buildings, and placed on their sides.

Primer

For priming surfaces to be waterproofed, there shall be used a mixture of the same asphalt used for the asphalt or plying cement and gasoline. The first-coat primer shall contain sufficient asphalt to give a distinct brown tint. The second-coat primer shall contain double the quantity of asphalt used in the first coat.

Asphalt or Plying Cement

Asphalt or plying cement shall be prepared by careful distillation and refining of heavy asphaltic petroleum. It shall be free from coal tar pitch or any of its products. It shall have the following properties resulting from the process of refining, without the aid or addition of any fluxing material:

Melting Point—150 to 170° F.—Ball and ring method.
Penetration at 32° F.—at least 10.
77° F.—25 to 35.
115° F.—75 to 100.
Ductility at 40° F.—at least 3.
77° F.—at least 22.
Specific Gravity at 77° F.—Greater than 1.
Solubility in Cold Carbon Disulphide—99.5 per cent minimum.
Ash—0.4 per cent maximum.
Flash Point—450° F.

Loss on heating 50 grams of material 5 hours at 325° F. shall not be greater than 0.5 per cent by weight.

Penetration at 77° F. after such heating shall be at least 90 per cent of the penetration at 77° F. before heating.

Ductility at 77° F. after such heating shall be at least 70 per cent of the ductility at 77° F. before heating.

Saturated Fabric

The fabric for reinforcing the membrane shall be a woven open mesh cotton fabric, containing before treatment no oils of any kind, and weighing not less than 5 ounces per sq. yd. The thread count of the fabric shall be not less than 18 and not more than 32, both with the warp and the filling. After treatment, the saturated fabric shall weigh not less than 14 ounces per sq. yd.

The fabric shall be treated by thorough saturation with the same asphalt as used for the asphalt or plying cement. The treatment shall be performed at a temperature not exceeding 275° F., and shall be accomplished by immersing the untreated fabric in the asphalt saturant solely, followed by pressure and heat. No oils, petroleum residues, or other bitumen solvents shall be used in the process. The meshes of the fabric shall not be closed or sealed by process of saturation.

Tensile strength tests of the treated fabric shall show the following tensile strengths per inch of width, both with the warp and the filling:

When tested by the "Strip" method.............................. 40 lb.
When tested by the "Grab" method.............................. 50 lb.

and the stretch of the fabric shall be as follows:

When tested by the "Strip" method............ Not less than 10 per cent
When tested by the "Grab" method............ Not less than 10 per cent

The treated fabric shall be flexible at temperatures between 0 and 250° F., and shall not flake nor crack when bent back on itself. It shall readily conform to uneveness in the surface to which it is to be applied, leaving no pockets, bridges nor air holes. It shall be easily bent into and over corners without injury. Covering of the fabric with talc, wood pulp or other substances which will tend to prevent a close adhesion between the plies or between the fabric and the asphalt or plying cement will not be permitted.

Elastic Cement

The elastic cement used in horizontal joints for sealing the waterproofing membrane to adjoining surfaces of concrete, steel, etc. shall be a homogeneous mixture of asphalt and asbestos. It shall be elastic at 0° F. and at 120° F. Its volume shall not be reduced by exposure to the air, but it shall be physically and chemically stable, unaffected by water, acids or alkalies in the solids it cements. No fluxing material shall be used in its preparation.

It shall have the following properties:

Melting Point--105 to 115° F.—Ball and ring method.

Penetration at 77° F.—at least 100.

Solubility in Cold Carbon Disulphide—99 per cent.

Ductility at 77° F.—at least 50.

The elastic cement shall be pliable, elastic and ductile between 20 and 120° F. It shall form a complete and permanent bond with the adjacent material, and form a joint which is impervious to water, yet will yield without breaking or cracking.

Workmen

The foreman or other workmen applying, heating or otherwise connected with waterproofing shall be thoroughly skilled in the work which they are to do.

Wet or Freezing Weather

Waterproofing shall not be done in wet weather nor at a temperature below 40° F.

Cleaning

Concrete surfaces to which waterproofing is to be applied shall be trowelled to a smooth finish, and all surfaces, whether steel or concrete, shall be swept broom clean.

Surfaces of concrete or steel coming in contact with the waterproofing shall be thoroughly cleaned of dirt, rust, loose particles, paint and grease. Gasoline shall be employed for removing paint and grease from the steel, freshening the surfaces of asphalt or plying cement where a junction of old and new is to be made or where a pocket of elastic cement is used between the steel and the fabric. Such cleaning shall be done in advance of the priming.

Surfaces of concrete and steel to be waterproofed shall be thoroughly dry, to prevent the formation of steam when the waterproofing materials are applied. Damp surfaces shall be covered with a 2-in. layer of hot sand, which shall be allowed to stand 1 to 2 hours, after which the sand shall be completely removed, uncovering the surfaces gradually as the waterproofing proceeds.

Non-injury of Waterproofing

During the progress of the work, care shall be taken to prevent injury to the waterproofing membrane by the passage of men, wheelbarrows or equipment.

Priming

Surfaces of concrete or steel to which waterproofing is to be applied, after being thoroughly dried, shall be given 2 coats of the primer previously described. These priming coats shall be thoroughly applied, and worked into all surfaces, to give a uniform coating. The first priming coat shall be thoroughly dry before the second priming coat is applied. The second priming coat shall be thoroughly dry before the waterproofing is applied. Priming shall be done immediately in advance of the application of the waterproofing

Membrane

The structure as a whole shall be waterproofed by the membrane system; this membrane is to consist of 4 layers of asphalt or plying cement and 3 layers of saturated fabric.

Application of Membrane

Application of the membrane shall be started at a low point and worked toward the crown. The fabric shall be laid so that water will flow over and not against the laps. The general arrangement of the waterproofing plies shall be as shown in Fig. 17.

On this clean, dry surface, which has been previously primed, there shall be applied hot, at a temperature not exceeding 350° F., with mops, a heavy mopping of asphalt or plying cement, using at least ⅓ gal. per sq. yd. Into this hot asphalt there shall be introduced a 14-in. width of fabric. The upper surface of this 14-in. width and adjacent concrete shall then be coated with asphalt and a second strip 25-in in width, completely lapping the first strip shall be placed. A third strip a full width of 36 in. shall be placed in a similar manner, completely lapping the first and second strips. The fourth and each successive strip shall be a full width of 36 in., and shall lap the preceding strip 25 in. leaving a section 11 in. wide for the full length of the strip in immediate contact with the asphalt or plying cement mopping applied to the concrete floor. The entire upper surface of the first and second strips and that portion of the upper surface of each succeeding strip which is to be lapped shall receive a thorough mopping of asphalt or plying cement as the membrane is placed. Each strip shall be laid smooth, without pockets, bridges or folds, and shall be thoroughly pressed into the underlying asphalt. This process shall be continued until the membrane consists of 3 layers of fabric with intervening moppings of asphalt or plying cement.

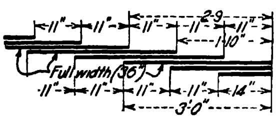


FIG. 17.—Diagram showing so-called "shingling" method of applying fabric in membrane waterproofing.

A final mopping of asphalt or plying cement shall then be given the entire upper surface of the membrane so placed, working the mops lengthwise of the strips. The completion of this mopping will make the membrane consist of three plies of fabric and four layers or moppings of asphalt or plying cement. Each mopping shall be complete, thorough, uniformly spread, and shall entirely conceal the texture of the surface and the weave of the waterproofing fabric. Under no circumstances shall one ply of fabric touch another or the concrete at any point, as there must be a mopping of asphalt or plying cement between the concrete and the fabric and between each two plies of fabric—that is, each ply of fabric must be completely encased in asphalt or plying cement.

At least ⅓ gal. per sq. yd. of asphalt or plying cement shall be used for each mopping, making at least 1⅓ gal. per sq. yd. of asphalt or plying cement for the finished membrane.

End laps of fabric shall be at least 9 in. and shall break joints.

Protection of Waterproofing Membrane

The waterproofing membrane shall be protected by a reinforced concrete protection. All concrete shall be composed of one part Portland cement, two parts sand and four parts broken stone or gravel. The broken stone or gravel used shall pass a ¾-in. screen and shall be held on a ½-in. screen. The upper surface of the concrete protection shall be trowelled to a smooth finish, sloped and formed.

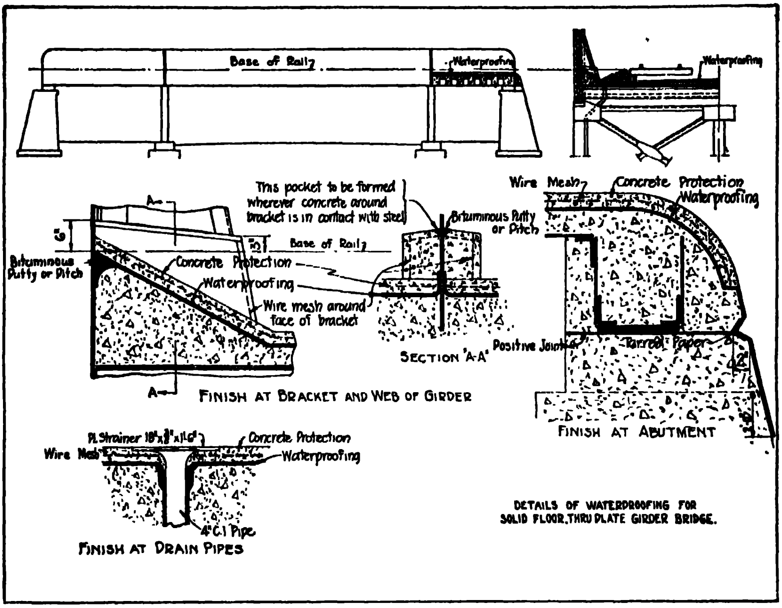

Base of Rail
Waterproofing
This pocket to be formed wherever concrete around bracket is in contact with steel
Bituminous Putty or Pitch
Base of Rail
Concrete Protection
Waterproofing
Wire mesh around face of bracket
Section 'A-A'
Finish at Bracket and Web of Girder
Wire Mesh
Concrete Protection
Waterproofing
Positive Joint
Tarred Paper
Finish at Abutment
Concrete Protection
Waterproofing
4" C.I. Pipe
Finish at Drain Pipes
Details of Waterproofing for Solid Floor, Thru Plate Girder Bridge.

Fig. 18.

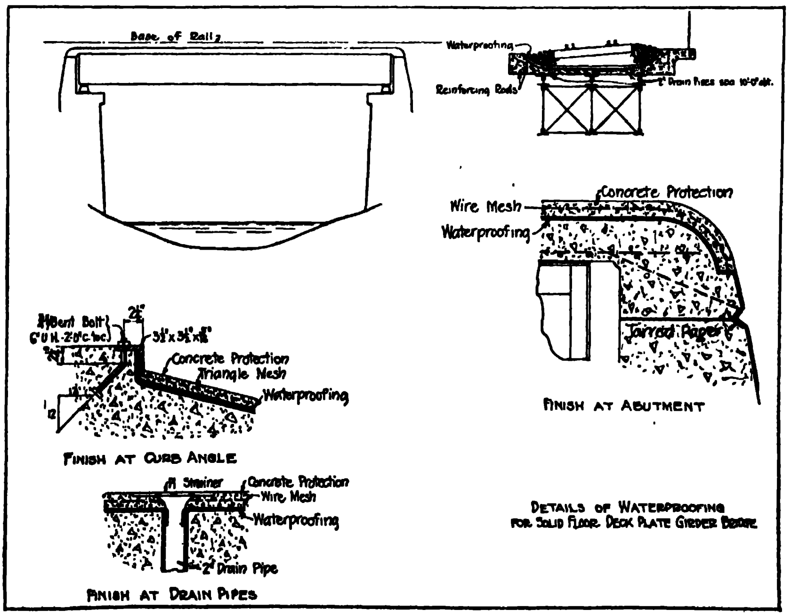

Base of Rail
Waterproofing
Reinforcing Rods
Concrete Protection
Wire Mesh
Waterproofing
Tarred Paper
Finish at Abutment
Concrete Protection
Triangle Mesh
Waterproofing
Finish at Curb Angle
Pl. Strainer
Concrete Protection
Wire Mesh
Waterproofing
2" Drain Pipe
Finish at Drain Pipes
Details of Waterproofing for Solid Floor Deck Plate Girder Bridge

Fig. 19.

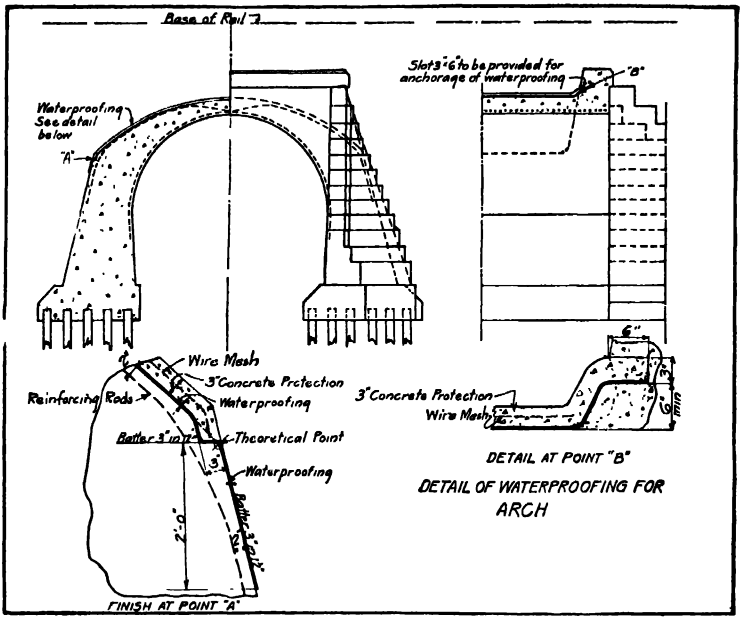

Base of Rail
Waterproofing See detail below
"A"
Slot 3"×6" to be provided for anchorage of waterproofing
"B"
Wire Mesh
3" Concrete Protection
Waterproofing
Reinforcing Rods
Batter 3" in 12"
Theoretical Point
Waterproofing
2'-0"
FINISH AT POINT "A"
6"
3" Concrete Protection
Wire Mesh
6" min
DETAIL AT POINT "B"
DETAIL OF WATERPROOFING FOR ARCH

FIG. 20.

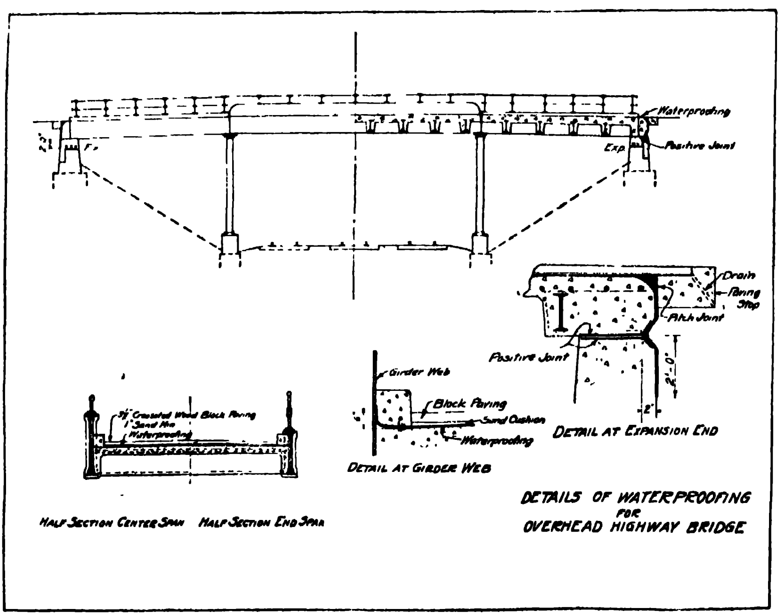

Waterproofing
Positive Joint
Exp.
Drain
Paving Stop
Pitch Joint
Positive Joint
2'-0"
DETAIL AT EXPANSION END
Girder Web
Block Paving
Sand Cushion
Waterproofing
DETAIL AT GIRDER WEB
HALF SECTION CENTER SPAN
HALF SECTION END SPAN
DETAILS OF WATERPROOFING FOR OVERHEAD HIGHWAY BRIDGE

FIG. 21.

Drainage

Drain pipes, down spouts and other appliances for carrying the water from the structure shall be provided.

14. Waterproofing Costs.—Membrane applied in accordance with the preceding specification will, so far as materials are concerned, cost 90 cts. to $1 per sq. yd. of finished membrane. The labor required in connection with the application of these materials will cost approximately $1 per sq. yd., making the total cost of the waterproofing membrane, in place, for materials and labor, approximately $2 per sq. yd. at present day prices.

SECTION 5

REINFORCED CONCRETE BUILDINGS[1]

GENERAL DESIGN

By Albert M. Wolf

1. Progress in Design.—Not over fifteen years ago reinforced concrete buildings were used very rarely, the lack of knowledge regarding concrete design and the prevalence of structural steel or timber and brick designs for buildings of all sorts mitigating against their use. The buildings built of steel or timber did not always give the best of service since, unless fireproofed, the steel frame building was just as vulnerable to damage by fire as a timber building. It is therefore not at all surprising that the use of reinforced concrete in building construction grew very rapidly after a few applications of the new material convinced the more advanced engineers, architects and owners of its particular advantages over the old types, as hereinafter set forth.

As was but natural, the first buildings built of reinforced concrete were modeled very closely after the other types of construction then in use—namely, timber and structural steel. As a result the monolithic character of concrete construction was ignored almost altogether and all possible economy was therefore not effected. The structures also were rendered unsightly by cracks developing in the members. After a time the principles of reinforced concrete became more widely known and more clearly interpreted and engineers began gradually to depart from the beaten paths of practice in other types and to treat reinforced concrete as a building material of an entirely different character from the then prevailing types, wood and steel.

The monolithic character of the material was soon recognized and advantage taken thereof in design of slabs, beams and girders, with the resultant saving in concrete and steel. This done, the next step was the endeavor to eliminate some of the many beams or joists to cut down on the cost of formwork and placement of steel. This marks the beginning of the decline of the beam and girder type in reinforced concrete construction; the intermediate beams so prevalent in the early designs were decreased in number, until in some cases the construction consisted of girders supported on columns with one or two intermediate beams in one direction with the slab reinforced transverse to the beams. In other cases where the spans were not so great, the beams were omitted entirely and the slabs carried directly on the girders framing into the columns. The next step was the elimination of the deep girders and providing the columns

[1] Fundamental principles of reinforced concrete design are not treated in this volume but may be found in the volume on "Structural Members and Connections" (see also report of Joint Committee in Appendix F of this volume).

with flaring capitals to act as supports for the slab, thus forming what is known as "flat slab" construction. The first building of this type built was the Johnson Bovey Company's building at Minneapolis, in 1906. This growth toward a more economical use of concrete and steel has been gradual and there still remain a few engineers and builders who have not been convinced that the flat slab type is as economical, strong and otherwise as desirable as the old type of beam and girder construction; but they are becoming fewer every day.

As is usually the case in any field enjoying a rapid growth, there have been devised several systems, some being modifications of others, or differing from them in minor details only, and for this reason only the two main types will be described here. The first system evolved was the multiple-way reinforced slab or "mushroom system," consisting essentially of a concrete slab carried on

Fig. 1.—Assembly floor, Pittsburgh Assembly Plant, Ford Motor Co.

columns with flaring capitals, the belts of bars in the bottom of the slab in the regions between column capitals being carried up into the top portion over the columns without bending, by passing them over a heavy framework of radial and circular bars formed in part by the bending down of the projecting ends of the column bars. In this type with all belts of bars passing over the column head in lines directly and diagonally between columns there are four layers of bars over the columns. The next system devised, which is essentially different from the first, is the two-way reinforced or Akme Flat Slab System (see Fig. 1). This system employs the flaring column capital, but the radial cantilever and circular hoop head frame used in the mushroom system is not used, or required. The reinforcement consists of rectangular belts of bars in two directions only, the main belts between columns being bent so as to be in the top portion of the slab between the column heads. The bars in the portion of the slab enclosed between the main bands are placed parallel to them and similarly bent, thus reinforcing the upper portion of the slab over the middle portion of the

main belts in transverse direction. The bars are bent previous to placing and held rigidly in place by supporting bars resting on concrete blocks, thus insuring their proper position in the finished slab. In this system there are only two layers of bars over the column heads, at right angles to each other. The first building built on this system was for the Hirsch, Stein & Company, at Hammond, Indiana, in 1908, designed for a live load of 400 lb. per sq. ft. A typical factory building of flat slab design is shown in Fig. 2.

Along with the development of the flat slab came what is commonly known as the joist type of concrete floor construction (see Fig. 3), consisting in general of concrete joists 4 or 5 in. wide and about 2-ft. centers, with clay tile, metal boxes or tile, or gypsum tile fillers between the joists and a very thin slab over these, the total thickness varying from 6 to 14 in., depending on spans and load. These joists are supported by girders framing into the columns. This type of floor has the

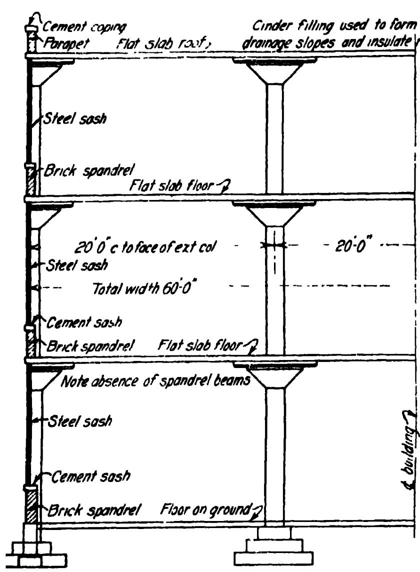


FIG. 2.—Half typical section of concrete flat slab factory.

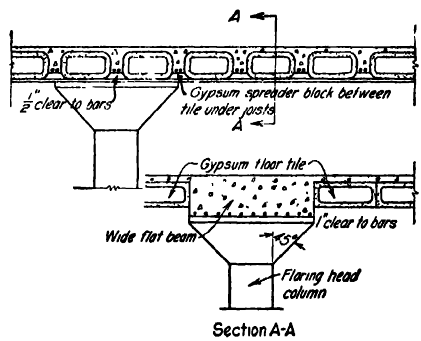


FIG. 3.—Cross-section of concrete joist and gypsum tile filler design with flat ceiling.

advantages of a flat ceiling and a considerable saving in dead weight (especially when metal or gypsum tile are used as fillers) over the solid concrete slab and, for long spans, oblong panels and relatively light loads, it is a very economical type of construction. The appearance of the ceiling is not as good as for the solid slab when clay or gypsum tile fillers are used and the ceilings are therefore generally plastered, while where metal tile fillers are used, plastering the ceiling is almost obligatory. The necessity of plastering cuts down the advantage of this type over the flat slab for ordinary spans, but for oblong panels and light loads, as for roofs of one-story buildings, it can still be made to show a saving over flat slab construction.

2. Economic Considerations of Design.—No fixed rules can be given for the most economical spacing of columns in reinforced concrete buildings since this is dependent upon the size and shape of the lot, the floor loads to be carried and the uses to which the building is to be put.

For large and important buildings several tentative designs with different column spacings should be made and the quantities of materials required for a typical panel computed, also the costs. Other things being equal, the column spacing giving the minimum cost should be used.

The cost of flat slab floors in buildings of several stories is in general less than that of beam and girder type floors owing to the simpler formwork required for the former. The expensive formwork necessary in beam and girder type buildings to care for changes in depth and widths of beams greatly increases their cost while adding nothing to the value of the finished structure. For the roofs of one-story buildings the concrete joist and filler block type of construction will prove cheaper than the flat slab, owing to the greater proportion of dead load to be carried in the latter type. In multiple story buildings with floors having different live loads, the flat slab proves very economical since the forms can be re-used in all stories, while with beam and girder construction the different sizes of members required, make this impossible without considerable alteration.

If a beam and girder design is used it will be found more economical to vary the depths of beams as required by the different floor loads and keep the widths constant. On account of the cost of changing forms it is sometimes more economical from a construction standpoint to use deeper beams than are actually required for the most economical percentage of steel, cutting down on the steel reinforcement to say 0.5 of one per cent; below this it is inadvisable to go. If the various spans are of different lengths it will generally be found most economical to choose a depth of slab, beam or girder suitable for the average span, thus preserving a uniform depth of beam or slab, and varying the strength as required by varying the amount of reinforcement.

The sizes of columns should be maintained as nearly uniform as possible to save on formwork, but it will seldom be found proper to use the same size column in three stories even if the percentage of steel is varied. Thus in a two-story building it will be found more economical to use the same size column in both stories, using a higher percentage of reinforcement in the lower story to take care of the stresses. The upper story columns will then be oversize which, especially in flat slab construction, is a good thing since they are thus better able to resist the bending stresses developed in them by unequal loading and the rigid connection with the slab.

In laying out flat slab buildings much can be saved on the cost by arranging the columns and spans so that all exterior spans (perpendicular to walls) are less than the interior ones by at least 18 in. or 2 ft. This will more nearly equalize the bending moments and not make the thickness of slab dependent upon the exterior spans, as is the case when the exterior spans are longer than the interior—in which case the slab would be too thick to be economical on the interior spans if made the same thickness throughout, which is best for appearance and economy of form construction.

Spans of from 18 to 20 ft. usually are the most economical for flat slab construction, as are the square or nearly square panels. The floor slabs should, if possible, project a distance beyond the center line between exterior columns to give sufficient resisting moment to carry the spandrel load in addition to the tributary floor load without using beams dropped below ceiling or raised above the floor, since these unduly raise the cost (see Fig. 2).

Finished cement floors are considerably cheaper than covering the slab with a wood floor and, where buildings are heated, there is little ground for the "time worn" argument that cement floors are unhealthful and cold to the feet of workmen. The fact that practically all of the concrete buildings built by the great automobile industry of the United States have cement finished floors should be convincing proof that they are entirely satisfactory.

By exposing the floor slabs at walls they serve the double purpose of window lintels and a part of the architectural treatment very satisfactorily (Fig. 2).

Just what size of building will prove the most economical or most satisfactory for the particular business in question depends upon several factors. First of all, if land is comparatively cheap, it will in general be found more economical to spread the building over more ground and decrease the number of stories than to build a four or five-story structure. This accounts for the great number of one- and two-story factories in small manufacturing towns, and the five- and six-story factories in the larger cities.

In a building of more than one story the cost of foundations and roof, which are necessary but unproductive adjuncts, is spread out over two or more floors and hence the cost per square foot of floor for these parts is decreased. As a general rule a two-story building will be found more economical than a single story structure and, for the same length of building and the same amount of floor space under one roof, the former will be a better lighted and ventilated structure. For ordinary manufacturing buildings, a width of from 60 to 80 ft., divided into three or four bays, will insure excellent light at all points, but, if a one-story building is used, such widths might require a building too long for the purpose intended and the area desired, and a wider building with saw-tooth skylights over the middle bays will be found the proper solution in so far as light is concerned, though more expensive to build.

Progressive manufacturers appreciate the value of equipping buildings with the most modern sanitary conveniences and welfare features and of constructing new factories of steel and concrete, which, in addition to being permanent, fire-resisting and sanitary, allow the provision of a maximum of window surface and ventilation, and the best possible distribution of natural and artificial light. Just how this can be obtained is briefly set forth in the following.

Advantages of Good Natural Lighting.—The advantages to be gained by arranging for the very best of natural lighting in a factory can be set down briefly as follows:

(1) Increased production.
(2) More accurate work.
(3) Fewer rejections by inspectors.
(4) Reduced number of accidents.
(5) More order and neatness in the shop.
(6) Easier supervision.
(7) Less lost and mislaid tools and materials.
(8) Less eye strain and sickness.
(9) Better morale in the shop force because of greater comfort and more cheerful surroundings.
(10) Reduced cost of artificial lighting.

Fundamental Considerations.—The three fundamental considerations of any method of lighting as set forth by the Wisconsin Industrial Commission in its Handbook on Industrial Lighting are Sufficiency, Continuity and Diffusion. With respect to the daylight illumination of interiors, sufficiency demands adequate window area; continuity requires (*a*) large

enough window area for use on reasonably dark days, (*b*) means for reducing the illumination when excessive due to direct sunshine, and (*c*) supplementary lighting equipment for use on particularly dark days; diffusion demands interior decorations that are as light in color as practicable for ceilings and upper portions of walls, and of a dull or mat finish in order that the light which enters the windows or that which is produced by lamps, may not be absorbed and lost on the first object that it strikes, but it may be returned by reflection and thus be used over and over again. Diffusion also requires that the various sources of light, whether windows, skylights or lamps, be well distributed about the space to be lighted. Light colored surroundings as here suggested result in marked economy, but their main object is perhaps not so much economy as to obtain a result that will be satisfactory to the human eye.

The following requirements may now be listed for good natural lighting:

(1) Each employee in the building should have adequate natural light to properly perform the task assigned him.

(2) Windows and skylights if used, should be spaced and located so that daylight conditions are fairly uniform over the working area.

(3) The daylight should be of such intensity that artificial light will be required in the building only at such times as it would naturally be considered necessary.

(4) The windows should be such as to free the daylight which enters the building from glare due to the sun's rays or light shining directly into the workman's eyes. Rough or ribbed glass and sometimes window shades will be necessary to attain this end.

(5) In order to obtain the greatest effectiveness of the natural lighting the ceilings and walls should be of a light color which will reflect rather than absorb the light rays. To render the lighting more restful to the eye and to make the decorations more serviceable the lower 3 or 4 ft. of walls or the dado should be a darker color, preferably factory green or other medium colors.

Ways of Lighting.—Natural lighting of buildings may be obtained in three ways:

(1) The most common, by the use of windows on the side of the building, to furnish diffused and direct light during a large portion of the day.

(2) By skylights or monitors located in the roof. These skylights are constructed in several forms, a common form being the sawtooth skylight with a north exposure which furnishes diffused light. Another kind is the horizontal or slightly hipped skylight, which furnishes direct light during a major portion of the day. Monitors—namely, raised portions of the roof with windows in the sides and sometimes with sash in the top—are a form of skylight which furnish diffused and direct lighting.

(3) By the use of prism glass so arranged on the side of a building as to take up the direct light from the sky, change its course and redirect it into the buildings.

The first method is of course the most common, since the second method is confined to the lighting of one-story structures, or to the top floors of multi-storied buildings, the lower floors of which can be lighted by windows only. The third method is used only where adjacent buildings are so close as to make impossible the adequate lighting of a building by reliance on the first method alone, and on account of its expense it is avoided whenever possible.

Effects of Width and Story Heights.—The modern multi-storied factory usually has a sash area equal to 60 to 85 per cent of the wall surface, depending on the architectural treatment and the character of the structure. For buildings up to 60 and 70 ft. in width and with proper story heights, these percentages of window area will give plenty of natural light without the use of prism glass.

The story height necessary for proper lighting depends, of course, on the width of the building, the depth of floor construction and on the proximity of adjacent buildings. Considering that the nearest building is at a distance equal to the width of the one under consideration, the following story heights will be adequate lighting for the various widths listed in buildings of flat slab construction.

Widths up to 50 ft. story heights 11 to 12 ft.

Widths of from 50 to 70 ft. story heights 12 to 14 ft.

Widths of from 70 to 90 ft. story heights 14 to 16 ft.

For buildings over 80 ft. in width an increase in the story height does not give commensurate increase in the lighting of the interior of the building, and greater widths are therefore not always economical or desirable from a lighting standpoint, unless the center

bays can be used for some purpose not requiring the best of natural lighting, or the size of the lot places some necessary restriction on the width.

If the building is too wide or the story heights too low, it may not be possible to depend entirely upon natural lighting even under the most favorable conditions, and hence manufacturing space will be lost or extra expense incurred by the use of artificial lighting. Under such conditions the employees near the windows have suitable daylight to work in and quite naturally those who have to depend on artificial light most of the time become envious and discontented and the cost of production quite naturally rises. Then also such lack of harmony among the workers lowers the quality of the product.

Since the upper portions of the windows are especially serviceable in lighting areas at some distance from the windows, and since they give a reduced illumination in proportion to their areas to the floor space near them where the light is always sufficient, it is especially desirable to place the windows as near the ceiling as possible. When the sun shines through windows so located the direct light must be reduced or diffused by the use of ribbed or hammered glass, or by window shades.

In factory buildings the best natural lighting is obtained when flat ceiling construction is used, since there are no beams or girders projecting below to interfere with the uniform distribution or reflection of the light.

If properly designed, a flat slab floor does not require a deepened spandrel beam, thus allowing the windows to be carried up to ceiling level, with the result that the maximum amount of lighting area is obtained.

Aids to Better Natural Lighting.—No matter how large the sash area may be and what type of glass is used, the amount of light entering the building will be greatly reduced by failure to keep windows and skylights clean at all times. While this means extra cost and maintenance, the money is well spent, inasmuch as a saving is made on the cost of artificial lighting. In addition to keeping the windows clean it is essential that the interior of the building—that is, the walls and ceilings be kept clean at all times and painted with a light colored paint which will reflect rather than absorb light. Practically all modern factories today are painted with a gloss paint or in some cases with a flat white paint. The latter retains its whiteness for a longer period, but gathers and holds dirt and dust more readily than the gloss paint, and furthermore will not stand the same amount of washing as the gloss paint.

Other colors than white should not be used for factory painting except the use of dark slate or green for a dado 4 or 5 ft. high around the walls and columns, and for push plates at doors. While the use of such a dado cuts down the lighting somewhat, it does not show finger marks and dirt which are sure to be prominent and uninviting on the lower portions of factory walls.

Along with the movement for better lighting and working conditions have come such changes as painting machinery white instead of the "conventional black." This means that the machines reflect rather than absorb light, eliminating shadows around them and also the chances of accidents to workers.

3. Design Loads.

3a. Live Loads (Floors).—Cities having building codes set forth the live loads on floors to be used in the design of buildings of different classes and the designer should be governed by them when making a design. As an example of good practice the following extract from the Seattle Building Code is given:

All floors shall be constructed to bear a safe live load per superficial square foot of not less than tne following amounts.

	Pounds
Public buildings	100
Detention buildings, in cells or wards	60
Churches, chapels, theatres, assembly halls or courtrooms with permanent seats	80
Lobbies, passageways, corridors and stairways of the same	100
Assembly halls with movable seats	100

	Pounds
Halls used for dancing, or roller skating	150
Lobbies, passageways, corridors and stairways of same	100
Stables	80
Dwellings, apartment houses, flat buildings and lodging houses	50
Class rooms in schools	60
Assembly rooms in schools	80
Office buildings and hotels, ground floor	125
For floors above the ground	75
Store buildings for light merchandise, ground floor	125
For floors above the ground	100
Store buildings for heavy merchandise, such as grocery stores or hardware stores	150
Warehouses	200
Factories and workshops, when the nature of the work is general	125
Machine shops, armories, drill rooms and riding schools	250

Floors in a building to be used for the sale, storage or heavy machinery, shall be proportioned to the load they may have to carry.

Chicago Code minimum live load requirements are as follows per square foot:

		Pounds
Class 1.	Commercial buildings not including department stores	100
Class 2.	Office buildings, hotels, clubs, lodging houses, and hospitals	50
Class 3.	Residences and hospitals for 20 or less patients, small garages	40
Class 4.	Churches, assembly halls, lodge halls	100
Class 5.	Public theatres	100
Class 6.	Tenements and apartment buildings	40
Class 7.	Department stores	100
Class 8.	Schools	75
Class 9.	Police stations, stairs	100

In the main the building codes of our larger cities, Chicago and New York and others have practically the same requirements as to live loads.

3b. Live Loads (Roofs).—Flat roofs should be designed for the dead load of slab and roofing and a snow (or live) load varying from 40 lb. per sq. ft. for Canada to 25 lb. per sq. ft. for southern latitudes. In the south the snow load will be very light if any, but the roof slab should be designed for a minimum of 25 lb. per sq. ft. live load to take care of construction loads and unusual loads such as may be occasioned by down spouts clogging with subsequent flooding of roof. The Chicago Ordinance requires roofs to be designed for a live load of 25 lb. per sq. ft.

For pitched roofs a wind load should also be considered in the design. The wind load, which acts horizontally, varies with the velocity of the wind. A pressure of 30 lb. per sq. ft. of vertical surface is usually assumed, Several formulas are in existence for determining wind pressure on inclined surfaces. Duchemin's formula which follows is preferred by many engineers as it is based upon carefully conducted experiments:

$$P = P_1 \frac{2 \sin A}{1 + \sin^2 A}$$

where P = normal pressure of wind in pounds per square foot of inclined surface.
P_1 = pressure of wind in pounds per square foot on a vertical surface.
A = angle of inclination of the roof.

Where wind load is considered in design an increase of 50 per cent in allowable stresses for combined live, dead and wind load stresses is allowed by good practice, but in no case should the section be less than required if wind forces are neglected.

3c. Dead Loads.—The following is a table of dead loads to be used in floor design:

Reinforced concrete	150 lb. per cu. ft.
Wood finished floor	4 to 6 lb. per sq. ft.
Screeds and nailing strips	2 lb. per sq. ft.
2-in. cinder concrete filling	20 lb. per sq. ft.
Plaster (one side partition)	5 lb. per sq. ft.
Suspended ceiling	10 lb. per sq. ft.
Cinders 1 in. thick	7 lb. per sq. ft.
Tile (see Arts. 7*e* and 7*f*)	

The dead loads encountered in roof design are summarized in the following table:

Material	Lb. per Sq. Ft.
Slate, 3/16 in. thick	7.25
Slate, 1/4 in. thick	9.60
Roman tiles, new style, 1 part	8
Oriental tiles, (improved)	11
Spanish tiles, new style, 1 part	8
Plain tile or clay shingle	11 to 14
Fancy tile laid in mortar	25 to 30
Ludowici tile	8
Add 10 lb. per sq. ft. for tiles laid in mortar.	
Pan tile	10
Flat tile, with mortar	20 to 30
Porous terra cotta roofing, 2 in. thick	12
Porous terra cotta roofing, 3 in. thick	15
Porous terra cotta roofing, 4 in. thick	19
Solid tile, 2½ in.	16
Copper roofing sheets	1.5
Copper roofing tiles	1.75
Tin, including one thickness of felt and paint	1
Felt and asphalt	1
5-ply felt and gravel	6
4-ply felt and gravel	5.5
3-ply ready roofing (ruberoid, elaterite, etc.)	1
Felt and gravel	8 to 10
Sheet zinc	1 to 2
Skylights, galvanized iron frame, 1/4-in. glass	4.5
Slylights, galvanized iron frame, 5/16-in. glass	5.0
Skylights, galvanized iron frame, 3/8-in. glass	6.0

Skylight and floor glass 13 × thickness in inches = wt. per sq. ft.

4. Floor Surfaces.

4a. Cement Finished Floors.—The great majority of concrete slabs in commercial and factory buildings have what are known as cement finished surfaces, put on integral with the slab or in some cases at some time after the structural slab has set up. The majority of building codes allows the cement finish to be considered as a part of the total slab thickness if placed before the concrete of the slab has set. Hence from the standpoint of economy as well as good design it is best to place the finish integral with the floor slab. (For further information regarding surfacing cement floors, see Sec. 4, Art. 9.)

4b. Tile Floors.—Various kinds of tile floors are used quite extensively in corridors, lobbies and vestibules in reinforced concrete buildings. These are: Cork tile made of compressed cork shavings; linoleum tile; rubber tile; quarry tile, usually 6 × 6 in.; ornamental tile; ceramic mosaic tile; marble tile ⅞ in. thick; and terrazzo tile.

In laying any tile on a reinforced concrete floor slab, allowance should be made for a setting bed of mortar at least as thick as the tile to be laid thereon.

4c. Asphalt Floors.—Where waterproof floors are required, an asphalt mastic floor about 2 in. thick is often laid on the concrete subfloor. Sand or quartzite is mixed with the asphalt to give a better wearing surface.

4d. Brick Floors.—Where heavy trucking occurs or where acid, hot water and cold water must be resisted, finished floors of vitrified shale brick laid in acid-proof and waterproof cement on a bedding course of cement mortar at least 1 in. thick give good service.

4e. Steel Plate Trucking Aisles.—Where very heavy and continuous trucking occurs along certain lines, steel plates are often anchored to the concrete floor to form the trucking aisles.

4f. Wood Block Floors.—Wood block floors have been used quite extensively in reinforced concrete factories. For foundries, machine shops and similar buildings, creosoted wood blocks 4 in. thick, set in asphalt, are used.

4g. Wood Floors.—Where finished wood floors of maple or oak are desired in reinforced concrete buildings, it is customary to set 2 × 3-in. or 2 × 4-in. beveled sleepers on the rough slab and anchored to the slab by clips or anchors set in the slab when poured. These sleepers are set 16 in. on centers and the space between is then filled with lean cinder concrete. On this a waterproof building paper with lapped joints should be laid if the slab rests on the ground. This is to prevent moisture from reaching the finished flooring. On ground floors it is essential that the sleepers be treated with a preservative to prevent dry rot. The finished wood flooring, usually 1⅛ × 3½-in. tongued and grooved material, is laid on the sleepers and blind nailed.

Another type of wood floor, designed to avoid dry-rot on ground floors, makes use of a layer of sand and coal tar mixed, spread 1¼ to 1½ in. thick on the base slab. This layer is leveled while still warm and soft and a layer of 2-in. plank embedded in it. On this the finished floor is laid, or sometimes an intermediate layer of ⅞-in. rough flooring is laid first and the finished flooring on top of this rough flooring, the different layers being placed at right angles to each other.

5. Roofings.

5a. Concrete Surfaces.—Although there are numbers of examples of cement finished concrete roofs on small buildings giving good service without leakage, the general practice among engineers and architects is overwhelmingly against this kind of roofing surface. The reason is that it is practically impossible to avoid cracks in the concrete owing to the wide changes in temperature to which the surface is subjected. Alternate expansion and contraction will sooner or later open up cracks in the protective cement finish and give rise to leaks.

A cement finish roof surface is satisfactory for the roofs of train sheds, reservoirs and similar structures where absolute dryness is not necessary.

5b. Asphalt Coating.—For unimportant concrete buildings a heavy coating of high melting point asphalt will often prove sufficient to waterproof the roof especially if the pitch is great enough. The coating should be renewed every year or two to insure watertightness.

5c. Pitch and Gravel Roofing.—A type of roofing used very widely on reinforced concrete buildings is the built-up pitch and gravel roof, the best known being the Barrett Specification Roof, specification for which is given below.

First.—Coat the concrete uniformly with specification pitch.

Second.—Over the entire surface lay four plies of specification tarred felt lapping each sheet 24½ in. over preceding one, mopping with specification pitch the full 24½ in. on each sheet, so that in no place shall felt touch felt.

Third.—Over the entire surface pour from a dipper a uniform coating of specification pitch, into which, while hot, embed not less than 400 lb. of gravel or 300 lb. of slag for each 100 sq. ft. The gravel or slag shall be from ¼ to ⅝ in. in size, dry and free from dirt.

General.—The felt shall be laid without wrinkles or buckles. Not less than 200 lb. of pitch shall be used for constructing each 100 sq. ft. of completed roof, and the pitch shall not be heated above 400° F.

5d. Asphalt Roofing.—Asphalt built-up roofing is being used more and more on concrete buildings. In this type of roofing no gravel or slag is used to protect the asphalt top coating as in the case of the pitch and gravel roof. Otherwise the manner of laying is similar except that asphalt felts and asphalt as a binder and cover are used instead of tar felts and coal tar pitch.

A specification for built-up asphalt roofing is given below:

First prime the concrete roof surface, when thoroughly dry, with asphalt primer, using approximately 1 gal. per 100 sq. ft. Over the concrete thus primed mop down approximately 30 lb. of asphalt per square, into which while still hot apply 2 layers of standard rag felt weighing at least 15 lb. per 100 sq. ft. Lap these sheets not less than 17 in. and mop same for full width of lap, using approximately 30 lb. of asphalt per square per mopping.

Over the 2 layers of felt thus laid, mop down approximately 30 lb. of asphalt per square, into which, while hot, apply 1 layer of cap-felt lapping sheets at least 2 in. Over this cap felt mop down approximately 30 lb. of asphalt per square.

All plies of the roofing should be turned up at walls and curbs to a height of not less than 8 in. and fitted into a flashing angle. Then a flashing strip of cap-felt should be inserted in the flashing angle, nailed and carried down on to the roof for at least 8 in. and thoroughly mopped onto the main roofing and covered with asphalt. This flashing should be counterflashed with copper.

Another type of asphalt roofing is built up using asbestos felt impregnated with asphalt instead of rag felt.

5e. Clay Tile.—Where architectural effect is desired, a clay tile covering is often used of concrete slabs, the tile being fastened to wood cant strips anchored to the slab. In order to insure watertightness the roof slab should first be covered with heavy slaters' felt. Clay tile can be had in various shades of red or green and in the form of Spanish, French, shingle and pan tile.

For methods of forming drainage slopes and insulating concrete roofs the reader is referred to the chapter on Special Roof Construction in this section.

6. Walls.—In reinforced concrete buildings the basement walls should be of reinforced concrete designed after the methods given in the chapter on Basement Walls in this section. The walls above grade are usually built of brick, the outer 4 in. usually being of face brick bonded every fifth course to the common brick backing by header-bricks or by the use of metal ties.

6a. Concrete Walls.—Concrete curtain walls for the upper stories of reinforced concrete buildings are not used very extensively owing to the difficulty of obtaining finished concrete surfaces which present a pleasing appearance without too great an expenditure.

When used in unimportant buildings, the concrete curtain walls can be made plain and are in general built independent of the floor construction—that is, the spandrel wall does not act as a beam to carry part of the floor load. Such walls should be anchored to the floor slabs and columns by means of stub bars and the joints keyed so as to make them watertight. The reinforcement should consist of at least 0.3 per cent of horizontal bars near the outer surface and about 0.2 per cent of vertical bars on low spandrels. On high curtain walls this latter amount should be increased to about 0.3 per cent.

6b. Brick Walls.—In the present-day reinforced concrete buildings, the exterior walls are in general merely brick and glass enclosing curtains supported on the concrete framework and do not assist in carrying the loads coming upon the structure as in the old type of wall-bearing construction. This means that the thickness of brick piers and spandrels can be the minimum allowed by the height and architectural treatment.

Where the exterior columns are of rectangular shape and form the piers, and the floor slabs are exposed as lintels over the windows in the story below the support of the brick spandrel walls is a simple matter, these walls being built up on the floor slab between the concrete piers or columns (see Fig. 4). Ordinarily a wall 9 in. thick will suffice for such construction unless the spandrels are over 5 ft. high when it is well to use a 13-in. wall to obtain greater stability. If the building is to be used for storage purposes wherein materials may be piled up against the spandrels, special attention should be given to the anchorage of the walls to columns and floor slabs either by keyways or stub bars anchored into the concrete and built into the brickwork.

If the architectural treatment is such that no concrete supporting members are exposed, then the problem of proper support for the brick and trim of the exterior walls is more difficult. This means that the concrete work must be so laid out that it is at all points at least the width of a brick (usually 4½ to 5 in.) behind the outer face of the wall, and some method of supporting the brickwork and trim over openings in the walls must be used which will not show on the face of the wall. For spandrels of this character, it is hardly feasible to use brick walls less than 13 in. thick—that is, the backing portion resting directly on the

concrete floor, should be at least 8 to 8½ in. thick to give the proper stability, since face brick is not generally bonded to the backing except by brick ties.

In Fig. 5 a spandrel wall of the character just mentioned is shown. It will be seen that the face brick row-lock course over the windows is supported on 4 in. × 3 in. × ⅜-in. angles anchored to the concrete floor slab with bent plate anchors spaced 18 in. on centers and that the backing of common brick,

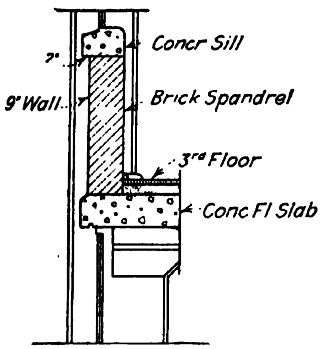


Fig. 4.—Nine-inch brick spandrel wall—concrete floor slab and column exposed.

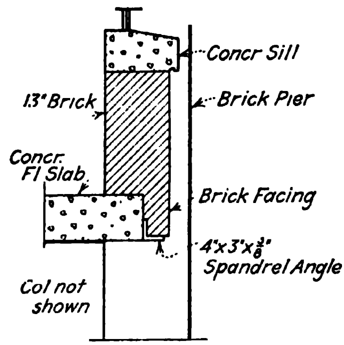


Fig. 5.—Brick spandrel wall and pier brick facing of lintel supported on spand angle.

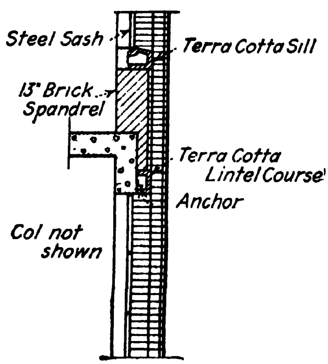


Fig. 6.—Brick spandrel wall and pier—terra cotta lintel course supported on spandrel angle.

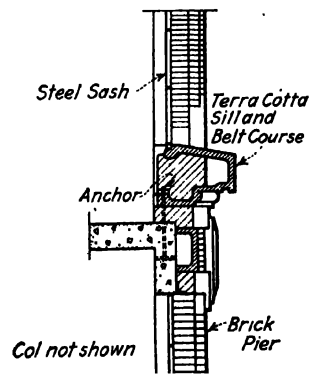


Fig. 7.—Terra cotta sill and belt course on brick spandrel anchored to concrete spandrel girder.

8½ in. thick, is carried directly on the concrete slab, the face brick course being tied to the backing with corrugated metal brick ties. A modified spandrel of this type is shown in Fig. 6. Terra cotta, it will be noted, is hung from the supporting angle, thus covering and protecting it from damage by fire and rust to which the angle is exposed when placed as shown in Fig. 5.

In either of the types of construction shown in Figs. 5 and 6, it is good practice to anchor the brickwork of the piers to the concrete columns, at least where the veneer is thin, by means of metal brick ties anchored into the columns. It is

also a wise precaution to provide supporting angles at floor lines for the brick facing of the piers. When wood column forms are used, the brick ties can be bent and tacked to the forms with the width of ties placed vertically so as to expose as little of the ties as possible to the falling concrete when the forms are filled, thus insuring better anchorage in the concrete. If metal column forms are used, the placing of brick ties in the columns is more difficult, but can be accomplished by tacking them to a thin wooden strip wired to the front portion of the form, through small holes provided for that purpose, the strip being pulled out with the removal of the form leaving the ties in place to be bent out into the brick joints.

When the architectural treatment necessitates the use of projecting belt courses at the top of spandrel walls, care should be exercised to see that the course is securely anchored to the spandrel, and further, that the center of gravity of the entire spandrel section lies in a plane well back of the facing course, or, in other words, in one intersecting the concrete slab, and not the spandrel angle. If deep spandrel girders are used, the wall may be anchored as shown in Fig. 7.

Where it is impossible to comply with the foregoing, the facing materials and projecting courses should be anchored to concrete beams raised above the floor as a part of the spandrel and reinforced to act as beams between columns with the load applied at the side. Such beams are subjected to bending in horizontal planes as well as vertically and should be well reinforced with vertical stirrups in addition to the main bars (see Art. **94** on Concrete Parapets).

6c. Tile Curtain Walls.—In concrete buildings whose purpose is utilitarian only, considerable saving can be made by using 8-in. clay tile walls below the sash and covering these with cement plaster.

7. Partitions.

7a. Concrete Partitions.—Reinforced concrete partitions are from the standpoint of fire resistance the logical thing to use in reinforced concrete buildings, but owing to the high cost of installation, excessive weight and difficulty of removal when alterations are desired, other types of fire-resisting partitions, such as brick, tile, gypsum block and metal lath and plaster are more commonly used. For vaults, stair and elevator well partitions which are practically permanent, 4 to 6-in. concrete partitions reinforced with 3/8 or 1/2-in. round bars 12 or 18 in. apart in both directions with rods well anchored in the floor slabs can be used to good advantage. Extra rods should be provided at openings, some being placed diagonally at the corners of openings. Concrete partitions can best be poured after the floors are in place so that it is necessary to leave slots in the floor slabs directly over where the partitions are to be located.

Partitions built of hollow concrete blocks or tile laid in cement mortar have been used to some extent and, while not as fire-resisting as a solid concrete wall, they possess the advantages of being much lighter and allowing changes to be made with less difficulty.

7b. Brick Partitions.—Brick partitions from 9 to 12 in. in thickness are often used in concrete buildings around elevator and stair walls and for dividing fire walls in large buildings. The latter class should never be less than 12 in. thick. Openings in such walls should be protected by fire doors with steel or channel frames to protect the edges of the openings.

7c. Plaster Partitions.—For unimportant partitions, such as for small offices, toilet rooms, etc., solid plaster partitions 2 in. thick may be used,

made of cement or prepared plaster on metal lath or plaster-board supported by steel channel studs set 12 to 16 in. on centers and anchored to floor and ceiling. This type of partition weighs about 17 lb. per sq. ft. and is less expensive than tile with the added advantage of lighter weight.

7d. **Steel Sash Partitions.**—For departmental and office walls where an unobstructed view is desired the steel sash partition serves to the utmost advantage. By its use small offices can be arranged along the outer walls without materially cutting down the natural lighting of the space beyond. These partitions are built of units similar to side wall steel sash units with vertical supporting members or mullions between. The lower portions of the units for a height of about 3 ft. are usually closed in with steel panels forming a wainscot while the upper portions are glazed with double strength glass. If wire glass is used, these partitions will resist considerable fire. Where desired, steel sash or tubular steel doors are put in between the mullions.

7e. **Clay Tile Partitions.**—Clay tile partitions are used extensively in reinforced concrete buildings around stair and elevator shafts, and for room partitions. The former should be at least 6-in. tile while the latter may be of 4-in. tile. Where the walls are to be plastered, the tile should be scored. Tile should be hard-burned clay tile free from cracks, blisters and warps so as to allow laying up the wall as nearly plumb as possible. Where wood trim is to be placed on tile partitions, wood nailing strips and grounds should be placed before the plastering is applied. In fireproof buildings, steel channel or sheet metal frames should be used for door openings.

The weights of standard tile partitions are given in the accompanying table:

Weight of Tile Partitions

Size of Tile (In.)	Weight per Square Foot (Lb.)	Weight per Square Foot Plastered Both Sides (Lb.)
3	13	21
4	15	23
6	22	30
8	28	36
10	34	42
12	35	43

As a general rule, a hard-burned tile weighs less than a porous or semi-porous tile, as the thickness of the material can be made less. Mortar for tile work should be composed of 1 part Portland Cement to 3 parts clean, sharp sand—lime not to exceed 10 per cent by volume.

7f. **Gypsum Tile Partitions.**—A partition used a great deal in office buildings and other fireproof buildings where frequent changes may be necessary is the gypsum block or tile partition. This type of partition is cheaper than clay tile and while not as fire-resistive, it possesses one great advantage—namely, that openings may be readily cut into it with a saw.

These gypsum blocks are made of calcined gypsum mixed with fiber and molded into solid or hollow blocks. The weights of gypsum block partitions are given in the following table:

WEIGHT OF GYPSUM BLOCK PARTITIONS

SIZE OF BLOCK	WEIGHT PER SQUARE FOOT (LB.)	WEIGHT PER SQUARE FOOT PLASTERED BOTH SIDES (LB.)
3-in. hollow	9.9	17.9
3-in. solid	12.4	20.4
4-in. hollow	13.0	21.0
5-in. hollow	15.6	23.6
6-in. hollow	16.6	24.6
8-in. hollow	22.4	30.4

8. Windows.—In a reinforced concrete building the only types of windows which should be given serious consideration are rolled steel sash and hollow metal sash and frames. When glazed with ¼-in. wire glass, such windows will prevent a fire from entering a building or keep it confined in the building if it occurs within.

Since the upper portions of the windows are especially serviceable in lighting areas at some distance from the side walls, it is essential that they be placed as close to the ceiling as possible. In flat slab construction the sash can be carried up to the underside of floor slab and a maximum of light obtained.

Details for anchoring of steel sash in brick walls, stone sills and concrete are shown in Fig. 8. These represent the best present-day practice.

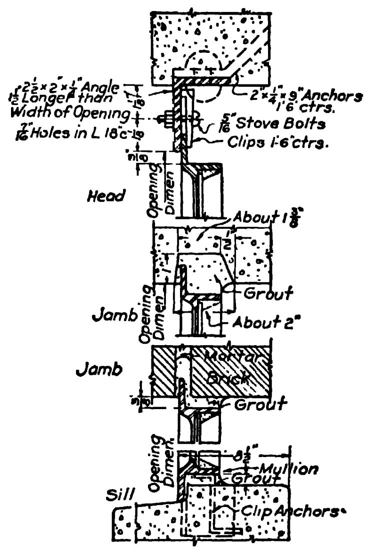


FIG. 8.—Typical detail for steel sash.

9. Doors.—The doors and door frames used in reinforced concrete buildings should be fire-resisting. For office portions of factories or office buildings hollow metal doors are best adapted since they present a fine appearance and can be given an excellent finish. The door frames should be of pressed metal. For factories, shops, warehouses, etc., tin clad doors, steel sash doors, rolling steel shutters, with steel channel frames for the openings, give excellent service. When such doors are used in exits they should be automatic closing. If glazed, the glass should be ¼-in. wire glass.

Factory elevator doors generally are of the Meeker or of the double-slide-up type, built entirely of metal and so counterbalanced as to operate very easily.

10. Floor Openings.—Often the manufacturing processes make necessary numerous openings in the floor of reinforced concrete buildings in addition to the regular stair, elevator and pipe shaft openings, and in making the detailed design it is essential that the presence of these openings be considered and the reinforcement placed accordingly. For openings up to 18 in. the bars can be spread apart and a few extra short bars placed at right angles over the gap thus formed. For larger openings special framing should be used around the openings as illustrated in Fig. 9 which shows the methods commonly employed in beam and

girder, tile and concrete, and flat slab construction. Floor openings should be protected against spread of fire through them.

11. Pipe Sleeves.—Where heating, sprinkler and plumbing system risers pass through the floor construction, sheet metal or pipe sleeves should be placed on the

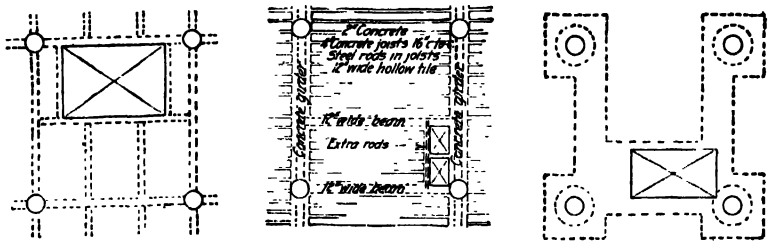
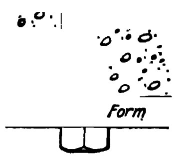
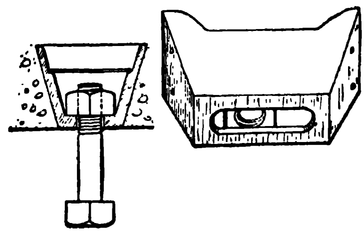


Fig. 9.

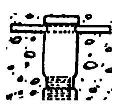
forms before pouring the concrete. Where such openings are necessary in regions where the compressive stresses in the concrete are relatively high, the sleeves should be of heavy pipe in order that they may sustain the compression.



Bolt insert. Bolt is removed before forms are taken down, leaving the nut in the concrete.

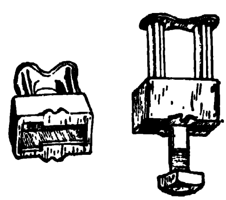


Wrialco insert.

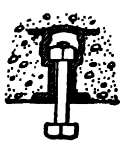


Kohler pressed steel insert.

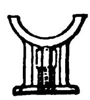
Barton steel spiral socket for lag screws.



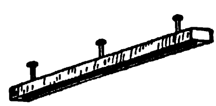
Dayton insert.



Security insert.



Havemeyer socket insert.



Truscon slotted insert.

Fig. 10.

At other points sheet metal sleeves (filled with sand or paper while pouring concrete) will suffice. After the pipes are placed in the sleeves, the openings should be closed with floor plates or filled so as to prevent passage of fire or water from one story to another through them.

12. Supports for Shafting, Machinery and Piping.—In a reinforced concrete building it is necessary to provide permanent means of supporting shafting, machinery and piping of various kinds on the ceilings of the various stories, at the time the floors are poured. This can best be done by placing metal inserts on the forms before placing the concrete. Various kinds of steel and malleable iron inserts are shown in Fig. 10.

For permanent piping, as for heating and sprinkler systems, the layout for inserts for pipe hangers should be made as soon as the system is designed and the inserts located on the floor forms accordingly. For shafting and machinery which may be changed at any time, it is best to provide inserts at a spacing of about 3 ft. in each direction. This allows great flexibility in layout of machinery and shafting.

13. Fire Protection.—In order to protect properly the contents of a reinforced concrete building from fire, an automatic sprinkler system should be provided. The sprinkler tank can be located above the roof of the building (25 ft. as a minimum) on a reinforced concrete (see Art. **96**) or steel tower, the building columns and footings being designed for the extra load.

At fire escapes and stair towers it will be well to provide standpipes and hose reels in addition to the sprinkler system. Where the contents are highly inflammable, it is good practice to provide chemical extinguishers at suitable locations.

FOUNDATIONS

By Albert M. Wolf

14. Investigation of Foundation Site.—Before any work is done on the design of the foundations for a building, all available data on the condition of the soil at the site of the building should be obtained. In large cities like New York and Chicago so much data on foundation conditions encountered in construction in the different sections is available that the designer can readily determine the most logical type of foundation, and the load the soil can safely carry, based on past experience. Should the building to be designed be much taller or otherwise materially different than existing structures in the immediate neighborhood, the engineer should carefully examine the various soil strata on the site and make load tests of the soil on which the footings are to rest. An investigation as to the loads on foundations assumed in the design of nearby buildings, and their condition after a period of service, will go a long way toward confirming the results of test loads on soil and the correctness of assumptions made by previous designers.

15. Methods of Investigation.—It is not the purpose here to enter into a lengthy discussion on foundation investigation and tests but rather to point out briefly the various methods and their limitations. For a more extensive discussion on the subject the reader is referred to the volume on "Foundations, Abutments and Footings."

15a. Auger Boring.—One of the simplest and cheapest methods of investigating soil conditions for relatively shallow depths is by auger boring.

An ordinary wood auger or a special large size auger will bring up reliable samples of the earth strata passed through, and with this data at hand and a knowledge of loads successfully used on the various strata in the locality, the design of footings for ordinary one- and two-story buildings can be made.

The auger can be used only in sand and clay soils and is useless where gravel strata are encountered. It can be used to advantage to ascertain the depth to bedrock (if near surface) or the location of gravel strata. Another usage to which the auger may be put is to investigate the strata below the level of a test pit or foundation bottom in order to determine the character of the underlying soil and confirm the assumptions as to design loads based on actual soil tests. For example, should an auger boring reveal a layer of wet sand, clay or silt within a few feet below the level of the foundation bed in a clay stratum which load tests showed could safely carry a load of 2 tons per sq. ft., it might prove disastrous to load the clay to this amount if there were any possibility of the soft layer below being disturbed by future building operations in the vicinity.

The auger boring does not allow the testing of the carrying capacity of the various strata and hence only acts as a guide to the judgment in arriving at design loads based on past experience with similar soils.

Where auger borings are made to a relatively great depth, they are very likely to prove unreliable, especially when the depth to bedrock is to be determined. The stratum directly over the rock is very likely to contain large stones or boulders and should the auger strike one of these, the operators are very likely to record bedrock at that depth.

An example of this stands out very vividly in the writer's experience where auger borings from several holes in the lot apparently disclosed bedrock at 85 ft. below the street. The first open well or "Chicago Caisson" being sunk to this depth disclosed boulders and silt and further excavation to a depth of 100 ft. opened up a stratum of quicksand and water (of considerable depth by rod test), which precluded the possibility of going down to bedrock without the use of pneumatic caissons, which were deemed out of question on account of the cost of installation. The design was accordingly changed by sealing the bottom of the wells with concrete and belling out the wells in a strata of hardpan above, in order to get the necessary bearing area.

15*b*. Rodding.—In cases where the upper strata of a building site are of soft material, and it is only desired to ascertain the depth to a solid stratum to determine the probable lengths of piles or the desirability of their use, a steel rod driven into the ground will give the desired information. If the rod can be driven down only a few feet, then a test pit should be dug, or an auger boring be made, to disclose the character of the harder stratum.

15*c*. Test Pit.—The most satisfactory method of investigating soil conditions at a building site is by digging one or more test pits of convenient size at one or more locations on the lot. These can be readily carried down to a depth of 10 or 15 ft. if necessary, unless ground water interferes. Test pits actually disclose to the eye the actual soil conditions, and allow the making of load tests at different levels. The cost of such methods of investigation is higher than for auger borings, but the test pit method is always more reliable.

15d. Wash Borings.—In hard soils the auger or rod cannot readily be used and wash borings are often resorted to. These are made by driving a 2 to 4-in. pipe into the soil and keeping a jet of water working inside the pipe. The water brings the material to the surface where it must be collected and the results tabulated to show the various strata.

The objection to wash borings is that in clay soils the clay is likely to be dissolved and the fine materials separated from the coarse by the action of the water, thus making the results appear different than the actual conditions. They are much cheaper than core borings and for most soils give accurate enough results to serve the purpose.

15e. Core Borings.—Where the importance of the work makes it necessary to ascertain the depth and nature of the bedrock, core borings made with a diamond drill are used. With these drills, consisting of a cutting tool made of diamond, shot or pieces of chilled cast iron, solid cores (about 1 in. in diameter) of the rock encountered are taken out and the results and specimens tabulated.

16. Load Tests on Soil.—Where data on the safe allowable loads on soils is not available, no important building should be erected without making a load test of the soil stratum on which it is proposed to rest the footings.

For ordinary cases a post having a base 1 ft. square supporting a load box at the top will be sufficient. However, for important structures, an area greater than 1 sq. ft. should be tested so as to approximate actual conditions more closely. This requires that the test apparatus be arranged to make use of the principle of the lever so as to reduce the amount of weight required on the test platform.

In testing the carrying capacity of a soil stratum the soil should be backfilled in the pit around a cylinder through which the test post passes in order that actual conditions in the proposed building may be closely approximated. To rest the test block on a large exposed area of the soil will tend to cause the soil to squeeze up around the block due to absence of the earth above and modify the results by indicating undue settlement and lower bearing capacity.

For a detailed description of proposed standard load testing apparatus for soils, the reader is referred to the Progress Report of the Special Committee to Codify Present Practice on the Bearing Value of Soils for Foundations, etc., published in the March, 1922 Proceedings of the American Society of Civil Engineers.

17. Bearing Values of Soils of Various Characteristics.—A building foundation should never be placed directly (*a*) on loam which is a mixture of organic matter in various stages of decomposition and sand, clay etc.; (*b*) silt, peat or soft marl as are often encountered near streams or lakes. In such cases the foundation should be carried down through the soft material to a harder stratum below by the use of piles or deep piers.

Because of the great variation in the characteristics of soils, no hard and fast rules can be given for the allowable bearing values of the different kinds of soil. A little water allowed to enter a clay stratum may so alter its character as to greatly reduce its load bearing capacity and in fact the presence of water greatly modifies the bearing value of all soils and is a danger sign for the designer.

Sand as generally found consists of fine particles of silica in pure form or mixed with various proportions of silt, loam, clay, vegetable matter, minerals and water. The presence of much of the last mentioned materials, except minerals, lowers the bearing value of the sand. As long as it is held in place by surrounding earth or other barriers, sand will safely sustain loads of from 2 to 4 tons in excavations of ordinary depth, but if mixed with clay and water, adjacent excavation and pumping is likely to cause it to run and thus undermine the footing. Coarse sand will sustain a greater load than fine.

Gravel as found in natural deposits is a mixture of stones (varying in size from 1 to 12 in.) and sand, and oftentimes having other substances such as loam, silt and clay mixed with it. Coarse gravel, if free from the last mentioned materials, can be safely loaded up to 6 tons per sq. ft. if found in thick layers.

Perhaps no soil has a greater variation in its make-up than that which we call clay, it being a mixture of silica and alumina with a great variety of impurities. These impurities and the presence of different amounts of moisture give rise to a great numer of different kinds of clay and make it almost impossible to fix any bearing value except after most extensive experience or actual tests. A dry clay which will support a relatively heavy load will, when wet, carry a relatively small load, and if the underlying stratum is on a slope, the wet clay when loaded may slide out. This characteristic of clay makes it a very treacherous material to deal with and unless well acquainted with its action under load the engineer should use the utmost caution in the design of foundations to rest upon it. Dry clay in thick beds will safely carry 4 tons or more per sq. ft., but the same clay when wet can be relied upon to carry only a very small fraction of that amount because of the great tendency to settle and squeeze out under load.

In Chicago where the great majority of the buildings except sky-scrapers rest on clay, the bearing value for footings is specified by ordinance as the result of years of experience, and is as follows:

(*a*) If the soil is a layer of pure clay at least 15 ft. thick, without admixture of any foreign substance other than gravel, it shall not be loaded to exceed 3,500 lb. per sq. ft. If the soil is a layer of pure clay at least 15 ft. thick and is dry and thoroughly compressed, it may be loaded not to exceed 4,500 lb. per sq. ft.

(*b*) If the soil is a layer of firm sand 15 ft. or more in thickness, and without admixture of clay, loam or other foreign substance, it may be loaded not to exceed 5,000 lb. per sq. ft.

(*c*) If the soil is a mixture of clay and sand, it shall not be loaded to exceed 3,000 lb. per sq. ft.

Hardpan is the name given mixtures of clay, sand and gravel usually found at considerable depths below the surface, some of them so hard that they can only be removed in small pieces by picks or chisels which require frequent sharpening. Where there is no danger of undermining, hardpan in thick beds can be safely loaded to 8 or 10 tons per sq. ft.

Rock in its natural bed, when not decayed, will safely carry almost any load. In fact the load applied will be limited by the strength of the concrete in the footings, except where the rock strata are inclined or likely to slide on their base. Soft or decayed rock will not support a greater load than gravel.

18. Allowable Foundation Loads.—As a summary of good practice the following allowable loads per square foot for various soils are given:[1]

[1] New York Building Code, 1916.

Wet clay	1 ton
Wet sand	2 tons
Firm clay	2 tons
Sand and clay mixed or in layers	2 tons
Fine and dry sand	3 tons
Hard dry clay	4 tons
Coarse sand	4 tons
Gravel	6 tons
Soft rock	8 tons
Hardpan	10 tons
Medium rock	15 tons
Hard rock	40 tons

19. Relative Allowable Pressure on Soil.—The bearing power of various soils has been discussed in a general way in Art. **17**, but in actual foundation design for a building it will be necessary to modify or discount these values in accordance with the height and character of loading of the building. That is, for the same soil conditions a lesser bearing value should be used for a high building than for a low (one or two-story) building, since any inequality in the loading of the building, or in the bearing value of the soil, might result in a serious overload on the soil in that section and possible failure due to unequal settling of the high structure, while the low one under the same circumstances might be little harmed. Then again, the relative bearing value of the same soil should be taken as less for a large multi-storied warehouse designed for heavy floor loads, than for a low (three- or four-story) building with comparatively light floor loads. Briefly, this means that any table of bearing values for various soils should be used with a great deal of discretion and modified to correspond with what experience has taught to be safe values for any certain district.

If the soil on which the footings are to rest is likely to flow under load, or be disturbed by other foundation work adjacent, or by seepage or drainage into sewers, special precautions should be taken to retain the soil. This is especially necessary where the foundation bed is of wet sand which might be pumped out in keeping the water out of excavations. Lines of sheet piling of concrete, steel, or wood, driven down to a depth below which any subsequent excavation is likely to go, will usually give the desired results.

For the footings of buildings without basements the footings need only be carried down below the maximum frost line. This rule holds good for footings on solid rock as well as in earth, since much damage can be done by the freezing of water which may find its way into fissures in the rock under footings. After getting below the frost line, in general it will not be economical to excavate deeper unless a soil of greater bearing capacity can be found at such depth as will make the saving in concrete and steel in the footing, due to the lesser area, greater than the extra cost of excavation. However, in compressible soils, such as various clays and in wet sands, the footings should be carried down below the line of possible danger of disturbance or lateral displacement of the soil by adjacent building operations, or to such depth that the weight of the soil above will prevent heaving at the periphery of footings under load.

The bearing power of the soil under many buildings has been improved by drainage of the foundation bed by means of lines of drain tile laid adjacent to the exterior or wall footings and slightly below the bottom of these footings. In this way any ground or surface water, which may find its way to the level of the bottom of the footings and tend to lower the bearing value of the soil by the attendant softening thereof, is conducted a way from the foundations. Where such drains are laid in sand, the joints of the tile should be carefully wrapped with burlap to prevent the entrance of the sand, for the movement of the sand might undermine the footings.

Sometimes heavy layers of sand or gravel have been placed in bottoms of excavations in poor soils in order to improve the bearing capacity, but this method is of extremely doubtful value if the undersoil is soft, for when put under load the tendency is for the added material to squeeze into the natural soil and so cause settlement. The better method of increasing the allowable bearing on compressible foundation beds such as clays, is to drive short piles as close to each other as possible over the foundation area, thereby compressing the soil and raising its bearing power.

20. Proportioning Footings.—The aim in all footing design where the foundation bed is at all compressible is to have the unit bearing pressure as nearly uniform under various conditions of loading as is possible, in order that the settlement, if any, may be uniform. The present-day methods of monolithic reinforced concrete construction greatly reduce the possibilities of unequal settlement since the strength of the connecting units—columns, slabs, beams, girders, or all combined—act as a stiff framework, transferring some of the load to adjacent column footings, where one footing tends to settle unequally, thus relieving the situation.

To have the bearing pressure under all footings uniform or very nearly so is impossible of attainment under all conditions of loading because of the fact that the interior columns carry a greater percentage of live load than the exterior columns. The problem, therefore, resolves itself into approximating equality of soil pressure by making the footing areas proportional to loads which include only a part of the live load—say 50 per cent or less, or none of the live load—depending on the character of the occupancy. This results in relatively larger footing areas for exterior columns (for a given bearing value) as compared with the interior, than would be the case if the full live load were considered in proportioning the areas of footings.

The loads to be considered on building foundations are: (1) The dead load of the building; (2) the live or movable loads to which the floors may be subjected; and (3) the wind loads. The latter loads are in general neglected on buildings having a width as great or greater than the height, or where the side walls are protected by other buildings. For very narrow and high buildings it is essential that the wind loads be considered, for in buildings only two or three bays wide the loads on the leeward footings may be considerably increased by wind loads and unless they are proportioned accordingly, unequal settlement is very likely to occur.

The dead load, or the weight of the structure itself including walls and partitions, can be readily computed. The maximum allowable live load can also be readily computed, but it is evident that a building very seldom carries the maximum allowable live load over the entire area of each floor at the same time.

Aisle spaces, unloaded areas, and partially loaded areas generally considerably reduce the actual live loads on the various floors, and it would therefore be wrong to proportion the columns and footings for the full allowable live load for which the floors may be designed.

The usual practice in building design, therefore, is to design the floor slabs for the full allowable live load per square foot, the girders for 85 per cent of this allowable, or assumed live load, and a further reduction is made on the amount of live load carried by the building columns. The reduction of live load for columns varies with different city ordinances, the idea in general being to design the columns as nearly as possible for the probable actual loads they will receive owing to the conditions mentioned in the preceding paragraph.

The Chicago Building Code fixes the amount of live load to be used in computing the live load carried by the columns as follows: For the roof, full live load; for the first floor below the roof, 85 per cent of the live load; and for each succeeding floor below, a further reduction of 5 per cent until a reduction of 50 per cent is reached, after which no further reduction is allowable. Not considering the roof load—as this is usually only 25 lb. per sq. ft.—the above reduction formula gives the live load carried to the footing as 67.5 per cent of the total allowable live load (based on the assumption that the allowable live load on all floors is the same).

Other column load reduction formulas are used, the more common being that recommended by the National Board of Fire Underwriters, namely:

In buildings more than five stories in height, the following reductions are permissible: For columns supporting roof and top floor, no reduction; for columns supporting each succeeding floor, a reduction of 5 per cent of the total of live load per floor may be made, but the total reduction shall not exceed 50 per cent.

No reduction of live load on columns shall be permitted in buildings where the assumed floor load is more than 120 lb. per sq. ft. and is likely to be permanent in character, as in warehouses, printing houses, machine shops, etc.

For structures carrying machinery, such as cranes, conveyors, printing presses, etc., at least 25 per cent shall be added to the stresses from live loads to provide for impact and vibrations.

These two latter requirements of the Underwriters' Code seem a little severe and increase the cost of a building considerably by the greater size of columns required.

The lower story column live load, as arrived at by the above reduction formulas, is the load for which the maximum stresses in the footing should be computed. For interior column footings this load should be used together with the dead load to find the area of footing required, using the maximum allowable bearing pressure on the soil in question.

After having found the footing area required, divide the sum of the dead load and 50 per cent of the lower story column live load (as reduced) by said area, and a new bearing value be will found. Now take the exterior column dead load plus 50 per cent of the lower story column live load (as reduced), and divide by the new bearing value found above. The result will be the area for exterior footings. In computing stresses in the exterior footings, however, the lower story column live load as reduced (not 50 per cent of said live load), plus the dead load, should be used.

Illustrative Problem.—Determine column footing areas for a three-story and basement building with floor loads of 200 lb. per sq. ft., roof load of 25 lb. per sq. ft., and panels 20 × 20 ft. Column loads are to be reduced in accordance with the Chicago Building Code. Maximum allowable soil pressure is 4,000 lb. per sq. ft., neglecting weight of footing.

Interior column load:

Dead load	193,000 lb.
Live load	202,000 lb.
Total	395,000 lb.

Dead load plus 50 per cent of live load = 294,000 lb.

Exterior column load:

Dead load	143,000 lb.
Live load	106,000 lb.
Total	249,000 lb.

Dead load plus 50 per cent of live load = 196,000 lb.

Interior column footing area required $= \frac{395,000}{4,000} = 99$ sq. ft. Make footing 10 ft. square = 100 sq. ft.

Pressure on soil for dead load plus 50 per cent live load $= \frac{294,000}{100} = 2,940$ lb. per sq. ft.

Area of exterior footing required $= \frac{196,000}{2,940} = 66.6$ sq. ft. Make footing 8 ft. 2 in. square.

In computing stresses in the footing, the pressure for full live and dead load must be taken. This equals $\frac{249,000}{66} = 3,740$ lb. per sq. ft. From this it will be noted that while under total live and dead load the pressure under the interior footings is 4,000 lb. per sq. ft. Under the exterior footings it will be only 3,740 lb., which indicates that the design is such as to preclude any marked difference in settlement of the exterior and interior footings. For dead load only the pressure under the exterior footing equals 2,166 lb. per sq. ft. and under interior column footing equals 1,930 lb. per sq. ft.

In cases where the assumed live load is very small—say 40 lb. per sq. ft.—it may be well to proportion the footings for dead load only, but in general it will be better practice to include a certain amount of the lower story column live load—say 30 per cent for buildings with light loads and 50 per cent or more for buildings with heavy live loads of a more or less permanent character.

21. Wall Footings.

21*a*. Plain Concrete.—In structures of the wall bearing type—that is, those where no exterior columns are used, the floors resting directly on the exterior brick or concrete walls—one to three stories high, stepped footings of plain concrete are generally used. To find the width of footing required, the total load per linear foot (with live load reduced) as hereinbefore recommended should be divided by the allowable soil pressure. The necessary width at the bottom of the footing is then obtained by using a sufficient number of courses or steps, each one projecting beyond the one above a distance equal to one-half the thickness of the course.

21*b*. Reinforced Concrete Wall Footings.—Where the allowable soil pressure is low or the wall load great, the width of footing required is likely to be so great as to make the use of plain concrete uneconomical. In such cases a reinforced footing should be used.

The bending moment at any section of such a footing at a distance x from the end (Fig. 11) will be $M = \frac{1}{2}wx^2$, where w is the uniform bearing pressure per linear foot of footing for a given width of section. Now if l is the extreme width of footing, and a is the thickness of the wall, the bending moment at the face of the wall which is the critical section will be

$$M = \frac{1}{8}w(l - a)^2$$

In designing reinforced concrete cantilever footings for walls, special consideration should be given to the ascertaining of bond stress and diagonal tension. In computing the maximum bond stress on bars, Talbot's tests show that the total external shear at the face of the wall should be used in the formula for unit bond stress. In computing the shear for diagonal tension, however, it should be taken on a line at a distance away from the wall equal to the effective depth of the footing. In view of this fact diagonal tension is quite likely to govern in the design of footings composed of a number of steps or with a sloped top, since the depth is generally less at a distance d from the face of the wall than at the wall. As a general rule it will be found better practice, from the standpoint of design and construction, to keep down the amount of diagonal tension reinforcement by making the footing courses relatively thick. In any event the reinforcement for diagonal tension should be bent to template and proper supports provided to ensure its proper placement and location in the finished work.

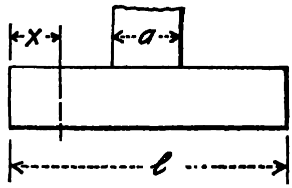


Fig. 11.

21c. Eccentric Loading.—Sometimes local conditions and lot line restrictions make it necessary to omit the footing projection on one side of the wall. This gives rise to a condition of eccentric loading with a pressure on the soil varying from a maximum on the side of the wall on which the footing is omitted, to a minimum at the other edge of the footing. Such a condition should be carefully investigated and the footing so proportioned as to keep the maximum pressure within the allowable safe load on the soil. In no case should the line of the resultant of pressure on the footing lie outside the middle third of the footing, for such a result indicates an uplift at one side of the footing.

22. Plain Concrete Column Footings.—From the results of tests and the uncertainty of the tensile strength of concrete, the best practice in the design of plain concrete footings would seem to be to so proportion them as to keep all bending stresses very low or practically negligible.

In a reinforced concrete footing the load is transmitted to the soil over its entire area by virtue of the deflection or deformation of the footing under load, while in a plain concrete footing on a hard soil or rock the load tends to distribute only over such area as lies within the base of a pyramid or cone formed by the lines of stress from the base of the column to the bottom of the footing. The general practice is therefore based on the assumption that the load is carried through the concrete at an angle of 30 deg. with the vertical. If all the projections lie outside of a line drawn at 30 deg. with the vertical from the edge of the column to the bottom of the footing, no bending stresses greater than the concrete is capable of resisting safely, will exist. The simplest form of a plain concrete

footing would therefore seem to be a pyramid or cone, but owing to the difficulty of holding the forms on footings of this shape, stepped or coursed footings are used with all projections lying outside of the above mentioned line of stress. To design a plain concrete footing on this basis, first find the area of footing required, and from one-half the width of the bottom course subtract one-half the column size, and divide the result by the tan 30 deg. This will give the required height of footing. Then divide the footing into as many vertical steps as desired keeping the projections entirely outside of a 30-deg. line with the vertical from the edge of the footing to the edge of the column. If this method is followed, the safe punching shear value of 120 lb. per sq. in. will never be exceeded in the footing.

In stepping off plain concrete footings the steps should be at least 12 in. high and preferably more. The area of the top course should be such as will allow the maximum bearing value on the footing concrete directly under the column base.

For footings on rock or on soil capable of sustaining relatively high unit loads, plain concrete footings should be used rather than reinforced concrete since, owing to the unyielding character of the foundation, the reinforced concrete footing could not act as designed.

Where excavation must be carried to a considerable depth below the ground floor line, it will often be found more economical to use plain concrete footings since no saving can be made in excavation, which generally makes for economy in reinforced footings. Then also, the footing concrete will usually be cheaper than the extra length of reinforced concrete column required if reinforced footings without plinth blocks are used.

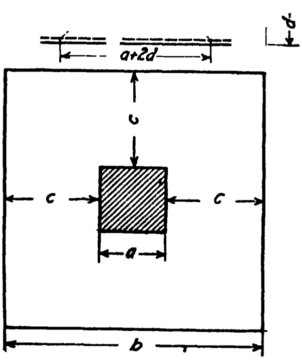


Fig. 12.

23. Advantages of Reinforced Concrete Footings.—Except in certain cases as mentioned above where plain concrete footings can be used to advantage, reinforced concrete footings will in normal times be found the most economical, since a saving in excavation, material, and weight of the footing itself can be made.

A study made by the writer a few years ago as to the relative cost of various shapes of reinforced concrete footings developed the conclusions that footings with sloping tops are more expensive to build, single course footings next, and a decreasing range of cost as more courses, for a given depth, were used in the footings.

24. Design of Isolated Column Footings of Reinforced Concrete.—The methods of design set forth by Professor A. N. Talbot of the University of Illinois in a bulletin (No. 67) of the Engineering Experiment Station as the result of a series of tests made at the University are now generally accepted as the proper ones to use in the design of the reinforced concrete footing and they will accordingly be briefly explained.

Referring to Fig. 12, the value of the bending moment in one direction is given by the formula

$$M = (\tfrac{1}{2}ac^2 + 0.6c^3)w$$

where w is the upward unbalanced pressure or soil reaction in pounds per square foot.

The width of footing in which the steel is effective in resisting moment equals

$$a + 2d + \tfrac{1}{2}(b - a - 2d)$$

The load producing punching shear equals

$$(b^2 - a^2)w$$

The unit punching shear equals

$$\frac{(b^2 - a^2)w}{4ad}$$

The load producing diagonal tension in one direction,

$$V = [b^2 - (a + 2d)^2]w$$

The critical vertical shearing stress becomes

$$v = \frac{V}{4(a + 2d)jd}$$

The bond stress

$$u = \frac{V}{mojd} = \frac{(ac + c^2)w}{mojd}$$

where m = number of bars, and o is the periphery of one bar.

The stresses recommended in connection with the above formulas are those of the Joint Committee:

f_s = 16,000 lb. per sq. in.

f_c = 650 lb. per sq. in. in 1 : 2 : 4 concrete.

Punching shear = 120 lb. per sq. in.

Shear as measure of diagonal tension = 40 lb. per sq. in.

Bond = 80 lb. per sq. in. for plain bars and 100 lb. for deformed bars.

Illustrative Problem.—Design an isolated footing to support an interior column 24 in. in diameter.

Lower story D.L. + L.L. = 375,000 lb.

D.L. + 50 per cent of L.L. = 280,000 lb.

Allowable pressure of soil, neglecting weight of footing = 4,000 lb. per sq. ft.

$$\text{Footing area required} = \frac{375{,}000}{4{,}000} = 93.75 \text{ sq. ft.}$$

Use footing 9 ft. 8 in. square.

The load producing punching shear equals

$$\frac{93.75 - 3.1}{93.75}(375{,}000) = 360{,}000 \text{ lb.}$$

$$\text{Depth for punching shear} = \frac{360{,}000}{(75.4)(120)} = 39.7 \text{ in.}$$

Use a 40-in. depth to steel with 4 in. below center of steel, making the total depth 44 in.

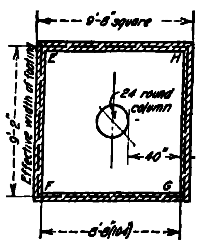


FIG. 13.—Load area producing shear.

The shear as a measure of the diagonal tension is measured at a distance from the face of the column equal to the depth of footing to center of steel, or 40 in. The area between the square formed by lines 40 in. from the face of the column and the edge of the footing produces shear on the vertical planes through line *EFGH* (Fig. 13).

$$\text{Total shear on } EFGH = \frac{93.75 - (8.67)^2}{93.75}(375{,}000) = 75{,}000 \text{ lb.}$$

Using a unit shear value of 40 lb. per sq. in., the depth necessary at the plane in question may be found. Thus

$$d = \frac{75{,}000}{(40)(4)(104)(0.875)} = 5.15 \text{ in.}$$

This is less than the minimum of 12 in. recommended for the thickness at the edge of the bottom course and hence the footing will not fail by diagonal tension.

The area of the top of the footing on which the column bears should be at least twice the area of the column, in which case a bearing of 600 lb. per sq. in. is allowable under Chicago Ordinance and 700 lb. per sq. in. under Joint Committee Rules. The balance of the load in the column must be transmitted to the footing by stub bars embedded for one-half their length in the footing. For the footing in question the top area should be at least 6.28 sq. ft. For practical purposes the projection beyond the column face should be at least 6 in. so that the column forms can be rested thereon without difficulty. This means that the top of the footing will be 3 ft. square. If sloping sides are used, they should be sloped from the 3-ft. square line to a line giving at least 1-ft. total depth of footing at edge. This will meet all shear requirements, as the above computation shows. If a stepped footing is used, a good practical rule to follow is to keep the steps outside of a line drawn from the center of the column to the edge of the base in the plane of the reinforcement (Fig. 14). The outline of a sloped footing is also shown in Fig. 14.

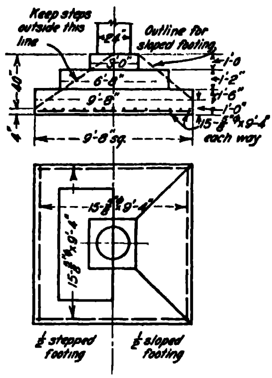


FIG. 14.—Design of isolated footing.

Testing for shear at the face of the next to bottom step for the stepped footing we have

$$\frac{\frac{93.75 - (6.67)^2}{93.75}(375{,}000)}{(4)(80)(0.875)(18)} = 39 \text{ lb. per sq. in.}$$

This indicates that the outline of the footing is such as will not require the use of stirrups to take care of the shear in the concrete. Except where local or special conditions demand it, footings should be designed so as to avoid the use of stirrups.

The bending moment in each set of rods is

$$M = [(1)(3.83)^2 + (0.6)(3.83)^3](4{,}000)(12) = 2{,}483{,}200 \text{ in.-lb.}$$

The area of steel required equals

$$A_s = \frac{2{,}483{,}200}{(16{,}000)(0.875)(40)} = 4.45 \text{ sq. in.}$$

The effective width of footing equals

$$24 + [(2)(40)] + 6 = 110 \text{ in.} = 9 \text{ ft. } 2 \text{ in.}$$

Inasmuch as no bars should be placed closer than 3 in. from the edge of the footing, they will be effective in resisting bending if placed in a band 9 ft. 2 in. wide. For the area of steel required in each band, fifteen ⅝-in. round bars will be ample.

The bond stresses on each set of bars

$$u = \frac{(360{,}000)(0.25)}{(1.96)(15)(0.875)(40)} = 87.5 \text{ lb. per sq. in.}$$

If deformed round bars are used with an allowable bond stress of 100 lb. per sq. in., the number and size of bars as selected will be satisfactory, while if plain bars are used, a larger number of smaller sized bars will be required to keep the bond stress within the allowable 80 lb. per sq. in. The details of the footing are shown in Fig. 14.

25. Rectangular Isolated Footings.—Lot line restrictions and the location of machinery foundations etc., often make necessary the use of rectangular isolated footings. The design of such footings follows the same procedure as for square isolated footings except that the bending moments produced on planes at right angles to each other are different, owing to difference of cantilever length.

Where the length of the footing is not greater than 50 per cent more than the breadth, the moments at the different sections (Fig. 15) are

$$M_{(1-1)} = (\tfrac{1}{2}\, ac^2 + 0.6c^2b)w$$
$$M_{(2-2)} = (\tfrac{1}{2}\, ab^2 + 0.6cb^2)w$$

Where the length exceeds the breadth of footing by more than 50 per cent, as shown in Fig. 16, the moments are

$$M_{(3-3)} = \left(\frac{wb^2}{2}\right)g$$

$$M_{(4-4)} = \left(\frac{ac^2}{2} + \frac{bc^2}{2}\right)w$$

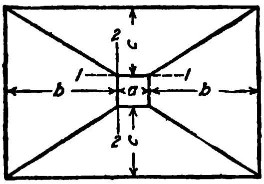


Fig. 15.

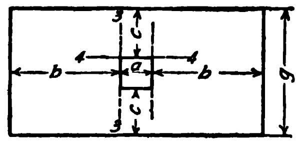


Fig. 16.

26. Combined Column Footings.—Combined column footings—that is, those where one or more exterior and interior columns are carried on a common footing—are not as economical as isolated footings, but where the exterior columns are located close to the property line, or where street restrictions do not allow the use of a symmetrical isolated footing, they must be used.

In such a footing, the exterior column in question is carried on a common footing with the adjacent interior column, the footing being of such size and shape as to give the required bearing area and to make the center of gravity of the loads coincide with the center of the upward reaction. This is essential in order that any settlement which might occur will be as nearly uniform as the character of the soil will allow, and also to avoid dangerous transverse stresses in columns. At corners of buildings it often becomes necessary due to the above mentioned restrictions, to place four columns on a combined footing, which may be solid or with a portion of the center omitted if the bearing value of the soil is relatively high.

Various shapes of combined footings can be used depending upon the relation of the column loads, the allowable projection of the footing beyond the respective column center lines, and whether or not relatively high shearing stresses are allowed. From the standpoint of economy the rectangular shaped footing with a greater thickness under columns (to take care of punching and diagonal shear) is the best. In such a footing fewer different lengths of bars are required, the transverse bending stresses are reduced to a minimum and also the amount of diagonal tension and direct moment reinforcement. The two latter reductions are possible because of the greater thickness of slab, in the same manner as the allowance for the flaring heads in flat-slab construction.

A combined footing of uniform thickness is not economical owing to the fact that the span producing moment must then be taken as the clear distance between columns, which gives a greater bending moment than if plinth blocks are used under the columns. Then also if the thickness of the footing satisfies the shearing stress requirements, the depth for moment will be excessive, or vice versa. A combined footing of uniform thickness should not be used except for small footings or where local conditions require.

The following example illustrates the design of a rectangular combined footing:

Illustrative Problem.—Design a combined footing to carry an interior column and an exterior column (19 ft. c. to c.) with loads as follows:

Column	Dead Load	Live Load	Total
Interior..........	193,000	202,000	395,000
Exterior............	143,000	106,000	249,000

The interior column is 24 in. in diameter and the exterior 2 × 3 ft., allowable load on soil 4,000 lb., weight of footing being neglected.

The area of footing required for the interior column $= \frac{395,000}{4,000} = 99$ sq. ft. Use 100 sq. ft.

With a footing of this size the pressure on the soil for dead load plus 50 per cent of the live load carried by the column $= \frac{294,000}{100} = 2,940$ lb. per sq. ft.

The exterior column footings should be proportioned for the dead load plus 50 per cent of the live load, hence the bearing value of 2,940 lb. should be used in determining the area required which equals $\frac{196,000}{2,940} = 66.6$ sq. ft.

Thus the total area required for an interior and an exterior column equals 166.6 sq. ft. Now the center of gravity of the combined footing must coincide with the center of gravity of the loads, and since the latter is located at $\frac{(294,000)(19)}{490,000} = 11.4$ ft. from center of exterior column (by taking moments about center of exterior columns), the footing must be $2(11.4 + 1) = 24.8$ ft. or 24 ft. 9½ in. long. Use 24 ft. 10 in in computations. The width required equals $\frac{(100 + 66.6)}{24.8}$ or 6.72 ft. Use a width of 6 ft. 9 in.

Since the total load on the interior column is 395,000 lb. and on the exterior column is 249,000 lb., the average pressure on the soil for full live load will be

$$W = \frac{644,000}{166.6} = 3,860 \text{ lb. per sq. ft}$$

This pressure will be used in the detail design of footing. In this, as in the other examples, the maximum allowable soil pressure has been assumed at 4,000 lb. per sq. ft. and the weight of footing neglected in the computations. In other words, the soil has been assumed to be capable of bearing the additional load due to the weight of the footing.

The negative moment due to the overhang of the footing beyond the center of interior column 1 will reduce the maximum moment in mid-portion of the footing and amounts to

$$M_{(1-1)} = (6.75)(4.83)\left(\frac{4.83}{2}\right)(12)(3,860) = 3,640,000 \text{ in.-lb.}$$

Maximum moment between columns 1 and 2 $= \frac{wl}{8} - \frac{1}{2}M_{(1-1)} =$

$$(3,860)(6.75)(19)^2(^{12}\!/_8) - 1,820,000 = 12,276,000 \text{ in.-lb.}$$

Depth required for moment

$$d^2 = \frac{M}{Kb} = \frac{12,276,000}{(108)(6.75)(12)} = 1,405 \qquad d = 37.6 \text{ in. say 38 in.}$$

The depth required for punching shear at interior column

$$d_1 = \frac{395,000 - (3.14)(3,860)}{(75.4)(120)} = 42.5 \text{ in.}$$

At exterior column (only three faces of column effective in punching shear)

$$d_2 = \frac{249,000 - (6)(3,860)}{(84)(120)} = 22.5 \text{ in.}$$

Taking into consideration the depths required for moment and punching shear, a depth of 38 in. to center of steel (42 in. total) will be used for the main footing slab and an extra block of concrete (6 in. thick and 6 ft. wide with a length equal to the width of footing: will be used under column 1.

Using the depth of 38 in. the area of steel required in the bottom of the slab at column 1 equals

$$A_s = \frac{3,640,000}{(16,000)(0.875)(38)} = 6.85 \text{ sq. in.}$$

Use sixteen ¾-in. rounds = 7.06 sq. in.

$$\text{Bond stress } u \quad \frac{(3,860)(4.83 - 1)(6.75)}{(16)(2.35)(0.875)(38)} \quad 80 \text{ lb. per sq. in.}$$

Plain bars will be satisfactory.

The area of steel required in top of slab between columns

$$A_s = \frac{12,276,000}{(16,000)(0.875)(38)} = 23 \text{ sq. in.}$$

Use twenty-three 1⅛-in. rounds = 23 sq. in. approximately. Bend these bars down into the footing slab at the end under column 2 to anchor them and to aid in resisting the tendency of the concrete to shear off on a line *a-a*, Fig. 17.

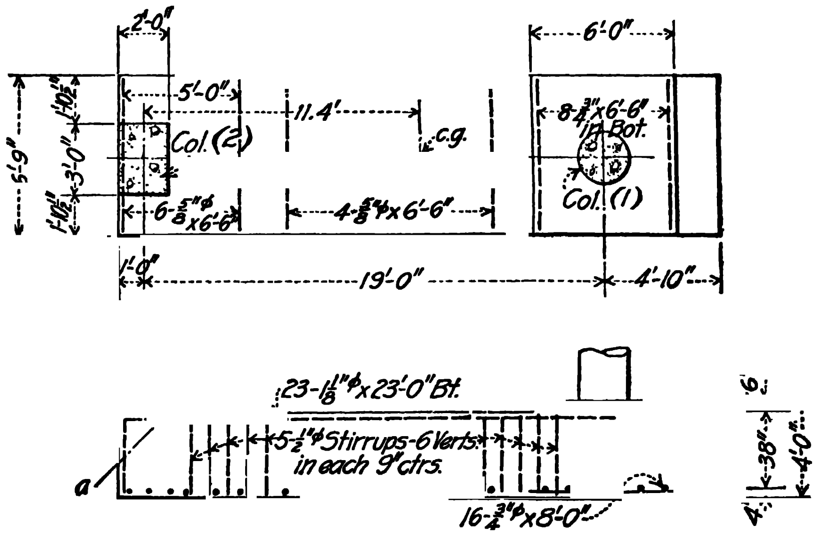


FIG. 17.

The shear at the side of columns 1 and 2 = 221,000 lb. Bond stress in bars equals

$$u = \frac{221,000}{(23)(3.53)(0.875)(38)} \quad 82 \text{ lb. per sq. in.}$$

Plain round bars will do.

The footing will now be investigated for shear. The shear as a measure of diagonal tension is taken at a line at a distance d from the face of the column, and equals (5.33) (6.75)(3,860) = 139,000 lb.

$$v = \frac{139,000}{(6.75)(12)(0.875)(38)} = 52 \text{ lb. per sq. in.}$$

This means that stirrups will have to be used in the footing.

$$A_s \text{ for stirrups} = \frac{139,000}{16,000}\left(\frac{2}{3}\right) = 5.8 \text{ sq. in. in a 38-in. length of footing.}$$

Five ½-in. round stirrups with six verticals each will give 5.85 sq. in. of steel. These will be spaced at 9 in. centers at each column.

The transverse bending in the footing should next be investigated.

At column 1:

$$M = \left(\frac{395,000}{2}\right)\left(\frac{6.75 - 2}{6.75}\right)\left(\frac{2.37}{2}\right)(12) = 1,975,000 \text{ in.-lb.}$$

$$A_s = \frac{1,975,000}{(16,000)(0.875)(44)} = 3.2 \text{ sq. in.}$$

Use eight ¾-in. round bars in a width of 6 ft., allowable effective width = 2 ft. plus 7 ft. 4 in. or 9 ft. 4 in.; 6 ft. is used since this is the width of a footing of 44-in. depth.

$$u = \frac{53,500}{(8)(2.35)(0.875)(44)} = 74 \text{ lb. per sq. in.}$$

Plain bars will be satisfactory.

At column 2:

$$M = \left(\frac{249,000}{2}\right)\left(\frac{6.75 - 2}{6.75}\right)\left(\frac{1.87}{2}\right)(12) = 983,600 \text{ in.-lb.}$$

$$A_s = 1.85 \text{ sq. in.}$$

Use six ⅝-in. rounds in a width of 5 ft. (column plus depth of footing = 62 in.).

Place four ⅝-in. rounds, as distributing bars, in the remaining distance between the two bands of transverse reinforcement above computed.

27. Cantilever Footings.—A combined footing is sometimes erroneously called a cantilever footing, since in the former the center of gravity of the column loads coincide with the center of gravity of the upward reaction, while

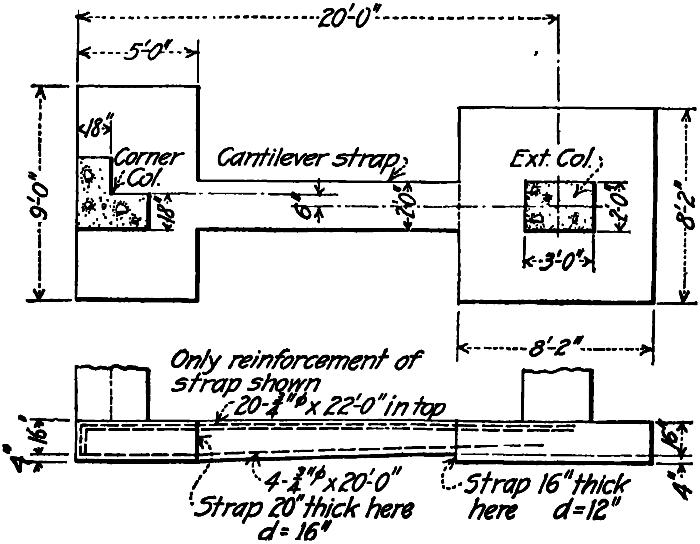


FIG. 18.

in the latter type two separate footings are tied together by a strap (see Fig. 18) which transfers the uplift caused by placing one column eccentric with the center of reaction of its footing. Cantilever footings are often used where property and street line conditions do not allow the placing of isolated footings and the footing area required would make a combined footing very narrow.

In designing a cantilever footing, the uplift caused at the footing (which is not eccentric with its column) is found by multiplying the full live and dead load on the exterior column (which is eccentric with respect to its footing) by the eccentricity of the latter footing, and dividing by the distance between the centers of gravity of the two footings. The exterior footing should be designed for the basement column live and dead load plus the amount of the uplift computed, in the same manner as any isolated footing, while the footing which is not eccentric with its column should be designed for its full basement column live and dead load neglecting the uplift.

The strap connecting the two footings and thus transferring the uplift must be designed to resist the bending moment produced in it by the eccentricity of the one column and the shear produced at the non-eccentric footing which counteracts the tendency of uplift. The bending moment equals the eccentric column load times the eccentricity with respect to footing. The width of strap to resist shear caused by uplift equals the uplift divided by the product of the effective depth of strap times the allowable shearing stress.

To eliminate stirrups a shearing stress of 40 lb. per sq. in. should be used in design of the strap. The bars, to resist the bending in the top of the strap, should be bent into hook form at the end under eccentric columns, to anchor the bars and also to provide extra shear reinforcement owing to the location of the column at the edge of the footing. In construction the strap beam should be built so as not to bear on the foundation bed which would complicate the action of the footing. This requirement can be met by excavating below the line of the bottom of the strap and building a bottom form to support the concrete. The form can later be removed and boards placed along the side of the strap to prevent earth filling in under it.

A typical design for a cantilever footing is shown in Fig. 18.

28. Continuous Footings.—There are in general two main classes of continuous footings, namely: (*a*) Those continuous in one direction under a row of columns; and (*b*) those continuous in two directions at right angles to each other.

Where they are continuous between columns in one row only, as is often the case for wall columns where it is necessary to keep the projection beyond the building to a minimum owing to building code or property line restrictions, they are usually called *continuous* footings; while if they cover the entire lot, or are composed of several strips at right angles to each other built monolithic and supporting all of the columns, they have generally been called *raft* footings. This latter name has been applied since such footings should be used only where the bearing power of the soil is very low and the function of the footing is literally to "float" the building on a raft covering the entire, or a large percentage of the ground area occupied by the building. They may also be used where the soil conditions require pile foundations, and the building loads are so heavy as to require a large number of piles which make it necessary to cover practically the entire building area with a "raft."

When soil conditions seem to warrant the use of continuous footings, the engineer should, by very careful study and tentative design, determine whether or not this type will be more economical and satisfactory than pile foundations, the next logical choice. Where investigation of the soil conditions shows a soil strata of greater carrying capacity, underlying the one in which the footings would be placed under ordinary circumstances, at a depth not to exceed 25 or 30 ft. below the latter, and there is any danger of the upper strata being disturbed or settling materially due to adjacent building operations, it will usually be found more economical and a more stable foundation secured by using concrete or wood pile foundations or circular piers carried down to the solid stratum in wood-sheeted wells.

Continuous footings are best adapted to clay soils of low bearing power where the tendency to unequal settlement due to unequal loading of various parts of the building can be counteracted by tying the entire structure together as a

large box, thus making adjoining portions of the footing aid the overloaded one in carrying the load. The action of such a footing under load is analogous to what tests show happens in a flat slab floor under unequal loading.

In cases where it is necessary to load the entire foundation area, the footing slab can be designed as a flat slab floor. In designing, however, it will be well to guard against undue bending stresses in the exterior columns produced by eccentricity of loading if the slab or mat does not project beyond the building lines sufficiently to balance the load. It should be remembered, however, that,

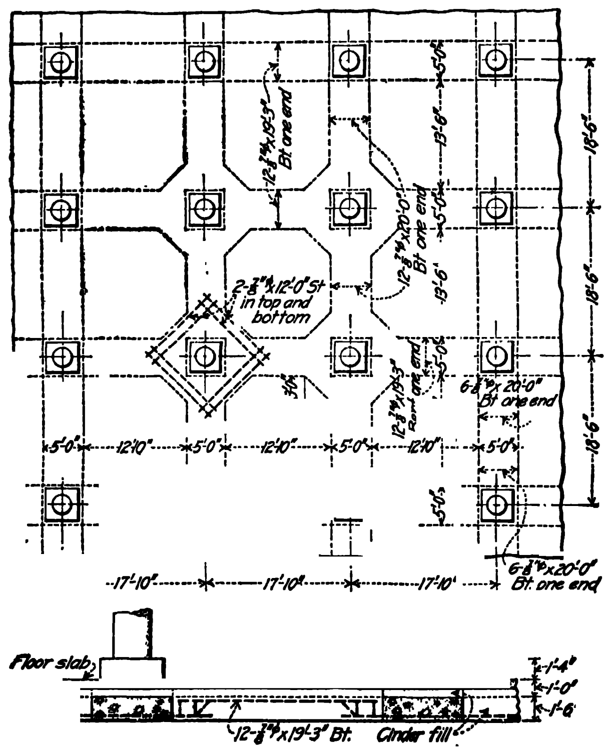


Fig. 19.

where the exterior columns act as buttresses or supports for the basement walls, the pressure against these will relieve or counteract a certain amount of the bending induced by the eccentricity of the footing load.

Instead of using the ordinary sloping column head, so common in flat slab construction, as a base for the column resting on the footing slab, it will be found more economical to use a square or oblong pedestal consisting of one or more courses of sufficient size and depth to meet the requirements of accepted flat slab design. For cases where the column spacing is over 20 ft., it may be found more economical to use a paneled slab for the mat—that is, one with greater thickness for the portions of the slab containing the main reinforcing running direct between columns.

A considerable saving in concrete may thus be effected by such footing slabs which should be designed by the same methods as used for "paneled ceiling" construction.

An example of a continuous footing supporting all columns of the building on several strips of reinforced concrete slabs at right angles to each other is shown in Fig. 19. Note the method of obtaining additional bearing area for columns supporting the sprinkler tank in addition to the regular building loads.

29. Pile Foundations.—The use of piles to support building foundations is usually resorted to (*a*) when a stratum of soft soil of from 10 to 30 ft. in thickness intervenes between the surface and a hard or unyielding stratum below into which the piles are driven; and (*b*) when no solid stratum can be reached at a reasonable depth and piles are driven deep enough into the soft material to develop a certain load carrying capacity by skin friction of the pile in the ground.

29*a*. Wood Piles.—Until recent years wood or timber piles were used exclusively in construction work but the rapid advances made in concrete practice have brought in the concrete pile which is now used very extensively. The kinds of timber best adapted for use as piles in building construction where permanence and durability are essential are: Oak, longleaf pine, Douglas fir, cedar, chestnut, redwood and cypress. Wood piles should be at least 6 in. in diameter at the point and 10 in. at the butt for piles not over 25 ft. long, and at least 12 in. for longer piles, with a uniform taper from butt to tip. A line drawn from the center of the butt to the center of the tip should lie within the body of the pile. Piles should be cut from sound trees when the sap is down, be peeled soon after cutting, the knots trimmed off, and should be close grained and solid, free from defects, such as injurious ring shakes, large and unsound or loose knots, decay or other defects. For further specifications for wood piles the reader is referred to the Manual of the American Railway Engineering Association.

For ordinary construction the length of piles used varies from 20 to 40 ft. depending on the character of the soil and the loads to be carried. When it is necessary to drive piles to a greater depth than 40 ft., splicing of piles must be resorted to and this requires great care to insure proper results.

29*b*. Pile Driving.—Piles can be driven by three different methods, namely: (*a*) By drop-hammer driver, a heavy weight raised in the leads and then dropped; (*b*) by steam-hammer driver, an improvement over the drop-hammer in that the hammer is automatically raised a comparatively short distance by steam pressure and then released and forced down by a combination of gravity and steam pressure; (*c*) by the use of a water jet to aid in displacing the earth at the foot of the pile and to lessen the friction on the pile as it is driven down by a drop or steam-hammer.

The steam-hammer has so many advantages over the drop-hammer driver that practically all piles in the larger cities are now driven by steam-hammers, the main advantages being: Reduced cost of driving, ease of guiding piles while driving, quicker driving, pile kept in motion thus reducing frictional resistance, and piles not broomed and split.

29*c*. Concrete Piles.—Concrete piles as used at the present time can be divided into two classes: (*a*) *Pre-cast*, and (*b*) *molded in place*. The former should always be reinforced and the latter are, in general, only when they are of more than ordinary length. Concrete piles are not dependent on ground water

level for durability as is the case with wood piles, and they have the further advantage of greater load-carrying capacity, carrying from 20 to 50 tons as compared with 10 to 20 tons for wood piles. This means that fewer piles, if made of concrete, are required to carry a given load. The excavation necessary for footing capping the piles is reduced for concrete piles since they can be capped above ground water level. This fact and the smaller footings necessary to spread the load over a smaller number of piles mean more rapid construction.

Pre-cast piles are of various shapes and types (some patented) and reinforced in different ways with longitudinal bars stayed by hoops or continuous spirals. They are cast in molds in a yard or at the site, allowed to cure for about 30 days and then driven like wood piles. The New York City Code requires that "the pile shall be not less than 8 in. at the bottom and not average less than 12 in. in thickness; shall not contain more than 4 per cent of steel reinforcement; that the length shall not exceed 20 times the average thickness, if driven to rock, nor 40 times if not driven to rock. When driven to rock the allowable load shall not exceed 500 lb. per sq. in. of concrete per average cross-section, and 5,000 lb. per sq. in. on the steel longitudinal reinforcement. When not driven to rock, the carrying capacity is determined by test."

The Chicago Code requirements for concrete piles are:

(*a*) Where concrete piles are used test piles shall be driven and loaded under the general direction of the Commissioner of Buildings.

(*b*) The allowable compression of concrete piles shall not exceed 400 lb. per sq. in. at a section 6 ft. from the surface of the ground in immediate contact with the pile.

(*c*) These tests shall conform to the following regulations: Tests shall be made on at least two piles in different locations and as directed by the Commissioner of Buildings; not less than three piles to be driven for each test; the pile to be loaded to be driven first, the second pile to be driven within 6 hr. of the driving of the first; the third pile to be driven within 20 to 24 hr. after the first. The two latter shall each be driven with centers not to exceed twice the greatest diameter of pile, from the center of the test pile.

(*d*) The tests shall not be started until at least 10 days after the piles to be loaded are driven, except that piles that have been cast and set up before driving may be tested as soon as practicable after driving. The piles shall be loaded with twice the proposed carrying load of the piles.

(*e*) The settlement shall be measured daily until a period of 24 hr. shows no settlement.

(*f*) One-half of the test load shall be allowed for the carrying load, if the test shows no settlement for 24 hr. and the total settlement has not exceeded $\frac{1}{100}$ in. multiplied by the test load in tons.

Molded-in-place piles are in general patented. One of the best known types is the Raymond pile which is formed by driving down a sheet steel shell by means of a mandrel and then withdrawing the mandrel and filling the shell with concrete. The permanent shell tends to prevent squeezing in of the earth while concrete is being placed. These piles may be reinforced if desired.

The Simplex pile is formed by driving down a closed steel pipe and withdrawing it as concrete is forced out at the bottom. The Pedestal pile is similarly made with a pedestal or footing of concrete at the bottom formed by driving the concrete out of the bottom of the drive pipe and at the same time compressing the earth at the end of the pile.

The reader is referred to the volume on "Foundations, Abutments and Footings" for a more detailed discussion of pile foundations.

29d. Safe Loads—Wood Piles.—In general, the maximum load allowed on wood piles is 20 tons. This is the maximum allowed by the New York Code, while the Chicago Code allows a maximum of 25 tons on a timber pile, but requires that the actual allowable safe load shall be determined by the Wellington formula which is

$$P = \frac{2wh}{s + 1} \text{ (drop-hammer driver)}$$

$$P = \frac{2wh}{s + 0.1} \text{ (steam-hammer driver)}$$

where P is the safe load in pounds, w the weight of hammer in pounds, h the fall of the hammer in feet, and s the penetration or sinking under last blow measured in inches.

Since the carrying capacity of piles must be determined by test, a tentative design must first be made allowing a certain load, say 20 or 25 tons per pile, and this must be revised in accordance with the driving records for piles in each footing. Sometimes this necessitates radical changes in the design as the work progresses.

29e. Footing Caps for Piles.—Footings on piles may be of the same general types as the plain and reinforced concrete footings previously described and illustrated, the difference in design being that instead of assuming a uniform bearing of the soil over the area of the footing, the reaction of each pile is considered as a concentrated load equal to the safe allowable load for the pile.

In figuring the load to be carried on the piles some building ordinances allow a further reduction in live load over the regular column live load reduction. Thus the Chicago Ordinance allows the use of the following percentages of the basement column live load to be used in computing the number of piles in footings:

For manufacturing, storage and mercantile buildings, 75 per cent.
For hotels, clubs, hospitals, residences, and apartment buildings, 50 per cent.
For churches, public halls, theaters, and schools, 25 per cent.

The majority of building codes require that the concrete footing capping the piles be carried down 6 in. below the top of the piles and that this concrete be neglected in computing the strength of the footing. Then, also, the spacing of the piles is usually limited to 2 ft. 6 in. for wood piles and 3 ft. for concrete piles, except where driven in staggered rows, when the spacing of rows may be reduced 2 or 3 in. for wood and concrete piles respectively. For wood piles, the cut-off line, or top, should be below the natural ground water level to avoid decay. Where the meeting of this requirement means added expense, concrete piles should be used.

In Fig. 20 are shown the most economical arrangements of piles and shapes of footings for same for various numbers of piles required.

In designing the footing, take the basement column load, reduce this if allowed by code (*viz.:* Chicago Code requires only 75 per cent of basement column live load as computed to be considered in determining the number of piles), and divide this load by the allowable load on one pile, say 20 tons. This

will give the number of piles required and the size and shape of footing can be determined using the pile spacings above mentioned and the diagrams in Fig. 20. The depth of footing required for punching shear at the periphery of the column should then be determined using the full basement column load and deducting the reaction of the pile or piles coming directly under the column. The depth required for the different courses, if the footing is stepped, can be found in the same way considering the course above in the same manner as for the column resting on the top course. The bending moment at the face of the column or at the edge of the top course can next be determined by considering only the reaction of piles within diagonal lines drawn from the center of the column to the corners of the footing. Where such diagonals cut piles, assume only one-half the reaction of that pile effective in producing moment in the quadrant in question.

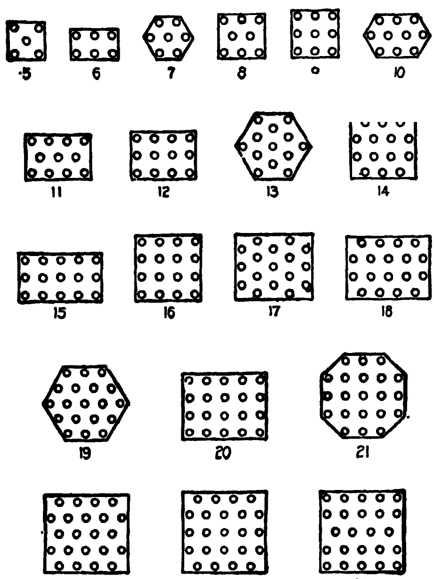


Fig. 20.—Arrangement of piles and shapes of footings.

30. Cofferdams.—In the placing of foundations where sand and water are to be encountered, cofferdams of sheet piling are used where the depth of excavation is not over 20 ft. These cofferdams are built up of sheet piling consisting of interlocking pieces of planking, steel sheeting, or concrete sheet piling generally driven down into the soil to a depth below the bottom of the footing and braced as the excavation proceeds.

Where the conditions are very bad, a double-wall cofferdam is used with clay packed between the walls to keep out the water. Because of its ability to resist earth pressure and stand repeated driving, steel sheet piling is used very extensively for building cofferdams in foundation work. It is also more economical than wood sheeting which, for a 10-ft. excavation, should be 2- or 3-in. plank and 6- to 8-in. plank for greater depths up to 20 ft.

31. Open Wells.—The open well method of excavating for pier foundations to be carried down to hardpan or rock through loam and clay strata has been used very extensively for foundations of high buildings in Chicago, St. Louis and Kansas City.

The wells are sunk either by (*a*) driving sheet piling around a template and then excavating the soil, called the *sheet piling* method; or (*b*) first excavating a depth of 4 or 5 ft. and then setting the lagging or sheeting, called the *sheeting* method. The first method is used where soft materials or quicksand are encountered and can be used to depths of about 60 ft. with economy. The second method, commonly known as the Chicago method, owing to its extensive use in Chicago, is best adapted to clay or a material which will stand up while excavation is going on, and has been used to depths of 125 ft.

The wells are usually circular in form since it is easier to brace the lagging, but the rectangular form can be used. The smallest well in which men can work to advantage is 3 ft. in diameter and the average size is about 6 ft. in diameter. In the Chicago method the lagging is braced by circular rings of steel which are removed as the well is filled with concrete, the lagging being left in the ground.

Open well piers are more economical for buildings of six or more stories (where a soft stratum overlays a harder stratum) than piles or spread footings. In such cases the piers are belled out in a conical form with a slope of 1 horizontal to 2 vertical and a bearing of 13,000 lb. per sq. ft. is allowed on the hardpan in Chicago. Where 1: 2: 4 concrete is used in the piers, a unit stress of 400 lb. per sq. in. on the concrete is allowed by the Chicago ordinance in determining the cross-sectional area of the pier to carry the basement column load. For 1: 3: 5 concrete 350 lb. per sq. in. is the unit stress allowed.

32. Pneumatic Caisson Foundations.—In the construction of tall office or monumental buildings which should be founded on rock, soft soil, quicksand

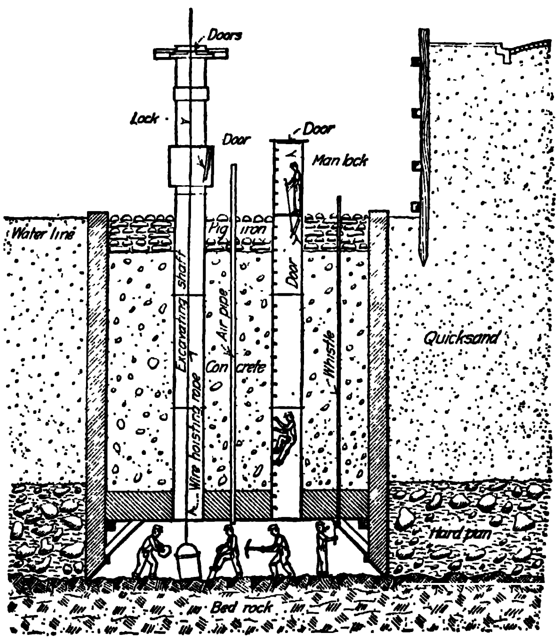


FIG. 21.—Pneumatic caisson sunk to bed rock.

and water are often encountered in the different strata as the work of excavating the pier wells to rock progresses. In such cases it is necessary to prevent the water and quicksand from being forced into the excavated well by putting the excavating chamber under air pressure and forcing the chamber down by weights or concrete put on above.

Caissons may be constructed of wood, steel, or reinforced concrete side walls and a roof to retain the air. In the larger size caissons two shafts are put through the roof with air locks near the top, one for the workers to enter and leave the caisson and the other for hoisting up the excavated material. As the material

is excavated from within the working chamber, the caisson is sunk either by weights applied or by the weight of the concrete placed above the roof which later acts as part of the concrete pier, or both, depending on the character of the soil (see Fig. 21).

To retain the earth as the working chamber is sunk, a wood, steel, or concrete cofferdam is built up on the caisson roof, these walls also serving as the form for the concrete of the pier as poured in on the caisson roof. The cutting edge of a pneumatic caisson does not have to be a sharp steel edge as commonly supposed; an oak timber on a wood caisson, or a 6 or 8-in. channel laid flat with the edges turned up on a steel or concrete caisson, form a good cutting edge. Heavy angles with one leg projecting down form a good cutting edge except when stones or boulders are encountered, in which case the angle is likely to become badly bent and useless as a cutting edge.

After the caisson reaches rock or the stratum on which the foundation is to rest, the working chamber and shafts are filled with concrete, making the pier solid.

While under normal conditions the air pressure maintained in the caisson should be sufficient to balance the earth or water pressure on the outside, there are times when the air pressure will be low so that the caisson walls must be designed to withstand the outside pressure. Then also, considerable strength is required to keep the caisson from failing due to unequal loading which occurs if it does not sink uniformly.

The concrete shaft built inside the caisson and the cofferdam above should be designed to carry the column load in the same way as given for open well piers.

BASEMENT WALLS

By Albert M. Wolf

33. Wall Supported Top and Bottom.—In basement walls where no windows occur and the span of walls between columns is greater than twice the height of walls, the most economical method of design is to consider the walls as vertical slabs supported at the top by the first floor slab and at the bottom by the footing of the wall spanning between column footings. With

p = equivalent fluid pressure of soil back of wall,
P = total pressure on back of wall,
h = height of wall,

the reactions at bottom and top of slab are:

$$R_2 = \frac{2P}{3} \text{ and } R_3 = \frac{P}{3}$$

where $P = \frac{ph^2}{2}$ (see Fig. 22).

At any depth h_1

$$M = R_2h_1 - \frac{ph_1^3}{6}$$

The maximum moment occurs at a depth equal to $0.58h$ and

$$M_{\max} = 0.064ph^3 \text{ (ft.-lb.)}$$

The slab should be designed for this moment with some negative moment reinforcing placed in the outer portion of wall at top if the wall is rigidly connected to the floor slab. Longitudinal distributing bars equal to 0.3 of 1 per cent should be provided to take care of shrinkage and temperature stresses and to support the vertical bars while concrete is being placed (see Fig. 23).

Since the wall footing must support the bottom of slab, it must be designed as a beam between column footings and to carry a load per lineal foot equal to $\frac{2P}{3}$ or $\frac{ph^2}{3}$. Where continuous for several spans, good practice is to design the beam for $\frac{WL^2}{12}$ $\left(\text{where } W = \frac{ph^2}{3}\right)$, providing the same amount of negative reinforcing in the outer part of wall footing at the column as at the inner part of wall footing at center of span.

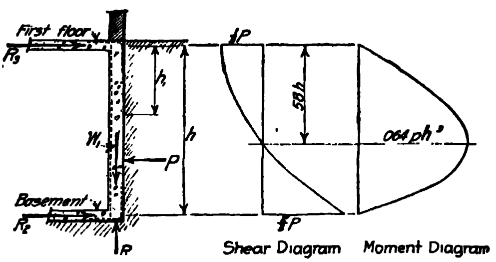


FIG. 22.

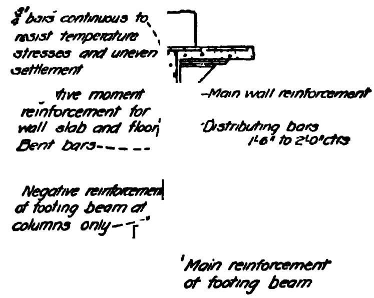


FIG. 23.—Wall supported top and bottom—wall footing acting as bottom support of slab.

34. Wall Supported by Columns.—In cases where the column footings are carried down to a considerable depth below the basement floor in order to find a suitable foundation for them, and windows are placed between first floor and top of wall, the wall can be most economically designed as a slab spanning between columns. Furthermore, if the soil at basement level is poor it may be advisable to dispense with the wall footing and support it (as a beam) at the column footings only. This requires that the wall be designed: (1) As a continuous beam between column footings to carry its own weight and (2) as a continuous slab, spanning between columns and sustaining the pressure of the earth fill.

To meet the first requirement will require only a nominal amount of steel in the bottom of the wall between footings and in the top at the columns. To find the bending moment the formula $M = \frac{WL}{12}$ should be used for intermediate spans and $\frac{WL}{10}$ for end spans, where W = the total weight of wall slab and L the clear span between footings plus 1 ft. The same amount of steel should be used for positive and negative moments. For practical reasons of placing and to resist shrinkage and temperature stresses, it is advisable to run the steel in top of wall continuously.

To find the stresses in the slab due to the earth pressure, it should be considered as divided into narrow horizontal strips of a width (measured vertically) of 1 ft. At the top of the wall the earth pressure is zero (except when the wall

supports a surcharge of earth) and at the bottom the pressure is ph. The pressure varies uniformly between these limits and the load per lineal foot of each slab strip (1 ft. high) can therefore be readily found. To find the bending moments the wall should be considered as fixed at columns and the formula $M = \frac{pL^2}{12}$ should be used for intermediate panels and $M = \frac{pL^2}{10}$ for end or corner panels.

The steel to resist the positive moments should be placed on the inside of the wall and extend into the columns. The steel to resist the negative moment at the columns should be placed near the outer face of the wall and extend at least one-quarter of the distance between columns beyond each column face.

Vertical steel to support the main or horizontal steel and to act as distributing bars should be used, spacing same at about 12 to 18 in. on centers depending on the height and thickness of wall.

With walls of this kind, the columns should be investigated to make sure that the bending induced in them by serving as the supports of a basement wall does not overstress the column section.

35. Two Way Reinforced Wall.—In high basements and with a column spacing of from 16 to 18 ft., with wall supported at the top, it will be found most economical to design the wall as a slab supported on four sides, assuming the proportion of the load carried by each span to be given by the formula

$$r = \frac{l}{b} - 0.50$$

where r = proportion of load carried by the shorter span; l = the longer span in feet; and b = the shorter span in feet.

36. Basement Wall with Railroad Track Adjacent.– In a great many industrial buildings one side of the building will be located adjacent to a railroad

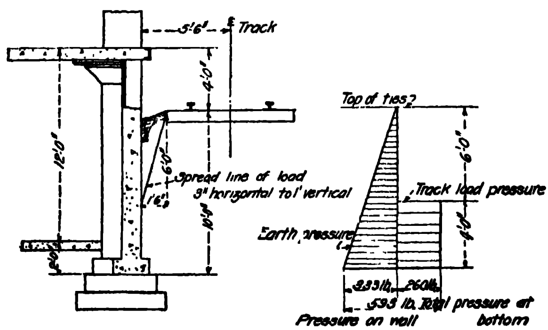


FIG. 24.—Diagram showing spread of track load.

track and in such cases the basement wall should be designed to resist the pressure of such additional load and the earth pressure. The amount of this track load producing pressure on the wall will depend upon the location of the track with respect to the face of the wall, since the usual assumption is that the spread of the track load through the earth from the ends of ties is on a slope of 3 in. horizontal to 1 ft. vertically. This means that with the track center 5 ft. 6 in. from the basement wall, almost standard for sidings, the track load would not cause pressure on the wall until a depth of 6 ft. had been reached (see Fig. 24). When the near-

est rail of track is more than $0.6h$ from the face of the wall, the effect of the track load on the wall is nil.

For Cooper's E-50 loading (most commonly used in design) the maximum load per lineal foot of rail is $\frac{25,000}{5} = 5,000$ lb. and the load per square foot with track center 5 ft. 6 in. from wall would be $\frac{5,000}{5.5} = 909$ lb. Now the live load thrust on the wall from a point 6 ft. below track level to the bottom of the wall would equal $0.286 \times 909 = 260$ lb. per sq. ft. By adding this constant live load pressure to that of the earth pressure varying from zero at ground line to a maximum at the bottom of wall, the total pressure per square foot at any point can be found and the wall designed for this pressure by the methods previously discussed.

CONCRETE COLUMNS

By Albert M. Wolf

37. Types of Columns.—Concrete columns as used in reinforced concrete buildings are of five principal types.

(1) Plain concrete columns or piers.

(2) Vertically reinforced concrete columns with vertical bars stayed or tired with hoops or ties.

(3) The most commonly used type—that is, concrete columns reinforced with spirals and vertical bars.

(4) Structural steel cores encased in concrete (not a concrete column in the strict sense of the word).

(5) Concrete columns with cast-iron cores, commonly called Emperger columns.

38. General Considerations of Column Design.—Unless otherwise specifically mentioned the formulas hereinafter given are for columns subject to direct axial compression. Now in actual building conditions it seldom occurs that a column or strut is subjected to direct compression alone but usually has some eccentric load in addition due to the unequal loading of the floors, rigidity of column and floor connections, and inaccuracies of construction. The formulas for direct compression and the limitations set on column sizes, amount of reinforcement, etc., however, place factors of safety on the design which actual practice has shown to be adequate to take care of the ordinary eccentric stresses.

For exterior columns in flat slab construction, columns supporting long span beams rigidly connected to the columns, columns supporting crane runway brackets and any other eccentrically applied loads, due allowance should be made for eccentric stresses in the design. The reader is referred to Arts. **46** and **47** of this section dealing with this subject and to the very complete discussion of bending and direct stress given in the volume on "Structural Members and Connections."

The length or rather the height of a compression member is assumed as the distance between planes at either end where lateral support is provided in at least two directions making an angle of not less than 75 deg. and not more than 105 deg. with each other. This means that in beam and girder construction where girders frame into the columns along one center line only, the unsupported length of column should be taken as the distance from top of floor slab to the underside

of the slab above. Where girders and beams frame into the columns the shallower set governs the length of column. In flat slab construction the flaring column heads stay the top of the column and the length is therefore assumed as the distance between the top of floor slab and the under side of the column capital.

The symbols used in the formulas that follow are as given in Appendix F with the following additions:

A' = loaded portion of area of member (considered as surrounded by unloaded portions).

p' = percentage of spiral reinforcement.

P' = total safe load on long column or strut.

P_c = load on concrete of composite column.

P_s = load on steel or cast iron of composite column.

r_a = permissible compressive stress on area A'.

R = least radius of gyration of net section of member or of structural steel or cast-iron core section in composite members.

The commonly used formulas for designing compression members are: For plain concrete piers, caissons, or walls,

$$\frac{P}{A} = f_c = 0.25f_c'$$

For columns or struts reinforced with ½ to 4 per cent vertical rods tied with not less than ¼-in. round rods at 8-in. centers,

$$\frac{P}{A} = f_c(1 - p + np)$$

For columns or struts reinforced with closely spaced spirals enclosing vertical rods,

Chicago Ordinance $$\frac{P}{A} = f_c(1 + 2.5np')(1 - p + np)$$

New York Ordinance $$\frac{P}{A} = f_c(1 - p + np) + 2f_sp'$$

1921 J. C. Report $$\frac{P}{A} = [300 + (0.1 + 4p)f_c'] (1 - p + np)$$

A. C. I. Specifications $$\frac{P}{A} = f_c[(1 + 4np') - p + np]$$

For composite columns or struts of structural steel and spiral-encased concrete core, the Joint Committee recommends

$$P = 0.25f_c'A + \left(18{,}000 - \frac{70h}{R} - 0.25f_c'\right)A_s$$

(The quantity in the parentheses must not exceed 16,000.)

For composite columns or struts of cast-iron and spiral-encased concrete core

$$P = 0.25f_c'A + \left(12{,}000 - \frac{60h}{R} - 0.25f_c'\right)A_s$$

(The quantity in parentheses must not exceed 10,000.)

For limiting conditions see the various specifications. For long columns and struts $\left(\frac{h}{R}\text{ exceeding } 40\right)$ the Joint Committee recommends

$$\frac{P'}{P} = 1.33 - \frac{h}{120R}$$

For loading over a portion only of the area of a column the compressive unit stress may be increased to the value given by the Joint Committee formula

$$= \frac{P}{A'} = 0.25f_c' \sqrt[3]{\frac{A}{A'}}$$

In arriving at the radius of gyration of reinforced concrete sections the reinforcement is considered to be of n times its actual area or as equivalent concrete area acting at the same centroid.

The methods of computing the loads carried by columns have been discussed in the chapter on Foundations and the reader is referred to Art. **20** therein.

39. Plain Concrete Columns or Piers.—Plain concrete columns should never be used where the height of the column exceeds four times its least dimension. This is the Joint Committe recommendation, and while some building codes allow a height of six times the least dimension, the use of plain concrete in piers of such height is not advisable. Neither is it advisable to use a plain concrete column where the load is eccentrically applied, thus causing bending as well as direct stress. In such a case the column should be reinforced with vertical bars and ties.

The Joint Committee formula for allowable unit stresses in concrete piers is

$$\frac{P}{A} = f_c = 0.25f_c'$$

40. Vertically Reinforced or Tied Concrete Columns.—While vertically reinforced (called rodded or tied) concrete columns have been the subject of heated arguments between engineers mainly as to their ability to carry loads under various conditions, nevertheless the facts are that a great number of columns of this type have and are giving very satisfactory service. On the other hand it is of course true that poorly designed and constructed rodded columns have been the cause or contributing causes of some concrete building failures. Careful investigation, however, will show that in the case of most failures the columns were either too slender, reinforced with small rods (under ⅝ in.), had ties spaced too far apart, had insufficient fireproofing or were constructed of poor concrete full of stone pockets and not properly proportioned, so as to give a very poor bond between concrete and steel. If care is used in design and construction, tied columns can be used to distinct advantage, especially for economy in exterior or wall columns when of rectangular section and for the top story columns of buildings where the columns carry the roof load only. In this latter case it will be found better practice to use a spiral at 6 or 8-in. pitch to stay the bars rather than loose ties.

Tied columns are cheaper than spirally reinforced concrete columns since for carrying compressive loads concrete is cheaper than steel. For the same reason the cheapest tied column is one using the minimum amount of steel—that is, ½ of one per cent. Then also, for a given load to be carried, the column with the richer mixture of concrete will be the cheaper. For practical reasons of construction it is best to keep the concrete mixture in columns the same as in floor construction since if 1 : 2 : 4 concrete is used in the latter, it will also form the part of the column through the floor and a workman's error may result in the entire column being of 1 : 2 : 4 instead of say 1 : 1 : 2.

Tied columns should have a minimum of 1½ in. of concrete fireproofing and preferably 2 in. over the vertical bars and in no case should ¼- in. ties be

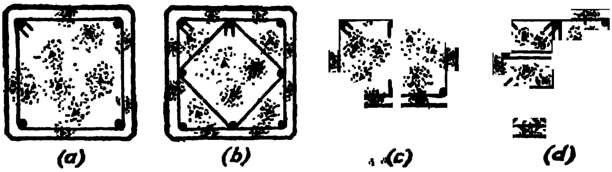
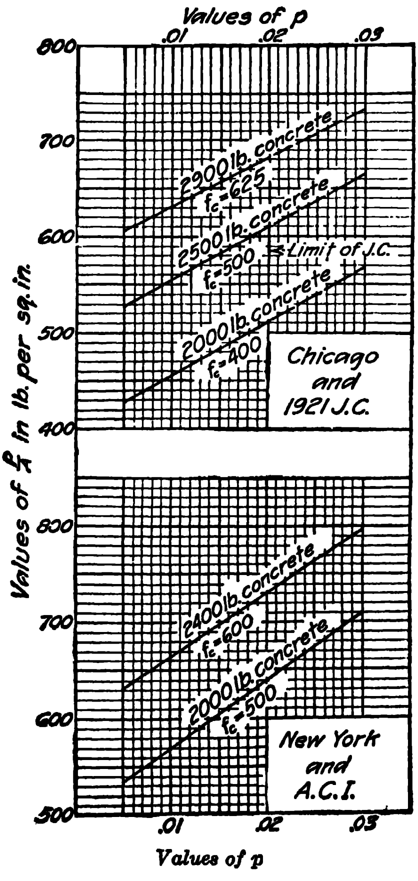


FIG. 25.—Arrangement of ties in square and rectangular columns.

spaced farther apart than 8 in. regardless of what the local ordinances may allow. For columns of large section and heavily reinforced, the area of the ties per foot

DIAGRAM 1.

TIED COLUMN DESIGN BY FOUR SPECIFICATIONS.



of column length on a section through the center of column should be at least 5 per cent of the area of vertical steel. This means that in large columns simple hoops extending around all the bars will not give sufficient area of ties, and cross

ties are required as shown in Fig. 25. The arrangement of ties shown in Fig. 25*d* should be avoided since the strength is considerably reduced where a number of parallel ties run through the column in the same plane.

In designing a tied column a core area is first assumed based on column outline allowed by the architectural design of walls (if a wall column) and the load to be carried. Then divide the column load (including the weight of column itself) by the assumed core area thus obtaining the average stress on the core. Then in the formula

$$\frac{P}{A} = f_c(1 - p + np)$$

the value of $\frac{P}{A}$ or the average stress on the core must equal the value

$$f_c(1 - p + np)$$

For any given allowable concrete stress f_c this can best be found by the means of graphs. Diagram 1, p. 206, contains such a graph for Chicago Ordinance requirements, the 1921 Joint Committe Report for 2,000, 2,500 and 2,900 lb. concrete, and theAmerican Concrete Institute and New York Codes for 2,000 and 2,400 lb. concrete.

TABLE 1.—CORE AREAS, PERIMETERS, AND CONCRETE VOLUMES FOR COLUMNS

Diameter		Core area (sq. in.)	ϕ col. perimeter	Volume (cu. ft. per ft.)		
Col. (in.)	Core (in.)		Ft. In.	Round	Octagonal	Square
14	10	78.5	3—8	1.07	1.12	1.36
	11	95.0				
16	12	113.1	4—2	1.40	1.47	1.78
	13	132.7				
18	14	153.9	4—8	1.77	1.86	2.25
	15	176.7				
20	16	201.0	5—2	2.18	2.30	2.78
	17	226.9				
22	18	254.4	5—9	2.64	2.78	3.36
	19	283.5				
24	20	314.1	6—3	3.14	3.31	4.00
	21	346.3				
26	22	380.1	6—9	3.69	3.89	4.70
	23	415.4				
28	24	452.3	7—4	4.28	4.51	5.45
	25	490.8				
30	26	530.9	7—10	4.91	5.17	6.25
	27	572.5				
32	28	615.7	8—4	5.58	5.89	7.11
	29	660.5				
34	30	706.8	8—11	6.30	6.64	8.03
	31	754.7				
36	32	804.2	9—5	7.07	7.45	9.00
	33	855.3				
38	34	907.9	10—0	7.88	8.30	10.02
	35	962.1				
40	36	1,017.8	10—6	8.73	9.20	11.10
	37	1,075.2				
42	38	1,134.1	11—0	9.62	10.15	12.25

TABLE 2.—VOLUME OF CONCRETE IN COLUMN SHAFTS AND COLUMN CAPITALS

Round columns								Square columns							
Dia. of col. shaft	Vol. of col. shaft	Diameter of column capital						Side of col. shaft	Vol. of col. shaft	Side of column capital					
		3′ 6″	4′ 0″	4′ 6″	5′ 0″	5′ 6″	6′ 0″			3′ 6″	4′ 0″	4′ 6″	5′ 0″	5′ 6″	6′ 0″
14	1.07	4.87	....	.....	.....	.....	.....	14	1.36	6.13					
16	1.40	4.63	7.61	.....	.....	.....	.....	16	1.78	5.70	8.90				
18	1.77	4.38	7.05	10.60	.....	.....	.....	18	2.25	5.26	8.37	12.41			
20	2.18	3.94	6.49	9.92	14.46	.....	.....	20	2.78	4.76	7.79	11.70	16.68		
22	2.64	3.49	5.93	9.23	13.58	.....	.....	22	3.36	4.24	7.15	10.97	15.80	22.24	
24	3.14	3.04	5.37	8.54	12.69	17.80	24.15	24	4.00	3.80	6.50	10.23	15.02	21.10	28.00
26	3.69	2.60	4.80	7.81	11.81	16.78	23.00	26	4.70	3.18	5.83	9.43	14.07	19.95	26.80
28	4.28	2.15	4.22	7.08	10.91	15.76	21.80	28	5.45	2.65	5.13	8.61	13.10	18.78	25.60
30	4.91	1.70	3.64	6.35	10.00	14.74	20.55	30	6.25	2.14	4.46	7.78	12.10	17.58	24.40
32	5.58	....	3.10	5.65	9.11	13.68	19.35	32	7.11	....	3.78	6.73	11.09	16.38	23.10
34	6.30	....	2.55	4.95	8.22	12.59	18.10	34	8.03	....	3.12	5.68	10.05	15.16	21.75
36	7.07	....	2.00	4.24	7.32	11.47	16.80	36	9.00	....	2.44	4.63	9.00	13.92	20.25
38	7.88	....	....	3.59	6.49	10.44	15.52	38	10.02	....	....	4.03	8.00	12.70	18.83
40	8.73	....	....	2.94	5.66	9.40	14.22	40	11.10	....	....	3.45	6.99	11.46	17.35
42	9.62	....	....	2.29	4.83	8.34	12.92	42	12.25	....	....	2.88	5.96	10.22	15.86
44	10.56	....	....	....	4.08	7.37	11.70	44	13.44	....	....	....	4.88	9.02	14.40
46	11.54	....	....	....	3.33	6.40	10.49	46	14.69	....	....	....	3.78	7.84	12.95
48	12.57	....	....	....	2.59	5.43	9.28	48	16.00	....	....	....	2.69	6.68	11.52

For octagonal columns and capitals add 6 per cent to values given for round columns and capitals.

Volume of concrete in column shaft is given in cubic feet per foot of height. For column capitals it is given in cubic feet and includes only the concrete in the capital outside the surface of the column enclosed in the capital.

TABLE 3.—AREAS AND WEIGHTS OF COLUMN RODS

Heavy figures = area Number of rods Light figures = weight per foot

Size (in.)	1	4	6	8	10	11	12	13	14	15	16	17	18	19	20
½φ	**0.1963** 0.668	**0.79** 2.67	**1.18** 4.01	**1.57** 5.34	**1.96** 6.68										
½□	**0.2500** 0.850	**1.00** 3.40	**1.50** 5.10	**2.00** 6.80	**2.50** 8.50										
⅝φ	**0.3068** 1.043	**1.23** 4.08	**1.84** 6.26	**2.46** 8.35	**3.07** 10.4	**3.38** 11.5	**3.68** 12.5								
¾φ	**0.4418** 1.502	**1.77** 6.01	**2.65** 9.01	**3.54** 12.0	**4.42** 15.0	**4.86** 16.6	**5.30** 18.1	**5.75** 19.6	**6.19** 21.1						
⅞φ	**0.6103** 2.044	**2.40** 8.18	**3.61** 12.3	**4.81** 16.4	**6.01** 20.4	**6.62** 22.5	**7.22** 24.6	**7.82** 26.6	**8.43** 28.7	**9.03** 30.7	**9.63** 32.8				
1φ	**0.7854** 2.670			**6.29** 21.4	**7.85** 26.7	**8.65** 29.4	**9.43** 32.0	**10.2** 34.7	**11.0** 37.4	**11.8** 40.0	**12.6** 42.7	**13.4** 45.4	**14.1** 48.0	**14.9** 50.7	
1□	**1.000** 3.400			**8.00** 27.2	**10.0** 34.0	**11.0** 37.4	**12.0** 40.8	**13.0** 44.2	**14.0** 47.6	**15.0** 51.0	**16.0** 54.4	**17.0** 57.8	**18.0** 61.2	**19.0** 64.6	
1⅛□	**1.266** 4.303				**12.7** 43.0		**15.2** 51.6	**16.4** 56.0	**17.7** 60.3	**19.0** 64.5	**20.2** 68.9	**21.5** 73.1	**22.8** 77.5	**24.0** 81.8	**25.3** 86.1
1¼□	**1.563** 5.312				**15.6** 53.1						**25.0** 85.0	**26.6** 90.3	**28.1** 95.6	**29.7** 101.0	**31.3** 106.2

Table 1 gives the core areas, perimeters, and weights per foot of round, square and octagonal concrete columns, while Table 2 gives the volumes of column capitals in flat slab construction as an aid in computing the column weights.

Entering the particular graph in Diagram 1 for the stress and code in question the percentage of vertical steel required can be found. Then by multiplying the core area by the percentage of vertical steel required the area of steel is obtained. Table 3 gives the areas and weights of column rods, of various sizes and numbers.

After the vertical steel has been determined, the number and size of ties should be determined in accordance with the requirements set forth above.

41. Columns Vertically and Spirally Reinforced.—A natural development and improvement on the tied column is that in which the ties to stay the vertical reinforcing bars are replaced by closely wound spiral reinforcement. Tests made on this type of column indicated that by preventing lateral or radial deformation and failure by diagonal shearing in columns under load, the spiral greatly increased the amount of vertical stress to which the concrete could be subjected with safety. The Joint Committee report takes account of this increased carrying capacity by allowing 55 per cent more stress on the concrete core of a spirally reinforced concrete column than on that of a column in which the reinforcement is stayed with ties. The spiral column has been the subject of many tests since the pioneer tests of Considére led him to establish his formula for spiral columns in which the spiral was considered as being 2.4 times as effective as the same amount of vertical steel—that is,

$$P = A\ f_c + A_s(n - 1)f_c + (2.4n - 1)A_s'f_c$$

and the spiral column is now almost universally used where high stresses are to be carried by the concrete.

A great variety of formulas for the design of spiral columns are to be found in the various city building ordinances and codes, some of which allow dangerously high stresses in the vertical steel. Those of the Chicago and New York Codes, the Joint Committee and the American Concrete Institute have been given above. Of these the Joint Committee formula is recommended for general use since it gives the most consistent results.

The Chicago code allows a variation in percentage of spiral reinforcement p' from a minimum of 0.005 to a maximum of 0.015, while the percentage of vertical steel p cannot be less than p' but may equal 0.005. While it is most economical to use the minimum percentages of spiral and vertical steel, that is, $p' = p =$ 0.005, the designer should avoid such a combination inasmuch as exceedingly high stresses result if low percentages of vertical steel are used owing to the fact that the shrinkage of the concrete in setting throws initial compression in the vertical steel and the smaller the amount of steel the greater will be the proportion of stress due to shrinkage. The same applies to the New York Code formula and the American Concrete Institute formula. It is accordingly recommended that the designer use a minimum of 1½ per cent of vertical steel if medium grade steel is used, and 1 per cent if hard grade bars are used.

Good practice indicates that a minimum of 1 per cent of spiral steel should be used (the Joint Committee makes this the maximum and minimum) and that the pitch be such that at least 1¼ in. clear is left between turns. Vertical bars should be set in one ring against the spiral and the number of verticals should

DIAGRAM 2.—PERCENTAGE OF REINFORCEMENT IN SPIRAL COLUMN BY 1921 JOINT COMMITTEE SPECIFICATIONS.

NOTE.—Percentage of Spiral Reinforcement equals one-fourth of percentage of vertical reinforcement in all cases.

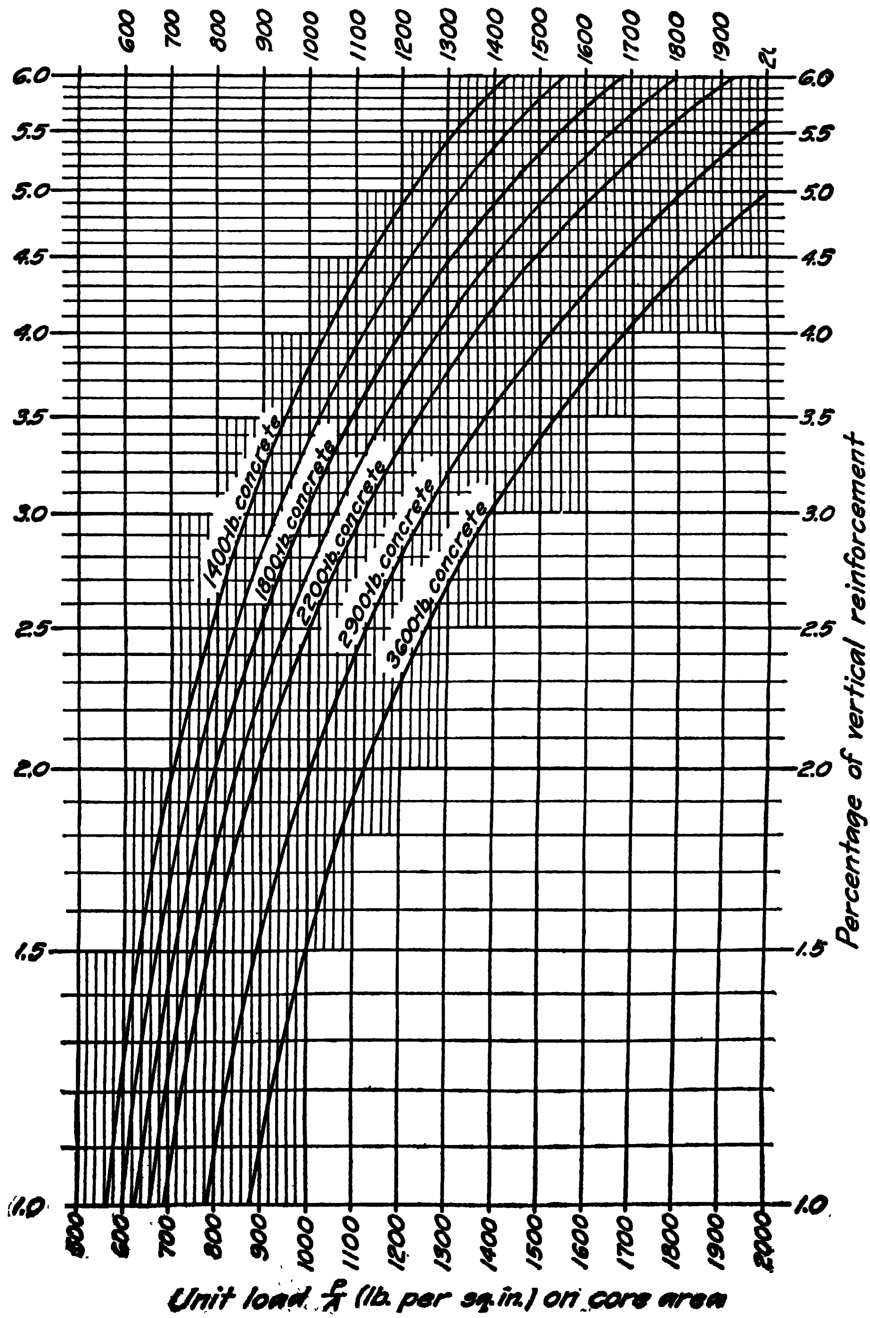
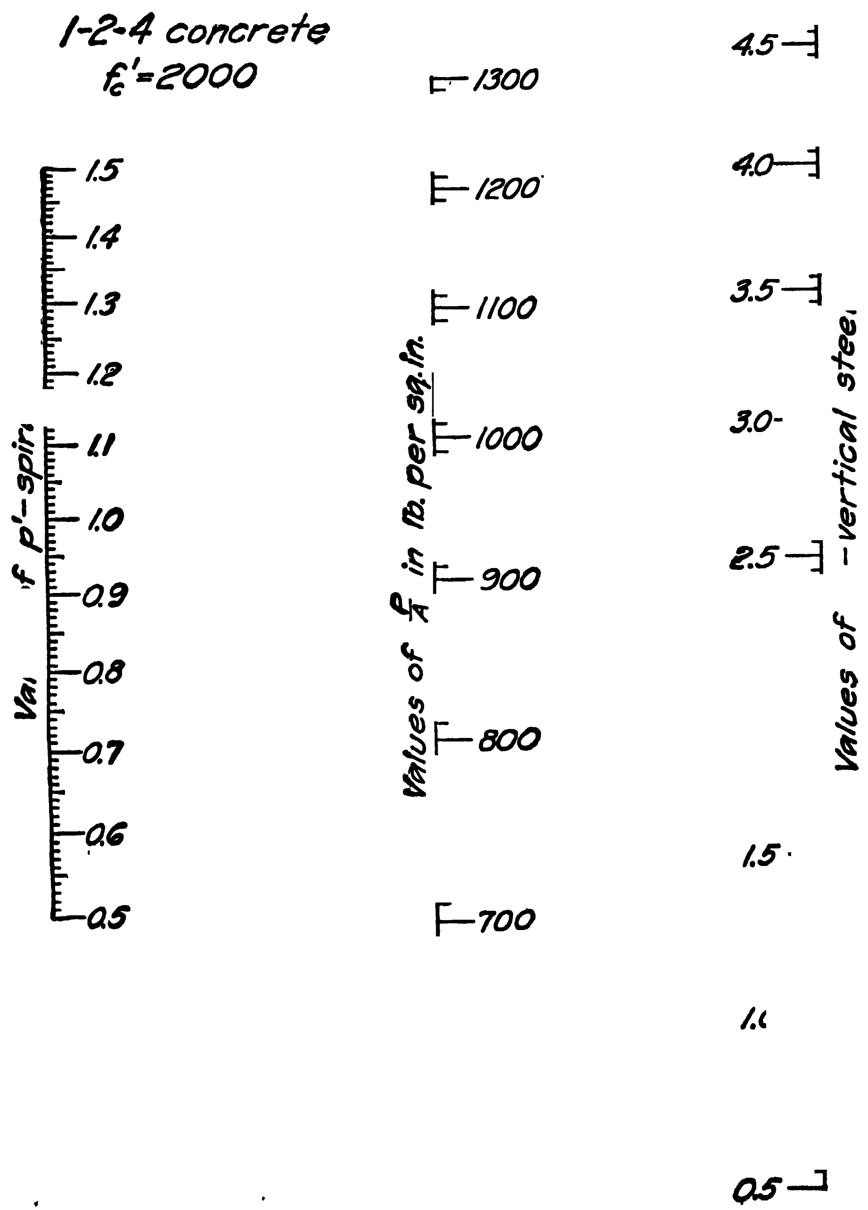


DIAGRAM 3.

CHICAGO SPIRAL COLUMN DESIGN[1]—1:6 CONCRETE.

NOTE.—Set straight edge on any two known quantities and read concurrent value of third quantity.

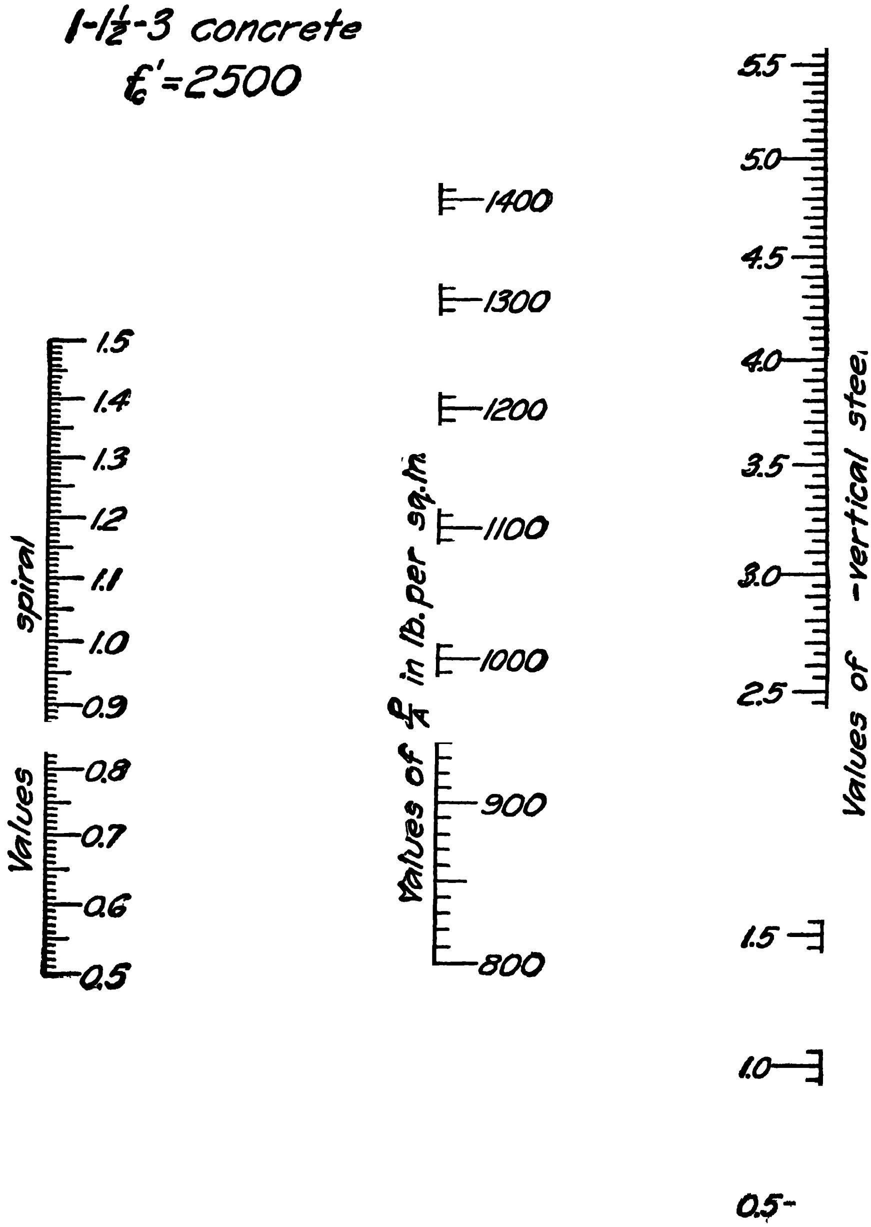


[1] Prepared by Gardner and Lindberg, Industrial Engineers, Chicago. Wallace Berger, Structural Engineer.

DIAGRAM 4.

CHICAGO SPIRAL COLUMN DESIGN[1]—1:4½ CONCRETE.

NOTE.—Set straight edge on any two known quantities and read concurrent value of third quantity.

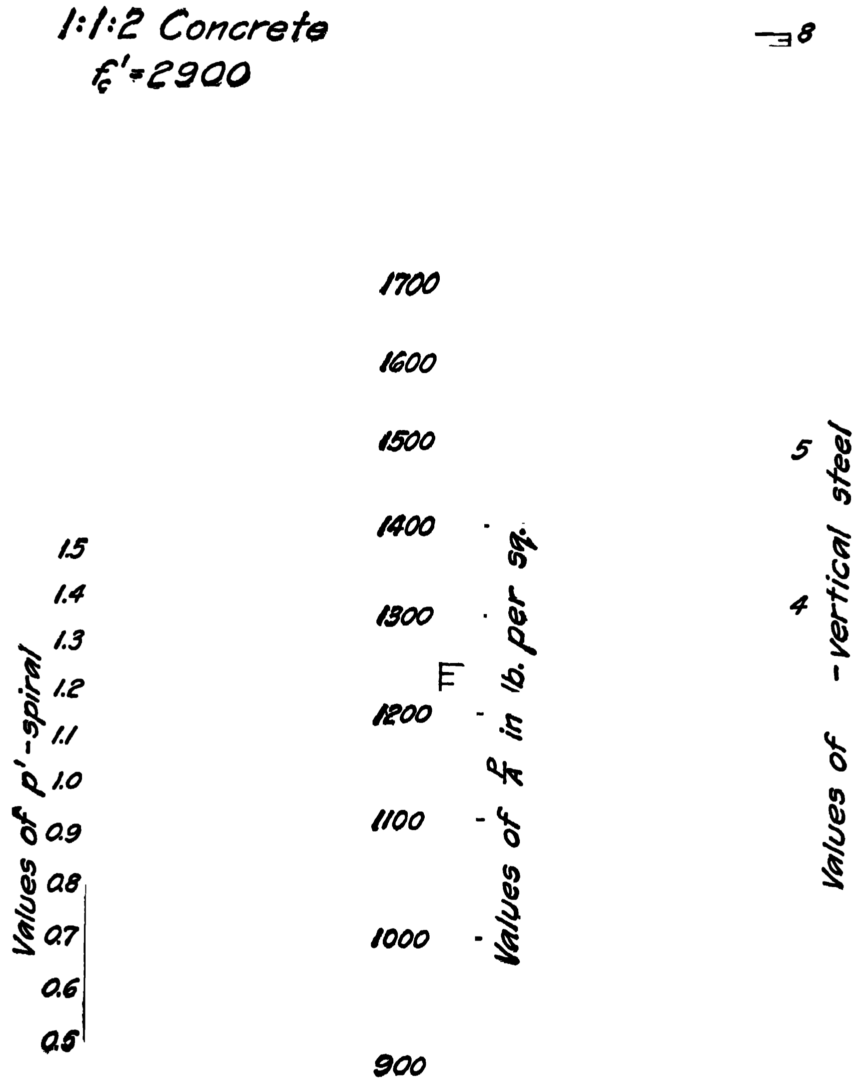


[1] Prepared by Gardner and Lindberg, Industrial Engineers, Chicago. Wallace Berger, Structural Engineer.

DIAGRAM 5.

CHICAGO SPIRAL COLUMN DESIGN[1]—1:3 CONCRETE.

NOTE.—Set straight edge on any two quantities and read concurrent value of third quantity

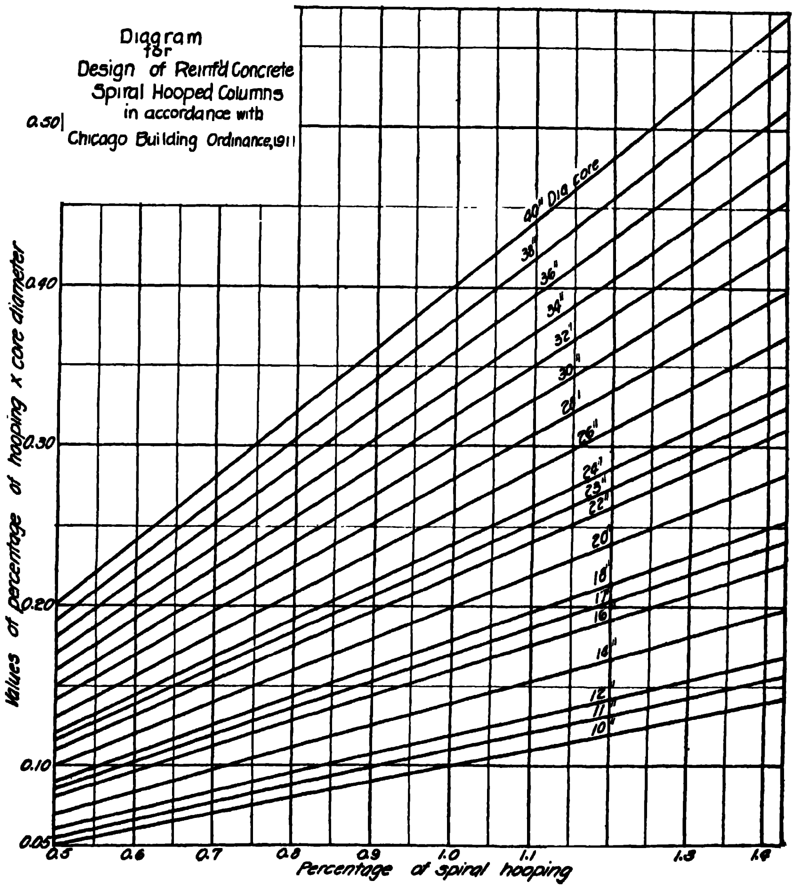


[1] Prepared by Gardner and Lindberg, Industrial Engineers, Chicago. Wallace Berger, Structural Engineer.

not exceed the diameter of the core in inches. This will allow sufficient space between the steel (verticals and spirals) to allow the concrete to flow freely around the steel and completely fill the form thus avoiding stone pockets which are a point of weakness. There should be at least 1½ in. of concrete outside of the spiral and preferably 2 in. for fireproofing in important buildings carrying heavy loads.

In designing a spiral column divide the load carried by the column, including its own weight, by the core area of an assumed column (meeting the requirements of span length of panel supported and unsupported length of column) thus obtaining

DIAGRAM 6.

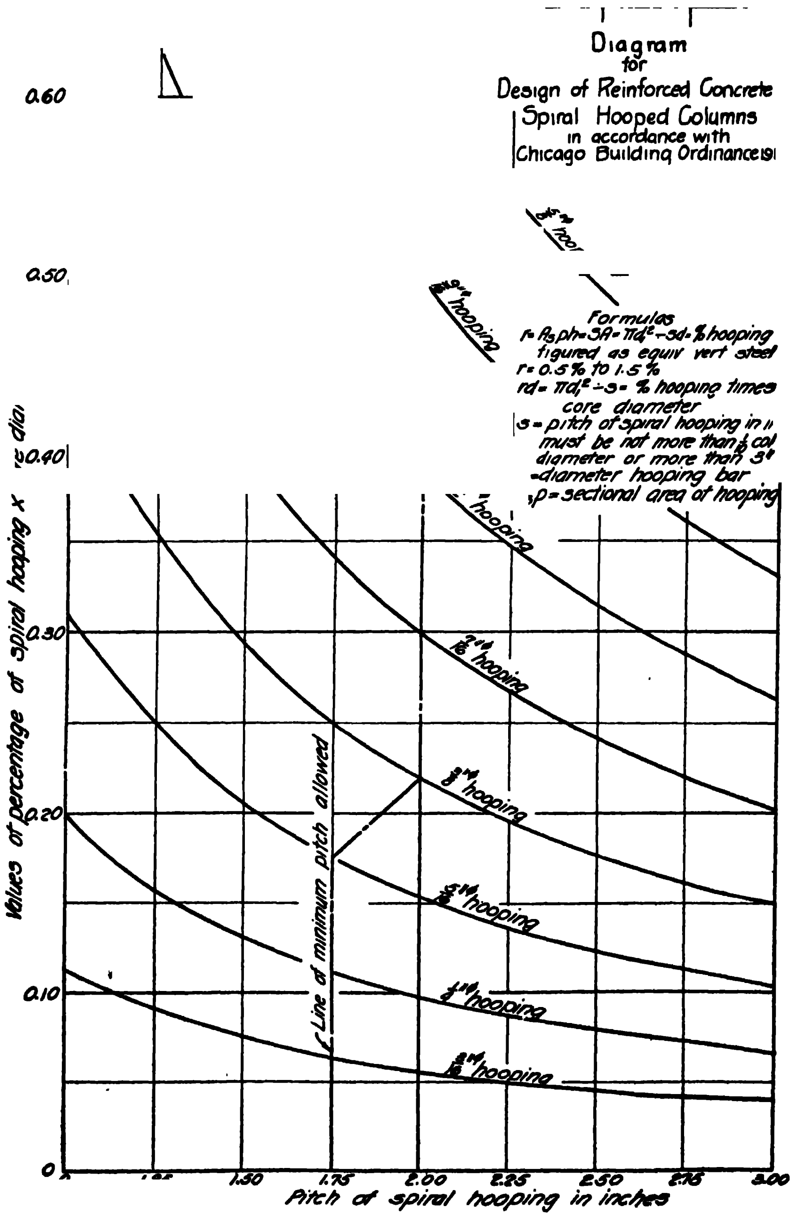


the average stress on the core in pounds per square inch. Then enter the graph for the particular formula and percentage of spiral being used with the stress given (Diagrams 2, 3, 4 and 5) and where the straight line connecting the two values intersects the graph for the vertical steel at the right ascertain the percentage of vertical steel required.

Multiply the core area by the percentage p of vertical steel required and obtain the area of vertical steel. The number of bars can then be ascertained from Table 3.

To find the spiral required enter Diagram 6 with the percentage of spiral given and follow the vertical line to its intersection with the line for the particular

DIAGRAM 7.



core diameter and on the left read the value of percentage of spiral times the core diameter. With this value enter Diagram 7 at the left and follow the

TABLE 4.—TABLES OF LENGTHS AND WEIGHTS IN POUNDS PER FOOT OF COLUMN FOR R. C. COLUMN SPIRALS

Core diameter of column (in.)	Spiral pitch 1¼ in.							Spiral pitch 1⅜ in.						
	Length of spiral (ft.)	Weight per foot of column						Length of spiral (ft.)	Weight per foot of column					
		3/16″φ	¼″φ	5/16″φ	⅜″φ	7/16″φ	½″φ		3/16″φ	¼″φ	5/16″φ	⅜″φ	7/16″φ	½″φ
10	25.1		4.2	6.6	9.4	12.8	16.8	22.9		3.8	6.0	8.6	11.7	15.3
12	30.2	2.8	5.0	7.9	11.4	15.4	20.2	27.4	2.6	4.6	7.2	10.3	14.0	18.4
14	35.2	3.3	5.9	9.2	13.2	18.0	23.5	32.0	3.0	5.4	8.3	12.0	16.4	21.4
16	40.2	3.8	6.7	10.5	15.1	20.5	26.9	36.6	3.4	6.1	9.6	13.7	18.7	24.3
17	42.7	4.0	7.1	11.2	16.0	21.8	28.5	38.8	3.6	6.5	10.1	14.6	19.8	25.9
18	45.2	4.3	7.6	11.8	17.0	23.1	30.2	41.1	3.9	6.9	10.7	15.4	21.0	27.4
20	50.3	4.7	8.4	13.1	18.9	25.7	33.5	45.7	4.3	7.6	11.9	17.2	23.4	30.5
22	55.3	5.2	9.2	14.4	20.8	28.3	36.9	50.3	4.7	8.4	13.1	18.9	25.7	33.6
23	57.8	5.4	9.7	15.1	21.7	29.5	38.5	52.5	4.9	8 8	13.7	19.7	26.8	35.1
24	60.3	5.7	10.1	15.7	22.6	30.8	40.2	54.8	5.2	9.2	14.3	20.6	28.0	36.6
26	65.3	6.1	10.9	17.1	24.5	33.4	43.6	59.4	5.6	9.9	15.5	22.3	30.4	39.7
28	70.4	6.6	11.8	18.4	26.4	36.0	47.0	64.0	6.0	10 7	16.7	24.0	32.7	42.7
30	75.4	7.1	12.6	19.7	28.4	38.5	50.2	68.5	6.4	11.5	17.9	25.8	34.9	45.7
32	80.4	7.6	13.4	21.0	30.2	41.1	53.7	73.0	6.9	12.2	19.1	27.4	37.4	48.8

NOTE.—Values below and to the left of heavy lines are in accordance with various city building ordinances as indicated:

— —— — —— — NEW YORK Maximum pitch ⅙ core piameter.

— — — — — — — CLEVELAND Maximum pitch ⅛ core diameter.

———————— CHICAGO Maximum pitch 1/10 core diameter.

The entire table is in accordance with Joint Committee Recommendation. Maximum pitch ¼ core diameter.

TABLE 4.—TABLES OF LENGTHS AND WEIGHTS IN POUNDS PER FOOT OF COLUMN FOR R. C. COLUMN SPIRALS.—(*Continued*)

Core diameter of column (in.)	Spiral pitch 1½ in. Length of spiral (ft.)	Spiral pitch 1½ in. Weight per foot of column 3/16″φ	1/4″φ	5/16″φ	3/8″φ	7/16″φ	1/2″φ	Spiral pitch 1⅝ in. Length of spiral (ft.)	Spiral pitch 1⅝ in. Weight per foot of column 1/4″φ	5/16″φ	3/8″φ	7/16″φ	1/2″φ
10	20.9		3.5	5.5	7.9	10.7	14.0	19.3	3.2	5.1	7.3	9.9	12.9
12	25.1	2.4	4.2	6.6	9.4	12.8	16.8	23.2	3.9	6.1	8.7	11.9	15.5
14	29.3	2.8	4.9	7.7	11.0	15.0	19.6	27.1	4.5	7.1	10.2	13.8	18.1
16	33.5	3.2	5.6	8.8	12.6	17.1	22.4	30.9	5.2	8.1	11.6	15.8	20.6
17	35.6	3.4	6.0	9.3	13.3	18.2	23.8	32.9	5.5	8.6	12.3	16.8	21.9
18	37.7	3.6	6.3	9.9	14.1	19.3	25.2	34.8	5.8	9.1	13.1	17.8	23.2
20	41.9	3.9	7.0	10.9	15.7	21.4	27.9	38.7	6.5	10.1	14.5	19.8	25.8
22	46.1	4.3	7.7	12.0	17.3	23.5	30.7	42.5	7.1	11.1	15.9	21.7	28.4
23	48.2	4.5	8.1	12.6	18.1	24.6	32.1	44.5	7.4	11.6	16.7	22.7	29.6
24	50.3	4.7	8.4	13.1	18.9	25.7	33.5	46.4	7.8	12.1	17.4	23.7	30.9
26	54.4	5.1	9.1	14.2	20.4	27.8	36.3	50.2	8.4	13.1	18.8	25.7	33.5
28	58.7	5.5	9.8	15.3	22.0	30.0	39.1	54.1	9.0	14.1	20.3	27.6	36.1
30	62.8	5.9	10.5	16.4	23.6	32.1	41.8	58.0	9.7	15.1	21.8	29.6	38.7
32	67.0	6.3	11.2	17.5	25.2	34.2	44.8	61.8	10.3	16.1	23.2	31.6	41.2

NOTE.—Values below and to the left of heavy lines are in accordance with various city building ordinances as indicated:

— — — — — NEW YORK Maximum pitch 1/6 core diameter.

— — — — — CLEVELAND Maximum pitch 1/8 core diameter.

———— CHICAGO Maximum pitch 1/10 core diameter.

The entire table is in accordance with Joint Committee Recommendation. Maximum pitch 1/4 core diameter.

TABLE 4.—TABLES OF LENGTHS AND WEIGHTS IN POUNDS PER FOOT OF COLUMN FOR R. C. COLUMN SPIRALS.—(*Continued*)

Core diameter of column (in.)	Spiral pitch $1\frac{3}{4}$ in.						Spiral pitch $1\frac{7}{8}$ in.				
	Length of spiral (ft.)	Weight per foot of column					Length of spiral (ft.)	Weight per foot of column			
		$\frac{1}{4}''\phi$	$\frac{5}{16}''\phi$	$\frac{3}{8}''\phi$	$\frac{7}{16}''\phi$	$\frac{1}{2}''\phi$		$\frac{5}{16}''\phi$	$\frac{3}{8}''\phi$	$\frac{7}{16}''\phi$	$\frac{1}{2}''\phi$
10	18.0	3.0	4.7	6.7	9.2	12.0	16.8	4.4	6.3	8.6	11.2
12	21.6	3.6	5.6	8.1	11.0	14.4	20.1	5.3	7.6	10.3	13.4
14	25.2	4.2	6.6	9.4	12.8	16.8	23.5	6.1	8.8	12.0	15.7
16	28.7	4.8	7.5	10.8	14.7	19.2	26.8	7.0	10.1	13.7	17.9
17	30.5	5.1	8.0	11.4	15.6	20.4	28.5	7.4	10.7	14.5	19.0
18	32.3	5.4	8.4	12.1	16.5	21.6	30.2	7.9	11.3	15.4	20.1
20	35.9	6.0	9.4	13.5	18.4	24.0	33.5	8.7	12.6	17.1	22.4
22	39.5	6.6	10.3	14.8	20.2	26.4	36.9	9.6	13.8	18.8	24.6
23	41.3	6.9	10.8	15.5	21.1	27.6	38.6	10.1	14.5	19.7	25.8
24	43.1	7.2	11.3	16.2	22.0	28.8	40.2	10.5	15.1	20.6	26.9
26	46.7	7.8	12.2	17.5	23.8	31.2	43.6	11.4	16.3	22.2	29.1
28	50.3	8.4	13.1	18.9	25.7	33.6	46.9	12.3	17.6	24.0	31.4
30	53.8	9.0	14.1	20.2	27.5	35.9	50.3	13.1	18.9	25.7	33.5
32	57.5	9.6	15.0	21.6	29.4	38.5	53.7	14.0	20.1	27.4	35.8
34	....	...	....	....	....	....	57.0	14.9	21.4	29.1	38.0
36	....	...	....	....	....	....	60.4	15.8	22.7	30.8	40.3

NOTE.—Values below and to the left of heavy lines are in accordance with various city building ordinances as indicated:

— — —— —— NEW YORK Maximum pitch $\frac{1}{6}$ core diameter.

— — — — — - - CLEVELAND Maximum pitch $\frac{1}{8}$ core diameter.

———————— CHICAGO Maximum pitch $\frac{1}{10}$ core diameter.

The entire table is in accordance with Joint Committee Recommendation. Maximum pitch $\frac{1}{4}$ core diameter.

TABLE 4.—TABLES OF LENGTHS AND WEIGHTS IN POUNDS PER FOOT OF COLUMN FOR R. C. COLUMN SPIRALS.—(*Continued*)

Core diameter of column (in.)	Spiral pitch 2 in.					Spiral pitch 2¼ in.				Spiral pitch 2½ in.				Spiral pitch 3 in.			
	Length of spiral (ft.)	Weight per foot of column				Length of spiral (ft.)	Weight per foot of column			Length of spiral (ft.)	Weight per foot of column			Length of spiral (ft.)	Weight per foot of column		
		5/16″φ	3/8″φ	7/16″φ	1/2″φ		3/8″φ	7/16″φ	1/2″φ		3/8″φ	7/16″φ	1/2″φ		7/16″φ	1/2″φ	9/16″φ
10	15.7	4.1	5.9	8.0	10.5	14.0	5.3	7.2	9.3	12.6	4.7	6.4	8.4	10.5	5.4	7.0	8.9
12	18.9	4.9	7.1	9.7	12.6	16.8	6.3	8.6	11.2	15.1	5.7	7.7	10.1	12.6	6.4	8.3	10.7
14	22.0	5.8	8.3	11.2	14.7	19.6	7.4	10.0	13.0	17.6	6.6	9.0	11.8	14.7	7.5	9.8	12.2
16	25.2	6.6	9.5	12.9	16.8	22.4	8.4	11.4	14.9	20.1	7.5	10.3	13.4	16.8	8.6	11.2	14.2
17	26.7	7.0	10.0	13.7	17.8	23.7	8.9	12.2	15.8	21.4	8.0	10.9	14.3	17.8	9.1	11.9	15.0
18	28.3	7.4	10.6	14.5	18.9	25.1	9.5	12.9	16.8	22.6	8.5	11.5	15.1	18.9	9.6	12.6	16.0
20	31.5	8.2	11.8	16.1	21.0	27.9	10.5	14.3	18.6	25.1	9.4	12.8	16.8	21.0	10.7	14.0	17.9
22	34.6	9.1	13.0	17.7	23.1	30.8	11.6	15.7	20.5	27.7	10.4	14.1	18.6	23.1	11.8	15.4	19.6
23	36.2	9.5	13.6	18.5	24.1	32.2	12.1	16.4	21.4	28.9	10.8	14.8	19.3	24.1	12.3	16.1	20.2
24	37.8	9.9	14.2	19.3	25.2	33.6	12.6	17.2	22.4	30.2	11.3	15.4	20.2	25.2	12.8	16.8	21.3
26	40.9	10.7	15.3	20.9	27.3	36.3	13.7	18.6	24.2	32.7	12.2	16.7	21.8	27.3	13.9	18.3	23.1
28	44.0	11.5	16.5	22.5	29 3	39.2	14.7	20.0	26.2	35.2	13.1	18.0	23.5	29.4	15.0	19.6	24.8
30	47.2	12.3	17.7	24.1	31.6	41.8	15.7	21.4	27.9	37.7	14.1	19.2	25.3	31.5	16.1	21.0	26.6
32	50.3	13.1	18.9	25.7	33.5	44.7	16.8	22.8	29.8	40.2	15.1	20.5	26.8	33.6	17.1	22.4	28.4
34	53.4	13.9	20.0	27.3	35.6	47.5	17.9	24.3	31.7	42.8	16.1	21.8	28.5	35.6	18.2	23.8	30.1
36	56.6	14.8	21.2	28.9	37.7	50.3	18.9	25.7	33.6	45.3	17.0	22.1	30.2	37.7	19.2	25.2	31.9

NOTE.—Values below and to the left of heavy lines are in accordance with various city building ordinances as indicated:

— — —— — — NEW YORK Maximum pitch 1/6 core diameter.

—————— — — CLEVELAND Maximum pitch 1/8 core diameter.

—————————— CHICAGO Maximum pitch 1/10 core diameter.

The entire table is in accordance with Joint Committee Recommendation. Maximum pitch 1/4 core diameter.

horizontal line to its intersection with the line for a suitable spiral to the right of the line of minimum pitch, then follow down vertically and read the pitch of spiral required.

The weights per foot of height of various spirals are given in Table 4 which also gives the length of wire for each foot of height.

42. Details of Reinforced Concrete Columns.

42a. Lap of Column Bars.—To transmit the stress from the bars in one column to the one below, the best practice requires that the bars from the lower story column be carried up far enough into the upper to transmit the stress from the upper bars to the lower ones by bond. Based on the Joint Committee allowance of 80 lb. per sq. in. for bond stress on plain bars the required lap of bars is 33 diameters where the number and size are the same for each story. Usually, however, the lower story column contains the greater number of bars and the length of lap should therefore be proportioned for the number of bars in the upper column. It is good practice to make the minimum lap 2 ft. to insure tying the two columns together.

Pipe sleeve connections for column bars are a relic of the early days of concrete and their use should be forbidden for ordinary construction since it is practically impossible to get the bars to bear completely on one another without pouring molten lead into the sleeve. This procedure is too expensive except where it is desired to place additional stories on a building and the conditions do not allow the projecting of the column bars much above the temporary roof slab. By the use of bars of relatively small size (up to ¾ in.) provision for future extension can be made by bending down the projecting bars into the cinder fill put on to provide the drainage slopes for the temporary roof.

Where the column sizes change in the different stories, the bars projecting into the upper column must be bent to bring them inside the spiral above, or if the number of bars in the upper column equals or very nearly equals the diameter of the core in inches, they should be bent sufficiently to bring them inside the line of vertical bars in the upper column. Bends in column bars should be made on a slope of 3 in. in 18 in. as a maximum. Where the offset in the column faces is over 4 in. it will be found better practice to use straight stub bars set in the top portion of the lower column rather than bend the main bars.

42b. Footing and Column Connection.—For columns reinforced with vertical bars and wire ties, the bearing stress at the footing—considering the entire area of the column effective (which is permissible since fire cannot damage the column at this point)—will usually be found to be less than the allowable bearing stress on the footing concrete. The load from the column can therefore be considered as transferred to the footing by direct bearing with a few stub bars placed in the footing so as to lap with column bars and anchor the column to the footing.

For columns reinforced with vertical bars and spirals the allowable unit compressive stresses are relatively high and the critical section for bearing will be the top course of the footing. In some cases the average bearing over the gross area of the column will be within the allowable for the top of the footing and the design can be made for direct bearing only as outlined above.

For heavily reinforced columns of rich concrete (1:1½:3 or 1:1:2 concrete) the unit stresses on the column core will be found so high that the average bearing

over the gross area of the column will exceed the allowable for 1:2:4 concrete and therefore the load must be transferred partly in bearing and partly by bond on the stub bars placed in the footing. In some cases the load may be transferred by direct bearing by putting in a top course of a richer mixture of concrete. Some designers make use of a spirally reinforced cap to transfer the load to the footing, designed in the same manner as the column but of larger diameter. This method, however, is rather uncommon in practice and not as economical as employing a richer mixture for a larger block of plain concrete with stub bars to transfer some of the load by bond.

In arriving at the number of stub bars necessary for transference of load partly by bond and partly by direct bearing, the amount of load transferred by bearing should first be determined. This load (P_1) equals the area of column multiplied by the allowable unit bearing. Then the load transferred by bond is

$$P_2 = \text{column load} - P_1$$

The number of bars required for bond is $m = \dfrac{P_2}{u o 33 d}$ for 1:2:4 concrete and $m = \dfrac{P_2}{u o 26 d}$ for 1:1½:3 concrete, where d = the diameter of the bars used.

If this computation shows that more stub bars are required than the number of vertical bars in the column, then a richer mixture should be used in the cap, thus increasing the bearing value and cutting down on the number of stubs required.

42*c*. Fabrication of Spirals.—Spirals below 18 in. diameter should have at least two spacing bars to insure the positive and uniform spacing of the spiral. From 18 in. to 30 in. diameter use three spacing bars and four for larger spirals.

All spirals should be fabricated with an extra turn top and bottom tight against the next turn, and the end of the spiral left projecting 6 in. or more into the core space. For beam and slab construction the spiral should extend practically to the top of floor slab, while in flat slab construction it may be stopped at the bottom of the depressed panel or column cap.

42*d*. Minimum Pitch of Spiral.—In order that stone pockets may be avoided in pouring spirally reinforced columns, the minimum pitch used for spirals should be 1½ in. for ¼-in. wire and 1¾ in. for ½-in. rod thus leaving a clear space of at least 1¼ in. between the turns.

42*e*. Maximum Pitch of Spiral.—Good practice indicates that the maximum pitch allowable in good design is 3 in. for spirally reinforced columns. For the top story columns of buildings the size of the column is usually limited by the restriction as to minimum diameter and not core stress. For such columns designed as rodded columns the writer has found it economical from a construction standpoint to use ¼-in. spirals at 6 or 8 in. pitch instead of wire ties. Such spirals cut the erection cost and make a much better column, than if wire ties are used since the light vertical bars are stayed more securely.

42*f*. Number of Vertical Bars, Spirally Reinforced Columns.—Some codes specify that the minimum number of bars to be used in a spirally reinforced column be eight, and others six. The limiting area of steel being the same, the smaller number of bars but of larger size would seem to the writer to result in a better column since the tendency to buckle between spiral turns is

thereby lessened. Good practice therefore places the minimum number of bars as eight ½-in. bars or six ⅝-in. bars, with the latter to be preferred.

The maximum number of bars allowable is dependent upon all bars being properly bonded in the concrete. Tests show that if placed too close together the strength of bars cannot be properly developed in bond, hence a limiting spacing of bars should be used in designing columns using high percentages of vertical steel. A very good rule to follow is: Keep the number of vertical bars equal to or less than the core diameter of the column expressed in inches. This means that the minimum spacing will be 3 1/7 in.

In no case should bars over 1¼ in. in size and preferably not over 1⅛ in. be used with such spacing.

42g. Details of Rodded Columns.—In the design of tied or rodded columns the ties should not be under ¼ in. in diameter and spaced not more than 8 in. apart with plenty of lap allowed at the ends and bent into the column so as to insure proper anchorage.

The bars in a rodded column should not be smaller than ⅝ in. in diameter and the minimum number of bars should be four.

With columns of relatively great width and those of "L" shape used frequently at corners of buildings the vertical bars should be uniformly distributed near the periphery and the ties so formed as to stay every vertical bar adequately at not less than 8 in. spacing.

For columns of rectangular section where the face is three to four times as great as the thickness, the design of ties requires special attention in order that the vertical rods may be prevented from buckling. This requires that the sectional area of ties in a length of 1 ft. be equal to at least 10 per cent of the area of the vertical steel at the section of maximum stress.

43. Concrete Columns with Cast-iron Cores.—A type of column, devised by the great German authority on concrete, Professor Emperger, while still relatively

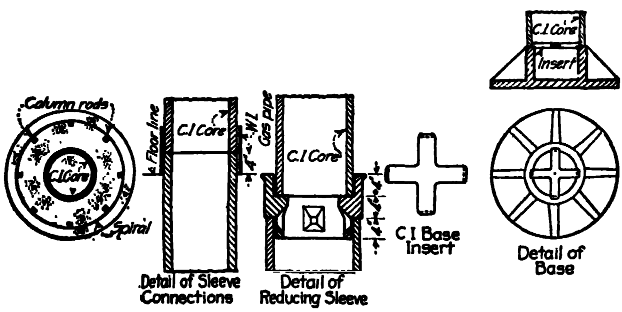


FIG. 26.—Emperger column.

new and untried bids fair to be used extensively in building construction. This column shown in Fig. 26 consists of a round cast-iron core filled with and surrounded by concrete reinforced near the periphery with vertical rods and a spiral. The cast-iron core, which is well suited to carrying a direct compressive load but not reliable in carrying eccentric loads with resulting tension, is thus reinforced and stayed by the surrounding shell of reinforced concrete.

As the result of tests made the U. S. Bureau of Standards has recommended the following formula for determining the ultimate strength of such columns:

$$f_s = 5{,}300(1 - p) + 63{,}000p - 240\frac{L}{d}$$

where f_s is the average stress per square inch on the area within the spiral, p is the percentage of cast iron used, L the length and d the diameter of the column in inches. At least 1 per cent of vertical steel and spiral should be used to fill core and encase it. The minimum thickness of casing should be 5 in.

As a result of its use the building departments of several cities have made rulings relative to the design of such columns. A formula which gives safe results as borne out by tests is

$$P = 1{,}120A_c + 11{,}200A_I$$

where P = safe load, A_c the effective concrete section inside of spiral with cast-iron core area deducted, A_I = sectional area of cast-iron core. The accompanying Table 5 prepared by W. Stuart Tait gives the safe loads for Emperger columns in accordance with the above formula.

The Joint Committee report formula is

$$P = 0.25f_c'A + \left(12{,}000 - \frac{60h}{R} - 0.25f_c'\right)A_s$$

The quantity in parentheses must not exceed 10,000, the diameter of the cast-iron core must not exceed one-half the core diameter of the concrete column and at least ½ of 1 per cent spiral must be used.

Columns of this type possess the same advantage of keeping down the column size as do steel core columns. The cast-iron cores and bases are shown in Fig. 26 and for economy the cores should be maintained at a uniform outside diameter in as many stories as possible to minimize the number of reducing sleeves required, the area of the cast iron being varied by changing the shell thickness. The cores should be provided with brackets to receive the load from beams where the concrete shell is near the minimum thickness of 5 in.

44. Structural Steel Columns Encased in Concrete.—In reinforced concrete buildings of over six or eight stories in height, depending upon the live load used, it will generally be found advisable to use structural steel column cores encased in concrete. The steel core is designed to carry the entire load while the concrete encasement is considered only as stiffening the column sufficiently to allow the use of higher unit stresses than for ordinary structural steel columns.

The Joint Committee recommends that for composite columns of structural steel and spiral encased concrete core the load allowed be

$$P = 0.25f_c'A + \left(18{,}000 - \frac{70h}{R} - 0.25f_c'\right)A_s$$

The quantity in parentheses must not exceed 16,000. If the steel core is of such shape as not to enclose and restrain the core concrete in a degree equal to a ½ per cent spiral, then a ½ per cent spiral must be used.

The Chicago Code and the American Concrete Institute ruling both provide that the steel core should be designed to take the entire load at a unit stress determined by the formula, $f_s = 18{,}000 - 70\frac{L}{r}$ but in no case shall the stress exceed 16,000 lb. per sq. in. In this formula L is the unsupported length, and

TABLE 5.—TABLE OF SAFE LOADS IN THOUSANDS OF POUNDS FOR EMPERGER COLUMNS

1-1-2 mix		Reinforcement				Cast-iron core																		
Column, diam.	Core, diam.	Spiral		Verticals		5-in diam.			7-in diam.				9¼-in diam.				11⅜-in diam.				13½-in diam.			
						Metal thickness			Metal thickness				Metal thickness				Metal thickness				Metal thickness			
(in.)	(in.)	Diam. (in.)	Pitch (in.)	No.	Size (in.)	½"	¾"	1"	½"	¾"	1"	1¼"	½"	¾"	1"	1¼"	¾"	1"	1¼"	1½"	¾"	1"	1¼"	1½"
16	12	3/16	1	8	⅜□	197	226	253																
18	14	¼	1⅜	8	½φ	243	272	299	275	320	362	400												
20	16	¼	1¼	8	½□	296	325	352	328	373	415	453	363	426	486	542								
22	18	¼	1⅛	8	⅝φ	356	385	412	388	433	475	513	423	486	546	602	535	615	687	755				
24	20	¼	1	8	⅝□	423	452	479	455	500	542	580	490	553	613	669	602	682	754	822	657	750	840	925
26	22	5/16	1 1/16	8	¾φ	....	....	....	528	573	615	653	563	626	686	742	675	755	827	895	732	823	913	998
28	24	5/16	1 5/16	8	¾□	....	....	....	608	653	695	733	643	706	766	822	755	835	907	975	812	903	993	1078
30	26	5/16	1¼	8	⅞φ	....	....	....	696	741	783	821	731	794	854	910	843	923	995	1063	900	991	1081	1166
32	28	5/16	1⅛	10	⅞φ	....	....	....	791	836	878	916	826	889	949	1005	938	1018	1090	1158	995	1086	1176	1261
34	30	⅜	1½	10	⅞□	....	....	....	....	939	981	1019	929	992	1052	1108	1041	1121	1193	1261	1098	1189	1279	1364
36	32	⅜	1⅜	10	1φ	....	....	....	....	....	1090	1128	1038	1101	1161	1217	1150	1230	1302	1370	1207	1298	1388	1473

r the least radius of gyration, both expressed in inches. This formula allows an increase of 2,000 lb. per sq. in. over the minimum of 14,000 lb. per sq. in. allowed for ordinary structural steel columns not encased in concrete. The American Concrete Institute ruling requires that the encasing concrete be reinforced with wire mesh or hoops weighing at least 0.2 lb. per sq. ft.

From the foregoing it will be noted that the Joint Committee formula allows more load on a composite concrete and structural steel column than the Chicago Ordinance or the American Concrete Institute Specifications in the amount of the load carried by the concrete core.

In Fig. 27 are shown several of the most common types of structural steel columns encased in concrete. The lattice and Gray type columns (Figs. 27*a* and 27*c*) are the best types since the concrete not only encases but fills the center of the column and is not separated into a number of relatively thin shells of concrete as is the case with the other types shown. The lattice and Gray columns, however, do not afford good connection for steel girders if any are required, but where the floors are entirely of reinforced concrete they are well adapted.

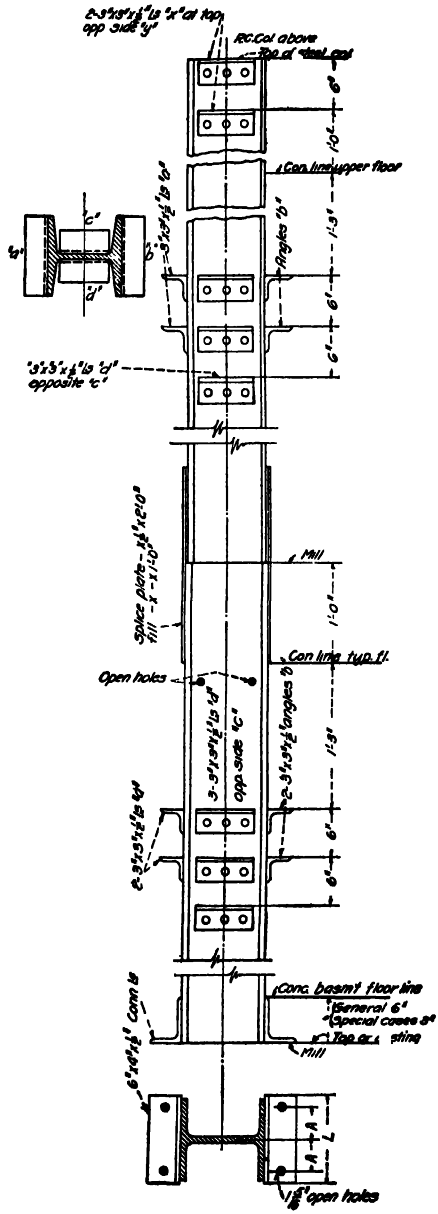
(a) (b)

FIG. 27.—Types of steel columns encased in concrete.

Concrete encased steel columns are more costly than reinforced concrete columns of the same capacity, but where large column sizes are objectionable, the extra cost is generally immaterial.

No matter what form of steel column core is used, a sufficient number of shelf angles should be provided on the column to transfer the entire floor load to the steel core with the shelf angle rivets figured as acting in single or double shear, as the case may be. The concrete shell should be assumed to carry no load. In flat slab construction it is a relatively easy matter to do this since the shelf angles can be placed below the line of floor slab and within the flaring column head, and ample space can be provided for several rows of angles. In beam and girder construction, however, no such latitude of placing shelf angles is provided and since they are limited to about one location near the bottom of the beam or girder it will usually be found necessary to provide bracketed shelf angles similar to those used in steel construction.

Another advantage of the lattice and Gray columns is that they permit of passing the negative reinforcing bars of flat slab or beam and girder construction through the column with very little interference, while with the "H" type columns, holes must be drilled in the flanges and webs to allow the placing of these bars. A typical detail of an "H" column used as a column core in flat slab construction with shelf angles to transmit load to core, and with holes for the top reinforcing bars to pass through, is shown in Fig. 28. Splices in steel cores should be made at a point at least 1 ft. above the finished floor line to allow the erection of the cores in single story heights, since it is difficult to stay columns of greater length when there are no steel girders.



Detail of H-Column

FIG. 28.—Detail for structural steel core in reinforced concrete building construction.

45. Long Concrete Columns.—It will be noted that formulas for vertically reinforced and spirally reinforced concrete columns do not consider the length of the column except in so far as a limitation of length of 10 times the diameter of the hooped core of spiral columns, and 15 times the least effective dimension of tied columns is placed on the Joint Committee formula. It would seem good practice to use the formulas given for columns having an unsupported length not over 15 times the least effective dimension. In flat slab construction the unsupported length can be assumed as the distance from the floor to the bottom of the column capital.

For columns whose unsupported length is greater than 15 times the least dimension, the unit stresses can be reduced by the following formula of the Los Angeles Building Ordinance:

$$f_{ac} = 1.6 - \frac{1}{25}\left(\frac{L}{d}\right)$$

where f_{ac} is the factor by which the ordinary stress is to be multiplied for columns where $\frac{L}{d}$ exceeds 15, d being the least dimension of the effective section.

While the Joint Committee recommends that for long columns where the value of $\frac{h}{r}$ exceeds 40, the allowable load P' be found by multiplying the allowable load P as found by the regular formula, by

$$1.33 - \frac{h}{120R} \text{ or } P' = P\left(1.33 - \frac{h}{120R}\right)$$

Thus if $\frac{h}{R} = 60$, the allowable load on the column would be

$$P' = 0.83P$$

46. Bending in Columns.—Where reinforced concrete columns support long span beams in one-story buildings, considerable bending is induced in the columns due to the flexure of the beam and the rigid monolithic connection between the column and the beam. The stresses thus set up must be provided for in the reinforcement in the top of the beam at the column and in the outside face of the column. The column should be designed to provide for the stresses due to bending in addition to those caused by the load carried.

In reinforced concrete flat slab buildings considerable bending is induced in the upper story columns due to the rigid connection between slabs and columns and some flat slab codes require that the exterior columns be designed to care for this bending. The Chicago Flat Slab Code requires that:

> Wall columns in skeleton construction shall be designed to resist a bending moment of $\frac{WL}{60}$ at floors and $\frac{WL}{30}$ at roof, where W is the total live and dead load on the whole panel and L is the length of the side of a square panel center to center of columns. The amount of steel required for this moment shall be independent of that required to carry the direct load. It shall be placed as near the surface of the column as practicable on the tension sides, and the rods shall be continuous in crossing from one side to another. The length of rods below the base of the capital and above the floor line shall be sufficient to develop their strength through bond, but not less than 40 diameters, nor less than ⅓ the clear height between the floor line and the base of the column capital.

If the interior columns in flat slab construction are of such size that the least dimension is not less than $\frac{1}{15}$ of the unsupported length or $\frac{1}{12}$ of the panel

length, the ordinary cases of bending induced in the columns due to unbalanced loading of panels will be taken care of.

The American Concrete Institute Ruling recommends that the bending in any column in flat slab construction be computed by the formula

$$M = 0.022w_1l_1(l_1 - qc)^2$$

where w_1 is the design live load, l_1 is the distance center to center of columns in one direction and l_2 in the other; q is $\frac{2}{3}$ for round column capitals and $\frac{3}{4}$ for square; c is the cap diameter.

For exterior columns the total live and dead load (w) should be used in the above formula instead of w_1. For top story columns the bending is considered as all applied at one section of the column, while for lower story columns the moment is divided between the upper and lower columns in proportion to their thickness. An allowance of 50 per cent increase in stresses for direct load and bending over those for direct load only is allowed.

47. Columns Supporting Craneway Brackets.—In industrial buildings of reinforced concrete it is often necessary to provide crane runways for traveling cranes, the runway girders being carried on brackets on the concrete columns.

In such a case the stresses in the columns due to directly applied loads are first ascertained and to these must be added the stresses produced by the bending induced in the columns due to the eccentrically applied crane load.

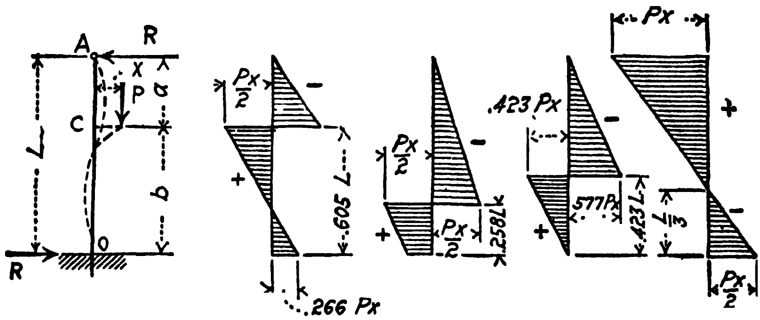


FIG. 29.

The maximum bending moment occurs at the load and depends upon the height at which the load is placed and the end conditions of the column. In the calculations the depth of the bracket is considered small in comparison with the length of the column. While not exactly correct, the error is on the safe side, since deepening the bracket decreases the bending induced in the column.

In reinforced concrete buildings two conditions of end support of columns are encountered, namely: (*a*) One end fixed and one free, and (*b*) both ends fixed. The former condition is closely approximated in one story structures on the top story columns supporting roof trusses or simply supported beams, while the condition (*b*) is the one to be considered in multi-storied buildings for all stories except the top one.

47a. Column Fixed at One End and Free to Turn at the Other.—The minimum value of the maximum bending moment for this condition equals $\frac{1}{4}Px$ and occurs when b is equal to $0.258L$ or $0.605L$ (Fig. 29). In Fig. 29 are

shown the bending moment diagrams for the conditions of b equal $0.258L$ and $0.605L$ and for making the moment at the base zero and the case when $b = L$.

The general formulas are

$$R = \frac{3Pxb}{2L^2}\left(2 - \frac{b}{L}\right)$$

Just above C (Fig. 29) $M = -Ra$

Just below C $M = Px - Ra$

At O $M_o = Px - \frac{3}{2}Px \quad \left(2 - \frac{b}{L}\right)$

47b. Columns Fixed at Both Ends.—The minimum value of the maximum bending moment occurs when b has values of $0.211L$, $0.500L$ and $0.789L$ and amounts to $\frac{1}{2}$ Px. These moment diagrams are shown in Fig. 30. The greatest value of the maximum moment occurs when the bracket is at the top or bottom of the column and equals Px.

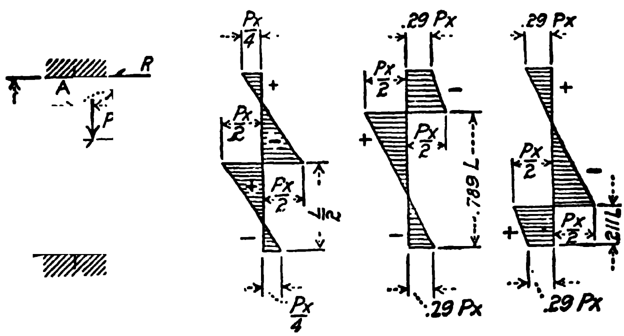


FIG. 30.

The following general formulas apply:

$$\text{At } A \quad M_A = Px\frac{b}{L}\left(2 - 3\frac{b}{L}\right)$$

$$R = \frac{6Pxb}{L^2}\left(1 - \frac{b}{L}\right)$$

Just above C $M = M_A - Ra$

Just below C $M = M_A - Ra + Px$

or $M = M_o + Rb$

$$\text{At } O \quad M_o = Px\left[1 - 4\left(\frac{b}{L}\right) + 3\left(\frac{b}{L}\right)^2\right]$$

48. Column Brackets.—The column brackets supporting crane runway girders should be designed for the most severe condition of loading possible and they should have such depth as to keep the shear on the effective section below 60 lb. per sq. in. The maximum load coming on a crane runway bracket is the maximum end reaction of loaded crane with the carriage as far to one side as possible.

Tests on various types of column brackets made by the Unit Construction Company of St. Louis are of interest and the results are here given.

Seven different types of brackets were tested, one unreinforced and the others reinforced with various combinations of three distinct types of bent bars designated X, Y and Z. The type X bars were short bars bent down at each end; type Y

bars were bent into a complete rectangle; and type Z were single bars bent to conform to the shape of the bracket, this being the usual detail for bending bracket bars. The specimens, with one exception, were tested at the age of six weeks.

John E. Conzelman, Chief Engineer, in speaking of these tests says: "As was to be expected, none of the specimens failed through tension in the steel. The failures were due to shearing or diagonal tensile stresses and insufficient bond. The action is similar to that which exists at the end of a beam; the condition is aggravated, however, by the difficulty of securing sufficient bond to develop high stresses in the steel.

"Although the conclusions that are derived from these tests are not final, yet the fact is brought out that the type of bracket ordinarily used is the least efficient of the forms tested. The failure is fairly sudden and the concrete is almost completely destroyed. The most satisfactory form is type Y, or closed loop. With a number of loops spaced close together the failure is slow and even after the concrete is partially destroyed, considerable load-carrying capacity exists. A study of these tests leads to the conclusion that the most efficient reinforcement would be the Y-type loop bars, some of which would be bent downward into the bracket."

The condition of the brackets tested was not strictly analogous to that of column brackets supporting crane runway girders, and this fact should therefore be considered in interpreting the results. The brackets tested were considerably shorter than would be used to support crane girders, which in general must be of sufficient size to give a clearance of at least 9 or 10 in. between the center line of rail (and girder) and the face of the column, and hence better bond would be secured on bars of the X and Z types. In the writer's opinion a combination of all three types makes an ideal reinforcement for a bracket, the type Y and type Z bars being figured to take care of the tension developed in the top; the type X bars to provide for diagonal tension stresses; and the type Z bars to prevent shrinkage cracks at the lower junction of column and bracket. By looping the type X bars around the type Z bars at the outside of the bracket the reinforcement for the same would be securely anchored into the column.

The following illustrative design of a bracket will be made on this basis.

Illustrative Problem.—Design a column bracket to support a concrete crane runway girder carrying a 5-ton crane, center of girder being 10 in. from face of column. Unit stresses = 16,000 lb. per sq. in. tension in steel; 12,000 lb. per sq. in. in web reinforcement; 650 lb. per sq. n. compression in concrete; 60 lb. per sq. in. shear as measure of diagonal tension allowed on effective section of bracket. This latter value is chosen because of the fact that it does not seem advisable to use vertical stirrups in brackets of ordinary dimensions, bent bars alone being used in addition to the concrete to resist diagonal tension stresses.

Assume maximum reaction of crane	24,000 lb.
Weight of runway girder	8,000 lb.
Total on bracket	32,000 lb.

Assume load concentrated at center line of runway girder, or 10 in. from face of column. Then

$$M = \frac{(32,000)(10)}{2} = 160,000 \text{ in.-lb.}$$

Shear on effective section will be limited to 60 lb. per sq. in. in order to avoid use of stirrups, the value of which are somewhat doubtful in this connection owing to the rapid change in depth. Assume width of bracket to be 18 in. Then

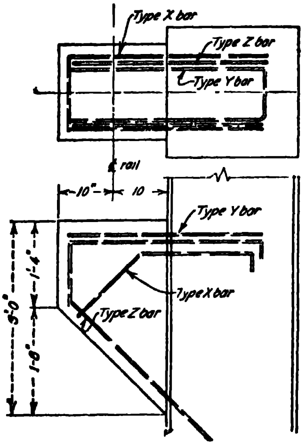


FIG. 31.—Crane bracket.

$$d = \frac{32{,}000}{(19)(60)(0.87)} = 33.6 \text{ in.}$$

Use total depth = 36 in. d to center of steel = 34.5

Steel required to resist bending moment equals

$$A_s = \frac{160{,}000}{(16{,}000)(0.87)(34.5)} = 0.33 \text{ sq. in.}$$

Use: One $\frac{3}{8}''\phi$ — Y type loop bar = 0.22 sq. in. area
Two $\frac{3}{8}''\phi$ — Z type bars = 0.22 sq. in.
Total area = 0.44 sq. in.

Steel required to resist diagonal tension

$$a_s = \frac{(\frac{3}{4})(0.7V)}{f_s jd} \text{ for 45 deg. shear bars}$$

$$a_s = \frac{(\frac{3}{4})(0.7)(32{,}000)}{(12{,}000)(0.87)(18)} = 0.08.$$

Use one $\frac{3}{8}''\phi$ type X bent at 45 deg., area 0.22 sq. in.

As shown in the detail of the bracket just designed (Fig. 31), the $\frac{3}{8}$-in. diameter bar to care for diagonal tension is looped around the type Z bars at the outer face of the bracket thus more completely tying the reinforcement and bracket together and anchoring it securely to the column.

FLOORS AND ROOFS—BEAM AND SLAB CONSTRUCTION

By W. J. Knight

49. Practical Considerations.—The successful engineer of today cannot attribute high recognition alone to a mastery of theory, without giving due credit to the great importance of possessing common sense and the knowledge of application in its many diversified forms. An engineer cannot be an efficient designer or consulting engineer unless he has a thorough knowledge of the successive operations of modern building construction, the cost, quality and suitability of building materials obtainable under different conditions, the cost of labor to execute a design, the comparative costs of the various structural arrangements, the point of view of the superintendent, and laborer, the importance of avoiding complicated designs when simple ones are just as efficacious, the principles of architectural practice, how to assemble drawings neatly, correctly and conscientiously, and last but not least, human nature and the technique of good salesmanship. One may possess a wealth of theoretical knowledge and yet be grossly deficient in the basic fundamentals which are indispensable to success.

An expert knowledge of design is of little consequence to an engineer, if the ability to impress prospective clients and secure work has been neglected for things of lesser value. A client first of all desires to know how much he can get for the least expenditure of money. He is interested in results and not in theoretical formulas. An engineer in order to combine theory, application and common sense must first of all know and appreciate his work. Impossible conclusions are too often entertained and demanded by many engineers who admittedly possess authority and knowledge more than their work portends. If a

1 : 1½ : 3 concrete mix be specified for all columns and the remaining parts of the structure require a 1 : 2 : 4 mix, it should occur to the designer that the intersection of floor slabs, beams, girders, etc., with columns, preclude the possibility of consistent compliance with this requirement. If the strength of a column is measured by its weakest section, then there is little logic in demanding or expecting fictitious conclusions.

If the locality in which a structure is to be erected cannot furnish aggregates of sizes and cleanliness approaching theoretical perfection such as prescribed by some authorities, it should be remembered that many of our large structures erected under such adverse conditions, disprove the advisability of adhering literally to laboratory results.

The engineer who confines his efforts exclusively to book knowledge, discounting the value of social intercourse, initiative and cooperation is destined to a future of intellectual misgivings, devoid of mediums through which to ply his art. Many engineers know how to design a building, but comparatively few are capable of adapting their knowledge to the essential requirements of economical arrangements and design. The engineer of the future will be in demand because of his theoretical, practical and business knowledge, great breadth in his understanding of men and affairs, rather than to things exclusively theoretical. The engineer, to be successful, must also cultivate diversified thinking. His intercourse with business men who finance and build structures should be confined more to matters of general recognized interest, costs of labor and materials, the maximum return on every dollar spent, rather than to questions pertaining to stresses and strains.

Before adopting a design for a structure the engineer must of necessity visualize the practical application of his work. Impossible arrangements of bars, stirrups and slab rods and the like, which often approach the difficulties of a puzzle, should be avoided. Because the engineer possesses authority is no logical reason why the erector's point of view, ventured in the light of possible conclusions, should be entirely neglected and discounted for things of lesser importance. The common laborer on the building often gives valuable suggestions to attentive receptive minds.

Two or more preliminary estimates of suitable floor arrangements should be made to ascertain comparative costs based on existing conditions and the cost of materials and labor, before proceeding with the final layout of a structure. It is often surprising to note the great saving that can be made in the ultimate cost of large and important buildings, by a possible shifting of beam and slab spans or rearrangement of columns and at the same time not detract from the efficiency and purpose of a structure.

The engineer to be of greatest benefit to a client or employer must also be familiar with the design and possibilities of structural steel, as well as have an expert knowledge of reinforced concrete. Engineers who possess initiative and a thorough understanding of their work, will often find it expedient, in many cases, to combine the use of structural steel and reinforced concrete and thereby perfect a much greater saving in cost, than could be accomplished by the exclusive use of the former material, for beams, girders and columns. The experience of many engineers of long, active service, reveals an astounding waste of structural materials in the design of a large percentage of the structures erected, due entirely

to unfamiliarity with the costs of building materials, a lack of theoretical knowledge, appreciation of comparative values and conscientious personal interest. The engineer who specializes in the design of structures assumes great responsibilities which many can hardly appreciate until some disastrous failure has occurred to emphasize the importance of good engineering practice.

50. Bar Supports and Spacers.—In the light of past experience the steel bars of a reinforced concrete structure cannot be accurately installed and maintained in position without the use of some device or devices that will serve the purpose of supporting the reinforcing bars the proper distance from the bottom face or surface of the concrete, spacing them the correct distances center to center and locking them in position to prevent subsequent displacement before and during concreting operations. This very important requirement is too often neglected and omitted in specifications for reinforced concrete work.

If, for example, ½-in. round rods reinforcing a slab are shown spaced 6½ in. center to center and a certain minimum thickness of concrete is given to insure fire-proofness, it will be found impossible for the contractor even to approach, with reasonable accuracy, the results intended, in the absence of some form of device or devices to make possible good, accurate workmanship.

To maintain in proper position the negative reinforcement in continuous slabs, has been, in the past, a great source of annoyance and dissatisfaction. Some engineers may contend that no failures have occurred as a direct result of neglecting this important feature in construction work. In this connection it must be realized that the factor of safety which fortunately exists in reinforced concrete, has very often concealed glaring incongruities of design and construction and has made possible the continued practice of many engineers and contractors who are not sufficiently skilled in the performance of their work. Dangerous defects can result from haphazard methods of placing steel bars by mechanics who are not intelligently disciplined in the execution of their work, or even trained to regard a plan other than as something incidental, relying on personal judgment and individual methods to place and secure reinforcement in one position or another.

The ultimate calculated strength of reinforced concrete buildings cannot be realized until some definite, tangible, practicable means of securing, supporting and spacing of slab and beam bars is universally adopted by engineers. Even an inch variation in the position of negative or positive reinforcement in the direction of the neutral plane, completely disturbs the theoretical accuracy of a design. Many engineers spend hours solving the more exact moment distribution in continuous beams and slabs, for the purpose of ascertaining accurate steel areas at different points, and yet the means to insure proper installation of the steel is too often a matter of remote concern.

While the Joint Committee, other committees and societies are conscientiously attempting to prepare reinforced concrete specifications on design for universal adoption, there appears to exist an unfortunate disposition on the part of engineers to neglect and discount the importance of increasing the efficiency of prevailing construction methods. A carefully executed design can be easily rendered a careless piece of work in the hands of contractors, who fail to appreciate that good design and careful intelligent construction methods are inseparable instruments of good service. The fabrication of structural steel at the building

site, by means of unskilled mechanics, would be considered suicidal in the light of good practice. The same general opinion will, no doubt, exist in the near future about reinforced concrete structures erected without the use of bar supports and spacers.

In continuous slab design very satisfactory results have been obtained by employing high chairs of proper height and spaced about 2 to 4 ft. on centers, parallel with the supports. The high chairs first receive a ⅜- or ½-in. rod extend-

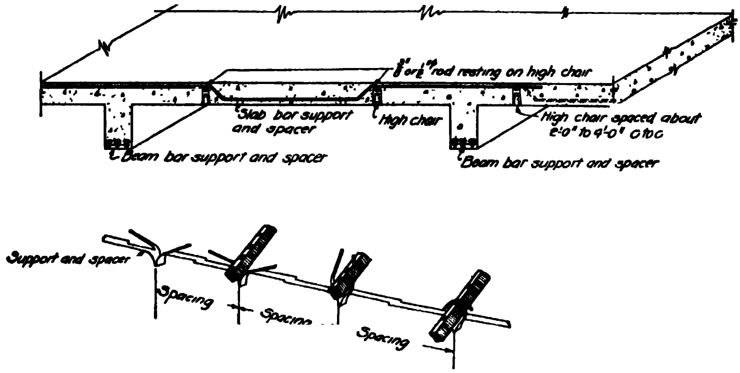


FIG. 32.

ing perpendicular to the main slab reinforcement, the bent-up ends of the latter resting upon the rod and chai supports (see Fig. 32). High chairs cost from 4 to 8 cts. each and consequently add little, if anything, to the cost of construction. The proper position of the bent-up portion of slab rods may also be obtained by wiring the rods to the under surface of wood screeds, placed along both sides of the beam (see Fig. 33). Screeds so placed will also serve the purpose of forming a gage by which the specified thickness of a slab may be properly maintained.

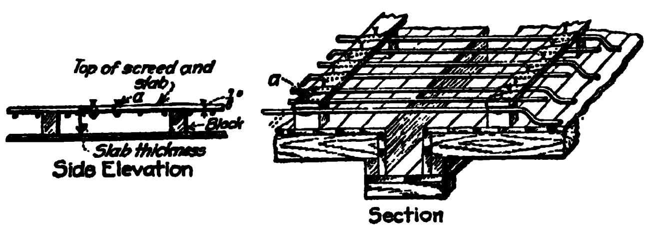


FIG. 33.

The rods of beams or girders should also be supported and spaced by means of mechanical devices (see Fig. 32). If the bond stress of concrete in-casing steel bars is figured, for example, at 100 lb. per sq. in., then a practicable means of actually obtaining this safe value in practice should be specified. Rods bunched together cannot be expected to give results compatible with rods properly separated.

When city building codes of the country specify the use of bar supports and spacers in the construction of every fireproof building, then engineers can

reasonably assume higher unit working stresses than now exist in the concrete and steel, and at the same time, be entirely consistent with the results obtained by the average present-day construction methods.

51. Arrangement and Bending of Slab Steel.—The steel bars of slabs may be bent and arranged several ways. Simple arrangements that require little interpretation and insure accurate results are always preferable to those that invite improper workmanship. Figure 34*a* illustrates an arrangement consisting of straight rods in the bottom and loose rods in the top over the supports. This arrangement eliminates, to a great extent, the cost of bending, but is objectionable on account of the difficulty of properly placing the loose rods in the top. This method has been employed on many occasions, but if it could be possible to ascertain the actual location of such rods, after the concrete had set, their assumed purpose to function as continuous reinforcement would be problematical. The partial

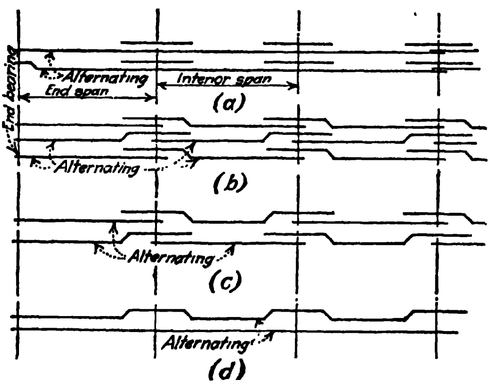


Fig. 34.

failure of a warehouse building in St. Joseph, Missouri, occasioned by the defective supports of a large elevated house tank above the roof, revealed a very interesting fact about loose rods over the supports. At least 50 per cent of these rods were found to be in the bottom of the slab, although the usual care was exercised to hold them in place during construction. Loose rods of this nature must be wired in place to screeds, metal high chairs, or placed after the slab has been poured to its full thickness. In the latter case the rods are given the responsibility of remaining in their proper positions, while concreters and blockmen are walking about engaged in screeding the concrete surface.

Figures 34*b* and *c* show arrangements which are used frequently in short and long span slabs. Both arrangements give the same amount of steel over the supports, as at the center of spans. The bending cost of bars in Fig. 34*b* is greater than the cost of arrangement given in Fig. 34*c*. The latter method is just as satisfactory as the former.

Rigid slab supports are particularly advantageous to short slab spans from 4 to 6 ft. due to the existing arch action. In such cases the question of providing for negative moment is not so vital a factor of design. Figure 34*d* shows

arrangement which is adaptable to short spans, where the steel area over the supports is equal to one-half the area at the center of span.

52. Screeds for Floor Slabs.—Various methods are employed by contractors to gage the proper thickness of floor slabs specified or shown on a plan. The specified thickness of a slab cannot be realized at the building unless contrivances similar to the commonly known screeds are constructed with depth equal to the depth of slab desired and installed at such intervals as will permit the blockmen to level the surface of the slab with a straight edge extending from one screed to the other (see Fig. 35).

Many reinforced concrete buildings which have been dismantled and removed to make space for more modern types of construction, have shown decided variations in the thicknesses of slabs originally specified and those actually obtained. The neglect of this very important feature in building construction has caused many discordant results. Until engineers and contractors realize that brick bats, isolated wood or concrete blocks and other unsatisfactory forms

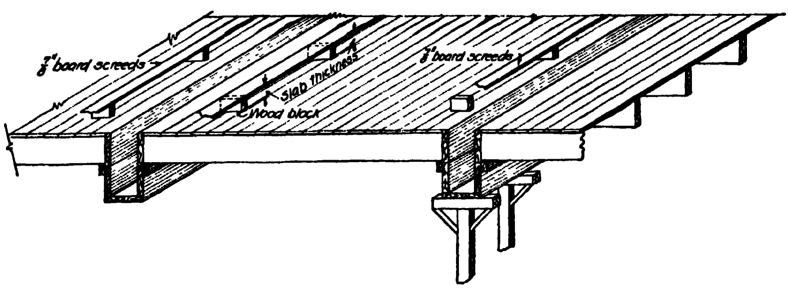


Fig. 35.

of gages, cannot even approximate accurate adherence to slab thicknesses required, then it must be expected that the practices of the past will continue a detriment to accurate workmanship.

53. Marking of Bent Rods.—The marking of bent rods in a clear, distinct and legible manner is another form of excellent insurance for both client and owner. Any form of cloth or linen tag marked or stamped with indelible pencil, ink or otherwise, has failed to render the service desired. Exposure to the elements, where rain, snow, dirt or mud are present, serves to eliminate the cloth tag as a valuable means of identifying bent rods. If the tags cannot withstand the abuses of handling and resistance to varying weather conditions, then their detachment from bent rods or bundles of identical bent rods, or the possibility of becoming illegible, promotes a spirit of carelessness on the part of iron workers, who, in the majority of cases will accept the way of least resistance, substituting other bent rods that may or may not be similar to actual requirements. The high prevailing cost of labor in the erection of buildings should encourage engineers to give due attention and study to construction methods and devices, which will serve to prevent repetition of effort, unnecessary expense, lost motion, delays and inefficiency. Tags made of non-corrosive metal give ideal service and should always be used in connection with the marking of straight and bent rods.

The segregation of straight rods for ready identification at the building site entails little difficulty, but the contrary is true in the case of bent rods. Good results may be obtained by giving bent rods of beams and girders the same numbers as given on the plans. For example B-1, B-2, or G-100, G-101, etc. The miscellaneous bent rods of slabs present a somewhat different problem. The following simple method has been used with success where employed and consists of stamping non-corrosive metal tags with numbers that designate each different bent rod, besides indicating by the first figure of the mark number the size of the rod. To illustrate: Reduce all merchantable bar sizes to fractions of eighths, the dividend of the fraction for each bar size always representing the first figure of the mark number as follows.

$\frac{1}{4}$ in. =	$\frac{2}{8}$ in. =	mark 200
$\frac{3}{8}$ in. =	$\frac{3}{8}$ in. =	mark 300
$\frac{1}{2}$ in. =	$\frac{4}{8}$ in. =	mark 400
$\frac{5}{8}$ in. =	$\frac{5}{8}$ in. =	mark 500
$\frac{3}{4}$ in. =	$\frac{6}{8}$ in. =	mark 600
$\frac{7}{8}$ in. =	$\frac{7}{8}$ in. =	mark 700
1 in. =	$\frac{8}{8}$ in. =	mark 800
$1\frac{1}{8}$ in. =	$\frac{9}{8}$ in. =	mark 900
$1\frac{1}{4}$ in. =	$\frac{10}{8}$ in. =	mark 1,000

Bent rods marked 400, 401, 402, etc., will indicate at once a $\frac{1}{2}$-in. rod, marks 900, 901, 902, etc., a $1\frac{1}{8}$-in. rod and so on. This system used in conjunction with non-corrosive metal tags, besides being simple, is most efficient when applied intelligently and consistently by workmen at the building or by steel mills engaged in bending rods from engineering shop drawings.

54. T-Beams Continuous at Both Supports.—The design of T-beams at the continuous ends over supports involves a multiplicity of complications if theoretical accuracy is desired. Between the points of inflection and the supports, a T-beam becomes a rectangular section in compression reinforced top and bottom. It may be stated with reasonable assurance that the majority of engineers proportion the sizes of T-beams exclusively for diagonal tension and positive bending moments, but neglect to consider the compressive stresses in the concrete and steel at continuous ends, after first assuming the most unfavorable distribution of loading that the particular member is likely to sustain. The successful operation of buildings designed, where this feature of design has been neglected, can be attributed to several causes, foremost among them may be outlined as follows: In the first place buildings for residential purposes, such as hotels and apartments are seldom, if ever, subjected to the live loads for which the various members are designed. Continuous T-beams in such buildings must have adjacent spans fully loaded simultaneously, in order that the maximum negative moment may be realized, assuming the usual formula $M = \frac{WL}{12}$ has been employed to obtain both positive and negative moments. It must also be understood that should the concrete of a beam or girder be excessively stressed at the continuous ends, this condition prevails for a very limited distance out from the edge of supports, due to the abrupt change from maximum negative moment to the

point of inflection. The Joint Committee has recommended that the rods in the bottom to support the continuous ends of beams be extended into adjacent spans to develop sufficient bond stress. After analyzing the opposing stresses in a fully continuous member the actual necessity of this provision may be reasonably questioned. Figure 36 represents a continuous beam showing straight rods in the bottom. The portion of the beam between *a* and *b*, *c* and *d* are in compression from neutral axis to the bottom of the member. Between *b* and *c* the member is in tension. Since the portion of bottom rods between *b* and *c* are in tension, it is difficult to conceive how the change from tensile to compressive stress at the ends would necessitate the development of sufficient bond stress beyond the center line of supports equal to the compressive stress in the rods, for the reason that the position of the ends in compression cannot be appreciably altered unless the tensile portion becomes distorted or elongated to

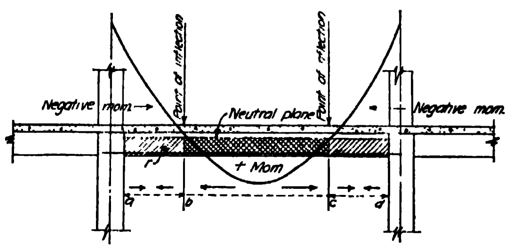
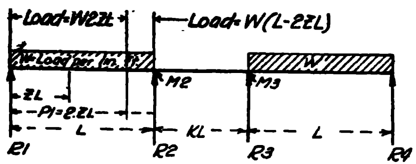


Fig. 36.

the extent of failure. The value of bottom rods at the ends of continuous beams is somewhat analogous to the action of longitudinal rods in columns subjected to test loads in a testing machine. In the latter case the longitudinal rods are not extended beyond the bearings to develop bond stress, and yet the absence of this feature is not considered a detriment to the ultimate carrying capacity of columns. The ends of straight rods in continuous ends of beams have the advantage of being restrained along their length between points of inflection.

55. Three Continuous Spans.—The space allotted to this chapter precludes an exhaustive analysis of moments in continuous beams of equal and different spans, resulting from the various loadings, applicable to the design of practical structures. In large hotel buildings the width of wings, in order to furnish a corridor and rooms along both sides, average about 46 to 48 ft. The corridor extending along the center of the wing compels, as a general rule, the use of two equal end spans and a short interior corridor span. This slab arrangement being a typical condition in buildings of this class and so often encountered by designers, Tables 6 and 7 have been prepared to illustrate that the more exact theorem of maximum positive and negative moments can be tabulated in such a manner that the reactions, points of contra-flexure and maximum moments may be readily derived for the separate loading of the two end spans, the center span, or the loading of all three spans, providing the two end spans are equal. The center span may be of any length.

TABLE 6



W = Load per linear foot in pounds
L = Span in feet
K = Ratio of center span to end span
$R1$, $R2$, $R3$, $R4$ = End reactions
P_1 = Distance from end support to point of inflection
ZL = Distance from end support to point of maximum positive moment
$M2$, $M3$ = Maximum negative moments at 2d and 3d supports for loading indicated

Formulas

$M2 = M3 = -C2 \cdot W \cdot L^2$

where $C2 = \dfrac{1}{4(2 + 3K)}$ = values in table

$R1 = R4 = W \cdot L \cdot X3$
where $X3 = (\frac{1}{2} - C2)$ = values in table
$R2 = R3 = W \cdot L \cdot X4$
where $X4 = 1 - X3$ = values in table

Maximum positive moment end spans

$$+M_o = \frac{W(2 \cdot ZL)^2}{8}$$

To find variable Z in $Z \cdot L$.

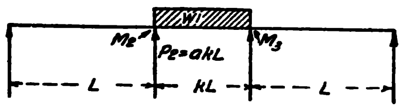
$$Z = X3 - \left[\left(\frac{W1}{W}\right)(X5)\right]$$

Values $X5$ found in Table 7 for loading on center span

NOTE.—This table must always be used jointly with Table 7 to find maximum negative moments and reactions. From this table maximum negative moments, reactions, etc., are derived from loading of end spans only, as shown. All values found to be added to values obtained by Table 7, where center span KL is loaded with dead load only, or dead plus live loads.

Values of K	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	1.20	1.40	1.60	1.80	2.00
Values of C2	0.0862	0.0781	0.0714	0.0658	0.0610	0.0568	0.0532	0.05	0.0446	0.0403	0.0368	0.0340	0.0313
Values of X3	0.4138	0.4219	0.4286	0.4342	0.4390	0.4432	0.4468	0.45	0.4554	0.4597	0.4632	0.4660	0.4687
Values of X4	0.5862	0.5781	0.5714	0.5658	0.5610	0.5568	0.5532	0.55	0.5446	0.5403	0.5368	0.5340	0.5313

TABLE 7



$W1$ = Load per linear foot in pounds on center span
$P2 = aKL$ = Distance from 2d or 3d support to point contraflexure of center-span, when positive moment exists

Formulas

$M2 = M3 = -C3 \cdot W1 \cdot L^2$

where $C3 = \dfrac{K^3}{4(2 + 3K)}$ = values in table

$R1 = R4 = -W1 \cdot L \cdot (X5)$

where $X5 = C3$ = values in table
$R2 = R3 = W1 \cdot L \cdot (X6)$
where $X6 = (\frac{1}{2}K) + (X5)$ = values in table

To find variable a in $P2 = aKL$

$$a = \pm \sqrt{0.25 - \left[\left(\frac{W}{W1}\right)\frac{(1 - 2Z)}{K^2}\right]} + 0.5$$

When $\left(\frac{W}{W1}\right)\frac{(1 - 2Z)}{K^2}$ is *greater* than 0.25, center span has no points of contraflexure and therefore no positive moment.
When this value is *less* than 0.25, center span has points of contraflexure, and therefore positive moment.

NOTE.—Other notations of Table 7 are the same as identical notations of Table 6.
NOTE.—This table must always be used jointly with Table 6 to find maximum negative moments and reactions.

Values of K	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	1.20	1.40	1.60	1.80	2.00
Values of $C3$	0.00233	0.0050	0.00893	0.0142	0.0209	0.0291	0.0388	0.0500	0.0771	0.1107	0.1506	0.1970	0.2500
Values of $X5$	0.00233	0.0050	0.00893	0.0142	0.0209	0.0291	0.0388	0.0500	0.0771	0.1107	0.1506	0.1970	0.2500
Values of $X6$	0.15233	0.2050	0.25893	0.3142	0.3709	0.4291	0.4888	0.5500	0.6771	0.8107	0.9506	1.0970	1.2500

In the preparation of Tables 6 and 7 due recognition is given to M. F. Marks.

To illustrate the adaptability of Tables 6 and 7 the following simple example of three equal spans will be applied in order to afford the reader less difficulty, should it be desired to check the results by the use of any other method commonly employed under similar circumstances.

Illustrative Problem.—Three continuous beams 10 × 18 in. of a hotel building of three equal spans are designed to support loads as indicated in Fig. 37. The two end spans are loaded with a live load of 50 lb. per sq. ft. of floor surface or 650 lb. per lin. ft. and dead load of 1,230 lb. per lin. ft. Find points of inflection, maximum positive and negative moments and reactions.

(1) Find reaction at support $R1$.

From Table 6, K is equal to one since the ratio of span lengths is equal to one.

From Table 6, $R1 = (WL)(X3) = (1{,}880)(18)(0.45) \qquad = +15{,}228$

From Table 7, $R1 = (-W1L)(X5) = -(1{,}230)(18)(0.05) = -1{,}107$

$R1 = +14{,}121$ lb.

(2) Find point of inflection A span.

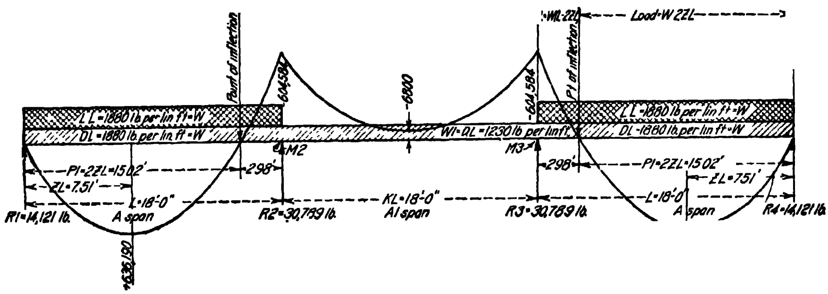


FIG. 37.

Inflection point from support $R1$ is equal to $P1 = \dfrac{(14{,}121)(2)}{1{,}880} = 15.02$ ft. or point maximum positive moment A span $= ZL$ where $Z = (X3) - \left[\left(\dfrac{W1}{W}\right)(X5)\right]$. Substituting values from Tables 6 and 7

$$Z = 0.45 - \left[\left(\frac{1{,}230}{1{,}880}\right)(0.05)\right] = 0.45 - 0.0327 = 0.4173$$

Point maximum moment $= (18)(0.4173) = 7.51$ ft., or point of inflection $= (2)(7.51) = 15.02$ ft.

(3) Having found the reaction $R1$ and the point of inflection, the maximum positive moment of A span may be found thus:

$$(1{,}880)(15.02)^2(1.5) = 636{,}190 \text{ in.-lb.}$$

Or to check this moment value by coefficients derived for three equal spans and the loading shown.

From dead load at point 0.4 of span from $R1$

$$(0.08)(1{,}230)(18)^2(12) = +\ 382{,}580$$

From live load $= (0.10)(650)(18)^2(12) \qquad +\ 252{,}720$

$+\ 635{,}300$ **in.-lb.**

(4) Find maximum negative moment $M2$ at support $R2$.

Table 6 moment from A span

$$M2 = M3 = -C2\ WL^2 = (0.05)(1{,}880)(18)^2(12) = -365{,}472$$

Table 7

$$M2 = M3 = -C3\ W1L^2 = (0.05)(1{,}230)(18)^2(12) = -239{,}112$$

$$M2 = M3 = \qquad -604{,}584 \text{ in.-lb.}$$

Or knowing the relative position of loads and point of inflection for span A this moment may be easily checked by taking moments about support $R2$.

$$
\begin{aligned}
&(7.51)(1{,}880)(2.98)(12) &&= 504{,}888\\
&\frac{(2.98)^2(1{,}880)(12)}{2} &&= 100{,}166\\
&M2 = M3 = &&- 605{,}054 \text{ in.-lb.}
\end{aligned}
$$

It may be seen from this simple analysis that maximum negative or maximum positive moments can be easily obtained by simple methods, after the point of inflection is found for any given loading.

Or checking by coefficients derived for three equal spans, and loading shown

$$
\begin{aligned}
&\text{From dead load } M2 = M3 = (-0.10)(1{,}230)(18)^2(12) &&= -478{,}220\\
&\text{From live load } M2 = M3 = (-0.05)(650)(18)^2(12) &&= -126{,}330\\
&M2 = M3 = &&-604{,}550 \text{ in.-lb.}
\end{aligned}
$$

(5) In this example the ratio of live to dead $= \dfrac{650}{1{,}230} = 0.53.$

When three equal spans are loaded, as in this example, and the ratio of live to dead loads is equal to 0.5, then the moment at center line of second or center span is equal to zero. When this ratio is *greater* than 0.50, then negative moment exists at center line of second span. When this ratio is *less* than 0.5 then positive moment exists at center line of second span.

Assuming similar loadings on three equal spans as in Fig. 37,

When live load = dead load then,
Point inflection A span $= 0.850\,L$
Point maximum positive moment A span $= 0.425\,L$

When live load = ½ dead load then,
Point inflection A span $= 0.835\,L$
Point maximum positive moment A span $= 0.4175\,L$

When live load = 2 dead load then,
Point inflection A span $= 0.866\,L$
Point maximum positive moment A span $= 0.433\,L$

When live load = 3 dead load then,
Point inflection A span $= 0.875\,L$
Point maximum positive moment A span $= 0.4375\,L$

It therefore follows that in this example, negative moment exists at the center line of the second span, and consequently this span has no point of inflection.

Whenever the ratio of live to dead is less than 0.5 for three equal spans, the points of inflection for center span may be found by the formula. Point inflection $P2 = aKL$ in which, from Table 7

$$\pm\sqrt{\frac{1}{4} - \left(\frac{W}{W1}\right)\left(\frac{1 - 2.Z}{K^2}\right)} + \frac{1}{2}$$

Where

$$
\begin{aligned}
\frac{W}{W1} &= \frac{1{,}880}{1{,}230} = 1.528\\
2.Z &= (2)(0.4173) = 0.8346\\
K &= 1
\end{aligned}
$$

Referring to Table 7 and explanations attending formula for finding a, and substituting values in

$$\left(\frac{W}{W1}\right)\left(\frac{1 - 2.Z}{K^2}\right) = (1.528)(1 - 0.8346) = -0.2527$$

Since this result is greater than 0.25, negative moment exists from center to center of supports, or

$$
\begin{aligned}
&\text{From dead load} = +M = (0.025)(1{,}230)(18)^2(12) &&= +119{,}556\\
&\text{From live load} = -M = (0.05)(650)(18)^2(12) &&= -126{,}360\\
&\text{Negative moment at center line } A1 \text{ span} = &&-6{,}804 \text{ in.-lb.}
\end{aligned}
$$

Illustrative Problem.—Consider the three spans as in above problem, uniformly loaded with 1,880 lb. per lin. ft. (see Fig. 38). Find points of inflection, maximum positive and negative moments and reactions.

(1) Find reaction at support $R1$.

From Table 6, K is equal to one.

From Table 6, $R1 = (WL)(X3) = +(1{,}880)(18)(0.45) = +15{,}228$

From Table 7, $R1 = -(W1L)(X5) = -(1{,}880)(18)(0.05) = -1{,}692$

Reaction $R1 =$ 13,536 in.-lb.

(2) Find point inflection A span.

$$P1 = \frac{(13{,}536)(2)}{1{,}880} = 14.4 \text{ ft.}$$

Hence point inflection is 3.6 ft. from reaction $R2$.

(3) To find maximum positive moment A span

$$\text{Moment} = (1{,}880)(14.4)^2(1.5) = +584{,}755 \text{ in.-lb.}$$

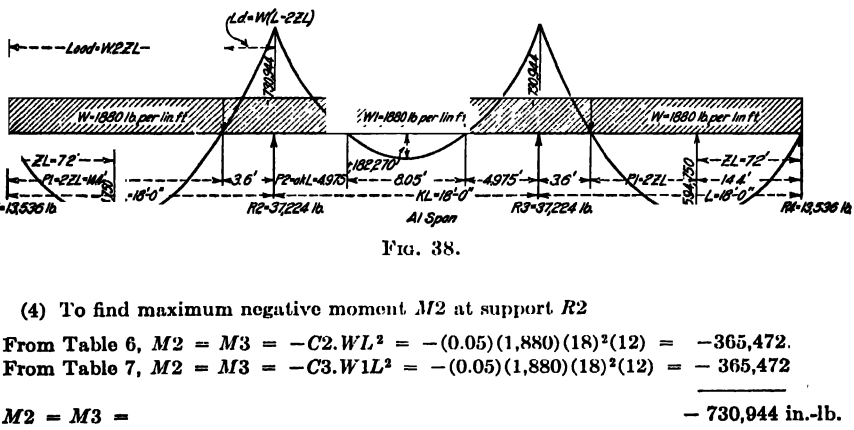


Fig. 38.

(4) To find maximum negative moment $M2$ at support $R2$

From Table 6, $M2 = M3 = -C2.WL^2 = -(0.05)(1{,}880)(18)^2(12) = -365{,}472.$

From Table 7, $M2 = M3 = -C3.W1L^2 = -(0.05)(1{,}880)(18)^2(12) = -365{,}472$

$M2 = M3 =$ $-730{,}944$ in.-lb.

(5) Point of inflection span $A1$.

Point inflection $P2 = aKL$.

$$a = \pm\sqrt{0.25 - \left(\frac{W}{W1}\right)\left(\frac{1 - 2.Z}{K^2}\right)} + 0.50$$

$$\frac{W}{W1} = \frac{1{,}880}{1{,}880} = 1. \qquad K = 1. \qquad Z = \frac{14.4}{(2)(18)} = 0.40$$

Substituting values in formulas

$$a = \pm\sqrt{0.25 - 0.20} + 0.5 = 0.2764$$

$$P2 = (0.2764)(1)(18) = 4.975 \text{ ft.}$$

(6) To find maximum positive moment $A1$ span.

$$\text{Moment} = (1{,}880)(8.05)^2(1.5) = +182{,}700 \text{ in.-lb.}$$

or from 3-moments theorem equals

$$M = +(0.025)(1{,}880)(18)^2(12) = +182{,}700 \text{ in.-lb.}$$

It will be found that Tables 6 and 7 apply for three continuous spans where the end spans are the same and center span of any length. Values of K are given from 0.3 to 2.0. Intermediate values of K may be found reasonably close for

practical purposes, by interpolation. Spans of this nature, applying particularly to slabs in hotel and apartment buildings, may be readily analyzed. When the end spans are, say 18 ft. 0 in. and center span 7 ft. 2 in., the average designer proceeds to approximate the negative moments, inflection points and reactions. These tables and formulas will also prove convenient for plotting the maximum moment curves, thus supplying information so frequently lacking in the average calculations of similar slab arrangements. A study of conditions represented in Figs. 37 and 38 will serve to visualize the effects of the different loadings on the points of inflection and intensity of positive and negative moments.

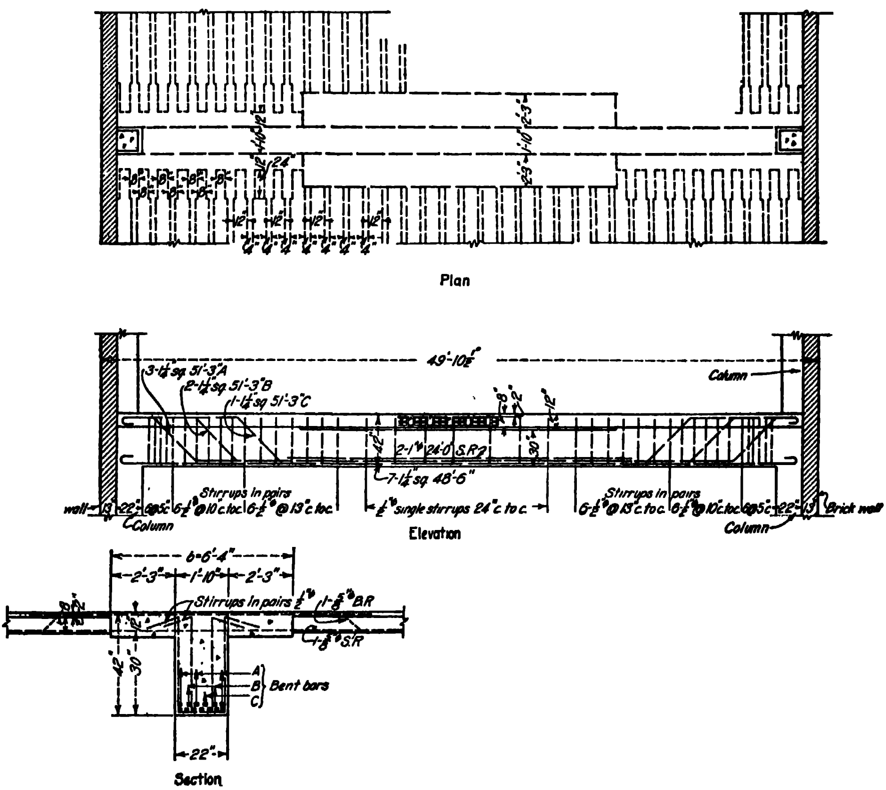


FIG. 39.

56. T-Beam Design.

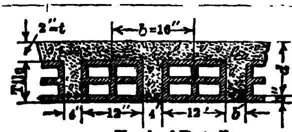
Illustrative Problem.—Long span T-beams 21 ft. 3 in. center to center support dead and live loads from floor panels, including the weight of beam, equal to 209,600 lb. (see Fig. 39). Span length for moment equals 45 ft. 9 in., f_s = 20,000 lb., f_c = 800 lb., and n = 15. Live load on floors 100 lb. per sq. ft. Determine size and reinforcement necessary. Slab design when $M = \frac{WL2}{12}$. For economic reasons combination hollow tile and concrete floor will be used (see Tables 8, 9 and 10).

TABLE

SAFE SUPERIMPOSED LOADS IN POUNDS PER SQUARE FOOT, FOR Unit Steel Stress = 16,000 lb.

One-way System

$M = \frac{WL}{12}$, $n = 15$	4″ × 12″ × 12″ Tile, 4″ Ribs, 16″c., 2″ Top					6″ × 12″ × 12″ Tile, 4″ Ribs, 16″c., 2″ Top				
	Weight Fl. per sq. ft. = 50 lb.					Weight Fl. per sq. ft. = 60 lb.				
	Concrete per sq. ft. 0.25 cu. ft.			Tile per sq. ft. 0.75–4″ Tile		Concrete per sq. ft. 0.292 cu. ft.			Tile per sq. ft. 0.75–6″ Tile	
Values p k j	.00276 .249 .918	.00351 .276 .908	.00491 .3172 .8943	.00625 .349 .884	.00767 .378 .874	.0025 .235 .921	.00351 .274 .909	.0045 .305 .90	.00548 .334 .893	.00697 .372 .887
Reinforcement each rib	2-3/8″φ	2-3/8″φ	2-1/2″φ	2-1/2″φ	2-5/8″φ	2-3/8″φ	2-1/2″φ	2-1/2″φ	2-5/8″φ	2-5/8″φ
Span in feet 10	44 *71	56 *103	78 *161	100 *215	122 *272	56 *157	79 *240	100 318	123 400	
11	40 *51	51 *76	71 *124	91 *169	111 *216	51 *119	71 *189	91 252	111 320	
12	37 *34	47 *56	65 *96	84 *134	102 *174	47 *91	66 *148	88 202	102 260	
13	34 *22	43 *41	60 *74	77 *107	94 *140	43 *68	60 *118	77 163	94 212	120 284
14		41 *28	56 *57	71 *85	88 *114	40 *51	56 *93	71 133	88 175	111 237
15			52 *44	67 *68	*82 93	38 *37	52 *73	66 108	82 145	104 199
16			49 *32	62 *53	77 *76		49 *57	62 88	76 119	98 167
17				*58 42	72 *61		46 *43	58 70	72 99	92 142
18				55 *32	68 *49		43 *32	56 57	68 82	87 119
19					65 *39			52 44	64 67	82 101
20									61 55	78 85
21									58 44	74 72
22										71 60
23										68 50
24										65 40
25										
26										
27										
28										
29										
30										
Resisting moment, in.-lb. (M_s)	16,230	20,410	28,120	35,360	42,930	28,980	40,010	50,460	61,370	77,590



Typical Detail

When value of k is less than $\frac{t}{d}$, Case I applies.
When value of k is greater than 0.3786, M_c controls.

When value of k is less than 0.3786, M_s controls.
*Indicates neutral axis in the flange.

NOTE: This table is based on $M = \frac{WL}{12}$. Top steel over support for negative M, same area A_s as for positive at center of span, top steel over supports extending 1/4 or 1/5 of span length. For end spans, when $M = \frac{WL}{10}$, use 5/6 of the combined superimposed load and dead weight of floor given.
For simple spans, when $M = \frac{WL}{8}$, use 2/3 of the combined table values as for end spans.

The unit shear $s = \frac{V}{b'jd}$ is given for each load value in small type.

COMBINATION TILE AND CONCRETE FLOORS
Unit Concrete Stress = 650 lb.

Continuous Spans

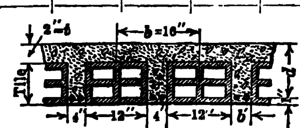
8″ × 12″ × 12″ Tile, 4″ Ribs, 16″c., 2″ Top					10″ × 12″ × 12″ Tile, 4″ Ribs, 16″ c., 2″ Top					12″ × 12″ × 12″ Tile, 4″ Ribs 16c., 2″ Top			
Weight Fl. per sq. ft. = 70 lb.					Weight Fl. per sq. ft. = 81 lb.					Weight Fl. per sq. ft. = 91 lb.			
Concrete per sq. ft. 0.334 cu. ft. Tile per sq. ft. 0.75–8″ Tile					Concrete per sq. ft. 0.375 cu. ft. Tile per sq. ft. 0.75–10″					Concrete per sq. ft. 0.417 cu. ft. Tile per sq. ft. 0.75–12″			
.00273 .250 .920	.00347 .280 .914	.00426 .310 .910	.00542 .350 .906	.00614 .372 .905	.00284 .264 .925	.00348 .294 .922	.00443 .334 .920	.00502 .358 .920	.0057 .381 .919	.00295 .283 .933	.00375 .324 .931	.00425 .348 .930	.00483 .372 .930
2-½″φ	2-½″φ	2-⅝″φ	2-⅝″φ	2-¾″φ	2-½″φ	2-⅝″φ	2-⅝″φ	2-¾″φ	1-¾″φ 1-¾″φ	2-⅝″φ	2-⅝″φ	2-¾″φ	1-¾″φ 1-¾″φ
78 320	100 424	123 533			100 529	122 665				122 801			
71 255	91 338	111 428			91 424	111 536				112 647			
65 201	83 273	102 349			83 343	102 437				102 529			
60 161	77 222	94 286			77 280	94 361	120 480			94 438	120 580		
56 129	71 182	88 237	111 320		71 231	88 300	111 403			88 365	112 488		
52 103	67 149	81 198	104 269		67 190	81 250	104 340	118 396		82 305	104 413		
49 82	62 123	77 165	98 228	110 267	62 158	77 211	97 289	110 338		77 258	98 352	110 410	
46 65	59 101	72 139	91 194	104 230	59 130	72 178	92 247	104 290		72 218	92 301	104 353	118 413
43 50	55 82	68 116	87 165	98 197	55 107	68 149	87 211	98 250	110 290	68 184	87 259	98 305	112 359
41 38	52 66	65 97	82 141	93 169	52 88	64 126	82 181	93 216	104 251	64 156	82 223	93 264	106 312
	50 53	61 81	78 120	89 146	50 71	61 106	78 156	88 188	99 219	61 132	78 192	88 229	100 273
	47 42	58 66	74 103	84 126	47 57	58 88	74 134	84 162	94 191	58 111	74 166	84 200	95 239
		56 55	71 87	80 108	45 45	56 73	71 115	80 141	90 167	56 93	71 143	80 174	91 210
		53 44	68 74	77 93	43 34	53 60	68 98	77 121	86 146	53 78	68 123	77 151	87 184
		51 34	65 63	73 80		51 48	65 83	74 106	82 127	51 64	65 106	73 132	83 162
			62 52	70 68		49 38	62 70	70 91	79 111	49 51	62 90	70 114	80 142
			59 42	67 58			60 59	68 78	76 97	47 41	59 75	68 99	77 124
				65 48			58 49	65 66	73 83	45 31	58 65	65 84	74 109
				63 40			56 40	63 55	70 72		56 54	63 72	72 95
								61 47	68 61		54 44	61 61	69 82
								59 38	66 52		52 35	59 51	67 71
52,000	65,810	80,400	101,890	115,200	81,400	99,570	126,460	143,140	1[illegible]9,100	119,008	151,240	171,000	194,270

TABLE

SAFE SUPERIMPOSED LOADS IN POUNDS PER SQUARE FOOT, FOR

One-Way System — Unit Steel Stress = 18,000 lb

$M = \frac{WL}{12}$, $n = 15$	4" × 12" × 12" Tile, 4" Ribs, 16"c., 2" Top					6" × 12" × 12" Tile, 4" Ribs, 16"c., 2" Top				
	Weight Fl. per sq. ft. = 50 lb.					Weight Fl. per sq. ft. = 60 lb.				
	Concrete per sq. ft. 0.25 cu. ft.			Tile per sq. ft. 0.75–4"		Concrete per sq. ft. 0.292 cu. ft.			Tile per sq. ft. 0.75–6"	
Values p, k, j	.00276 .249 .918	.00351 .276 .908	.00491 .3172 .8943	.00625 .349 .884	.00767 .378 .874	.0025 .235 .921	.00351 .274 .909	.0045 .305 .900	.00548 .334 .893	.00697 .372 .887
Reinforcement each rib	2-⅜"φ	2-⅜"φ	2-½"φ	2-½"φ	2-⅝"φ	2-⅜"φ	2-½"φ	2-½"φ	2-⅝"φ	2-⅝"φ
Span in feet 10	49 *87	63 *122	88 *187			63 *184	88 *288	112 365		
11	45 62	58 *92	80 *146	102 *197		58 *142	80 *219	102 291		
12	41 *45	52 *70	74 *115	95 *157	115 *202	52 *110	74 *174	94 235	115 300	
13	38 *31	49 *52	69 *90	86 *126	106 *164	49 *73	68 *140	86 191	106 246	
14		45 *38	63 *72	81 *102	99 *134	15 *64	63 *112	80 157	98 204	
15		41 *27	60 *55	75 *82	92 *111	41 *48	59 *89	75 129	92 170	117 231
16			55 *43	70 *67	86 *92	39 *35	55 *71	70 106	86 142	110 196
17			52 *32	66 *54	81 *75		51 *56	66 87	81 119	103 166
18				63 *43	76 *61		49 *44	62 71	77 99	98 143
19				59 *33	73 *50		47 *34	59 57	72 83	92 121
20								56 46	68 69	87 103
21								53 36	65 57	84 89
22									62 46	79 75
23									59 38	76 63
24										73 53
25										70 44
26										67 37
27										
28										
29										
30										
Resisting moment, in.-lb. (M_s)	18,180	22,940	31,620	39,780	48,300	32,600	45,030	56,650	69,010	87,280



Typical Detail

When value of k is less than $\frac{t}{d}$, Case I applies.
When value of k is greater than 0.3846, M_c controls.

When value of k is less than 0.3846, M_s controls.
*Indicates neutral axis in the flange.

Note: This table is based on $M = \frac{WL}{12}$. Top steel over support for negative M same area A_s as for positive at center of span, top steel over supports extending ¼ or ⅓ of span length.
For end spans, when $M = \frac{WL}{10}$, use ⅚ of the combined superimposed load and dead wt. of floor given.
For simple spans, when $M = \frac{WL}{8}$, use ⅔ of the combined table values as for end spans

The unit shear $v = \frac{V}{b'jd}$ is given for each load value in small type.

9

Combination Tile and Concrete Floors
Unit Concrete Stress = 750 lb.

Continuous Spans

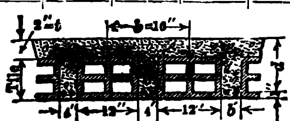
8" × 12" × 12" Tile, 4" Ribs, 16" c. 2" Top					10" × 12" × 12" Tile, 4" Ribs, 16" c., 2" Top					12" × 12" × 12" Tile, 4" Ribs, 16" c., 2" Top			
Weight Fl. per sq. ft. = 70 lb.					Weight Fl. per sq. ft. = 81 lb.					Weight Fl. per sq. ft. = 91 lb.			
Concrete per sq. ft. 0.334 cu. ft. Tile per sq. ft. 0.75–8"					Concrete per sq. ft. 0.375 cu. ft. Tile per sq. ft. 0.75–10"					Concrete per sq. ft. 0.417 cu. ft. Tile per sq. ft. 0.75–12"			
.00273 .250 .920	.00347 .280 .914	.00426 .310 .910	.00542 .350 .906	.00614 .372 .905	.00284 .264 .925	.00348 .294 .922	.00443 .334 .920	.00502 .358 .920	.0057 .381 .919	.00295 .283 .933	.00375 .324 .931	.00425 .348 .930	.00483 .372 .930
2-½"φ	2-½"φ	2-⅝"φ	2-⅝"φ	2-¾"φ	2-½"φ	2-⅝"φ	2-⅝"φ	2-¾"φ	1-¾"φ 1-¾"φ	2-⅝"φ	2-⅝"φ	2-¾"φ	1-¾"φ 1-¾"φ
88 370	112 485				112 606								
80 293	103 388				102 487								
74 235	94 315	115 401			94 395	115 502							
68 190	86 258	106 331			86 325	106 416				106 504			
64 154	80 214	98 277			80 269	98 347				98 422			
59 125	75 177	92 232	117 312		75 224	92 293	117 393			92 355			
55 101	70 147	86 195	110 267		70 188	86 247	110 336			86 301	110 408		
52 82	66 123	81 165	104 228	117 266	66 157	81 209	104 288	117 337		81 256	103 350		
49 66	62 101	76 139	97 196	111 230	62 131	77 178	97 248	110 292		76 220	97 303	110 354	
47 51	59 84	72 117	93 169	105 200	59 109	73 152	92 215	104 254	119 298	72 187	93 263	105 309	
44 39	56 69	69 99	88 145	99 173	56 90	69 129	87 185	99 221	113 262	69 160	87 228	99 269	113 318
	53 55	66 84	84 125	95 150	53 74	66 109	84 161	94 193	107 229	66 137	83 199	94 237	107 280
	51 45	63 70	80 107	90 131	51 61	62 93	79 140	90 168	103 202	63 116	80 172	90 207	103 247
	49 35	60 58	77 93	86 114	49 49	59 77	76 122	86 148	98 178	59 99	76 150	86 182	98 219
		58 47	73 79	83 98	47 39	57 65	73 104	83 128	94 157	57 83	73 130	83 159	94 193
		55 39	70 68	79 85		55 54	70 90	79 112	90 138	55 69	70 112	79 139	90 172
			67 57	77 73		53 43	67 77	76 97	87 121	53 58	67 98	76 122	86 151
			65 48	75 64		51 34	65 66	73 85	83 107	51 48	65 84	74 107	84 134
			63 40	71 54			62 55	71 73	80 94	49 37	63 72	71 93	80 118
			60 32	68 45			60 46	68 63	78 82		60 61	68 80	77 103
				66 38			58 38	66 53	75 71		58 51	66 69	76 91
58,570	74,030	90,450	114,620	129,600	91,570	112,010	142,260	161,030	182,740	133,960	170,140	192,370	213,560

TABLE

SAFE SUPERIMPOSED LOADS IN POUNDS PER SQUARE FOOT, FOR
Unit Steel Stress = 20,000 lb.

One-Way System

$M = \frac{WL}{12}$	4" × 12" × 12" Tile, 4" Ribs, 16"c., 2" Top					6" × 12" × 12" Tile, 4" Ribs, 16"c., 2" Top				
	Weight Fl. per sq. ft. = 50 lb.					Weight Fl. per sq. ft. = 60 lb.				
$n = 15$	Concrete per sq. ft. 0.25 ft. Tile per sq. ft. 0.75–4" Tile					Concrete per sq. ft. 0.292 cu. ft. Tile per sq. ft. 0.75–6"				
Values p	.00276	.00351	.00491	.00625	.00767	.0025	.00351	.0045	.00548	.00697
Values k	249	.276	.3172	.349	.378	.235	.274	.305	.334	.372
Values j	.918	.908	.8943	.884	874	.921	.909	.900	.893	.887
Reinforcement each rib	2-3/8"φ	2-3/8"φ	2-1/2"φ	2-1/2"φ	2-5/8"φ	2-3/8"φ	2-1/2"φ	2-1/2"φ	2-5/8"φ	2-5/8"φ
Span in feet 10	55 *102	70 *141	98 *213			70 *211	98 *315			
11	50 *75	64 *108	89 *168	113 *224		64 *164	89 *250	113 *330		
12	45 *55	58 *83	82 *133	105 *180		58 *129	82 *200	104 *268		
13	42 *40	51 *63	76 *106	96 *146	117 *184	54 *99	76 *162	96 *219	118 *280	
14		50 *48	70 *85	90 *119	108 *152	50 *78	70 *131	89 *181	109 *233	
15		46 *35	66 *67	83 *97	101 *126	46 *60	65 *106	83 *150	102 *195	
16			61 *53	78 *80	94 *104	43 *46	61 *86	78 *124	95 *164	122 *224
17			58 *41	73 *65	89 *87	41 *34	57 *69	73 103	90 139	114 191
18				70 *53	83 *72		55 *56	69 86	85 117	109 165
19				66 *42	79 *59		52 *44	65 *70	80 99	102 141
20					75 *49			62 58	76 83	97 121
21					71 *39			59 47	72 70	93 105
22									69 58	88 90
23									66 49	85 77
24										81 66
25										78 56
26										75 48
27										
28										
29										
30										
Resisting moment, in.-lb. (M_s)	20,200	25,490	35,130	44,200	52,860	36,100	50,030	62,940	76,690	96,980



Typical Detail

When value of k is less than $\frac{t}{d}$, Case I applies.
When value of k is greater than 0.375 M_c controls.

When value of k is less than 0.375 M_s controls.
* Indicates neutral axis in the flange.

NOTE: This table is based on $M = \frac{WL}{12}$. Top steel over supports for negative M same area A_s as for positive at center of span, top steel over supports extending to 1/4 or 1/5 of span.

For end spans, when $M = \frac{WL}{10}$, use 5/6 of the combined superimposed load and dead wt. of floor given.

For simple spans, when $M = \frac{WL}{8}$, use 2/3 of the combined table values, as for end spans.

The unit shear $v = \frac{V}{b'jd}$ is given for each load value in small type.

10

COMBINATION HOLLOW TILE AND CONCRETE FLOORS
Unit Concrete Stress = 800 lb.

Continuous Spans

8″ × 12″ × 12″ Tile, 4″ Ribs, 16″ c., 2″ Top					10″ × 12″ × 12″ Tile, 4″ Ribs, 2″ Top					12″ × 12″ × 12″ Tile, 4″ Ribs 16″ c., 2″ Top			
Weight Fl. per sq. ft. = 70 lb.					Weight Fl. per sq. ft. = 81 lb.					Weight Fl. per sq. ft. = 91 lb.			
Concrete per sq. ft. 0.334 cu. ft. Tile per sq. ft. 0.75–8″					Concrete per sq. ft. 0.375 cu. ft. Tile per sq. ft. 0.75–10″					Concrete per sq. ft. 0.417 cu. ft. Tile per sq. ft. 0.75–12″			
.00273 .250 .920	.00347 .280 .914	.00426 .310 .910	.00542 .350 .906	.00614 .372 .905	.00284 .264 .925	.00348 .294 .922	.00443 .334 .920	.00502 .358 .920	.0057 .381 .919	.00295 .283 .933	.00375 .324 .931	.00425 .348 .930	.00483 .372 .930
2-½″φ	2-½″φ	2-⅝″φ	2-⅝″φ	2-¾″φ	2-½″φ	2-⅝″φ	2-⅝″φ	2-¾″φ	1-¾″φ 1-¾″φ	2-⅝″φ	2-⅝″φ	2-¾″φ	1-¾″φ 1-¾″φ
98 419							To find reinforcement and moment for any other width						
89 333	114 439				113 550		of rib than 4″, multiply moment and steel area A, each by distance center to center of ribs and divide by						
82 269	104 358				104 448		16, total lb. per sq. ft. remaining same. The unit shear for any other width of rib = 4″ divided by width of rib × shear sq. in. in table × distance c.c.						
70 219	96 295	118 376			96 370	118 471				118 570	ribs divided by 16.		
71 179	89 245	109 315			89 308	109 395				109 479			
65 147	83 204	102 265			83 258	102 334				102 405			
61 120	78 171	96 224	122 304		78 218	96 283	122 382			96 345			
58 99	73 144	90 191	115 261		73 183	90 241	115 329			90 295	115 399		
56 81	69 120	85 162	108 225	123 263	69 155	85 207	108 285	122 333		85 254	108 347		
52 65	66 101	80 138	103 195	117 230	66 130	81 178	102 248	116 291		80 218	103 302	117 353	
49 51	62 84	77 118	98 169	110 200	62 109	77 152	97 215	110 254	122 289	77 188	97 263	110 309	
47 41	59 69	73 101	93 147	105 174	50 91	73 130	93 188	105 223	116 254	73 162	93 231	105 273	119 321
	57 58	70 86	89 127	100 153	57 77	69 112	88 164	100 196	111 224	69 139	89 201	100 240	114 285
	55 47	67 72	85 111	96 134	54 63	66 95	85 144	96 173	106 199	66 120	85 177	96 212	109 253
		64 60	81 96	92 117	52 52	64 81	81 125	92 151	101 175	64 102	81 155	92 187	104 225
		61 51	78 83	88 102		61 69	78 109	88 133	98 156	61 87	78 135	88 165	100 201
		59 42	75 71	85 89		59 57	75 94	85 117	94 138	59 74	75 119	85 146	96 178
			72 61	83 79		57 47	72 82	81 103	90 121	57 63	72 103	82 129	93 159
			70 52	79 68			69 70	79 90	87 107	54 51	70 90	79 113	89 141
			67 43	76 58			67 60	76 79	84 95	52 42	67 78	76 99	86 125
				73 50			65 51	73 68	81 83		65 67	73 87	84 111
65,080	82,260	100,500	127,360	144,000	101,750	124,460	158,070	178,920	197,220	148,850	189,050	218,750	242,840

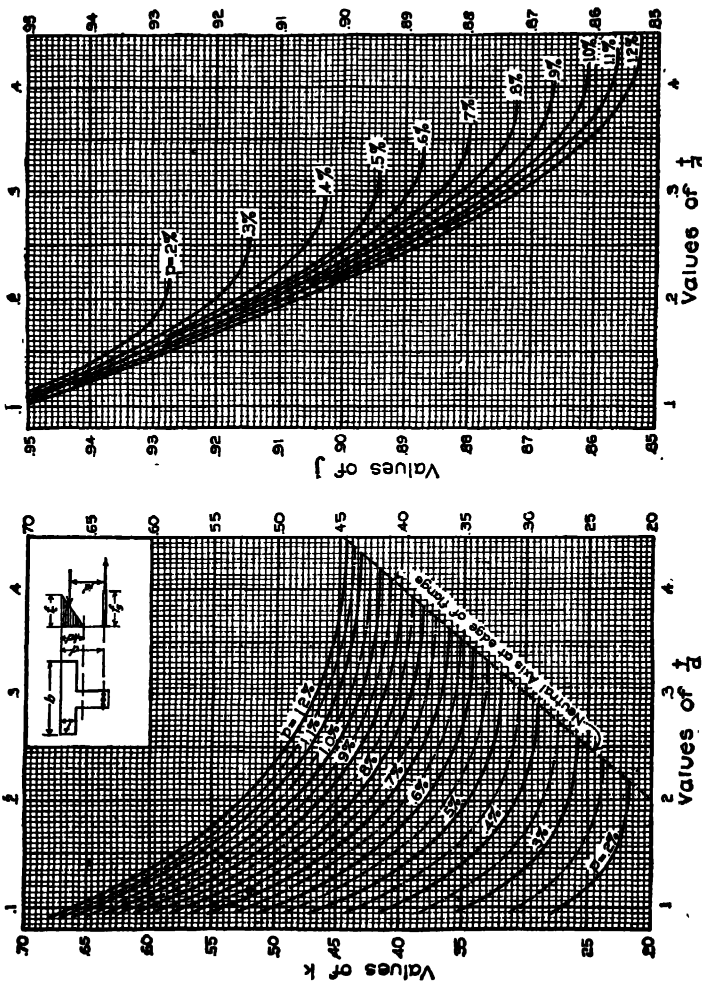
The combination slab will be designed for the following superimposed loads in pounds per square foot.

Live load =	100
Monolithic concrete finish	5
Total superimposed load per square foot	105

Referring to Table 10 of combination hollow tile and concrete joist when $f_s = 20{,}000$, $f_c = 800$, and $n = 15$, for a 21-ft. span and superimposed load of 105 lb. per sq. ft., panels

DIAGRAMS 8 AND 9

Use for T-beams. Values of k and j for various percentages of steel. Based on $n = 15$.



having 8 × 12 × 12-in. tile, 4-in. ribs and 2-in. top, weighing 70 lb. per sq. ft. will give the strength desired.

Assuming supporting girder to be 1 ft. 9 in. wide the actual load on each rib is

$$(105 + 70)(21.25 - 1.75)(1\tfrac{1}{3}) = 4{,}550 \text{ lb.}$$

$$M = \frac{(4{,}550)(21.25)(12)}{12} = 96{,}690 \text{ in.-lb.}$$

Referring to resisting moments at bottom of Table 10 it is found that the panel selected is sufficient for positive moment and that k is approximately 0.30 and j = 0.91. Two ⅝-in. round rods will be found sufficient for each rib.

$$\text{Checking back, } p = \frac{0.614}{(9)(16)} = 0.00426, \frac{t}{d} = \frac{2}{9} = 0.222$$

Diagrams 8 and 9, when $p = 0.00426$ and $\frac{t}{d} = 0.222$, give $k = 0.310$ and $j = 0.91$. Substituting values

$$f_s = \frac{M}{A_s.j.d.} = \frac{96,690}{(0.614)(0.91)(9)} = 19,230 \text{ lb. per sq. in.}$$

$$f_c = \frac{f_s k}{n(1 - k)} = \frac{(19,230)(0.310)}{(15)(1 - 0.310)} = 576 \text{ lb. per sq. in.}$$

For practical reasons it is not desirable to use stirrups to resist diagonal tension in panels of tile and concrete construction, on account of the difficulties incident to placing and holding them in proper position. It is good practice to assume one vertical rib of each tile to act with the concrete of each rib in resisting diagonal tension. Due to the solid concrete section required for T of beam (see Fig. 39), the actual reaction for each rib for a distance equal to ½ the span length of beam will be

$$\frac{(105 + 70)(21.25 - 6.33)(1\tfrac{1}{3})}{2} = 1,741 \text{ lb.}$$

Then shear measured on a vertical section will be

$$v = \frac{V}{bjd} = \frac{1,741}{(5)(\tfrac{7}{8})(9)} = 45 \text{ lb. per sq. in.}$$

When the shear is 45 lb. per sq. in. stirrups will not be required. Since the solid concrete T of beam extends out from the face of the beam a distance of 2 ft. 3 in. or a total of 3 ft. 2 in. from center line of beam it is at once understood that the negative moment at this point will not require investigation. The steel bars of each rib are bent as shown.

Design T-beam supporting panels (Fig. 39). The live load carried by beam will be assumed at 85 per cent of 100 lb. or 85 lb. per sq. ft.

Total D.L. panels = (75)(21.25)(44.0) =	70,120
Total L.L. panels = (85)(21.25)(44.0) =	79,470
D.L. beam assumed	60,000
Total uniform load	209,590 lb.

The span center to center of columns is 45 ft. 10½ in.

Maximum moment $M = (209,590)(45.87)(1\tfrac{1}{2}) = 14,420,800$ in.-lb

Assuming a section 22 × 42 in deep and d = 39 in., then

$$v = \frac{V}{bjd} = \frac{104,800}{(22)(\tfrac{7}{8})(39)} = 140 \text{ lb. per sq. in.}$$

For a beam section of this size a total shear of 140 lb. per sq. in. will be considered satisfactory and hence the section of beam to remain as assumed.

The approximate steel area will be

$$A_s = \frac{14,420,800}{(0.88)(39)(20,000)} = 21.01 \text{ sq. in.}$$

Now the approximate width of T required (assuming $p = 0.0075$ when $f_c = 800$ and $f_s = 20,000$) is $b = \frac{21.01}{(0.0075)(39)} = 71.8$ in. Should the bottom of flange be at the neutral axis of the section, the assumption for $p = 0.0075$ would give the correct width b for an extreme fiber stress of 800 lb.

In this design b = 6 ft. 4 in. will be selected for trial. Assuming t = 12 in. then

$$\frac{t}{d} = \frac{12}{39} = 0.3077$$

$$p = \frac{21.01}{(76)(39)} = 0.00709$$

Knowing values for $\frac{t}{d}$ and p, k and j will be found in Diagrams 8 and 9 when n = 15.

$$k = 0.365 \text{ and } j = 0.883$$

$w = \frac{M}{C} - D$

w = Safe superimposed loads in lb. per sq. ft.
M = moment in inch-pounds.
D = dead weight of slab per sq. ft.
l = span in feet.
$C = \frac{l^2}{12} \times 12$

TABLE 11

SOLID CONCRETE SLABS

1-2-4 MIX

SAFE SUPERIMPOSED LOADS IN POUNDS PER SQUARE FOOT

Continuous Spans. Unit Stress Steel = 16,000 lb. per sq. in., Medium Steel

Extreme Fiber Stress Concrete = 650 lb. per sq. in.

$n = 15$

Min. elastic limit = 33,000 lb. per sq. in.
Min. ult. strength = 55,000 lb per sq. in.

Moment in inch-pounds	Thickness of slab (inches)	Area of steel per 12 in. of width at 16,000 lb. per sq. in.	Corresponding size and spacing of round and square bars (inches)				M (in.-lb.) $= \frac{wl^2}{12} \times 12$																	Weight of slab per sq. ft.	Effective depth (in.)	Position of neutral axis (kd)
			Rounds		Squares		Span in feet																			
			ϕ	c.c.	ϕ	c.c.	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
5,160	3	0.185	¼ ⅜	3¼ 7	¼ ⅜	4 9	285	168	105	68	42													38	2	0.757
8,060	3½	0.231	⅜	5¾	¼ ⅜	3¼ 7¼	460	279	180	121	82	56	36											44	2½	0.947
11,610	4	0.277	⅜ ½	4¾ 8½	⅜	6	676	414	273	187	131	93	66	46	30									50	3	1.136
15,800	4½	0.323	⅜ ½	4 7¼	⅜ ½	5¼ 9¼	931	575	382	266	190	138	101	73	53	36								57	3½	1.325
18,140	4¾	0.346	½	6¾	⅜ ½	4¾ 8½	1073	666	444	310	223	164	121	90	66	47	33							60	3¾	1.420
20,640	5	0.369	½ ⅝	6½ 10	⅜ ½	4½ 8		763	510	358	260	192	143	107	80	59	43	29						63	4	1.514
23,300	5¼	0.392	½ ⅝	6 9½	⅜ ½	4¼ 7¾		866	581	410	298	221	167	129	96	72	53	37						66	4¼	1.609
26,120	5½	0.415	½ ⅝	5¾ 8¾	⅜ ½	4 7¼		976	657	464	339	254	192	147	112	86	64	47	33					69	4½	1.704
29,110	5¾	0.438	½ ⅝	5¼ 8½	½ ⅝	6¾ 10¾			736	522	383	288	219	169	130	100	76	57	42	29				72	4¾	1.798
32,250	6	0.461	½ ⅝	5 8	½ ⅝	6½ 10			821	583	429	322	248	191	149	116	90	68	51	36				75	5	1.893
35,560	6¼	0.484	½ ⅝	4¾ 7½	½ ⅝	6¼ 9¾			910	648	477	361	277	216	169	132	103	80	61	45	32			78	5¼	1.988
39,030	6½	0.508	⅝ ¾	7¼ 10½	½ ⅝	6 9¼			1002	714	528	400	308	241	189	149	117	92	70	53	38			82	5½	2.082

Values for p, k, and K when $f_c = 650$ lb. per sq. in. $f_s = 16{,}000$ lb. per sq. in. $n = 15$

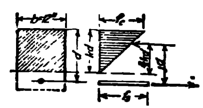
$$p = \frac{\frac{1}{2}}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right)} = 0.00769$$

$$k = \sqrt{2pn + (pn)^2} - pn = 0.3786$$

$k = 0.3786$ when $p = 0.00769$ and $n = 15$

$$K = \frac{M}{bd^2} = f_s p\left(1 - \frac{k}{3}\right) = 107.51 \text{ by steel}$$

$$K = \frac{M}{bd^2} = \tfrac{1}{2} f_c k\left(1 - \frac{k}{3}\right) = 107.51 \text{ by concrete}$$



(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
42,650	6¾	0.531	⅝ ¾	7 10	½ ⅝	5½ 8¾				786	582	442	341	267	211	167	133	104	81	62	47	33		85	5¾	2.177
46,440	7	0.554	⅝ ¾	6½ 9½	½ ⅝	5½ 8½				860	638	485	376	296	235	187	149	118	93	73	55	41	28	88	6	2.272
54,510	7½	0.600	⅝ ¾	6 8¾	½ ⅝	5 7¾				1018	758	579	451	356	284	229	184	148	119	94	74	57	42	94	6½	2.461
63,220	8	0.646	⅝ ¾	5¾ 8¼	⅝ ¾	7¼ 10½					888	681	532	423	339	274	223	181	147	119	95	75	58	100	7	2.650
72,570	8½	0.692	⅝ ¾	5¼ 7½	⅝ ¾	6¾ 9¾					1026	789	619	493	397	322	263	216	176	144	117	94	74	107	7½	2.840
82,570	9	0.738	⅝ ¾	5 7¼	⅝ ¾	6½ 9¼						906	713	569	460	375	308	254	210	173	142	116	93	113	8	3.029
93,210	9½	0.784	¾ ⅞	6¾ 9¼	⅝ ¾	6 8½						1032	813	651	528	432	357	295	245	204	168	139	114	119	8½	3.218
104,500	10	0.831	¾ ⅞	6¼ 8¾	¾ ⅞	8 11							920	738	601	493	408	339	283	236	198	164	136	125	9	3.407
$\frac{l^2}{12} \times 12 = C =$							16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400			

Note.—This table is based on $M = \frac{wl^2}{12}$. Top reinforcement for negative M same area A_s as for positive M at center of span, top steel over supports extending to ¼ or ⅓ of span. For end spans, when $M = \frac{wl^2}{10}$, use ⅚ of the combined superimposed and dead weight of floor given. For simple spans, when $M = \frac{wl^2}{8}$, use ⅚ of the combined table values as for end spans.

TABLE 12

SOLID CONCRETE SLABS

1-2-4 MIX

SAFE SUPERIMPOSED LOADS IN POUNDS PER SQUARE FOOT

Continuous Spans. Unit Stress Steel = 18,000 lb. per Sq In., Hard Grade Steel

Extreme Fiber Stress Concrete = 750 Lb. per Sq. In.

$n = 15$

$w = \frac{M}{C} - D$

w = safe superimposed loads in lb. per sq. ft.
M = moment in inch-pounds
D = dead weight of slab per sq. ft.
l = span in feet.
$C = \frac{l^2}{12} \times 12.$

Min. elastic limit = 50,000 lb. per sq. in.
Min. ult. strength = 75,000 lb. per sq. in.

Moment in inch-pounds	Thickness of slab (inches)	Area of steel per 12 in. of width at 18,000 lb. per sq. in.	Corresponding size and spacing of round and square bars (inches): Rounds ϕ	Rounds c.c.	Squares ϕ	Squares c.c.	M (in.-lb.) $= \frac{wl^2}{12} \times 12$; Span in feet: 4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Weight of slab per sq. ft.	Effective depth (in.)	Position of neutral axis (kd)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
6,030	3	0.192	1/4 3/8	3 7	1/4 3/8	4 8¾	339	203	130	86	56	37												38	2	0.77
9,430	3½	0.240	3/8	5½	1/4 3/8	3 7	545	333	218	149	104	73	50	34										44	2½	0.962
13,580	4	0.288	3/8 1/2	4½ 8¼	3/8 1/2	5¾ 10½	799	493	327	227	162	118	86	62	44									50	3	1.154
18,480	4½	0.337	3/8 1/2	4 7	3/8 1/2	5 9	1098	682	456	320	231	171	128	95	71	52	37							57	3½	1.346
21,210	4¾	0.361	1/2	6½	3/8 1/2	4½ 8¼	1268	789	529	373	271	202	152	115	88	65	48	34						60	3¾	1.442
24,140	5	0.385	1/2 5/8	6 9½	3/8 1/2	4¼ 7¾		903	608	429	314	235	178	136	105	80	61	44						63	4	1.538
27,250	5¼	0.409	1/2 5/8	5¾ 9	1/2 5/8	7¼ 11½		1024	691	491	359	270	206	159	124	95	73	55	40					66	4¼	1.635
30,550	5½	0.433	1/2 5/8	5½ 8½	1/2 5/8	7 10¾		1153	780	555	408	308	237	184	143	112	87	67	51	36				69	4½	1.731
34,040	5¾	0.457	1/2 5/8	5¼ 8	1/2 5/8	6½ 10¼			874	623	460	349	268	210	165	129	101	79	61	45	33			72	4¾	1.827
37,720	6	0.481	1/2 5/8	5 7½	1/2 5/8	6¼ 9¾			973	695	514	391	302	236	187	148	118	93	73	55	42			75	5	1.923
41,580	6¼	0.505	1/2 5/8	4¾ 7¼	1/2 5/8	6 9¼			1077	771	572	435	338	266	210	168	134	107	85	66	50	37		78	5¼	2.019
45,840	6½	0.529	5/8 3/4	7 10	1/2 5/8	5¾ 9			1185	849	631	481	374	295	235	188	151	121	96	76	59	44		82	5½	2.115

Values for p, k, & K when f_c = 750 lb. per sq. in. f_s = 18,000 lb. per sq. in. n = 15

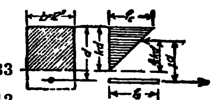
$$p = \frac{\frac{1}{2}}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right)} = 0.0080138$$

$$k = \sqrt{2pn + (pn)^2} - pn = 0.3846$$

$k = 0.385$ when $p = 0.008013$ and $n = 15$

$$K = \frac{M}{bd^2} = f_s p\left(1 - \frac{k}{3}\right) = 125.74 \text{ by steel}$$

$$K = \frac{M}{bd^2} = \tfrac{1}{2} f_c k\left(1 - \frac{k}{3}\right) = 125.74 \text{ by concrete}$$



(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
49,880	6¾	0.553	{ 5/8 3/4	6½ 9½	½ 5/8	5½ 8½				933	695	531	413	327	262	210	170	136	109	87	69	53	40	85	5¾	2.211
54,320	7	0.577	{ 5/8 3/4	6½ 9¼	½ 5/8	5¼ 8				1020	761	583	455	361	289	234	189	153	124	100	80	62	48	88	6	2.308
63,750	7½	0.625	{ 5/8 3/4	6 8½	5/8 3/4	7½ 10¾				1207	902	693	543	433	349	283	232	190	155	126	102	82	65	94	6½	2.500
73,930	8	0.673	{ 5/8 3/4	5½ 7¾	5/8 3/4	7 10					1055	813	640	511	413	338	277	229	188	156	128	105	85	100	7	2.692
84,870	8½	0.721	{ 5/8 3/4	5 7¼	5/8 3/4	6½ 9¼					1219	941	742	594	482	395	326	270	224	187	154	128	105	107	7½	2.885
96,570	9	0.769	{ 3/4 7/8	7 9½	5/8 3/4	6 8¾						1079	853	685	558	458	379	316	264	221	185	154	128	113	8	3.077
109,010	9½	0.817	{ 3/4 7/8	6½ 8¾	3/4 7/8	8¼ 11¼						1227	971	782	638	526	438	366	307	258	217	183	153	119	8½	3.269
122,220	10	0.865	{ 3/4 7/8	6 8¼	3/4 7/8	7¾ 10½							1097	885	724	598	499	418	352	297	252	213	181	125	9	3.461
$\frac{l^2}{12} \times 12 = C =$							16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400			

NOTE.—This table is based on $M = \frac{wl^2}{12}$. Top reinforcement for negative M same area A_s as for positive M at center of span, top steel over supports extending to ¼ or ⅓ of span. For end spans, when $M = \frac{wl^2}{10}$, use ⅚ of the combined superimposed and dead weight of floor given. For simple spans, when $M = \frac{wl^2}{8}$, use ⅔ of the combined table values as for end spans.

TABLE 13

SOLID CONCRETE SLABS

1-2-4 MIX

SAFE SUPERIMPOSED LOADS IN POUNDS PER SQUARE FOOT

Continuous Spans. Unit Stress Steel = 20,000 lb. per Sq. In., Hard Grade Steel

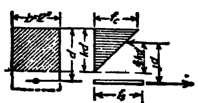
Extreme Fiber Stress Concrete = 800 Lb. per Sq. In.

$n = 15$

$w = \frac{M}{C} - D$

w = safe superimposed loads in lb. per sq. ft.
M = moment in inch-pounds
D = dead weight of slab per sq. ft.
l = span in feet.
$C = \frac{l^2}{12} \times 12.$

Min. elastic limit = 50,000 lb. per sq. in.
Min. ult. strength = 75,000 lb. per sq. in.

Moment in inch-pounds	Thickness of slab (inches)	Area of steel per 12 in. of width at 20,000 lb. per sq. in.	Corresponding size and spacing of round and square bars (inches): Rounds ϕ	Rounds c.c.	Squares ϕ	Squares c.c.	M (in.-lb.) $= \frac{wl^2}{12} \times 12$ — Span in feet: 4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Weight slab per sq. ft.	Effective depth (in.)	Position of neutral axis (kd)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
6,300	3	0.180	{ 1/4 3/8	3 1/4 7 1/4	1/4 3/8	4 9 1/4	356	214	137	91	60	40	Values for p, k, & K when f_c = 800 lb. per sq. in. f_s = 20,000 lb. per sq. in. n = 15											38	2	0.75
9,844	3 1/2	0.225	3/8	5 3/4	1/4 3/8	3 1/4 7 1/2	571	350	229	157	110	78	54	37					$p = \frac{\frac{1}{2}}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right)} = 0.0075$					44	2 1/2	0.938
14,175	4	0.270	{ 3/8 1/2	4 3/4 8 3/4	3/8	6 1/4	836	517	344	239	171	125	92	67	48				$k = \sqrt{2pn + (pn)^2} - pn = 0.375$					50	3	1.125
19,294	4 1/2	0.315	{ 3/8 1/2	4 7 1/2	3/8 1/2	5 1/4 9 1/2	1149	715	479	337	244	181	136	102	77	57	41		k = 0.375 when p = 0.0075 and n = 15					57	3 1/2	1.313
22,145	4 3/4	0.338	1/2	7	3/8 1/2	5 8 3/4	1324	826	555	392	286	213	161	123	94	71	53	38	$K = \frac{M}{bd^2} = f_s p\left(1 - \frac{k}{3}\right)$ = 131.25 by steel					60	3 3/4	1.406
25,200	5	0.360	{ 1/2 5/8	6 1/2 10	3/8 1/2	4 1/2 8 1/4		945	637	451	331	248	189	145	112	86	66	49	$K = \frac{M}{bd^2} = \frac{1}{2}f_c k\left(1 - \frac{k}{3}\right)$ = 131.25 by concrete					63	4	1.50
28,445	5 1/4	0.383	{ 1/2 5/8	6 9 1/2	3/8 1/2	4 1/4 7 3/4		1072	724	515	378	285	218	169	132	102	79	60	45					66 69	4 1/4 4 1/2	1.594 1.688
31,894	5 1/2	0.405	{ 1/2 5/8	5 3/4 9	3/8 1/2	4 7 1/4		1207	817	582	429	325	250	195	152	120	94	73	56	41						
35,535	5 3/4	0.428	{ 1/2 5/8	5 1/2 8 1/2	1/2 5/8	7 11			915	653	483	367	283	222	175	138	109	86	67	51	38			72	4 3/4	1.781
39,375	6	0.450	{ 1/2 5/8	5 1/4 8	1/2 5/8	6 1/2 10 1/4			1019	729	540	411	319	250	198	158	126	100	79	61	47			75	5	1.875
43,407	6 1/4	0.473	{ 1/2 5/8	5 7 3/4	1/2 5/8	6 1/4 9 3/4			1128	808	600	458	356	281	223	179	143	115	92	72	56	42		78	5 1/4	1.969
47,644	6 1/2	0.495	{ 1/2 5/8	4 3/4 7 1/4	1/2 5/8	6 9 1/2			1241	890	662	506	394	312	249	200	161	130	104	83	65	50		82	5 1/2	2.062

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)
52,070	6¾	0.518	⅝ ¾	7 10¼	½ ⅝	5¾ 9				978	729	558	435	345	277	223	181	146	118	95	76	59	45	85	5¾	2.156
56,700	7	0.540	⅝ ¾	6¾ 9¾	½ ⅝	5½ 8½				1069	798	612	479	381	306	248	201	164	133	108	87	69	54	88	6	2.25
66,544	7½	0.585	⅝ ¾	6¼ 9	½ ⅝	5 8				1264	946	728	571	456	368	300	246	202	166	136	111	90	72	94	6½	2.438
77,175	8	0.630	⅝ ¾	5¾ 8¼	⅝ ¾	7¼ 10¾					1106	853	672	538	436	357	294	243	201	167	138	114	93	100	7	2 625
88,594	8½	0.675	⅝ ¾	5¼ 7¾	⅝ ¾	6¾ 10					1277	987	779	625	508	417	345	287	239	200	166	138	114	107	7½	2.813
100,800	9	0.720	⅝ ¾	5 7¼	⅝ ¾	6½ 9¼						1131	895	720	587	483	401	335	281	236	198	166	139	113	8	3.00
113,794	9½	0.765	¾ ⅞	6¾ 9¼	⅝ ¾	6 8¾						1286	1019	821	671	554	462	387	326	275	232	196	165	119	8½	3.188
127,575	10	0.810	¾ ⅞	6½ 8¾	¾ ⅞	8¼ 11¼							1151	929	761	630	526	442	373	316	269	228	194	125	9	3.375
$\frac{l^2}{12} \times 12 = C =$							16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400			

Note.—This table is based on $M = \frac{wl^2}{12}$. Top reinforcement for negative M same area A_s as for positive M at center of span, top steel over supports extending to ¼ or ⅕ of span. For end spans, when $M = \frac{wl^2}{10}$, use ⅚ of the combined superimposed and dead weight of floor given. For simple spans, when $M = \frac{wl^2}{8}$, use ⅔ of the combined table values as for end spans.

TABLE 14.—STRENGTH OF SOLID SLABS

For Various Percentages of Steel when (f_s = 16,000, f_c = 650), (f_s = 18,000, f_c 750) and (f_s = 20,000, f_c = 800)

Ratio n = 15

Above heavy line M_s controls. Below heavy line M_c controls.

Slab thickness (inches)	Effective depth (inches)	Reinforcement (inches) Round	Reinforcement (inches) Square	Sectional area A_s (12 in. wide)	bd sq. in.	p	k	j	Moment (inch-pounds) f_s = 16,000, f_c = 650, n = 15	Moment (inch-pounds) f_s = 18,000, f_c = 750, n = 15	Moment (inch-pounds) f_s = 20,000, f_c = 800, n = 15
		(Centers)	(Centers)								
4	3	½-10½ ⅜- 6	⅜- 7¾	0.22	36	0.0061	0.346	0.885	9,350	10,510	11,690
4	3	½-10 ⅜- 5½	⅜- 7	0.24	36	0.0067	0.358	0 881	10,150	11,420	12,690
4	3	½- 9½ ⅜- 5¼	⅜- 6¾ ½-12	0.25	36	0.0069	0.363	0.879	10,550	11,860	13,190
4	3	½- 9 ⅜- 5	⅜- 6½ ½-11½	0.26	36	0.0072	0.369	0.877	10,950	12,310	13,680
4	3	½- 8½ ⅜- 4¾	⅜- 6 ½-10¾	0.28	36	0.0078	0.380	0.873	11,650	13,200	14,330
4	3	½- 8 ⅜- 4½	⅜- 5¾ ½-10¼	0.29	36	0.0081	0.385	0.872	11,780	13,600	14,500
4	3	½- 7½ ⅜- 4¼	⅜- 5½ ½- 9¾	0.31	36	0.0086	0.394	0.869	12,020	13,870	14,790
4	3	½- 7 ⅜- 4	⅜- 5 ½- 8¾	0.34	36	0.0094	0.407	0.864	12,340	14,240	15,190
4½	3½	½- 9 ⅜- 5	⅜- 6½ ½-11½	0.26	42	0 0062	0.348	0.884	12,870	14,480	16,090
4½	3½	½- 8½ ⅜- 4¾	⅜- 6 ½-10¾	0.28	42	0.0067	0.358	0.881	13,810	15,540	17,270
4½	3½	½- 8 ⅜- 4½	⅜- 5¾ ½-10¼	0.29	42	0.0069	0.363	0.879	14,270	16,060	17,840
4½	3½	½- 7½ ⅜- 4½	⅜- 5½ ½- 9¾	0.31	42	0.0074	0.372	0.876	15,210	17,110	19,010
4½	3½	½- 7 ⅜- 4	⅜- 5 ½- 8¾	0.31	42	0.0081	0.385	0.872	16,040	18,510	19,740
4½	3½	½- 6½ ⅝-10	⅜- 4¾ ½- 8¼	0.36	42	0.0086	0.394	0.869	16,360	18,870	20,130
4½	3½	½- 6 ⅝- 9½	⅜- 4¼ ½- 7¾	0.39	42	0.0093	0.406	0.865	16,780	19,360	20,650
5	4	½-10 ⅜- 5½	⅜- 7	0.24	48	0.0050	0.320	0.893	13,720	15,430	17,150
5	4	½- 9½ ⅜- 5¼	½-12 ⅜- 6¾	0.25	48	0.0052	0.324	0.892	14,270	16,060	17,840
5	4	½- 9 ⅜- 5	½-11½ ⅜- 6½	0.26	48	0.0054	0.329	0.891	14,830	16,680	18,530
5	4	½- 8½ ⅜- 4¾	½-10¾ ⅜- 6	0.28	48	0.0058	0.337	0.888	15,910	17,900	19,890
5	4	½- 8 ⅜- 4½	½-10¼ ⅜- 5⅜	0.29	48	0.0060	0.344	0.885	16,430	18,480	20,530
5	4	½- 7½ ⅜- 4½	½- 9¾ ⅜- 5½	0.31	48	0.0065	0.354	0.882	17,500	19,690	21,870
5	4	½- 7 ⅜- 4	½- 8¾ ⅜- 5	0.34	48	0.0071	0.367	0.878	19,100	21,490	23,880
5	4	½- 6½ ⅝-10	½- 8¼ ⅜- 4¾	0.36	48	0.0075	0.374	0.875	20,160	22,680	25,200
5	4	½- 6 ⅝- 9½	½- 7¾ ⅜- 4¼	0.39	48	0.0081	0.385	0.872	20,950	24,170	25,780
5	4	½- 5½ ⅝- 8½	½- 7 ⅝-11	0.43	48	0.0090	0.402	0.866	21,720	25,060	26,740

TABLE 14.—STRENGTH OF SOLID SLABS.—(*Continued*)

ab inc	Eff (in	Reinforcement (inches) Round (Centers)	Square (Centers)	ctional a 12 in. wid					Moment (inch-pounds) $f_s = 16{,}000$ $f_c = 650$ $n = 15$	$f_s = 18{,}000$ $f_c = 750$ $n = 15$	$f_s = 20{,}000$ $f_c = 800$ $n = 15$
5½	4½	½- 9 ⅜- 5	½-11½ ⅜- 6½	0.26	54	0.0048	0.315	0.895	16,750	18,850	20,940
5½	4½	½- 8½ ⅜- 4¾	½-10¾ ⅜- 6	0.28	54	0.0052	0.324	0.892	17,980	20,230	22,480
5½	4½	½- 8 ⅜- 4½	½-10¼ ⅜- 5¾	0.29	54	0.0054	0.329	0.891	18,600	20,930	23,260
5½	4½	½- 7½ ⅜- 4½	½- 9¾ ⅜- 5½	0.31	54	0.0057	0.335	0.889	10,840	22,320	24,800
5½	4½	½- 7 ⅜- 4	½- 8¾ ⅜- 5	0.34	54	0.0063	0.350	0.884	21,640	24,350	27,050
5½	4½	½- 6½ ⅝-10	½- 8¼ ⅜- 4¾	0.36	54	0.0067	0.358	0.881	22,840	25,690	28,540
5½	4½	½- 6 ⅝- 9½	½- 7¾ ⅜- 4¼	0.39	54	0.0072	0.369	0.877	24,630	27,700	30,780
5½	4½	½- 5½ ⅝- 8½	½- 7 ⅝-11	0.43	54	0.0080	0.384	0.872	26,450	30,370	32,550
5½	4½	½- 5 ⅝- 7¾	½- 6½ ⅝-10	0.47	54	0.0087	0.396	0.868	27,150	31,320	33,410
6	5	½- 8½ ⅜- 4¾	½-10¾ ⅜- 6	0.28	60	0.0047	0.312	0.896	20,070	22,580	25,090
6	5	½- 8 ⅜- 4½	½-10¼ ⅜- 5¾	0.29	60	0.0048	0.315	0.[illegible]	20,760	23,360	25,960
6	5	½- 7½ ⅜- 4¼	½- 9¾ ⅜- 5½	0.31	60	0.0052	0.324	0.892	22,120	24,890	27,650
6	5	½- 7 ⅜- 4	½- 8¾ ⅜- 5	0.34	60	0.0057	0.335	0.889	24,180	27,200	30,230
6	5	½- 6½ ⅝-10	½- 8¼ ⅜- 4¾	0.36	60	0.0060	0.344	0.885	25,490	28,670	31,860
6	5	½- 6 ⅝- 9½	½- 7¾ ⅜- 4½	0.39	60	0.0065	0.354	0.882	27,520	30,960	34,400
6	5	½- 5½ ⅝- 8½	½- 7 ⅝-11	0.43	60	0.0072	0.369	0.877	30,170	33,940	37,710
6	5	½- 5 ⅝- 7¾	½- 6½ ⅝-10	0.47	60	0.0078	0.380	0.873	32,350	36,930	39,810
6	5	½- 4½ ⅝- 7	½- 5¾ ⅝- 9	0.52	60	0.0087	0.396	0.868	33,510	38,670	41,240
6½	5½	½- 8 ⅜- 4½	½-10¼ ⅜- 5¾	0.29	66	0.0044	0.303	0.899	22,940	25,810	28,680
6½	5½	½- 7½ ⅜- 4¼	½- 9¾ ⅜- 5½	0.31	66	0.0047	0.312	0.896	24,440	27,500	30,550
6½	5½	½- 7c. ⅜- 4c.	½- 8¾ ⅜- 5	0.34	66	0.0052	0.324	0.892	26,690	30,020	33,360
6½	5½	½- 6½ ⅝-10	½- 8¼ ⅜- 4¾	0.36	66	0.0055	0.331	0.890	28,220	31,720	35,240
6½	5½	½- 6 ⅝- 9½	½- 7¾ ⅜- 4¼	0.39	66	0.0059	0.340	0.887	30,440	34,250	38,050
6½	5½	½- 5½ ⅝- 8½	½- 7 ⅝-11	0.43	66	0.0065	0.354	0.882	33,370	37,550	41,720
6½	5½	½- 5 ⅝- 7¾	½- 6½ ⅝-10	0.47	66	0.0071	0.367	0.878	36,310	40,850	45,390

TABLE 14.—STRENGTH OF SOLID SLABS.—(*Continued*)

Slab thickness (inches)	Effective depth (inches)	Reinforcement (inches) Round (Centers)	Reinforcement (inches) Square (Centers)	Sectional area A_s (12 in. wide)	bd (sq. in.)	p	k	j	Moment (inch-pounds) $f_s = 16{,}000$, $f_c = 650$, $n = 15$	Moment (inch-pounds) $f_s = 18{,}000$, $f_c = 750$, $n = 15$	Moment (inch-pounds) $f_s = 20{,}000$, $f_c = 800$, $n = 15$
6½	5½	½– 4½ ⅝– 7	½– 5¾ ⅝– 9	0.52	66	0.0079	0.382	0.873	39,340	44,910	48,420
6½	5½	½– 4 ⅝– 6¼	½– 5 ⅝– 8	0.59	66	0.0089	0.400	0.867	40,900	47,190	50,350
6½	5½	⅝– 5½ ¾– 8	½– 4½ ⅝– 7	0.67	66	0.0102	0.418	0.861	42,460	48,900	52,260
7	6	½– 7½ ⅜– 4¼	½– 9¾ ⅜– 5½	0.31	72	0.0043	0.300	0.900	26,784	30,120	33,480
7	6	½– 7 ⅜– 4	½– 8¾ ⅜– 5	0.34	72	0.0047	0.312	0.896	29,240	32,900	36,560
7	6	½– 6½ ⅝–10	½– 8¼ ⅜– 4½	0.36	72	0.0050	0.320	0.893	30,860	34,720	38,580
7	6	½– 6 ⅝– 9½	½– 7¾ ⅜– 4¼	0.39	72	0.0054	0.329	0.891	33,360	37,530	41,700
7	6	½– 5½ ⅝– 8½	½– 7 ⅝–11	0.43	72	0.0060	0.344	0.885	36,530	41,100	45,670
7	6	½– 5 ⅝– 7¾	½– 6½ ⅝–10	0.47	72	0.0065	0.354	0.882	39,800	44,770	49,740
7	6	½– 4½ ⅝– 7	½– 5¾ ⅝– 9	0.52	72	0.0072	0.369	0.877	43,780	49,250	54,720
7	6	½– 4 ⅝– 6¼	½– 5 ⅝– 8	0.59	72	0.0082	0.387	0.871	47,330	54,600	58,250
7	6	⅝– 5½ ¾– 8	½– 4½ ⅝– 7	0.67	72	0.0093	0.406	0.865	49,310	56,890	60,690
7	6	⅝– 5 ¾– 7¼	½– 4 ⅝– 6¼	0.74	72	0.0103	0.421	0.860	50,830	58,650	62,560
7½	6½	½– 7 ⅜– 4	½– 8¾ ⅜– 5	0.34	78	0.0044	0.303	0.899	31,790	35,760	39,740
7½	6½	½– 6½ ⅝–10	½– 8¼ ⅜– 4½	0.36	78	0.0046	0.309	0.897	33,580	37,780	41,980
7½	6½	½– 6 ⅝– 9½	½– 7¾ ⅜– 4¼	0.39	78	0.0050	0.320	0.893	36,220	40,750	45,270
7½	6½	½– 5½ ⅝– 8½	½– 7 ⅝–11	0.43	78	0.0055	0.331	0.890	39,800	44,780	49,750
7½	6½	½– 5 ⅝– 7¾	½– 6½ ⅝–10	0.47	78	0.0060	0.344	0.885	43,260	48,670	54,070
7½	6½	½– 4½ ⅝– 7	½– 5¾ ⅝– 9	0.52	78	0.0067	0.358	0.881	47,640	53,600	59,560
7½	6½	½– 4 ⅝– 6¼	½– 5 ⅝– 8	0.59	78	0.0076	0.376	0.875	53,690	60,400	66,720
7½	6½	⅝– 5½ ¾– 8	½– 4½ ⅝– 7	0.67	78	0.0086	0.394	0.869	56,420	65,100	69,440
7½	6½	⅝– 5 ¾– 7¼	½– 4 ⅝– 6¼	0.74	78	0.0095	0.409	0.864	58,230	67,190	71,660
7½	6½	¾– 6¼ ⅞– 8½	⅝– 5½ ¾– 8	0.85	78	0.0109	0.430	0.857	60,720	70,060	74,730
8	7	½– 6½ ⅝–10	½– 8¼ ⅜– 4½	0.36	84	0.0043	0.300	0.900	36,290	40,820	45,360

TABLE 14.—STRENGTH OF SOLID SLABS.—(*Continued*)

Slab thickness (inches)	Effective depth (inches)	Reinforcement, (inches) Round	Square	Sectional area A_s (12 in. wide)	bd (sq. in.)	p	k	j	Moment (inch-pounds) $f_s = 16{,}000$, $f_c = 650$, $n = 15$	$f_s = 18{,}000$, $f_c = 750$, $n = 15$	$f_s = 20{,}000$, $f_c = 800$, $n = 15$
		(Centers)	(Centers)								
8	7	½- 6 ⅝- 9½	½- 7¾ ⅜- 4¼	0.39	84	0.0046	0.309	0.897	39,180	44,080	48,980
8	7	½- 5½ ⅝- 8½	½- 7 ⅝-11	0.43	84	0.0051	0.322	0.893	43,010	48,380	53,760
8	7	½- 5 ⅝- 7¾	½- 6½ ⅝-10	0.47	81	0.0056	0.333	0.889	46,800	52,650	58,500
8	7	½- 4½ ⅝- 7	½- 5¾ ⅝- 9	0.52	84	0.0062	0.348	0.884	51,180	57,920	64,350
8	7	½- 4 ⅝- 6¼	½- 5 ⅝- 8	0.59	84	0.0070	0.365	0.878	58,020	65,270	72,520
8	7	⅝- 5½ ¾- 8	½- 4½ ⅝- 7	0.67	84	0.0080	0.384	0.872	63,990	73,610	78,760
8	7	⅝- 5 ¾- 7¼	½- 4 ⅝- 6¼	0.74	84	0.0088	0.398	0.867	65,940	76,090	81,160
8	7	¾- 6¼ ⅞- 8½	⅝- 5½ ¾- 8	0.85	84	0.0101	0.419	0.860	68,860	79,450	84,750
8½	7½	½- 6c. ⅝- 9½	½-7¾c ⅜- 4¼	0.39	90	0.0043	0.300	0.900	42,120	47,390	52,650
8½	7½	½- 5½ ⅝- 8½	½- 7 ⅝-11	0.43	90	0.0048	0.315	0.895	46,180	51,950	57,730
8½	7½	½- 5 ⅝- 7¾	½- 6½ ⅝-10	0.47	90	0.0052	0.324	0.892	50,310	56,600	62,890
8½	7½	½- 4½ ⅝- 7	½- 5¾ ⅝- 9	0.52	90	0.0058	0.337	0.888	55,410	62,340	69,260
8½	7½	½- 4 ⅝- 6¼	½- 5 ⅝- 8	0.59	90	0.0066	0.356	0.881	62,370	70,170	77,970
8½	7½	⅝- 5½ ¾- 8	½- 4½ ⅝- 7	0.67	90	0.0074	0.372	0.876	70,430	79,230	88,040
8½	7½	⅝- 5 ¾- 7¼	½- 4 ⅝- 6¼	0.74	90	0.0082	0.387	0.871	73,950	85,230	91,010
8½	7½	¾- 6¼ ⅞- 8½	⅝- 5½ ¾- 8	0.85	90	0.0094	0.407	0.864	77,140	89,010	94,940
8½	7½	¾- 5¾ ⅞- 7¾	⅝- 5¼ ¾- 7¼	0.92	90	0.0102	0.420	0.860	79,240	91,430	97,520

The steel bars selected will consist of

Thirteen 1¼-in. square = (13)(1.563) = 20.32
Two 1-in. round = (2)(0.785) = 1.57

21.89 sq. in. total area.

Then

$$f_s = \frac{14{,}420{,}800}{(0.883)(39)(21.89)} = 19{,}130 \text{ lb. per sq. in.}$$

$$f_c \quad \frac{(19{,}130)(0.365)}{(15)(1 - 0.365)} = \frac{6{,}982}{9.53} \quad 733 \text{ lb. per sq. in.}$$

The point of third bend occurs 10 ft. 0 in. from center line of support, or 0.22 of span length center to center of columns. At this point when $M = \frac{WL}{8}$, 68 per cent of the steel area is required for moment. At the point of second bend, 7 ft. 0 in. from center line of

support or 0.15 of span length, 50 per cent of steel is required. At first bend from support 33 per cent of steel is required for moment. It is readily noted that the rods remaining straight in the bottom at different points satisfy moment onditions.

Seven 1¼-in. square rods will be selected to remain straight extending into supports. The bond stress is

$$u = \frac{104{,}800}{(35)(\frac{7}{8})(39)} = 88 \text{ lb. per sq. in.}$$

The distance from support to the point beyond which no stirrups are needed, assuming $v_1 = 50$, is $x_1 = \frac{(v - v_1)L}{2v}$. Substituting $x_1 = \frac{(140 - 50)44}{(2)(140)} = 14.14$ ft.

The total shear to be taken by all stirrups (neglecting the value of bent rods) in one end of beam is

$$V_1 = \frac{(v - v_1)bx_1}{2} = \frac{(140 - 50)(22)(14.14)(12)}{2} = 168{,}000 \text{ lb.}$$

Assuming ½-in. round stirrups arranged in pairs, the value of each pair = 2 × 2 × 0.196 sq. in. × 12,000 lb. = 9,400 lb. or $\frac{168{,}000}{9{,}400}$ = 18 pairs ½-in. round stirrups at each end. The closest spacing required at each end near support will be

$$S = \frac{A_s f_s}{(v - v_1)b} = \frac{9{,}400}{1{,}980} = 4.75, \text{ say 5 in. center to center.}$$

The spacing of stirrups to resist diagonal tension should be limited to $\frac{d}{2}$ or ³⁹⁄₂ = 19½ in. The arrangement of stirrups can now be made as follows:

$$\left.\begin{array}{l}\text{6 pairs ½-in. round 5-in. center}\\ \text{6 pairs ½-in. round 10-in. center}\\ \text{6 pairs ½-in. round 13-in. center}\end{array}\right\}\text{each end}$$

Engineers who have had long experience in the design of reinforced concrete develop a quick perception of proper proportion. Many complicated arrangements, such as often occur in hotels, sometimes require from 500 to 1,000 different designs of beams. Under such conditions it is obviously impossible for the engineer to resort to the more accurate theories of design. In the first place he has not the time at his disposal, nor the inclination to engage in long calculations, when for all practical purposes, almost identical results may be obtained by approximate methods and formula .

57. Solid Slab Design.—Tables of solid slabs, giving the safe superimposed loads for any span and reinforcement required, will prove of inestimable value to any busy engineer (see Tables 11, 12, 13 and 14).

Illustrative Problem.—A solid slab having a span of 12 ft. 0 in. center to center of supports, and fully continuous, is required to support a superimposed load of 175 lb. per sq. ft. Find slab thickness and reinforcement.

When f_s = 16,000 lb., f_c = 650, n = 15.

Referring to Table 11, the nearest tabulated load for the span and load desired is 189 lb., requiring a 6½-in. slab and reinforcement given. To be more exact,

$$M = \frac{(175 + 82)(12)^2(12)}{12} = 37{,}008 \text{ in.-lb.}$$

The value of 6½-in. slab and reinforcement selected is good for 39,030 in.-lb.

Referring to Table 14, a 6½-in. slab with ½ in. round 5 in. has a resisting moment of 36,310 in.-lb. or with ½ in. round 4½ in. a resisting moment of 39,340. In practice the latter would be selected without any further calculations.

FLOORS AND ROOFS—FLAT SLAB CONSTRUCTION

By Albert M. Wolf

A flat slab floor can be defined as one in which the slab of concrete is carried directly by columns with or without flaring capitals and without the use of the ordinary deep members such as beams or girders.

58. Types of Flat Slab Floors.—The types of flat slab construction ordinarily used can be classified from the standpoint of column and ceiling details as follows:

(1) Column construction, in which the floor slab usually of relatively short spans is carried directly on the columns without the use of column capitals or drop plates.

(2) Cap construction in which the floor slab is carried on the flaring column capitals but without drop plates.

(3) Drop construction, same as for Type (2) except that the slab surrounding the column is thickened by means of a square or rectangular drop.

(4) Paneled ceiling construction same as Type (3) except that the portion of slab in center of panel is reduced in thickness leaving the portions directly between columns of greater thickness. Used mostly for long spans or for special architectura treatment.

(5) Tile construction, an adaptation of paneled ceiling construction in which the center portion of the slab is of tile and concrete joist construction of the same depth as the rest of slab or shallower as required.

The first four types are illustrated in Fig. 40.

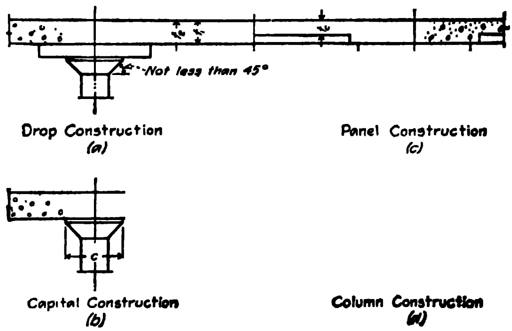


FIG. 40.—Types of flat slabs.

59. Types of Flat Slab Reinforcement.—Flat slab floors can also be classified as to systems depending upon the arrangement of the reinforcing.

(1) *Four-way System.*—Reinforcement extends directly between columns in both directions and also diagonally from column to column.

(2) *Three-way System.*—Reinforcement runs from column to column which are arranged at the corners of triangles.

(3) *Two-way System.*—The main reinforcement extends directly from column to column in both directions with auxiliary belts of bars parallel to them in space between.

(4) *Ring System.*—The main reinforcement consists of a series of rings or continuous flat spirals at the columns and center of panel. Sometimes the remaining portions are reinforced by groups of straight rods or by additional rings.

60. Advantages of Flat Slab Construction.—The advantages of flat slab construction over the beam and girder type are:

(1) Greater load-carrying capacity for a given amount of concrete and steel and lessened danger of collapse under overload due to the tendency of the entire floor to act together.

(2) Greater fire-resisting qualities owing to the fact that the ceiling offers no sharp edges or corners as in beam construction which spall off and weaken the floor or cause failure.

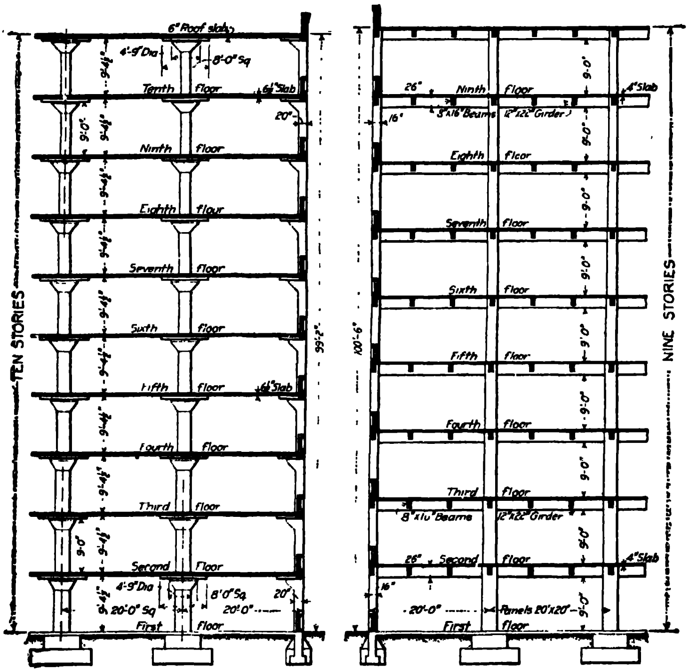
(3) Absence of beams permits the maximum spacing of sprinkler heads and hence the greatest efficiency.

(4) The flat ceiling permits of the most economical arrangement of piping for sprinklers and heating and lighting outlets.

(5) Flat slab floors can be designed without spandrel beams projecting below the ceiling line thus allowing the window to extend to the ceiling, thus eliminating the shadows caused when beams are used and resulting in better natural as well as artificial lighting.

(6) Better ventilation since no air pockets are formed nor is the circulation of air retarded.

(7) Formwork for both floor slabs and columns is simplified and cheaper than for beam construction.



FLAT SLAB CONSTRUCTION BEAM AND GIRDER CONSTRUCTION

FIG. 41.

(8) The above advantage allows the work to proceed with greater speed thus saving in cost and interest on money invested during construction.

(9) A flat slab floor allows a maximum storage capacity with no waste space as occurs between beams in beam and girder construction. This allows of a considerable saving in floor to floor heights (if the same effective story height is maintained or a corresponding increase in clear story height). This amounts to about 12 to 18 in. per story or a saving of one story in eight or ten, or an extra story for the same total height (see Fig. 41).

61. Types of Buildings for Which Flat Slab Construction is Best Adapted.— Flat slab construction is best adapted to warehouses, factories, assembling plants, garages, large wholesale and retail stores and railroad terminals. Where the

loads to be carried are relatively light and the spans of varying lengths as in hotels, office buildings and the like, it will be found more economical to use beam and slab construction or beam and tile and concrete joist construction.

62. Requirements for Economy in Flat Slab Construction.—To make for economy a flat slab floor should (*a*) have columns spaced from 18- to 22-ft. centers uniformly throughout the building; (*b*) be designed for a live load of at least 100 lb. per sq. ft. or more; and (*c*) be free from large openings which necessitate beams or deepened slabs.

63. Description of Flat Slab Systems.—As previously noted, flat slab floors are generally classified as to systems depending upon the details and arrangement of reinforcement. This has given rise to various schemes of reinforcing which are known by various trade names.

63*a*. Mushroom System.—The earliest practical system of flat slab construction used very extensively is the Mushroom System as developed by C. A. P. Turner. This system is a four-way reinforced floor slab in which the column bars are sometimes bent down into the slab around the column head in radial directions, with additional rods bent in the form of rings laid upon these radials thus forming a spider-web to provide additional reinforcement at the column head and support the slab steel.

In other designs of this system instead of bending the column bars, special elbow rods were bent down into the column head for the same purpose and a flat spiral placed thereon.

Being one of the earliest commercial systems, it gained a very wide usage, and while in some particular cases the concrete sections were entirely too light, in general the floors give as good service as could be expected from a new system of construction which was being improved upon continually. The majority of mushroom designs are of the true flat slab type—that is, no drop panel is used around the column capital. The column heads used were in general of smaller diameter and greater pitch than is now considered good practice.

63*b*. Akme System.—A two-way system of simple construction and used very widely on a great variety of buildings is the Akme system of girderless floor construction as developed by the Condron Company, Structural Engineers of Chicago, and shown in the accompanying photograph (Fig. 42).

This system consists of main belts of reinforcing bars extending directly between columns and so bent as to reinforce the bottom portion of the slab between columns and the upper portion around the columns. All bars are bent before placing and are held in place by raising bars placed on pre-cast concrete blocks. The center portion of the panel and a portion of the slab directly over the main belts between the columns is reinforced with belts of bent bars placed at right angles to each other and supported in a manner similar to that just described for the main belts. The portion of these head belts in the top of slab over the main bands reinforces the slab for stresses which exist in this portion due to the cross bending on the main belts. The rules for the design of this system as worked out by the Condron Company, together with the design standard sheets are given herewith. These design standards are followed in all cases except where local building ordinances have more severe requirements.

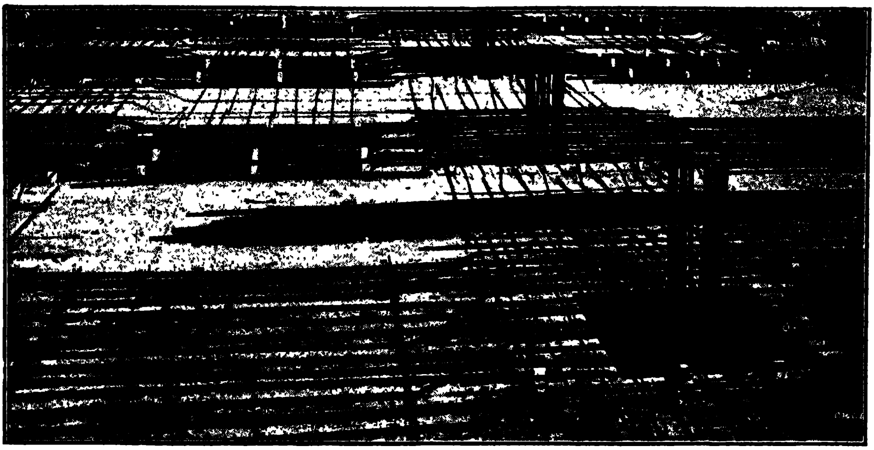


FIG. 42.

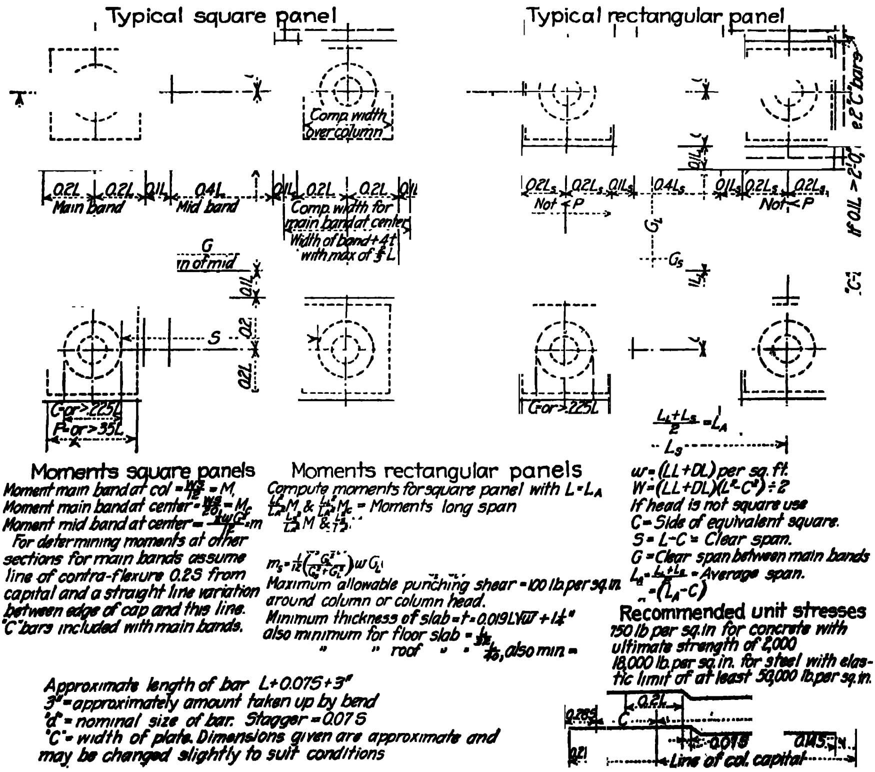


FIG. 43.—Akme system. Drop panel type. Design standards, Condron Co.

Rules for the Design of Girderless Floors

(To Accompany Akme Design Standards.) (Figs. 43 and 44.)

The term girderless floors as herein used refers to flat slabs of uniform or varying thickness supported without beams or girders on columns having flaring heads.

Flat-Slab Type.—In this type the slab thickness is uniform between column heads.

Drop-Panel Type.—In this type the lower face of the slab is dropped so as to increase the thickness of the slab above the column head. The lateral dimensions of this portion of the slab, which is usually made square, should be not less than $0.35L$.

Paneled-Ceiling Type.—This type may conform in general to either of the above types with the exception that the slab is reduced in thickness in the central portion of the panel.

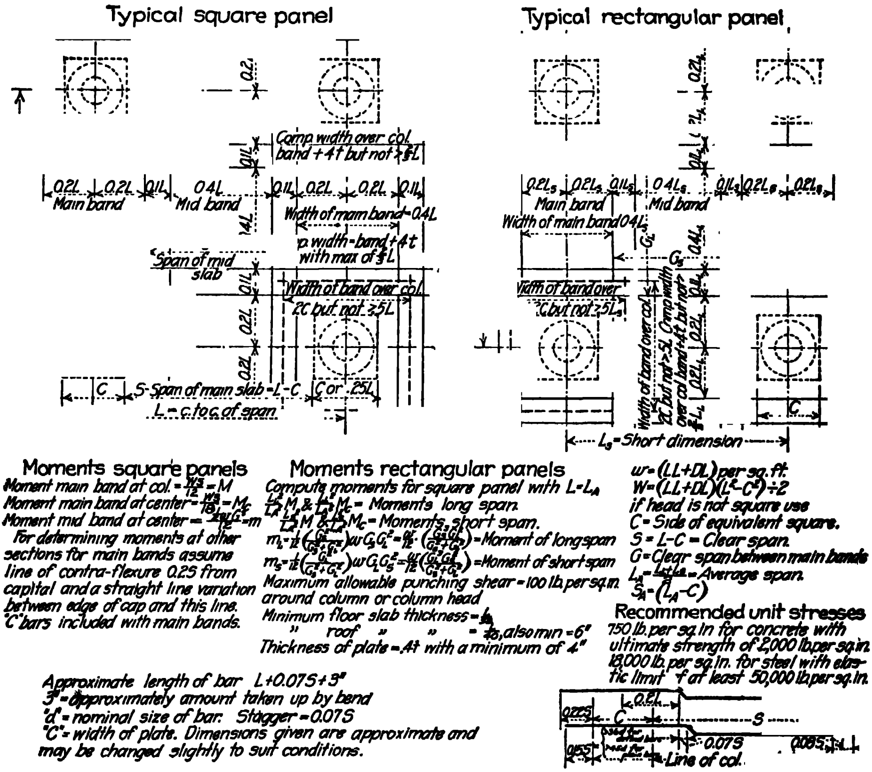


FIG. 44.—Akme system. Flat slab design standards, Condron Co.

Columns.—The diameter or side of any interior concrete column shall be not less than $\frac{1}{15}$ of the panel length or $\frac{1}{12}$ of the clear story height, except that for columns supporting roofs only this dimension shall be not less than $\frac{1}{18}$ of the panel length. In any case the diameter or side of the column shall be not less than 12 in.

Bending in Columns.—Exterior or wall columns supporting floors or roofs shall be designed to resist, in addition to direct load, 40 per cent of the negative bending moment for exterior floor panels or 80 per cent for exterior roof panels.

Column Head.—The diameter of the column head, measured where it intersects the underside of the slab, should be approximately $0.235L$, but may vary to suit conditions. It shall have a vertical face below the slab of $1\frac{1}{2}$ in., below which the surface of the head shall have a slope of 45 deg. to the vertical face of the column shaft. If other shapes of column head are used, the surface of the same shall nowhere fall inside of the surface of the above defined conical head. Heads may be round, octagonal, or square.

If round or octagonal heads are used, the diameter of head to be used in the slab calculations shall be the side of an equivalent square. Where a square plate is used as part of the column head and its lateral dimension is within the 45-deg. slope of the conical head, the size of said square plate shall be used as the diameter of column head in making slab calculations, provided the thickness of said plate is equal to or greater than one-half the thickness of the slab and not less than 4 in.

Slab Thickness.—The minimum thickness of the slab (except in paneled-ceiling type) shall be not less than $\frac{L}{32}$ for floors and $\frac{L}{40}$ for roofs, nor less than given by the following formula

$$t = 0.019L\sqrt{w} + 1\tfrac{1}{4} \text{ in.}$$

where t = total slab thickness in inches; L = panel length in feet; and w = total live and dead load in pounds per square foot.

In the paneled-ceiling type the thickness of the enclosed panel shall be not less than $\frac{1}{32}$ of its clear span.

Drop Panel.—The depth of drop panel where used shall be determined by using its width at the section considered as the full width to resist compression resulting from negative moment.

Panel Strips.—For purposes of computation each panel of the slab is to be divided into two sets of strips called A (main slab strips) and B (mid-slab strips). Strips A extend from column to column and have a width equal to $\frac{L}{2}$ and strips B occupy the space between strips A, and likewise have a width of $\frac{L}{2}$.

Reinforcement in strips A shall be placed symmetrically about column centers for a width of approximately $0.4L$ at mid-span and approximately $0.5L$ over columns. The width for compression shall be taken as the width of the belts of reinforcement, plus four times the thickness of the slab, but shall not exceed $\frac{2}{3L}$. The width of main belts of reinforcement over the columns shall not exceed twice the width of the column head.

Bending-Moment Coefficients, Interior Panels.—For the flat-slab type the negative bending moment taken at a cross-section of each strip A at the edge of a column head shall be $\frac{WS}{12}$. The positive bending moment taken at a cross-section of each strip A midway between column supports shall be $\frac{WS}{18}$. The positive and negative bending moments taken at a cross-section of each strip B at the middle of the panel on the center of the columns respectively, shall be $\frac{\frac{w}{2}G^3}{12}$.

For the drop-panel type the corresponding moments at the above-mentioned section shall be $\frac{WS}{12}$, $\frac{WS}{20}$ and $\frac{\frac{w}{2}G^3}{12}$.

For paneled-ceiling type the moment coefficients shall be the same as for the flat-slab type.

For determining moments at other sections of main strips A, the line of contraflexure shall be assumed to be at a distance equal to $\frac{L}{4}$ from the center of column with a straight-line variation of moment between the edge of the head and the said line of contraflexure.

In the above W = one-half total live and dead load on the panel, exclusive of the area over the column head; S = the clear span in feet between column heads; w = total live and dead load per square foot; and G = the clear distance in feet between main belts of bars at the section midway between columns.

Bending-Moment Coefficients, Exterior Panels.—For exterior panels without cantilever overhang, where wall columns with flaring heads or brackets are used, and for other spans not continuous over both supports, the positive bending moment coefficients shall be increased 20 per cent.

When bearing walls or piers and girders are substituted for the above wall columns with flaring heads or brackets, compute the moments for the exterior panels of such construction by assuming the distance from the face of the column head to the inside face of wall or girder as S; and the distance between the first interior main belt and the inside face of wall or girder as G.

Oblong Panels.—For oblong panels the moments shall first be determined for an assumed square panel with sides equal to the mean of the length and breadth of the oblong panel. The moments thus found for strips A shall be multiplied by the ratio of the square of the span in question, and the square of the span of the assumed square panel, and the moments thus found used in determining the steel required in strips A.

The moments for strips B shall be computed as follows: The load carried by the long and short span strips B shall be in the proportion of the ratio of the square of the short span to sum of squares of the long and short spans, and the ratio of the square of the long span to sum of squares of long and short spans respectively. The moments shall then be found as for square panel using the proportion of w carried by the span in question instead of $\frac{w}{2}$.

When the length of panel does not exceed the breadth by more than 5 per cent, all computations may be made on the basis of a square with sides equal to the mean of the length and breadth. The rules given herein shall not be used for rectangular panels in which the length exceeds $\frac{4}{3}$ of the breadth, but special consideration shall be given to such cases.

Stresses in Steel and Concrete.—The stresses shall be calculated on the basis of the straight-line formula, neglecting the tension value of the concrete. The depth of the slab for calculation of stresses shall be taken as the distance from the compressive face to the center of gravity of the belt of reinforcement in a given strip. The tensile stress in steel reinforcement should not exceed 16,000 lb. per sq. in. for structural steel grade nor 18,000 lb. per sq. in. for high-carbon deformed bars. The maximum allowable compression in the concrete shall not exceed 750 lb. per sq. in. The allowable punching shear on the perimeter of the column head shall not exceed 100 lb. per sq. in. Where governing ordinances or laws require lower allowable unit stresses, such unit stresses shall be substituted for the above.

Walls and Openings.—Where necessary, slabs shall be thickened or girders or beams shall be used under walls and around openings to carry concentrated loads.

Placing of Reinforcement.—Reinforcement shall be rigidly held in its designed position while pouring concrete. The bars in the upper portion of the slab should be rigidly supported by frames or transverse bars resting on concrete blocks of proper height. Bars in the lower portion of the slab should be raised from the forms and held in proper position, preferably by a continuous combined spacing and raising device. The lateral spacing of bars shall not exceed $1\frac{1}{2}$ times the thickness of the slab, nor more than 12 in.

Bars shall be bent to conform to the bending diagrams shown in Figs. 43 and 44 and shall be so placed in the slab that they will not be nearer than $\frac{3}{4}$ in. from the face of the concrete.

64. Cantilever Flat Slab Floors.—The Concrete Steel Products Company, Consulting Engineers, Chicago, have developed and used very extensively a four-way system of floor known as the *cantilever flat slab construction.* The belts of reinforcing bars run direct and diagonally between columns being held up in the top of slab at the columns by lead rods carried on concrete raising blocks. In the bottom portion of slab between columns the bars are accurately spaced and held in proper position by bar spacers.

This system is used for the true flat slab type of floor or the drop panel type with the latter predominating owing to the greater stiffness and economy of materials. The earlier designs of this company employed radial rods and column bars bent down into slab, also ring bars around column heads, but this method of design was discontinued after extensometer tests showed it to be inefficient. A typical view of the reinforcement for this system is shown in Fig. 45.

65. Simplex System.—A four-way system of flat slab construction which is similar to the cantilever flat slab construction with certain added details for

holding the bars in place, is the Simplex system developed by the Concrete Steel Co. of New York. The details are shown in Fig. 46.

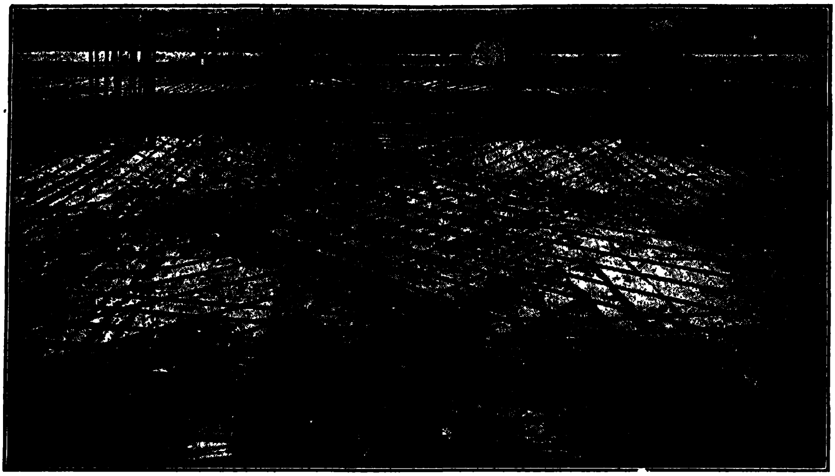


FIG. 45.

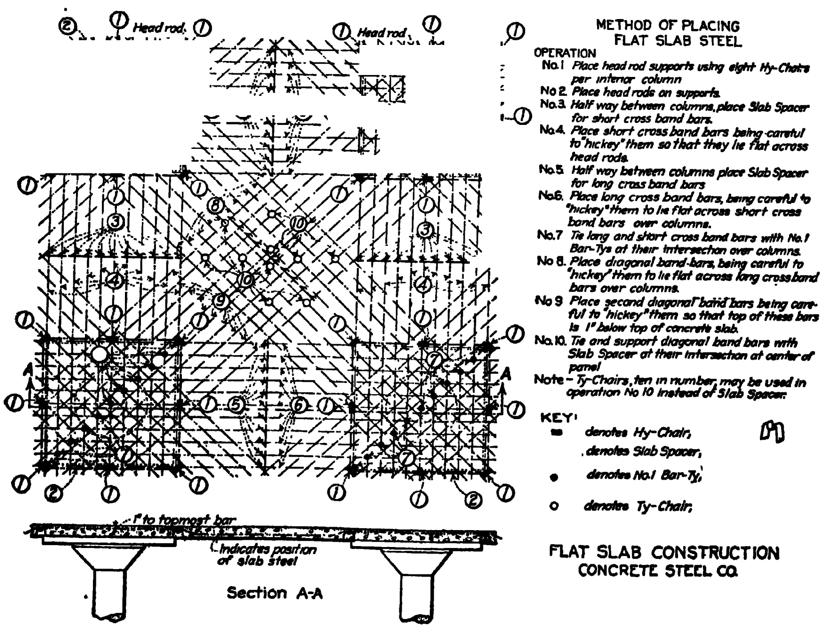
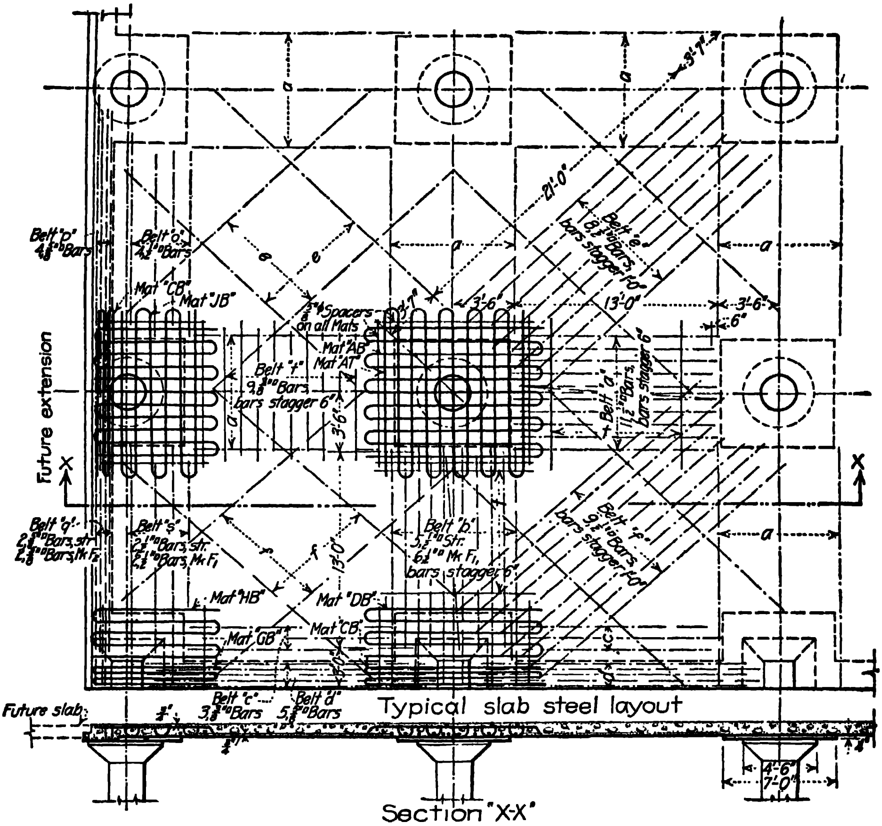
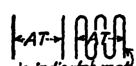


FIG. 46.

66. Barton Spider Web System.—A combination of the four-way and two-way systems of reinforcement quite widely used is the Barton Spider Web



Second floor slab steel schedule							
Mk	No. of Belts	Detail of belt				Width of Belt	Remarks
		No. of Bars	Size	Length	Mk		
a	19	11	[illegible]	12'-0"		7'-6"	
b	12	5	[illegible]	12'-6"		7'-6"	alternate bent & str't.
		6	[illegible]	15'-6"	F_1		
c	15	3	[illegible]	12'-0"		1'-6"	
d	11	5	[illegible]	16'-6"		1'-6"	
e	18	8	[illegible]	20'-0"		7'-6"	
f	28	9	[illegible]	20'-0"		7'-6"	
o	2	4	[illegible]	12'-0"		3'-6"	
t	35	9	[illegible]	9'-0"		8'-0"	top bars



Notes: The last letter of the mark of a layer of column head steel ('s indicates mat) gives the lo... of same. "B" indicates bottom layer: "T" indicates top layer. All layers of column head steel or mats are called for like other belts of bars, i.e. at right angles with bars.

Fig. 47.

System. The slab reinforcement is composed of small diameter rods running from column to column and not continuous, while the negative reinforcement over the column heads generally consists of two layers of bars at right angles to each other bent in the shape of continuous hairpin loops with the looped ends bent down to rest on the forms and thus support the mat in the proper position. Sometimes, instead of using these fabricated mats for column head reinforcement, two layers of loose, short bars supported on concrete blocks and raising bars are used, with zig-gag stirrups hanging from the ends of the mat bars to support the ends of bars in the direct bands. In this system short straight bars are placed in top of slab between columns to take care of the negative crossbending which experience

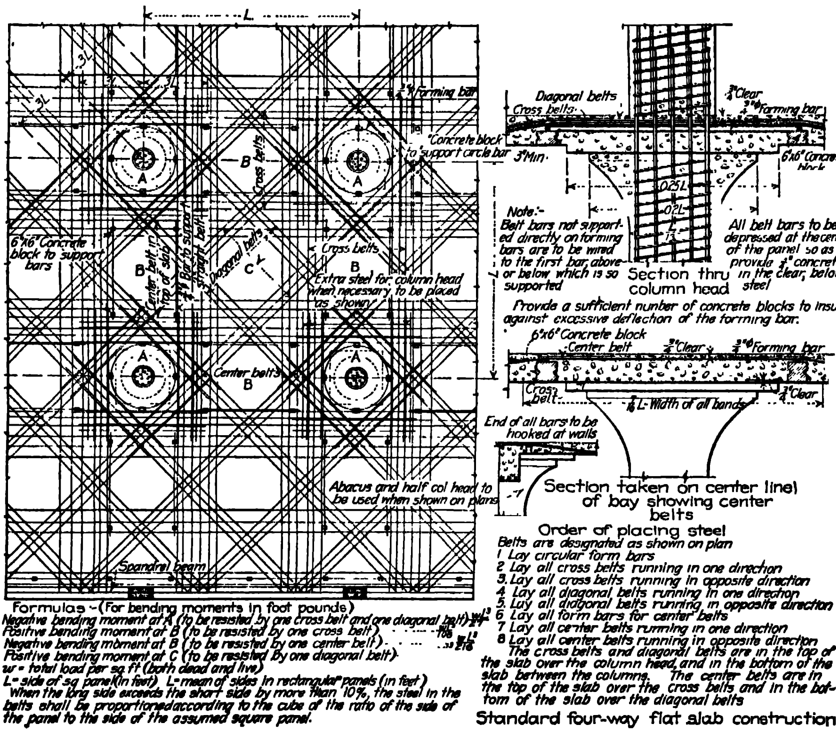


Fig. 48.

has shown exists over the main direct bands of bars midway between columns (see discussion on economy of systems, Art. **82**). A detailed drawing of this system is shown in Fig. 47.

67. Corr-Plate Floor.—A two-way type of flat slab which, in general, is the same as the Akme system except in minor details of the arrangements of belts of bars and the computation of moments, is the Corr-plate floor. The method of computation and arrangement of bars was developed primarily from a series of tests on rubber models of a floor panel.

68. Watson System.—The Watson system developed by Wilbur J. Watson and Company of Cleveland, Ohio, is a combination of the four-way and two-way systems inasmuch as mid-panel belts at right angles to each other and not passing

over the columns are used. The details of reinforcement and methods of computation are shown in Fig. 48.

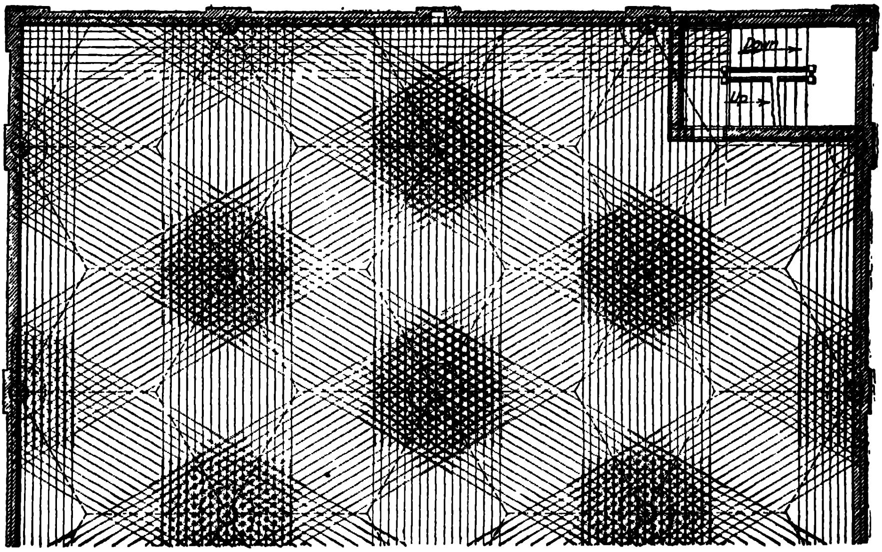


Fig. 49.

69. Morrow Three-Way System.—A three-way reinforced floor system in which the columns are located at the apices of equilateral triangles, and the belts

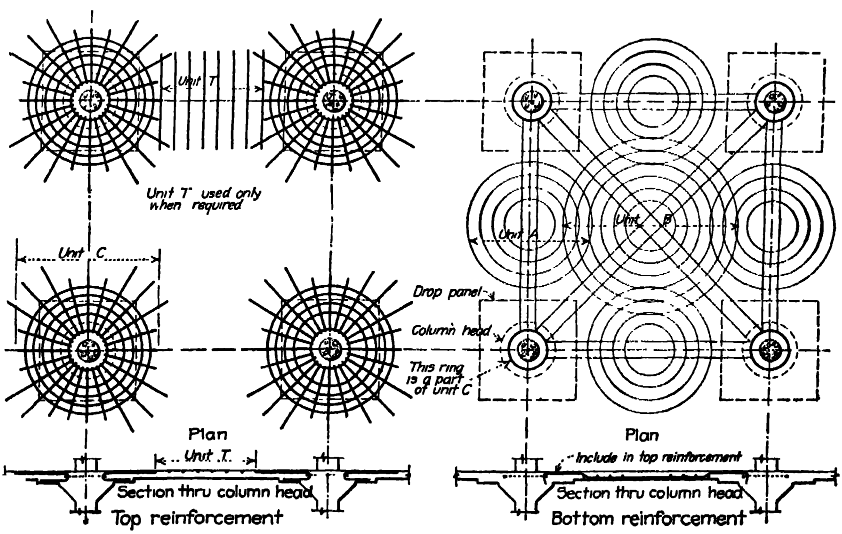


Fig. 50.

of bars run directly between columns with all belts of equal length, has been developed and patented by David W. Morrow, Cleveland, Ohio. This system has

been used extensively in buildings by the Cleveland Railway Company, since by the arrangement of columns right angle turns are avoided, and in passing from one aisle to another it is only necessary to turn through an angle of 60 deg., thus making the movement of cars, trucks or conveyors handling long material very easy. The same advantage occurs when used in garage buildings. The general layout of a three-way floor is shown in Fig. 49.

70. S-M-I System.—A type of flat slab floor which differs materially from any of the other systems described is the S-M-I or Smulski system in which circumferential and radial reinforcement units are used, with only a small amount of

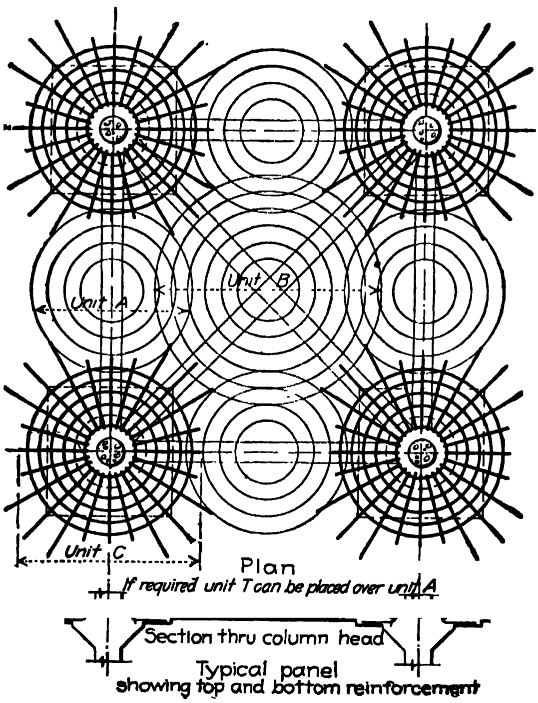


Fig. 51.

steel running directly and between columns. This system, patented by Edward Smulski of Boston, has been used mainly in the Atlantic States.

The following description of the system by Mr. Smulski and Figs. 50, 51 and 52 give an excellent idea of the system.

The reinforcement of a typical interior panel, fully illustrated in Figs. 50 and 51, consists of three types of units:

(1) Unit *C* at the column head composed of rings and radial bars in the shape of hair pins, the upper prong of which resists tension while the lower prong resists compression (see Fig. 52).

(2) Unit *A* between columns consisting of two trussed bars and rings.

(3) Unit B in central portion consisting of four diagonal trussed bars and rings.

Units *T* are sometimes used as shown in "Top Reinforcement," Fig. 50.

The radial bars are provided with a semicircular hook of sufficient dimensions to transfer the stresses into the concrete by bond and bearing. The center ring which they sometimes engage keeps them in place and forms an additional factor of safety.

The trussed bars of units A and B are bent up near the points of inflection and carried near the top and parallel to the surface of the slab to the column head, where they engage the center ring. The bent portion resists shear and binds the column-head section to the rest of the slab. The straight portion of the trussed bars in the center of the slab and at the column head resists tension due to the positive and negative bending moments respectively.

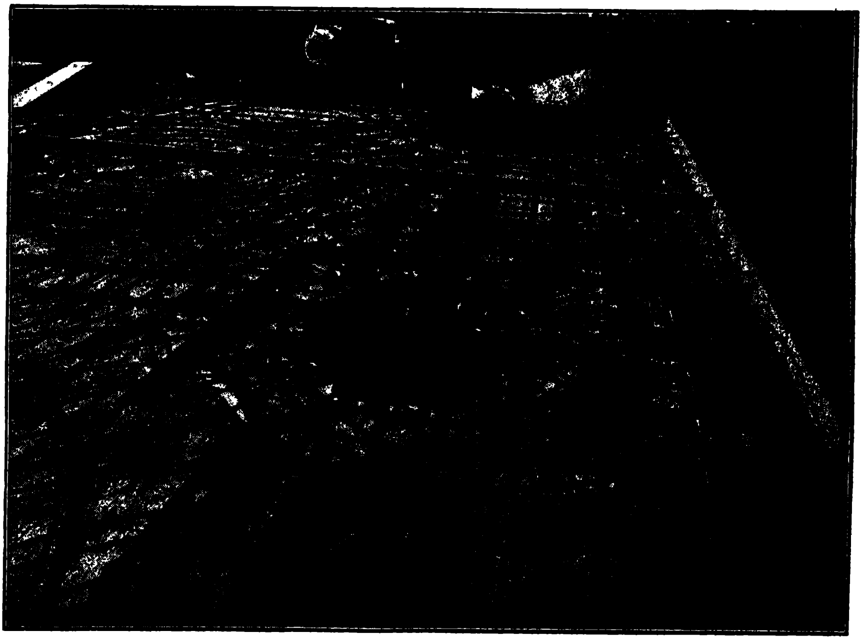


Fig. 52.

The trussed bar extends into the column head a sufficient distance beyond the point of maximm stress (that is, the edge of the column head), to develop in combination with the hook their full tensile strength. The ring which they engage serves to distribute the bearing stresses laterally onto a large area of concrete.

Position of Units.—Units A and B are placed near the bottom while unit C is near the top of the slab.

Compression Reinforcement.—By introduction of compression reinforcement in the shape of lower prongs of the radials, the slab is stiffened at the support, and the compression stresses in concrete reduced. If desired, therefore, it is possible to omit the drop panel at the column head and use an altogether flat ceiling. This is often desirable either for the sake of appearance or to simplify shafting or piping.

Secondary Reinforcement.—Sometimes to prevent cracks on the top of the slab between columns, additional secondary reinforcement consisting of short straight bars, and called units *T*, is used. These bars are usually placed after the concrete of the slab is poured.

The reinforcement is laid out on the basis that after deflection under load a flat slab floor panel assumes a composite shape, namely, that of an umbrella over the columns and the shape of a saucer in the central portion. This means that the lines of equal deflection are, roughly speaking, circular, and the stresses act perpendicular to these lines of equal deflection. The best method of resisting these stresses is by placing the bars perpendicular to the lines of equal deflection, or by enclosing them by means of a ring. The radials and trussed bars of this system are perpendicular to the lines of equal deflection while the rings either intersect them at 90 deg. or they enclose same and prevent the enclosed concrete from spreading.

The main advantage of this system over the others described is the economy of steel, but this is in a measure offset by the extra cost of bending the relatively large hoop bars, and the additional cost of placing the steel.

71. Flat Slab Codes.—The flat slab codes and rulings representing best practice are the Chicago Ruling, Joint Committee Specifications, and the American Concrete Institute Specifications for flat slab construction. The Joint Committee Specifications are reproduced in Appendix F and the reader is referred thereto.

72. General Notes on Flat Slab Design and Construction.

72*a*. Small Openings.—Where small openings are required in the floor slabs for pipes, etc., they can safely be put in without changing reinforcement if it is not necessary to move any of the bars. Pipe sleeves of heavy wrought-iron pipe up to 6 in. in diameter can be put through the column capitals to allow for passage of sprinkler and heating pipes or downspouts. The heavy sleeve is used to take care of the compression which must be resisted due to the omission of the concrete.

72*b*. Large Openings.—For large openings as are required for stairs, elevators, pipe shafts, etc., special framing is required. Wherever possible the slab around the opening should be deepened sufficiently to give the necessary resisting moment. The deepening of the slab on lines directly between columns and adjacent to the opening is better practice than to use deep beams which in conjunction with the adjoining flat slab portions give rise to complicated stress action.

72*c*. Columns.—Experience indicates that if the column diameters used in flat slab floors are at least $\frac{1}{12}$ of the span center to center of columns, the bending moments developed in the columns will be amply taken care of.

In no case should the column capital used be greater than three times the column diameter nor should it have a slope from slab to column shaft of less than 45 deg. with the horizontal. The Chicago code limits the size of capital to $0.225L$ while the American Concrete Institute ruling sets the minimum size at $0.2L$ and places no limit on the maximum size. It would seem advisable not to use a capital greater than $0.25L$ in any design.

At exterior columns in drop-panel construction a half drop panel should be used. The column capital should be a half capital if no spandrel beam is used,

but in the great majority of cases this is impossible on account of the use of square or rectangular column sections. These only allow the use of column bracket projecting out toward the center of building. This increases the span of the spandrel strip accordingly and due allowance should be made in the design as indicated in the examples hereinafter given above.

In pouring flat slab floors the columns should be cast to the bottom of capital and a period of 24 hr. allowed to elapse before pouring the capital and slab. This construction joint should not be made at top of capital.

72*d*. Thickness of Slab.—Experience gained in the construction of a large number of reinforced concrete flat slab buildings shows that many of the cases of unsatisfactory results can be laid to the use of relatively thin slabs and consequent high percentages of reinforcing steel to obtain the necessary strength. It has been noted that flat slab floors giving the best satisfaction in actual use and the best results in tests (least deflection recorded) are those which are under-reinforced—that is, relatively low percentage of steel used and the slab made thicker to give the necessary resisting moment. This keeps the compressive stresses in the concrete relatively low, making the strength of the slab dependent on the steel which is much more likely to be uniform in strength than the concrete.

As is evident, the thickness of slab depends upon the distance between supports, the condition of supports (that is, whether the panel is an interior or exterior one), the live load to be carried, the size and shape of the column capital, the thickness of slab at the column capital, the thickness of slab at the column (that is, whether or not a dropped panel is used over the column capital), and the percentage of reinforcement.

The formulas now recommended for the thickness of flat slabs are based on equating expressions for bending moment and resisting moment at any particular section, then solving for depth required to steel, and then adding a constant, 1 or 1½ in., to obtain the total thickness of the slab. The Joint Committee recommends for a slab without dropped panels (and column capitals not less than 0.2 of span center to center of columns) a minimum thickness of slab in inches, $t = 0.024L\sqrt{w} + 1\frac{1}{2}$ in., where L is the panel length in feet and w the total live load and dead load in pounds per square foot; for a slab with dropped panels, $t = 0.02L\sqrt{w} + 1$ in.; and for the dropped panel section having a width of $0.4L$

$$t = 0.03L\sqrt{w} + 1\tfrac{1}{2} \text{ in.}$$

It should be remembered that the above formulas are based on unit stresses of 16,000 lb. per sq. in. in steel and 650 lb. per sq. in. in the concrete and that if higher unit stresses are employed the constants in the first part of formula can justly be decreased, thus for $f_s = 18{,}000$ lb. and $f_c = 700$ lb., for slabs with drop panels, $t = 0.19L\sqrt{w} + 1$ in.

In these times of extreme high cost of steel, designs in which relatively thick slabs are used will be found more economical, and Joint Committee recommendations, or as modified for higher unit stresses, can therefore be adopted as good practice.

For practical reasons the construction of slabs having a total thickness of less than 6 in. is not to be recommended even though the formulas above cited

may give values of t less than this for light loads and relatively short spans. Another limitation, first specified in the Chicago Ruling on Flat Slab Construction and now recommended by the Joint Committee and American Concrete Institute to guard against the use of very thin slabs for light loads with resulting expressive deflection, is that in no case shall the slab thickness be less than $1/32$ of the panel length for floors and $1/40$ for roofs.

Besides these limitations the designer must make sure that the allowable punching shear (100 lb. per sq. in.) is not exceeded at the periphery of the column capital and also at the periphery of the dropped panel, if it is used. From the number of limitations placed on the thickness of flat slabs, it is readily apparent that this factor is a vitally important one to the proper execution of flat slab construction.

72e. Drop Panel.—The minimum size of drop panel will be determined by the diagonal tension along the edge of the drop. The thickness should not be less than $2/3$ of the thickness of the slab at center. The width of the drop should preferably be $0.4L$ with a minimum of $0.3L$ and a maximum of $0.5L$.

72f. Flat Slab Thickness.—The thickness of a flat slab where no drop panels are used is determined by the unit compressive stress at the column capital over the full width of column strip or by the unit shearing stress along the periphery of the column capital whichever requires the greater thickness.

The use of compression steel in the bottom of the slab over the column instead of a drop panel is not economical and gives rise to high shearing stresses and hence is not advisable.

72g. Tile Fillers.—For floors carrying relatively light loads tile fillers are sometimes used in the center portion of panels, but they should never be used in the drop panel area or within $0.2L$ from the center lines between columns.

72h. Details of Reinforcement.—Radial head bars and circumferential ring bars used in the earlier four-way designs have proven to be inefficient and are now little used. Experience shows that the slab at the ring bar tends to form cracks along the bar extending deeply into the slab. This may cause failure by shearing or continued increasing deflection and opening up of the cracks.

Experience and actual tests have shown the desirability of bending all reinforcing bars at predetermined points rather than depending on the bars to droop from top of slab at the columns or over the center portions of the main band into the bottom between columns or at the center of panel. This drooping is in general too gradual to get the bars in the positions to properly resist the stresses soon enough.

The points at which to bend down steel from top of slab are determined by the moment curves and the diagonal tension just beyond the edge of the drop panel. The Akme Design Standards for bending down of bars represent the best practice. They also provide for the proper length of bar to insure sufficient embedment beyond the point of inflection for bars in bottom of slab. In general it will be found most economical to bend bars at one end only—that is, so one end will be in the top of slab and the other in the bottom portion. The use of bars bent at both ends requires the use of a considerable number of straight bars in the bottom of the slab between columns which is not to be recommended.

In four-way designs the bars should not be lap-spliced at the sections of maximum moment, unless each bar end is carried to the quarter point of the adjacent span and lap counted as two full bars in tension.

72*i*. Support and Spacing of Bars.—Owing to the relatively thin slabs used in flat slab construction it is of the utmost importance that the reinforcing steel be securely held in place during concreting operations to insure its position in the finished slab as intended by the designer.

Broadly speaking, the steel should be so placed as to adequately reinforce all regions of tensile stress with sufficient length of bars to insure anchorage beyond the lines of critical stress. This means that at the column head section where the moments are greatest and subject to considerable variation under different conditions of loading on surrounding panels, the steel should be kept well up in the top of the slab over such an area as will provide steel at all points within the maximum possible position of the line inflection from the column center.

Records of tests made on slabs where bars were "merely drooped" show that the bars were very much out of place in some cases. In the Western Newspaper Union Building, recently wrecked to make way for the Chicago Union Station, in making a test of one of the floors before wrecking, it was found that bars in the center of the panel were as much as 2½ in. above the bottom of the slab, and at the column head section (very close to column) the lower layer of bars was in some cases as much as 3½ in. below top of slab (measurements made by the writer), mainly, no doubt, because the bars were drooped and not bent before placing. Bending may increase the cost of construction somewhat, but the owner can well afford to pay the additional cost because of the fact that the resulting structure will be much stronger. The designer should therefore make his plans in such a way as to provide definite bending details for each and every bar, so that when placed on spacing bars in bottom of slab, and when raising bars and blocks in top portion of columns and over many belts between columns, the actual position in the finished work will be within a fraction of an inch of the point assumed in design.

72*j*. Flat Slab Supported on Exterior Brick Walls.—The Chicago Ruling on Flat Slab Construction now requires that for wall-bearing panels the positive moments in the outer sections of wall panels be increased 50 per cent over that specified for interior panels and the positive moments at the inner section 50 per cent, and all walls to be pilastered opposite interior columns. Slabs designed on this basis have been proved satisfactory, but it must be admitted that, all things considered, it is better to have the exterior supports of the slab monolithic concrete columns rather than brick piers or walls. However, it will sometimes be found more desirable to eliminate the wall columns even if the cost of the slab is increased, because of the fact that little or no restraint is offered by the bearing walls, and for this reason it is well to have the definite specification as the moments to be used under such conditions.

To consider the inside face of a bearing wall the equivalent of a column center line (that is, the span as the distance from the inside of wall to center line of first interior row of columns), is manifestly in error since no support such as afforded by column capitals is provided inside of the wall line. To the writer it seems

entirely logical to consider the inside face of the wall as corresponding to the edge of the column capital (if one were used), or in other words, to consider the span of slab as the distance from the center line of first interior row of columns to a line one-half the width of column head (interior) beyond the inside face of the wall. By using this assumed span with the moment coefficients increased 20 per cent, the same as for ordinary exterior panels, entirely satisfactory results will be obtained. In such design the bearing wall takes the place of the spandrel belt of reinforcing as noted under the heading arrangement of reinforcement.

The writer has seen plans of two flat slab buildings in which the exterior walls, instead of exterior columns, supported the slabs, and in spite of this the bars in the exterior panels were arranged in practically the same manner as the interior panels. In other words, the brick piers were assumed to produce the same action in the slab as rigid reinforced concrete columns with flaring heads. Needless to say, this action failed to materialize, and the floors collapsed. In one case, the failure was attributed by engineers to poor brickwork, and in the other to faulty form construction, notwithstanding the fact that the failure undoubtedly occurred after the forms were removed. In both cases, however, the arrangement of steel had much to do with the results. Obviously, it is not correct to place bands of reinforcing bars parallel and adjacent to a bearing wall, and expect them to make up for the absence of columns. In such cases the wall must be considered as the edge of the direct band of reinforcement which would be placed there if exterior columns were used, with the inner edge of column capital corresponding to the inside face of wall.

72*k*. Brackets on Columns.—When brackets instead of column heads are used on exterior columns, as is now quite often the case where these columns are rectangular in shape and exposed as part of the architectural treatment, due allowance should of course be made in computing bending moments in case the brackets are not of the same shape and proportions as the interior column capitals, and also in the arrangement of steel and length of bars, particularly in the two-way system of reinforcement.

73. Rectangular Panels.—Since flat slab construction is most economical when the panels are approximately square, the designer should endeavor to arrange the column spacing so as to meet this requirement. Most codes are agreed that the methods of design should not apply where the long side of the panel is $\frac{1}{3}$ longer than the short side.

The American Concrete Institute and Akme moment formulas are so expressed as to apply directly to rectangular panels, while the Chicago Code has provisions for the computation of rectangular panels which do not agree with tests or good practice. This is due to the fact that in computing the moments for the long span the design is made the same as for a square panel of dimensions equal to the longer span.

In rectangular panels the slab thickness can be based on the average panel length when this is equal to or greater than 0.9 of the longer panel dimension. The computations for sections parallel to the short dimension of panel determine the drop thickness and should be made first.

74. Comparison of Different Flat Slab Rules and Codes.—The principal features of flat slab design as set forth by the most important flat slab rulings are given in Tables 15 and 16.

TABLE 15.—DESIGN PROVISIONS FOR FOUR-WAY SQUARE INTERIOR FLAT SLAB PANELS WITH DROPS AND CAPITALS. BASED UPON 2,000-LB. CONCRETE AND $n = 15$

	A.C.I.	Chicago	New York	1921 Joint Com.
Concrete stress, f_c	750	700	650	800[1]
Steel stress, f_s	16–18,000	16–18,000	16,000	16–18,000
Fixed col. capital		$0.225L$[2]	$0.225L$	
Formula No.	(64)	(65)	(64)	(64)
Slab thickness	$t_1 = 0.2L\sqrt{w} + 1''$	$t_1 = 0.0227\sqrt{W}$	$t_1 = 0.2L\sqrt{w} + 1''$	$t_1 = 0.2L\sqrt{w} + 1''$
Slab + drop, t_2	$<1.67t_1$	$<1.67t_1$	$>1.33t_1$	
Min. drop width	$0.3L$	$0.33L$	$0.33L$	$0.33L$
Shearing stress[7] (edge of capital)	100#/□″	120#/□″		
Diagonal tension[8] (edge of drop)	60#/□″	60#/□″		
Formula No.	(66)	(67)	(68)	(69)
Total moment coeff.[3]	$M_0 = 0.09wl_1 (l_2 - qc)^2$	$M_0 = WL/16$	$M_0 = WL/17$	$M_0 = 0.09wL \left(l - \frac{2c}{3L}\right)^2$
$-M_c$ Column strip[4]	50–55% M_0[6]	$WL/30$	$WL/32$	51–57% M_0
$+M_c$ Column strip	18–20% M_0[6]	$WL/80$	$WL/100$	18–20% M_0
$-M_m$ Middle strip[5]	18–20% M_0[6]	$WL/120$	$WL/133$	18–20% M_0
$+M_m$ Middle strip	10–12% M_0[6]	$WL/120$	$WL/100$	7–9 % M_0

[1] Maximum fiber stress computed by special formula taking into account variation in intensity of stress across the column strip.

[2] See Chicago code for special formula for flat slab on columns without capitals or drops.

[3] The moment coefficients in table are for four-way type only (see Table 16 and various specifications for coefficients for other types).

[4] Effective steel area, Chicago = one direct + one diagonal band. Other specifications = one direct + component of two diagonal bands.

[5] Effective steel area, Chicago = one diagonal band. Other specifications = component of two diagonal bands.

[6] The A. C. I. specification allows 10 per cent of total moment to be assigned to sections by designer. The percentages tabulated are proper for four-way type with drop. One hundred per cent of M_0 must be used on the 4 sections.

[7] Figured on vertical section through edge of column capital.

[8] Figured on vertical section of depth jd on edge of drop. Figures in parentheses are formula numbers for text reference.

TABLE 16.—DESIGN PROVISIONS FOR TWO-WAY SQUARE INTERIOR FLAT SLAB PANELS WITH DROPS AND CAPITALS. BASED ON 2,000-LB. CONCRETE AND $n = 15$

	A. C. I.	Chicago	New York	1921 Joint Com.
Total moment coeff.	(66) $M_0 = 0.09wl_1 (l_2 - qc)^2$	(70) $M_0 = WL/15$	(68) $M_0 = WL/17$	(69) $M_0 = 0.09\,wL \left(1 - \frac{2c}{3L}\right)^2$
$-M_c$ Column strip	50–55%	$WL/30$	$WL/32$	47–53%
$+M_c$ Column strip	18–20%	$WL/60$	$WL/80$	19–21%
$-M_m$ Middle strip	14–16%	$WL/120$	$WL/133$	14–16%
$+M_m$ Middle strip	14–16%	$WL/120$	$WL/133$	14–16%

The allowable concrete and steel fiber stresses, the size of column capital, the slab and drop thicknesses, the width of the drop and the allowable shearing stresses are the same for two-way as given for four-way in Table 15.

TABLE 17

SQUARE INTERIOR FLAT SLAB PANEL

American Concrete Institute Specifications

f_s = 18,000 f_c = 750 n = 15 c = 0.225L

Side of panel	Side of drop		Diam. of cap		Slab thickness	Slab & drop	Approximate weight of slab steel	Four-way type: Steel area column strip	Four-way type: Rods required, Column strip	Four-way type: Rods required, Middle strip		Two-way type: Steel area column strip	Two-way type: Rods required, Column strip	Two-way type: Rods required, Middle strip	
								$-M_c$	$+M_c$	$+M_m$	$-M_m$	$-M_c$	$+M_c$	$+M_m$	$-M_m$
Ft.	Ft.	In.	Ft.	In.	In.	In.	#/□′	□″				□″			
50 #/□′ superimposed roof load															
16	4	10	3	6	6	8½	1.10	1.86	10–3/8 ϕ	10–5/16 ϕ	11–1/4 ϕ	1.70	10–3/8 ϕ	11–5/16 ϕ	11–5/16 ϕ
17	5	2	4	0	6	8½	1.20	2.25	11–3/8 ϕ	12–5/16 ϕ	10–1/4 □	2.17	11–3/8 ϕ	13–5/16 ϕ	13–5/16 ϕ
18	5	6	4	0	6	8½	1.35	2.67	14–3/8 ϕ	14–5/16 ϕ	10–5/16 ϕ	2.48	14–3/8 ϕ	11–3/8 ϕ	11–3/8 ϕ
19	5	8	4	6	6	8½	1.50	3.13	16–3/8 ϕ	12–3/8 ϕ	11–5/16 ϕ	2.89	12–3/8 □	12–3/8 ϕ	12–3/8 ϕ
20	6	0	4	6	6	8½	1.65	3.70	18–3/8 ϕ	14–3/8 ϕ	13–5/16 ϕ	3.40	14–3/8 □	14–3/8 ϕ	14–3/8 ϕ
21	6	4	4	6	6½	9	1.80	4.10	16–3/8 □	15–3/8 ϕ	14–5/16 ϕ	3.85	12–1/2 ϕ	12–3/8 □	12–3/8 □
22	6	8	5	0	6¾	9¼	1.90	4.75	13–1/2 ϕ	13–3/8 □	12–3/8 ϕ	4.45	13–1/2 ϕ	14–3/8 □	14–3/8 □
23	7	0	5	0	7	9¾	2.05	5.20	15–1/2 ϕ	15–3/8 □	13–3/8 ϕ	4.90	15–1/2 ϕ	15–3/8 □	15–3/8 □
24	7	4	5	6	7¼	10¼	2.20	5.68	16–1/2 ϕ	17–3/8 □	14–3/8 ϕ	5.37	16–1/2 ϕ	17–3/8 □	17–3/8 □
25	7	6	5	6	7½	10¾	2.30	6.20	18–1/2 ϕ	18–3/8 □	16–3/8 ϕ	5.90	18–1/2 ϕ	14–1/2 ϕ	14–1/2 ϕ
26	7	10	6	0	8	11¼	2.40	6.90	19–1/2 ϕ	15–1/2 ϕ	17–3/8 ϕ	6.50	19–1/2 ϕ	15–1/2 ϕ	15–1/2 ϕ
100 #/□′ superimposed floor load															
16	4	10	3	6	6	8	1.45	2.59	10–3/8 □	10–3/8 ϕ	10–5/16 ϕ	2.40	10–3/8 □	10–3/8 ϕ	10–3/8 ϕ
17	5	2	4	0	6½	9	1.60	3.06	12–3/8 □	11–3/8 ϕ	11–5/16 ϕ	2.84	12–3/8 □	12–3/8 ϕ	12–3/8 ϕ
18	5	6	4	0	6¾	9¼	1.70	3.54	13–3/8 □	13–3/8 ϕ	12–5/16 ϕ	3.32	13–3/8 □	11–3/8 □	11–3/8 □
19	5	8	4	6	7¼	9¾	1.85	4.00	15–3/8 □	14–3/8 ϕ	14–5/16 ϕ	3.78	11–1/2 ϕ	12–3/8 □	12–3/8 □
20	6	0	4	6	7½	10	2.00	4.65	12–1/2 ϕ	13–3/8 □	11–3/8 ϕ	4.40	12–1/2 ϕ	13–3/8 □	13–3/8 □
21	6	4	4	6	8	10½	2.15	5.20	14–1/2 ϕ	14–3/8 □	12–3/8 ϕ	4.90	14–1/2 ϕ	15–3/8 □	15–3/8 □
22	6	8	5	0	8¼	11	2.25	5.80	16–1/2 ϕ	16–3/8 □	14–3/8 ϕ	5.65	16–1/2 ϕ	17–3/8 □	17–3/8 □
23	7	0	5	0	8¾	11½	2.40	6.50	17–1/2 ϕ	17–3/8 □	15–3/8 ϕ	6.20	17–1/2 ϕ	13–1/2 ϕ	13–1/2 ϕ
24	7	4	5	6	9	12	2.55	7.10	19–1/2 ϕ	14–1/2 ϕ	17–3/8 ϕ	6.80	15–1/2 □	15–1/2 ϕ	15–1/2 ϕ
25	7	6	5	6	9½	12¾	2.65	7.63	16–1/2 □	15–1/2 ϕ	15–3/8 □	7.36	16–1/2 □	16–1/2 ϕ	16–1/2 ϕ
26	7	10	6	0	9¾	13½	2.80	8.40	18–1/2 □	17–1/2 ϕ	16–3/8 □	8.00	18–1/2 □	18–1/2 ϕ	18–1/2 ϕ

150#/□′ superimposed floor load													
16	5 0	3 6	6	8¾	1.90	3.24	13-⅜ □	13-⅜ φ	12-⅝ φ	3.03	13-⅜ □	10-⅜ □	10-⅜ □
17	5 2	4 0	6½	9½	2.10	3.65	15-⅜ □	11-⅜ □	10-⅜ φ	3.43	11-½ φ	12-⅜ □	12-⅜ □
18	5 6	4 0	6¾	10	2.20	4.20	12-½ φ	13-⅜ □	11-⅜ φ	3.98	12-½ φ	13-⅜ □	13-⅜ □
19	5 10	4 6	7¼	10½	2.35	4.65	14-½ φ	14-⅜ □	12-⅜ φ	4.40	11-½ □	11-½ φ	11-½ φ
20	6 2	4 6	7½	11	2.50	5.20	16-½ φ	12-½ φ	11-⅜ □	5.00	12-½ □	12-½ φ	12-½ φ
21	6 6	4 6	8	11½	2.65	5.84	17-½ φ	13-½ φ	12-⅜ □	5.55	13-½ □	13-½ φ	13-½ φ
22	6 10	5 0	8¼	12	2.80	6.55	15-½ □	14-½ φ	14-⅜ □	6.25	15-½ □	15-½ φ	15-½ φ
23	7 2	5 0	8¾	12½	3.00	7.30	17-½ □	16-½ φ	15-⅜ □	7.00	14-⅝ φ	16-½ φ	16-½ φ
24	7 6	5 6	9	13¼	3.15	8.00	18-½ □	17-½ φ	16-⅜ □	7.65	15-⅝ φ	14-½ □	14-½ □
25	7 10	5 6	9½	14	3.30	8.65	17-⅝ φ	15-½ □	18-⅜ □	8.30	17-⅝ φ	16-½ □	16-½ □
26	8 4	6 0	9¾	14½	3.45	9.70	18-⅝ φ	17-½ □	14-½ φ	9.30	18-⅝ φ	17-½ □	17-½ □
200#/□′ superimposed floor load													
16	5 4	3 6	6½	9¼	2.20	3.76	11-½ φ	11-⅜ □	10-⅜ φ	3.54	11-½ φ	12-⅜ □	12-⅜ □
17	5 10	4 0	6¾	9¾	2.40	4.35	12-½ φ	13-⅜ □	11-⅜ φ	4.10	12-½ φ	14-⅜ □	14-⅜ □
18	6 2	4 0	7¼	10¼	2.60	4.92	14-½ φ	11-½ φ	13-⅜ φ	4.66	11-½ □	11-½ φ	11-½ φ
19	6 6	4 6	7½	10¾	2.80	5.53	16-½ φ	12-½ φ	11-⅜ □	5.25	13-½ □	13-½ φ	13-½ φ
20	6 10	4 6	8	11½	2.95	6.10	18-½ φ	13-½ φ	12-⅜ □	5.80	14-½ □	14-½ φ	14-½ φ
21	7 2	4 6	8¼	12	3.10	6.70	20-½ φ	15-½ φ	14-⅜ □	6.40	13-⅝ φ	15-½ φ	15-½ φ
22	7 6	5 0	8¾	12¾	3.25	7.50	17-½ □	16-½ φ	15-⅜ □	7.20	15-⅝ φ	13-½ □	13-½ □
23	7 10	5 0	9¼	13¼	3.40	8.30	16-⅝ φ	14-½ □	17-⅜ □	8.00	16-⅝ φ	14-½ □	14-½ □
24	8 4	5 6	9½	13¾	3.55	9.30	17-⅝ φ	15-½ □	14-½ φ	8.90	17-⅝ φ	16-½ □	16-½ □
25	8 8	5 6	10	14½	3.70	10.00	19-⅝ φ	17-½ □	15-½ φ	9.65	19-⅝ φ	18-½ □	18-½ □
26	9 2	6 0	10½	15	3.90	11 00	21-⅝ φ	18-½ □	16-½ φ	10.70	21-⅝ φ	16-⅝ φ	16-⅝ φ
250#/□′ superimposed floor load													
16	5 8	3 6	7	9¾	2.40	4.25	12-½ φ	12-⅜ □	11-⅜ φ	4.00	12-½ φ	13-⅜ □	13-⅜ □
17	6 2	4 0	7½	10½	2.60	4.80	13-½ φ	14-⅜ □	12-⅜ φ	4.50	13-½ φ	11-½ φ	11-½ φ
18	6 6	4 0	7¾	10¾	2.80	5.55	16-½ φ	12-½ φ	11-⅜ □	5.30	12-½ □	12-½ φ	12-½ φ
19	6 10	4 6	8¼	11½	3.00	6.10	14-½ □	13-½ φ	12-⅜ □	5.80	14-½ □	13-½ φ	13-½ φ
20	7 2	4 6	8½	12	3.20	6.90	15-½ □	14-½ φ	13-⅜ □	6.55	13-⅝ φ	12-½ □	12-½ □
21	7 6	4 6	9	12¾	3.40	7.70	17-½ □	16-½ φ	11-½ φ	7.30	14-⅝ φ	13-½ □	13-½ □
22	8 0	5 0	9½	13½	3.55	8.50	19-½ □	14-½ □	12-½ φ	8.20	16-⅝ φ	15-½ □	15-½ □
23	8 4	5 0	10	14	3.70	9.40	17-⅝ φ	15-½ □	13-½ φ	9.10	17-⅝ φ	16-½ □	16-½ □
24	8 8	5 6	10½	14¾	3.90	10.20	19-⅝ φ	16-½ □	14-½ φ	9.80	19-⅝ φ	14-⅝ φ	14-⅝ φ
25	9 2	5 6	11	15¼	4.05	11.30	20-⅝ φ	18-½ □	16-½ φ	11.00	20-⅝ φ	16-⅝ φ	16-⅝ φ
26	9 8	6 0	11¼	15¾	4.20	12.30	22-⅝ φ	17-⅝ φ	17-½ φ	12.00	22-⅝ φ	17-⅝ φ	17-⅝ φ
300#/□′ superimposed floor load													
16	6 0	3 6	7½	10¼	2.70	4.65	13-½ φ	10-½ φ	11-⅜ φ	4.45	13-½ φ	10-½ φ	10-½ φ
17	6 6	4 0	7¾	10¾	2.90	5.40	15-½ φ	11-½ φ	10-⅜ □	5.10	15-½ φ	12-½ φ	12-½ φ
18	6 10	4 0	8¼	11¼	3.05	6.13	17-½ φ	12-½ φ	12-⅜ □	5.86	13-½ □	13-½ φ	13-½ φ
19	7 2	4 6	8¾	12	3.25	6.75	15-½ □	14-½ φ	13-⅜ □	6.43	15-½ □	11-½ □	11-½ □
20	7 6	4 6	9¼	12¾	3.40	7.65	16-½ □	15-½ φ	11-½ φ	7.30	14-⅝ φ	13-½ □	13-½ □
21	7 10	4 6	9¾	13½	3.60	8.40	18-½ □	17-½ φ	12-½ φ	8.00	15-⅝ φ	14-½ □	14-½ □
22	8 6	5 0	10	14	3.80	9.40	20-½ □	15-½ □	13-½ φ	9.10	17-⅝ φ	16-½ □	16-½ □
23	9 0	5 0	10½	14½	4.00	10.40	19-⅝ φ	16-½ □	14-½ φ	10.00	19-⅝ φ	14-⅝ φ	14-⅝ φ
24	9 4	5 6	11	15¼	4.20	11.30	20-⅝ φ	18-½ □	16-½ φ	10.90	20-⅝ φ	16-⅝ φ	16-⅝ φ
25	9 8	5 6	11½	15¾	4.40	12.50	22-⅝ φ	19-½ □	17-½ φ	12.00	15-¾ φ	17-⅝ φ	17-⅝ φ
26	10 0	6 0	12	16½	4.60	13.50	24-⅝ φ	18-⅝ φ	15-½ □	13.00	17-¾ φ	18-⅝ φ	18-⅝ φ

Table 18.—Akme System Drop Panel

Panel	Live load	Slab thickness	Size, plate	Thickness, plate	Head Square	Head Round	Clear span S	Comp. width at col.	Comp. width at center	Main slab mom. at head
16′ × 16′ c. to c. cols	150	6″	5-9″	2½″	3′-6″	4′-0″	12′-6″	5′-9″	8′- 6″	342,900
	200	6½″	.....	3″					8′- 8″	430,000
	250	6¾″		3″					8′- 9″	510,000
	300	7¼″		3″					8′-11″	595,000
17′ × 17′	80	6″		2½					8′- 6″	236,500
	150	6½″	6′-0″	2½	3′-9″	4′ 3″	13′-3″	6′-0″	8′-11″	422,500
	200	6¾″		3″					9′- 0″	520,000
	250	7¼″		3″					9′- 2″	623,500
	300	7¾″		3″			....	.....	9′- 4″	725,000
18′ × 18′	80	6″		2½″			...	.	8′- 9″	283,000
	150	6¾″	6′-3″	3″	4′-0″	4′-6″	14′-0″	6′-3″	9′- 6″	505,000
	200	7″		3″					9′- 7″	621,000
	250	7½″		3½″				... .	9′- 9″	742,000
	300	8″		3½″				.....	9′-11″	862,500
19′ × 19′	80	6″		2½″				. ...	9′- 3″	334,000
	150	7¼″	6′-9″	3″	4′-3″	4′-9″	14′-9″	6′ 9″	9′-11″	611,000
	200	7½″		3″					10′- 0″	746,000
	250	8″		3¾″					10′- 2″	889,000
	300	8½″		4″					10′- 4″	1,030,000
20′ × 20′	80	6″		3″			.....	.....	9′- 6″	394,000
	150	7½″	7′-0″	3½″	4′-6″		15′-6″	7′-0″	10′- 6″	718,000
	200	7¾″		3¾″					10′- 7″	875,000
	250	8½		3¾″					10′-10″	1,047,000
	300	9″		4″					11′- 0″	1,210,000
21′ × 21′	80	6″	.... .	3″					10′- 0″	456,000
	150	8″	7′-6″	3½″	4′-9″	5′-3″	16′-3″	7′-6″	11′- 2″	852,500
	200	8¼″	.....	3½″					11′- 3″	1,035,000
	250	8¾″	.....	3¾″					11′- 5″	1,223,000
	300	9½″	.....	4″					11′- 8″	1,430,000
	80	6¼″	.....	3″					10′- 7″	540,000

Note.—Above designs are made in accordance with pages 2A and 2C of Akme Design Standards. panels only. Moments for exterior spans to be increased 20 per cent perpendicular to spandrel. In- and supporting bars not included in weights of steel per square foot of floor. These average 0.15 lb.

* Larger bar in lower layer only.

TYPE, QUANTITIES—TYPICAL DESIGNS

Main slab mom. at center	Mid slab mom.	Width of mid and main belts	Weight of steel per square foot	Reinforcement main belt		Reinforcement mid belt
				Center	Add. bars at head	
205,700	97,000	6′-6″	1.97	14½″ × 17′-6″	2⅜″ × 9′-0″ *1½″ × 9′-0″	12⅜″ × 17′-6″
258,000	120,800	....	2.12	15½″ × 17′-6″	2⅜″ × 9′-0″ *1½″ × 9′-0″	13⅜″ × 17′-6″
306,000	143,000	....	2.38	11⅝″ × 17′-6″	2⅜″ × 9′-0″ *1⅝″ × 9′-0″	14⅜″ × 17′-6″
357,000	168,000	....	2.66	12⅝″ × 17′-6″	2½″ × 9′-0″ *1⅝″ × 9′-0″	9½″ × 17′-6″
142,000	66,500	....	1.32	9½″ × 17′-6″	2⅜″ × 9′-0″ *1½″ × 9′-0″	8⅜″ × 17′-6″
253,900	124,700	6′-9″	2.15	10⅝″ × 18′-6″	2½″ × 9′-9″ *1⅝″ × 9′-9″	8½″ × 18′-6″
312,000	152,500	....	2.50	12⅝″ × 18′-6″	2½″ × 9′-9″ *1⅝″ × 9′-9″	9½″ × 18′-6″
374,100	189,000	....	2.71	13⅝″ × 18′-6″	2½″ × 9′-9″ *1⅝″ × 9′-9″	10½″ × 18′-6″
435,000	214,000	....	2.85	14⅝″ × 18′-6″	2½″ × 9′-9″ *1⅝″ × 9′-9″	10½″ × 18′-6″
170,000	83,400	....	1.48	11½″ × 18′-6″	2⅜″ × 9′-9″ *1½″ × 9′-9″	10⅜″ × 18′-6″
303,000	145,000	7′-3″	2.14	11⅝″ × 19′-6″	2½″ × 10′-3″ *1⅝″ × 10′-3″	8½″ × 19′-6″
372,500	178,500	....	2.55	13⅝″ × 19′-6″	2½″ × 10′-3″ *1⅝″ × 10′-3″	10½″ × 19′-6″
445,000	213,400	....	2.90	15⅝″ × 19′-6″	2½″ × 10′-3″ *1⅝″ × 10′-3″	11½″ × 19′-6″
517,500	249,000	....	3.10	16⅝″ × 19′-6″	2½″ × 10′-3″ *1⅝″ × 10′-3″	12½″ × 19′-6″
200,000	96,500	....	1.66	13½″ × 19′-6″	2⅜″ × 10′-3″ *1½″ × 10′-3″	12⅜″ × 19′-6″
367,000	184,000	7′-6″	2.27	12⅝″ × 20′-6″	2½″ × 11′-0″ *1⅝″ × 11′-0″	10½″ × 20′-6″
447,000	223,500	....	2.73	15⅝″ × 20′-6″	2½″ × 11′-0″ *1⅝″ × 11′-0″	11½″ × 20′-6″
533,000	266,000	....	3.00	16⅝″ × 20′-6″	2½″ × 11′-0″ *1⅝″ × 11′-0″	13½″ × 20′-6″
618,000	311,000	....	3.24	12¾″ × 20′-6″	2½″ × 11′-0″ *1¾″ × 11′-0″	14½″ × 20′-6″
236,000	118,300	....	1.81	15½″ × 20′-6″	2⅜″ × 11′-0″ *1½″ × 11′-0″	14⅜″ × 20′-6″
430,000	211,000	8′-0″	2.47	14⅝″ × 21′-6″	2½″ × 11′-6″ *1⅝″ × 11′-6″	11½″ × 21′-6″
525,000	256,000	....	2.76	16⅝″ × 21′-6″	2½″ × 11′-6″ *1⅝″ × 11′-6″	12½″ × 21′-6″
627,000	308,000	....	3.06	12¾″ × 21′-6″	2½″ × 11′-6″ *1¾″ × 11′-6″	14½″ × 21′-6″
727,000	356,000	....	3.46	14¾″ × 21′-6″	2½″ × 11′-6″ *1¾″ × 11′-6″	15½″ × 21′-6″
274,000	134,000	....	2.03	12⅝″ × 21′-6″	2⅜″ × 11′-6″	16⅜″ × 21′-6″
511,000	244,000	8′-6″	2.55	15⅝″ × 22′-6″	4½″ × 12′-0″	12½″ × 22′-6″
620,000	296,000	....	3.14	13¾″ × 22′-6″	4½″ × 12′-0″ *1¾″ × 12′-0″	14½″ × 22′-6″
734,000	351,000	....	3.36	14¾″ × 22′-6″	4½″ × 12′-0″ *1¾″ × 12′-0″	15½″ × 22′-6″
859,000	429,000	....	3.58	15¾″ × 22′-6″	4½″ × 12′-0″ *1¾″ × 12′-0″	16½″ × 22′-6″
324,000	154,500	....	2.11	13⅝″ × 22′-6″	4⅜″ × 12′-0″	17⅜″ × 22′-6″

The panels for 80 lb. live load are for roofs where an average of 9 in. cinder fill is used. Use for interior crease length of bent bars one foot for exterior spans. All bars to be deformed rounds. Raising per sq. ft. of floor for spans shown on this sheet.

TABLE 18.—AKME SYSTEM DROP PANEL

Panel	Live load	Slab thickness	Size, plate	Thickness, plate	Head: Square	Head: Round	Clear span S	Comp. width at col.	Comp. width at center	Main slab mom. at head
22′ × 22′ o. to o. cols.	150	8¼″	7′-9″	3½″	5′-0″	5′-6″	17′-0″	7′-9″	11′- 6″	985,000
	200	8½″	.....	3½″	.....	.....	.....	.....	11′- 7″	1,194,000
	250	9¼″	.....	4″	.....	.....	.....	.....	11′-10″	1,428,000
	300	9¾″	.....	4½″	.....	.....	.....	.....	12′- 0″	1,645,000
	80	6¾″	.....	3″	.....	.....	.....	.....	11′- 0″	640,000
23′ × 23′	150	8½″	8′-0″	4″	5′-3″	6′-0″	17′-9″	8′-0″	12′- 1″	1,140,000
	200	9″	.....	4″	.....	.....	.....	.....	12′- 3″	1,395,000
	250	9¾″	.....	4″	.....	.....	.....	.....	12′- 6″	1,660,000
	300	10¼″	.....	4½″	.....	.....	.....	.....	12′- 8″	1,910,000
	80	7″	.....	3″	.....	.....	.....	.....	11′- 7″	745,000
24′ × 24′	150	9″	8′-6″	4″	5′-6″	6′-3″	18′-6″	8′-6″	12′- 6″	1,325,000
	200	9½″	.....	4″	.....	.....	.....	.....	12′- 8″	1,610,000
	250	10″	.....	4½″	.....	.....	.....	.....	12′-10″	1,885,000
	300	10¾″	.....	5″	.....	.....	.....	.....	13′- 1″	2,187,000
	80	7¼″	.....	3½″	.....	.....	.....	.....	11′-11″	860,000
25′ × 25′	150	9¼″	8′-9″	4½″	5′-9″	6′-6″	19′-3″	8′-9″	13′- 1″	1,515,000
	200	9¾″	.....	4½″	.....	.....	.....	.....	13′- 3″	1,835,000
	250	10½″	.....	5″	.....	.....	.....	.....	13′- 6″	2,175,000
	300	11¼″	.....	5″	.....	.....	.....	.....	13′- 9″	2,510,000
	80	7½″	.....	3½″	.....	.....	.....	.....	12′- 6″	992,000
26′ × 26′	150	9¾″	9′-0″	4½″	6′-0″	6′-9″	20′-0″	9′-0″	13′- 9″	1,740,000
	200	10¼″	.....	4½″	.....	.....	.....	.....	13′-11″	2,100,000
	250	11″	.....	5″	.....	.....	.....	.....	14′- 2″	2,480,000
	300	11¾″	.....	5½″	.....	.....	.....	.....	14′- 5″	2,860,000
	80	7¾″	.....	3½″	.....	.....	.....	.....	13′- 1″	1,133,000
27′ × 27′	150	10″	9′-6″	5″	6′-0″	6′-9″	21′-0″	9′-6″	14′- 1″	2,000,000
	200	10½″	.....	5″	.....	.....	.....	.....	14′- 3″	2,415,000
	250	11½″	.....	5″	.....	.....	.....	.....	14′- 7″	2,870,000
	300	12¼″	.....	5½″	.....	.....	.....	.....	14′-10″	3,310,000
	80	8¼″	.....	4″	.....	.....	.....	.....	13′- 7″	1,333,000

NOTE.—Above designs are made in accordance with pages 2A and 2C of Akme Design Standards. panels only. Moments for exterior spans to be increased 20 per cent perpendicular to spandrel. In- and supporting bars not included in weights of steel per square foot of floor. These average 0.12 lb.
* Larger bar in lower layer only.

75. Tables for Flat Slab Design.—The accompanying Tables 17 to 19 give slab thicknesses, column capital, and drop details and reinforcement for different sized panels and live loads for the different rules and codes.

Table 17 prepared by Mr. A. R. Lord is for the American Concrete Institute Specifications.

Type, Quantities—Typical Designs.—(*Continued*)

Main slab mom. at center	Mid slab mom.	Width of mid and main belts	Weight of steel per square foot	Reinforcement main belt		Reinforcement mid belt
				Center	Add. bars at head	
591,000	295,000	8′–9″	2.77	12¾″ × 23′–6″	4½″ × 12′–9″ *1¾″ × 12′–9″	13½″ × 23′–6″
716,000	356,000	....	3.26	14¾″ × 23′–6″	4½″ × 12′–9″ *1¾″ × 12′–9″	16½″ × 23′–6″
856,000	427,000	....	3.48	15¾″ × 23′–6″	4½″ × 12′–9″ *1¾″ × 12′–9″	17½″ × 23′–6″
987,000	492,000	....	3.70	16¾″ × 23′–6″	4½″ × 12′–9″ *1¾″ × 12′–9″	18½″ × 23′–6″
384,000	191,000	....	2 28	14⅝″ × 23′–6″	4½″ × 12′–9″	11½″ × 23′–6″
685,000	333,000	9′–3″	2.97	14¾″ × 24′–9″	4½″ × 13′–0″	14½″ × 24′–9″
837,000	407,000	....	3.28	15¾″ × 24′–9″	4½″ × 13′–0″ *1¾″ × 13′–0″	16½″ × 24′–9″
905,000	484,000	....	3.66	16¾″ × 24′–9″	4⅝″ × 13′–0″ *1¾″ × 13′–0″	12⅝″ × 24′–9″
1,145,000	555,000	....	4 04	18¾″ × 24′–9″	4⅝″ × 13′–0″ *1¾″ × 13′–0″	13⅝″ × 24′–9″
447,000	218,000	....	2.45	16⅝″ × 24′–9″	4½″ × 13′–0″	12½″ × 24′–9″
795,000	402,000	9′–6″	3.10	15¾″ × 25′–9″	4½″ × 14′–0″	16½″ × 25′–9″
964,000	486,000	....	3.51	17¾″ × 25′–9″	4½″ × 14′–0″ *1¾″ × 14′–0″	18½″ × 25′–9″
1,132,000	572,000	....	3.94	18¾″ × 25′–9″	4⅝″ × 14′–0″ *1¾″ × 14′–0″	14⅝″ × 25′–9″
1,312,000	664,000	....	4.20	20¾″ × 25′–9″	4⅝″ × 14′–0″	14⅝″ × 25′–9″
516,000	263,000	....	2.55	17⅝″ × 25′–9″	4½″ × 14′–0″	14½″ × 25′–9″
910,000	449,000	10′–0″	3.23	16¾″ × 26′–9″	4⅝″ × 14′–6″	11⅝″ × 26′–9″
1,101,000	543,000	....	3.70	18¾″ × 26′–9″	4⅝″ × 14′–6″ *1¾″ × 14′–6″	13⅝″ × 26′–9″
1,305,000	643,000	....	4.06	15⅞″ × 26′–9″	4⅝″ × 14′–6″	14⅝″ × 26′–9″
1,506,000	745,000	....	4.33	16⅞″ × 26′–9″	4⅝″ × 14′–6″	15⅝″ × 26′–9″
596,000	326,500	....	2.80	13¾″ × 26′–9″	4½″ × 14′–6″ 1¾″ × 14′–6″	17½″ × 26′–9″
1,044,000	507,000	10′–6″	3.34	17¾″ × 27′–9″	4⅝″ × 15′–0″ 1¾″ × 15′–0″	12⅝″ × 27′–9″
1,260,000	612,000	....	3.90	20¾″ × 27′–9″	4⅝″ × 15′–0″ 1¾″ × 15′–0″	14⅝″ × 27′–9″
1,488,000	723,000	....	4.20	16⅞″ × 27′–9″	4⅝″ × 15′–0″ 1⅞″ × 15′–0″	15⅝″ × 27′–9″
1,716,000	835,000	....	4.45	17⅞″ × 27′–9″	4⅝″ × 15′–0″ 1⅞″ × 15′–0″	16⅝″ × 27′–9″
680,000	332,000	....	2.77	15¾″ × 27′–9″	4½″ × 15′–0″ 1¾″ × 15′–0″	16½″ × 27′–9″
1,200,000	590,000	10′–9″	3.70	20¾″ × 28′–9″	4⅝″ × 15′–6″	14⅝″ × 28′–9″
1,449,000	712,000	....	4.23	17⅞″ × 28′–9″	4⅝″ × 15′–6″	16⅝″ × 28′–9″
1,722,000	845,000	....	4.50	18⅞″ × 28′–9″	4⅝″ × 15′–6″ 1⅞″ × 15′–6″	17⅝″ × 28′–9″
1,986,000	975,000	....	4.75	19⅞″ × 28′–9″	4⅝″ × 15′–6″ 1⅞″ × 15′–6″	18⅝″ × 28′–9″
800,000	395,000	....	3.00	16¾″ × 28′–9″	4½″ × 15′–6″ 1¾″ × 15′–6″	18½″ × 28′–9″

The panels for 80 lb. live load are for roofs, where an average of 9 in. cinder fill is used. Use for interior crease length of bent bars one foot for exterior spans. All bars to be deformed rounds. Raising per sq. ft. *of floor for spans shown on this sheet.*

Table 18 is for the Akme Design Standards for two-way flat slab construction prepared under the direction of Albert M. Wolf, 1917, then Prin. Ass't. Engineer in Charge of Design, Condron Company, Structural Engineers, Chicago.

Table 19 is for the Corr-plate floor design as developed by the Corrugated Bar Company.

TABLE 19

Live load, lb.	Size of panel	Slab	Column head	Cap	20 bands	30 bands	40 bands	Extra bars over column	Reinforcement per sq. ft., lb.	Concrete per sq. ft., cu. ft.
	fs = 18,000 lb.				Corr. plate floor panels				fc = 700 lb.	
40		5″			16-⅜φ@ 6″c/c	12-⅜φ@ 8″c/c	16-⅜φ@11½″c/c	12-⅜φ-8′-0″	1.61	0.463
150		6″	42″		16-½φ@7″c/c	12-⅜φ@7″c/c	20-⅜φ@9″c/c	12-½φ-8′-0″	2.19	0.544
200	16′-0″×16′-0″	6½″	mini-	6′-6″×6′-6″×2″	16-½φ@7″c/c	16-⅜φ@6″c/c	20-⅜φ@8″c/c	14-½φ-8′-0″	2.34	0.585
250		6½″	mum		16-½φ@6″c/c	12-½φ@8″c/c	16-½φ@11½″c/c	14-½φ-8′-0″	2.72	0.583
300		7″			16-⅝φ@8″c/c	12-½φ@8½″c/c	16-½φ@11″c/c	10-⅝φ-8′-0″	3.25	0.621
40		5½″	42″		16-⅜φ@6″c/c	16-⅜φ@8″c/c	16-⅜φ@8″c/c	14-⅜φ-9′-0″	1.55	0.508
150		6½″	48″		12-⅝φ@10″c/c	12-½φ@10″c/c	16-½φ@12″c/c	14-½φ-9′-0″	2.55	0.590
200	18′-0″×18′-0″	7″	mini-	7′-3″×7′-3″×2″	12-⅝φ@9″c/c	16-½φ@9″c/c	16-½φ@11″c/c	12-⅝φ-9′-0″	2.83	0.631
250		7½″	mum		12-⅝φ@8″c/c	16-½φ@8″c/c	20-½φ@10″c/c	12-⅝φ-9′-0″	3.01	0.672
300		8″			16-⅝φ@7″c/c	16-½φ@8″c/c	16-½φ@ 11½″c/c	14-⅝φ-9′-0″	3.18	0.711
40		6″	42″	8′-0″×8′-0″×2″	16-½φ@7″c/c	16-⅜φ@7″c/c	28-⅜φ@9″c/c	12-½φ-10′-0″	2.02	0.539
150		7″	54″	8′-0″×8′-0″×2″	12-⅝φ@8½″c/c	16-½φ@8½″c/c	24-½φ@11″c/c	12-⅝φ-10′-0″	2.88	0.638
200	20′-0″×20′-0″	7½″	mini-	8′-0″×8′-0″×2″	16-⅝φ@7″c/c	16-½φ@7½″c/c	24-½φ@11″c/c	14-⅝φ-10′-0″	3.19	0.677
250		8″	mum	8′-0″×8′-0″×2″	20-⅝φ@7″c/c	16-½φ@7½″c/c	24-½φ@9″c/c	14-⅝φ-10′-0″	3.45	0.715
300		8½″		8′-0″×8′-0″×3″	20-⅝φ@7″c/c	12-⅝φ@10″c/c	16-⅝φ@13″c/c	14-⅝φ-10′-6″	3.64	0.767
40		6½″	48″	8′-0″×8′-0″×2″	16-½φ@7″c/c	16-½φ@10″c/c	20-½φ@12¼″c/c	18-½φ-11′-0″	2.47	0.580
150		8″	60″	8′-0″×8′-0″×2″	16-⅝φ@8″c/c	16-½φ@8″c/c	28-½φ@10¼″c/c	16-⅝φ-11′-0″	3.09	0.720
200	22′-0″×22′-0″	8½″	mini-	8′-0″×8′-0″×2″	20-⅝φ@7″c/c	20-½φ@7″c/c	28-½φ@9″c/c	18-⅝φ-11′-0″	3.53	0.758
250		8½″	mum	8′-0″×8′-0″×2½″	20-⅝φ@6½″c/c	16-⅝φ@9½″c/c	20-⅝φ@12″c/c	18-⅝φ-11′-0″	3.88	0.761
300		9″		8′-0″×8′-0″×3″	24-⅝φ@6″c/c	16-⅝φ@9″c/c	20-⅝φ@11½″c/c	18-⅝φ-11′-0″	4.12	0.805
40		7″	48″	9′-6″×9′-6″×2″	20-⅝φ@9″c/c	16-½φ@9″c/c	20-½φ@12¼″c/c	14-⅝φ-12′-0″	2.73	0.623
150		8½″	63″	9′-6″×9′-6″×2″	20-⅝φ@7″c/c	16-⅝φ@10¼″c/c	20-⅝φ@13″c/c	22-⅝φ-12′-0″	3.64	0.766
200	24′-0″×24′-0″	9″	mini-	9′-6″×9′-6″×3″	20-¾φ@9″c/c	16-⅝φ@9″c/c	20-⅝φ@12¼″c/c	20-⅝φ-12′-0″	4.09	0.819
250		9½″	mum	9′-6″×9′-6″×3½″	20-¾φ@9″c/c	16-⅝φ@9″c/c	20-⅝φ@12¼″c/c	20-⅝φ-12′-0″	4.09	0.863
300		10″		9′-6″×9′-6″×4″	20-¾φ@8″c/c	16-⅝φ@8″c/c	28-⅝φ@11″c/c	20-⅝φ-12′-0″	4.52	0.908

NOTE.—These designs are for panels continuous on all four sides. They are designed to carry in addition to the live loads, a finish load of 15 lb. per sq. ft. Reinforcement per square foot includes spacers and chairs. Concrete per square foot includes caps and heads.

These tables are given here as a handy check on designs and for estimating purposes. They are only prepared for square interior panels and should be used accordingly.

76. Akme System—Typical Calculations.[1]—A set of typical calculations for the Akme System of two-way girderless floor construction is given herewith together with a plan showing the arrangement of bars (Fig. 53).

Typical Panel Computations

Head = 20 × 0.225 = 4 ft. 6 in. round is equivalent to 4 ft. 0 in. square
Drop plate 20 × 0.35 = 7 ft. 0 in. square

Use 8 in. slab
Live load = 200 lb.
Dead load = 100 lb.
300 lb.

Live load = 200 lb. per sq. ft. f_s = 18,000 lb. per sq. in.
f_c = 650 lb. per sq. in. mid span.
f_c = 750 lb. per sq. in. adj. col. heads.

S = clear span = 20 ft. − 4 ft. = 16 ft. 0 in.

Area panel = 400 sq. ft.

$$W = \frac{300(400 - 16)}{2} = 57{,}600 \text{ lb.}$$

$M = (57{,}600)(16)(^{12}/_{12}) = 921{,}600$ in.-lb. $\qquad G = 20 - (0.4L) = 12$ ft.

$M_c = (57{,}600)(16)(^{12}/_{20}) = 552{,}960$ in.-lb.

$m = m_c = (^{80}/_{8})(12)^3(^{12}/_{12}) = 259{,}200$ in.-lb.

Comp. width over col. 84 = in.
Comp. width of ctr. $0.4L + 4t$ = 128 in.

d (center) = 8 in. − ($\frac{3}{4}$ + $\frac{5}{8}$) = $6\frac{5}{8}$ in.

Assume thickness of drop $3\frac{3}{4}$ in.
d for high bars = $11\frac{3}{4}$ in. − $1\frac{1}{8}$ in. = $10\frac{5}{8}$ in.
d for low bars = $10\frac{5}{8}$ in. − $\frac{3}{4}$ in. = $9\frac{7}{8}$ in.

$$K \text{ center} = \frac{552{,}960}{(128)(6\frac{5}{8})^2} = 91.5 \qquad p = 0.57$$

$A_s = (128)(6\frac{5}{8})(0.0057) = 5.01$ sq. in.
12—$\frac{3}{4}$ in. ϕ = 5.30 sq. in.

$$K \text{ high bars} = \frac{921{,}600}{(84)(10\frac{5}{8})^2} = 97 \qquad p = 0.61$$

$A_s = (84)(10\frac{5}{8})(0.0061) = 5.42$ sq. in.
12—$\frac{3}{4}$ in. + 2—$\frac{1}{2}$ in. ϕ = 5.68 sq. in.

$$K \text{ low bars} = \frac{921{,}600}{(84)(9\frac{7}{8})^2} = 112.5 \qquad p = 0.71$$

$A_s = (84)(9\frac{7}{8})(-.0071) = 5.90$ sq. in.
12—$\frac{3}{4}$ in. + 3—$\frac{1}{2}$ in. = 5.88 sq. in.

Mid slab bars: d = 8 in. − 1 in. = 7 in.

$$A_s = \frac{259{,}200}{(18{,}000)(0.9)(7)} = 2.28 \text{ sq. in.}$$

12—$\frac{1}{2}$ in. ϕ = 2.35 sq. in.

Typical Exterior Panel

20 × 19-ft. panel, average span = 19 ft. 6 in.

S = 19 ft. 6 in. − (2 ft. + 2 ft. 3 in.) = 15 ft. 3 in.
G = 19.5 ft. − 8 ft. = 11.5 ft.
Area panel = 380 sq. ft.

$$W = \frac{300(380 - 16)}{2} = 54{,}600 \text{ lb.}$$

$M = (54{,}600)(15.25)(^{12}/_{12}) = 832{,}650$ in.-lb.
$M_c = M \times 0.6 = 499{,}600$ in.-lb.
$m = m_c = (^{80}/_{8})(11.5)^3(^{12}/_{12}) = 228{,}000$ in.-lb.

[1] Prepared under the direction of Albert M. Wolf, 1917, then Prin. Ass't. Engineer in Charge of Design, Condron Company, Structural Engineers, Chicago.

Bending Details

Type of bar round deformed

Mark	No. Req.	Size	Length	A	B	C	D
M_{101}		3/4"	21'6"	10'4"	5 3/4"	10"	10'4"
M_{102}		3/4"	21'6"	10'4"	6 1/2"	10"	10'4"
M_{103}		3/4"	22'6"	10'4"	6 1/2"	10"	11'4"
M_{104}		3/4"	16'9"	5'8"	6 1/2"	10"	10'3"
M_{105}		3/4"	25'6"	10'3"	1'1 1/2"	1'9"	13'6"
M_{106}		5/8"	16'9"	5'8"	5 3/4"	10"	10'3"
P_{101}		1/2"	21'6"	10'4"	6 1/2"	10"	10'4"
P_{102}		1/2"	21'6"	10'4"	6"	10"	10'4"
P_{104}		1/2"	16'9"	5'8"	6"	10"	10'3"

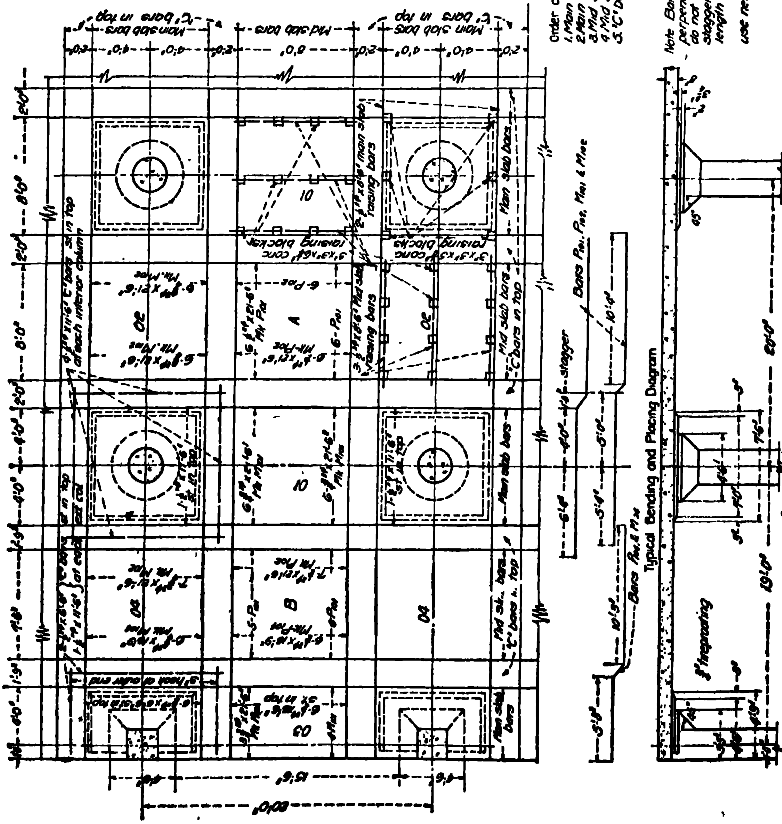


FIG. 53.

For strip 04 $\begin{cases} M_c = (499{,}600)(1.2) = 599{,}500 \text{ in.-lb.} \\ M = (832{,}650)(1.2) = 999{,}200 \text{ in.-lb.} \end{cases}$

$K \text{ center} = \dfrac{599{,}500}{(128)(6\frac{7}{8})^2} = 99 \quad p = 0.62$

$A_s = (128)(6\frac{7}{8} \text{ in.})(0.0062) = 5.45$ sq. in.

13—$\frac{3}{4}$ in.ϕ = 5.74 sq. in.

$K \text{ col.} = \dfrac{999{,}200}{(84)(10\frac{5}{8})^2} = 105$

$p = 0.665$

$A_s = (84)(10\frac{5}{8})(0.00665) = 5.95$ sq. in

13—$\frac{3}{4}$ in. + 2—$\frac{1}{2}$ in.ϕ = 6.12 sq. in

Mid slab parallel to spandrel:

$$A_s = \frac{22{,}800}{(18{,}000)(0.9)(7)} = 2.01 \text{ sq. in.}$$

11—$\frac{1}{2}$ in.ϕ = 2.16 sq. in.

Mid slab perpendicular to spandrel:

$$A_s = \frac{(228{,}000)(1.2)}{(18{,}000)(0.9)(7)} = 2.42 \text{ sq. in.}$$

13—$\frac{1}{2}$ in.ϕ = 2.55 sq. in.

For computations on spandrel strip 03 see Art. **80***a*.

77. Four-Way Slab Design.—The following examples of four-way reinforced flat slab construction by W. Stuart Tait are based upon the American Concrete Institute Ruling.

77*a*. American Concrete Institute Ruling.—The diagram, Fig. 54, together with the following notes, is a summary of this proposed ruling. It is inserted so that designers may easily follow the examples worked out later. The general notation is given at the end of *Appendix F*.

Slab Thickness.—t shall not be less than $0.02L\sqrt{w} + 1$ in., nor less than $\frac{L}{32}$ for floors and $\frac{L}{40}$ for roofs.

Design Moments.—Numerical sum of positive and negative moments shall not be less than $0.09\, wl_1(l_2 - qc)^2$. The report allows a slight variation in the distribution of this total moment. A reasonble division of this moment in percentage is shown in Fig. 54. Note that a slightly different distribution applies in the case of drop construction from that in cap construction. Corresponding moments shall be figured at right angles to those shown in Fig. 54. The moments shown in Fig. 54 are calculated for a value of the cap diameter $c = 0.225L$, and are for interior panels.

For exterior panels the negative moment at the first row of interior columns and the positive moments at the center of the exterior panels on sections parallel to the wall shall be increased 20 per cent over those specified for interior panels. The negative moment at the exterior column parallel to the wall shall not be less than 50 per cent of that for the interior panel.

Shear.—The shearing stress which is used as a measure of diagonal tension stress is calculated on a width equal to $\frac{L}{2}$, and the formula used in this calculation is $v = \dfrac{0.25W}{bjd}$ for cap construction, and $v = \dfrac{0.30W}{bjd}$ for drop construction. Punching shear at the edge of the drop and at the column cap is calculated by

multiplying the total panel load occurring outside the area under consideration by 1.25 and dividing this load by the perimeter of the cap (or drop as the case may be) and by d.

Columns.—Both interior and exterior columns shall be designed for bending. The moment in a column shall not be less than $0.022\ w_1l_1\ (l_2 - qc)^2$ where w_1 is the designed live load. In the case of exterior columns, the total dead and live load (w) should be used in the above formula instead of w_1. For top story

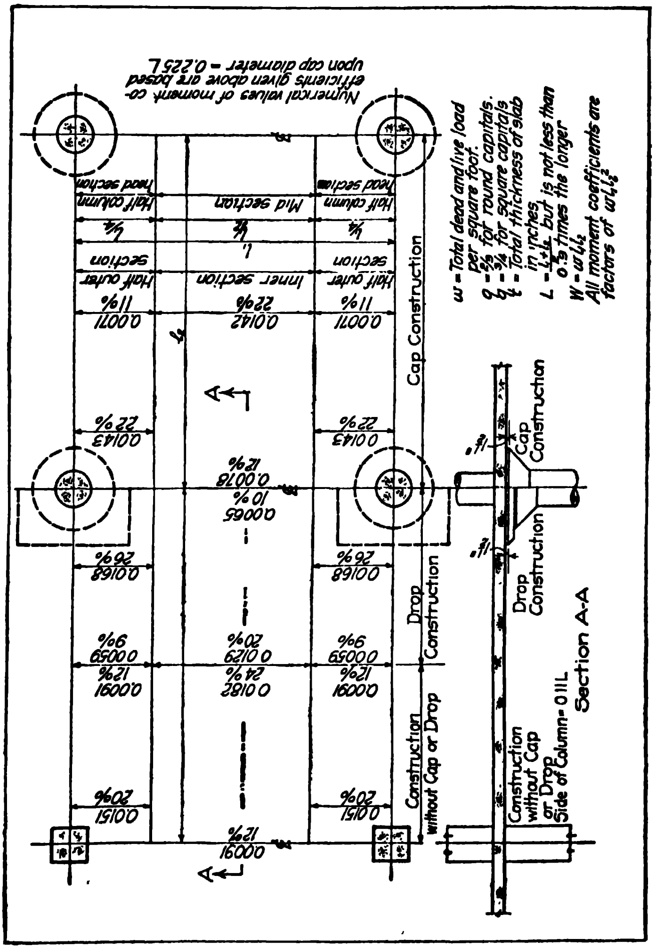


FIG. 54.

columns, this amount is all applied at one section of the column. For columns continuous through the story above, the moment is to be divided between the upper and lower column in proportion to their stiffness. Stress used in calculations for direct load and bending may exceed the direct load stresses allowed by 50 per cent.

Stresses.—In the examples worked out, the stresses recommended by the Joint Committee based on 1:2:4 gravel concrete (see *Appendix F*) are used as

follows: f_c for positive moment = 650 lb. per sq. in., f_c for negative moment = 750 lb. per sq. in., f_s = 16,000 lb. per sq. in., shear as a measure of diagonal tension = 40 lb. per sq. in. on plain concrete. Punching shear = 100 lb. per sq. in.

77b. Example of Design—Drop Construction, Four-Way Arrangement.—Take a panel 20 ft. square for a live load of 300 lb. per sq. ft., with cement finish laid with the slab.

Live load = 300 lb. $t = 0.02L\sqrt{w} + 1 = (0.02)(20)(\sqrt{415}) + 1 = 9.15$ in.

Dead load = 115 lb. t not less than $\frac{L}{32} = 7.5$ in.

w = 415 lb.

Use 9¼-in. slab. d for outer section = 9.25 − 1.25 = 8.00 in. (one layer of steel)

Fireproofing 1 in.

d for inner section = 9.25 − 1.50 = 7.75 in. (two layers of steel)

Column capital = $0.225L$ = 4 ft. 6 in.

M − column head section = $0.0336wl_1 \times l_2^2 = (0.0336)(415)(20)(20)^2(12)$(in.-lb.)

$M = Kbd^2$. $b = 0.3L$ = 6 ft. 0 in. $K = 134$

$$d^2 = \frac{(0.336)(415)(20)(20)^2(12)}{(134)(6)(12)} = 139 \text{ in., or } d = 11.8 \text{ in.}$$

= 11.8 + 1.00 + 1.00 (4 layers steel) = 13.8 in. Use 14 in. Slab = 9¼ in. Drop = 14 − 9¼ = 4¾ × 6 ft. 0 in. × 6 ft. 0 in. Note with 14-in. thickness, d at column becomes 12 in. (see later increase).

$$\text{Shear at column} = \frac{0.3W}{bjd} = \frac{(0.3)(415)(20)(20)}{(120)(0.86)(12)} = 40 \text{ lb.}$$

$$\text{Punching shear at edge of drop} = \frac{(415)(400 - 36)(1.25)}{(4)(72)(7.25)} = 90 \text{ lb.}$$

$$\text{Punching shear at edge of capital} = \frac{(415)(400 - 16)(1.25)}{(\pi)(54)(12)} = 99 \text{ lb.}$$

From this, it is noted that both punching shear and diagonal tension stress are within the limits prescribed.

M − column head section (see above) = 1,340,000 in.-lb.

$$A_s = \frac{1{,}340{,}000}{(0.86)(16{,}000)(12)} = 8.12 \text{ sq. in.}$$

M − mid. section = $0.0065wl_1 \times l_2^2$

$= (0.0065)\ (415)\ (20)\ (20)^2\ (12) = 258{,}000$ in.-lb.

$$A_s = \frac{258{,}000}{(0.86)(16{,}000)(8.00)} = 2.34 \text{ sq. in.} = 12\text{—}\tfrac{1}{2}\text{-in. round bars.}$$

M − at outer section = $0.0118wl_1 \times l_2^2$

$= (0.0118)\ (415)\ (20)\ (20)^2(12) = 470{,}000$ in.-lb.

$$A_s = \frac{470{,}000}{(0.86)(16{,}000)(8.00)} = 4.27 \text{ sq. in.} = 22\text{—}\tfrac{1}{2}\text{-in. round bars.}$$

d^2 required $= \frac{M}{Kb} = \frac{470{,}000}{(108)(120)} = 36.2$ in. $d = 6$ in., where we have 8.00 in.

M − at inner section = $0.0129\ wl_1 \times l_2^2$

In this design we are using the four-way arrangement of steel, and consequently each bar in each diagonal band cuts the inner section line at 45 deg. The A. C. I. ruling specifies that the sectional area of bars, crossing any section at an angle multiplied by the sine of the angle between these bars and the section may be considered as effective. Now we have two diagonal bands of rods, so the effective area of steel to resist the moment at the inner section = 0.7 × 2 bands of rods = 1.4 bands. Therefore

$$A_s \text{ each diagonal band} = \frac{514{,}000}{(0.86)(16{,}000)(8.0)(1.4)} = 3.45 = 18\text{—}\tfrac{1}{2}\text{-in. round bars.}$$

We, therefore, have the following reinforcing for the interior panels:

Direct bands............22—½-in. rounds = 4.30 sq. in.
Diagonal bands..........18—½-in. rounds = 3.53 sq. in.
Across direct bands.......12—½-in. rounds = 2.35 sq. in.

If general practice is followed, and we bend up all bars at the column, we have 4.3 + (1.4) (3.53) = 9.24 sq. in. effective, and we found above that 8.12 sq. in. were required at the column head section.

Exterior Panel.—In case the exterior panel is the same size as the interior, for which the design above is shown, the moment at the first interior column would be increased by 20 per cent and becomes 1,340,000 × 1.2 = 1,610,000 in.-lb. To resist this increased moment the depth of the drop or the width must be increased. For the sake of uniformity, it is good practice to make all drops the same size and to let all other interior drops be governed by the size of the first interior. If b is kept 6 ft. 0 in.,

$$d^2 = \frac{1{,}610{,}000}{(134)(72)} = 167 \quad d = 12.92 \text{ in., say 13 in.}$$

The drop then becomes 13 + 2 − 9¼ = 5¾ in. This increase in the interior column drop thickness would permit less steel to be used at the column section, but it is better practice to allow the number of rods given above to remain, as short bars should be avoided.

$$\text{Now } A_s \text{ at first int. column} = \frac{1{,}610{,}000}{(0.86)(16{,}000)(13)} = 9.0 \text{ sq. in.}$$

A_s direct band normal to wall becomes = (1.2) (4.3) = 5.1 sq. in. = 26—½-in. round bars.

A_s diagonal band in the exterior panels = (1.2) (3.45) = 4.14 = 21—½-in. round rods.

If we therefore bend up all bars to top of slab at the first interior column from the exterior span, we have 5.1 + (1.4) (4.1) = 10.8 sq. in. which is satisfactory. Since the moment is the same on each side of the column the extra bars in the exterior panel must continue past the first interior column to the quarter point of the next span. This is shown in diagram, Fig. 55.

The A. C. I. ruling specifies a bending moment at the column head section parallel to the wall at the exterior column of 50 per cent of the interior column head section moment, *i.e.*, $0.0168wl_1l_2^2$ = 670,000 in.-lb. The area of the steel which must be provided to resist the moment across this section = $\frac{670{,}000}{(0.86)(16{,}000)(12)}$ = 4.06 sq. in. We have available, 5.1 + (1.4) (4.1) = 10.8 sq. in. which is more than is required. It is good practice to allow about one-half the

bars in the exterior direct band to pass through in the bottom of the slab, and the other half to be bent up to top of slab. The moment which this steel is resisting occurs at the edge of the column capital and the distance from this point to the end of the bars is usually ample to develop 16,000 lb. in the steel in bond. It is, however, good practice to bend the ends of some of the bars down into the column or beam.

Now the moment in a direction normal to this is one-half the moment of an interior column head section, since there exists but one-half a section along the

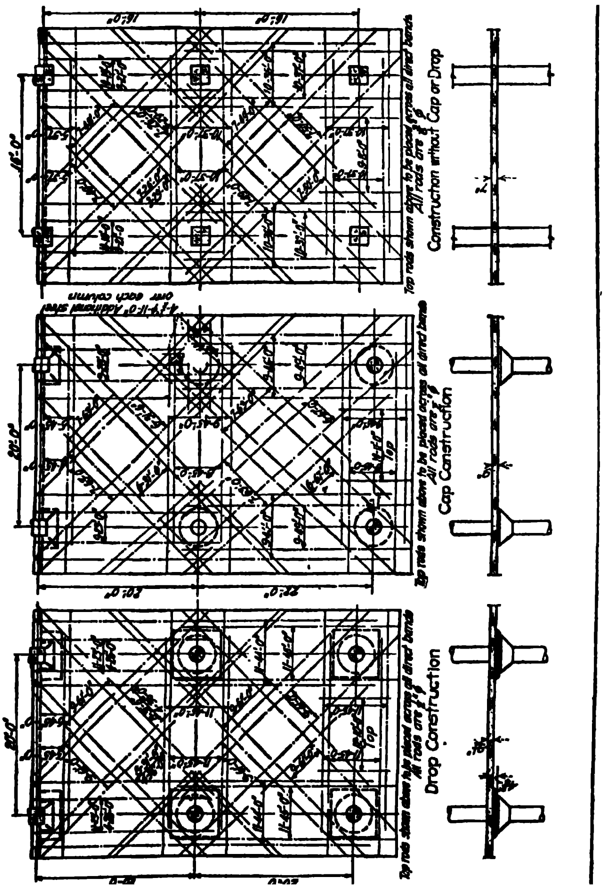


Fig. 55.

wall. Therefore, the cap and drop construction will be similar to that used at an interior column. The steel required at this section = 4.06 sq. in. We have available 2.15 + (0.7)(4.1) = 5.02 sq. in. So as to provide sufficient imbedment to develop the bars in the exterior diagonal bands, it is generally advisable to continue the ends of the bars along the wall a short distance. The steel arrangement for this design is shown in Fig. 55. Note that one-half of the bars in each band only are broken at each column. This is a recommendation of the A. C. I.

Column Moments.—The ruling specifies a bending moment of $0.022w_1l_1(l_2 - qc)^2$ for interior columns. In this case $M = 0.0158w_1l^3 = (0.0158)(300)(20)^3(12) = 455{,}000$ in.-lb. Top-story interior columns should be designed for this moment combined with the direct load. The lower-story columns, if of equal size above and below the floor considered, should be designed for half of this moment. If of different sizes, the moment should be divided directly as the stiffness of the columns, *i.e.*, in proportion to the value $\frac{I}{h}$ for each column, where I is the moment of inertia and h the height of the column. Similarly the exterior column moment

$$M = 0.0158wl^3 = (0.0158)(415)(20)^3(12) = 620{,}000 \text{ in.-lb.}$$

must be provided for. Note particularly that the A. C. I. ruling allows an extreme fiber stress combining direct load and bending 50 per cent greater than the direct stress allowed for columns. While no ruling or ordinance is distinct on this point, it is the writer's opinion that in designing columns for direct load and bending, the entire concrete section may be considered. His reason for this is the fact that we are not required to deduct any portion of the concrete in a beam in designing for negative bending and the lower side of a beam at the supports is just as liable to damage from fire as is the column it rests upon.

77c. Example of Design—Cap Construction, Four-Way Arrangement.—In the previous example the design was accompanied by many explanations but in this case these will be eliminated, as they would simply be repetition. Take a panel 20 × 22 ft. for a live load of 100 lb. per sq. ft. with a maple floor finish laid on sleepers with a cinder concrete fill between.

Live load = 100 $\qquad t = 0.02L\sqrt{w} + 1 = (0.02)(21)(\sqrt{232}) + 1 = 7.4$ in.

Floor finish = 20 $\qquad t$ not less than $\frac{L}{32} = 7.85$ in.

Dead load = 112 $\qquad$ Column cap = (0.225)(21) = 4 ft. 9 in.

$w = 232$ lb.

M – column head section $= 0.0286wl_1 \times (l_2)^2 = (0.0286)(232)(20)(22)^2(12)$ (in.-lb.)

$$d^2 = \frac{(0.0286)(232)(20)(22)^2(12)}{(134)(0.5)(20)(12)} = 47.5 \quad d = 6.90 \text{ in.}$$

t required = 6.9 + 1 + 1 (4 layers of steel). Use 9-in. slab.

d at column head section = 9 – 2 = 7 in.

d at mid-section and outer section = 9 – 1.25 = 7.75 in.

d at inner section = 9 – 1.5 = 7.5 in.

$$\text{Max. shear at column } \frac{0.25w}{bjd} = \frac{(0.25)(232)(20)(22)}{(120)(0.86)(7)} \text{ } 35 \text{ lb. per sq. in.}$$

$$\text{Punching shear at column} = \frac{(232)(440 - 17.7)(1.25)}{(\pi)(57)(7)} = 98 \text{ lb. per sq. in.}$$

From this we find that 9-in. slab satisfies the shear requirements.

$$A_s \text{ at column head section across span } l_2 = \frac{(0.0286)(232)(20)(22)^2(12)}{(0.86)(16{,}000)(7)} = 7.95 \text{ sq. in.}$$

$$A_s \text{ at column head section across span } l_1 = \frac{(0.0286)(232)(22)(20)^2(12)}{(0.86)(16{,}000)(7)} = 7.22 \text{ sq. in.}$$

$$A_s \text{ at outer section across span } l_2 = \frac{(0.0142)(232)(20)(22)^2(12)}{(0.86)(16{,}000)(7.75)} = 3.6 \text{ sq. in.} =$$

18—½-in. round rods.

A_s at outer section across span $l_1 = \dfrac{(0.0142)(232)(22)(20)^2(12)}{(0.86)(16,000)(7.75)} = 3.27$ sq. in. = 17—½-in. round rods.

A_s at inner section both directions $= \dfrac{(0.0142)(232)(21)(21)^2(12)}{(0.86)(16,000)(7.5)} = 3.53$ sq. in.

A_s each diagonal band $= \dfrac{3.53}{1.4} = 2.52 =$ 13—½-in. round rods.

If all bars are bent up to top of slab at column, the steel we have available across span $l_2 = 3.6 + 3.53 = 7.13$ sq. in. The steel required = 7.95 sq. in. We must therefore provide $7.95 - 7.13 = 0.82$ sq. in. or 4—½-in. round bars extra. The steel available across $l_1 = 3.27 + 3.53 = 6.8$ sq. in., and we require 7.22 sq. in. We must therefore provide in this direction 0.42 sq. in. For the sake of uniformity we will add 4—½-in. round bars in each direction and these bars will be made 11 ft. 0 in. in length.

The exterior panels and the bending moments in the column will be found and treated in a manner similar to the case where drop construction was used. It must be borne in mind, however, that the bending moment at the first row of interior columns will have to be increased across the section parallel to the wall. Since we must maintain the same thickness of slab, namely, 9 in., it will be necessary to introduce compression steel in this direction provided the exterior span is the same as the interior. It is convenient where the layout permits, to slightly reduce the exterior span so that the moment at the first interior column head section is the same as the others. This has been done in Fig. 55 which is a plan of this design.

77*d*. Example of Design Where Neither Drop Nor Cap Are Used.—It will have been noted that a smaller percentage of the total bending moment was used at the column head section in the case of cap construction than in the case of drop construction. This is on account of the fact that drop construction is slightly stiffer than cap construction at the supports. Now if in addition we eliminate the capital we have still a smaller amount of stiffness at the column section. Accordingly, a slightly smaller percentage of the total moment may be used at the column head section. A satisfactory distribution of moments for the four-way arrangement is shown in Fig. 54. Square columns are generally used in this design, as partitions fit up to them better than other shapes. The writer has found that a square column having a size of $0.11L$ usually proves economical and satisfactory. The bending moment coefficients shown in Fig. 54 for this class of design are based on this value. Designers will find it economical to maintain the same size of columns for a number of stories in this design. In most cases of designs of this class, the writer has maintained one size of column throughout the structure, simply varying the mix and steel for the increased loads. Take a panel 16 ft. square for a live load of 50 lb. per sq. ft., a partition load of 25 lb., plaster ceiling, and cement finish 1½ in. thick.

Live load	= 50	$\dfrac{L}{32} = 6$ in.
Partitions	= 25	$0.02L\sqrt{w} + 1 = 5.35$ in.
Plaster ceiling	= 8	
Cement finish	= 18	
Dead load (7-in. slab)	= 88	
	189 lb.	

Minimum column size = (0.11) (16) = 20 in. square. We will use 24 in. square as it will be found later that this size is necessary on account of punching shear.
Fireproofing below steel = 1 in.
Fireproofing above steel = ½ in. (Note cement finish above the structural slab).

d at column head section	= 7–½–¾	= 5.75 in.
d at outer section	= 7–1–3/16	= 5.80 in.
d at mid-section	= 7–½–3/16	6.30 in.
d at inner section	= 7–1–⅜	5.62 in.

M—column head section = $0.0302wl_1 \times l_2^2$

$$= (0.0302)(189)(16)(16)^2(12) = 278{,}000 \text{ in.-lb.}$$

Had this design been for cap construction, the diameter of the cap would have been about $0.225L$, or 3.6 ft., and b used in the resisting moment at the column head section would have been $\frac{L}{2}$, or 8 ft. In this case we have a column 1.8 ft. in width and we should therefore use a width of beam = 8.00 − (3.6 − 1.8) = 6.2 ft., or 74 in. Another good rule is to limit b to the width of the column plus $8t$. In this case we would have 24 + 56 = 80 in. In this case we will use the smaller value, namely, 74 in. At the column head section, then $d^2 = \frac{278{,}000}{(134)(74)} = 28.0$ in. and d = 5.28 in., so the 7-in. slab assumed above is satisfactory.

$$\text{Shear at column} = \frac{0.25w}{bjd} = \frac{(0.25)(189)(16)(16)}{(74)(0.86)(5.75)} = 33 \text{ lb. per sq. in.}$$

$$\text{Punching shear} = \frac{(189)(256 - 3)(1.25)}{(4)(24)(5.75)} = 108 \text{ lb. per sq. in.}$$ (slightly in excess of the allowable)

The 7-in. slab is satisfactory for shear.

$$A_s \text{ at column head section} = \frac{278{,}000}{(0.86)(16{,}000)(5.75)} = 3.52 \text{ sq. in.}$$

$$A_s \text{ at mid-section} = \frac{(0.091)(189)(16)^3(12)}{(0.86)(16{,}000)(6.3)} = 0.98 \text{ sq. in.} = 9\text{—⅜-in. round rods.}$$

$$A_s \text{ at outer section} = \frac{(0.182)(189)(16)^3(12)}{(0.86)(16{,}000)(5.8)}\ 2.12 \text{ sq. in.} = 20\text{—⅜-in. round rods.}$$

$$A_s \text{ at inner section} = \frac{(0.182)(189)(16)^3(12)}{(0.86)(16{,}000)(5.62)} = 2.18 \text{ sq. in.}$$

$$A_s \text{ each diagonal band} = \frac{2.18}{1.4} = 1.56 \text{ sq. in.} = 14\text{—⅜-in. round rods.}$$

We have available at the column head section, 2.12 + 2.18 = 4.30 sq. in., where we require 3.52 sq. in. provided all the bars are raised to the top of the slab at the column. It would be good policy, however, to run the excess steel, *i.e.* 4.30 − 3.52 = 0.78 sq. in. or 7—⅜-in. round bars in the direct band, through in the bottom of the slab and raise the remaining 13 bars at the column.

The exterior panel may be designed in a manner similar to that given under drop construction. In this case, as in the case of cap construction, it is desirable where the layout permits to decrease the size of the exterior panel. Unless this can be done it will usually be necessary to determine the slab thickness by using the bending moment for the column head section which applies to the first interior columns. Where relatively thin slabs are used, as in this case, compressive

reinforcement to provide the increased resisting moment necessary at the first row of interior columns is very inefficient. In the above case, the moment at the first tier of interior columns is such that a depth of 5.8 in. is required. We have practically this depth available due to the fact that punching shear governed the slab thickness. In cases where the increased moment warrants the addition of compression steel or increased slab thickness at the first interior tier, increasing the slab will usually be found to be the most economical. Figure 55 is a plan of this design.

Other arrangements of steel, such as the two-way or combinations of two and four-way, should be treated in the same manner as the above. Some of the systems of reinforcing prefer slightly different distributions of the total bending moments from those shown in Fig. 54. The distribution shown will, however, give satisfactory results.

78. Floors with Unequal Adjoining Spans.[1]—In flat slab construction, as in any form of design where we have continuity over a number of unequal spans,

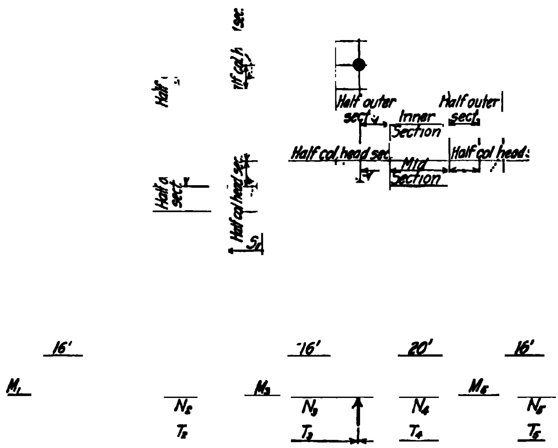


FIG. 56.

the correct bending moments must be obtained by applying the Theorem of Three Moments. In flat slab construction since the moments used are empirical we cannot apply the theorem directly but must increase or decrease the bending moment coefficients used for equal spans by applying certain factors to these moments. The following is a method of applying the Theorem of Three Moments and obtaining the factors referred to for the case shown in Fig. 56.

In this layout we have a series of panels 20 ft. in length and varying from 16 to 25 ft. in width. While the arrangement is somewhat irregular, it will be noted that the length of the panels is in no case greater than 1.33 times the breadth. We will assume that drop construction is to be used and that the column caps are $0.225L$ in diameter. The bending moment coefficient for uniform spans, then, will be as shown in Fig. 54. As explained previously

$$M_1 = \tfrac{1}{2} M \text{ for interior column head section}$$
$$= 0.0168\, wl_1 \times l_2^2 = (0.0168)\,(w)\,(X)\,(T_1)^2$$
$$\text{or per foot of width} = 0.016wT_1^2 \qquad (1)$$

[1] By W. Stuart Tait.

Now applying Clapeyron's theorem, we have

$$M_1T_1 + 2M_2(T_1 + T_2) + M_2T_2 = \frac{-w(T_1^3 + T_2^3)}{4} \tag{2}$$

$$M_2T_2 + 2M_3(T_2 + T_3) + M_3T_3 = \frac{-w(T_2^3 + T_3^3)}{4} \tag{3}$$

$$M_3T_3 + 2M_4(T_3 + T_4) + M_4T_4 = \frac{-w(T_3^3 + T_4^3)}{4} \tag{4}$$

$$M_4T_4 + 2M_5(T_4 + T_5) + M_5T_5 = \frac{-w(T_4^3 + T_5^3)}{4} \tag{5}$$

Now substituting in these equations for T_1, T_2, etc., we have values per foot width in the direction of span X as follows:

Equation (1) becomes $M_1 = -4.3w$

(2) becomes $82M_2 + 25M_3 = -4861.9w$

(3) becomes $25M_2 + 82M_3 + 16M_4 = -4930.2w$

(4) becomes $16M_3 + 72M_4 + 20M_5 = -3024w$

(5) becomes $20M_4 + 88M_5 = -3024w$

Solving these simultaneous equations:

$M = -4.3w$ $\quad$ $M_4 = -24.9w$

$M_2 = -46.8w$ $\quad$ $M_5 = -28.6w = M_6$

$M_3 = -41w$

Now find the positive moments N_1, N_2, etc., as follows:

$$N_1 = \frac{wl^2}{8} - \tfrac{1}{2}(M_1 + M_2) \qquad N_3 = -0.95w$$

$N_1 = 6.47w$ $\quad$ $N_4 = 23.25w$

$N_2 = 34.23w$ $\quad$ $N_5 = 3.40w$

The quantities M_1, M_2, etc., and N_1, N_2, etc., are the bending moments per foot of width at their respective points shown in the diagram and are for one-way construction. Now, if we obtain a value for N_1 similar to the above but for a series of spans equal to T_1, and also a value for N_2 for a series of spans T_2, etc., and designate these values by Q_1, Q_2, etc., we can by dividing N_1 by Q_1, N_2 by Q_2, etc., obtain factors C_2, C_4, etc., which are the coefficients giving the influence of the adjoining unequal spans upon the bending moments for equal spans. By solving equations (1) to (5) for equal spans and writing Q_1, Q_2, etc., for N_1, N_2, etc., we find

$$Q_1 = 0.066wT_1^2 = 0.066w \times 16^2 = 16.9w$$
$$Q_2 = 0.035wT_2^2 = 0.035w \times 25^2 = 21.8w$$
$$Q_3 = 0.043wT_3^2 = 0.043w \times 16^2 = 11.0w$$
$$Q_4 = 0.041wT_4^2 = 0.041w \times 20^2 = 16.4w$$
$$Q_5 = 0.042wT_5^2 = 0.042w \times 16^2 = 10.8w$$

$$\text{Now } C_2 = \frac{N_1}{Q_1} = \frac{6.47w}{16.9w} = 0.384$$

$$C_4 = \frac{N_2}{Q_2} = \frac{34.23w}{21.8w} = 1.57$$

$$C_6 = \frac{N_3}{Q_3} = \frac{-0.95w}{11.0w} = -0.09$$

$$C_8 = \frac{N_4}{Q_4} = \frac{23.25}{16.4} = 1.42$$

$$C_{10} = \frac{N_5}{Q_5} = \frac{3.4w}{10.8w} = 0.315$$

By means of these coefficients we may determine the correct moments across the spans T_1, T_2, etc., for the inner and outer sections. Take the outer section in span T_1. By referring to Fig. 54 we find that on this section

$M = 0.0118wl_1l_2^2$, which in this case

$= 0.0118wXT_1^2 \times 1.2$ for an exterior panel with equal spans adjoining

$= 0.0118wXT_1^2 \times C_2 \times 1.2$ for unequal panels

$= (0.0118)\ (w)\ (20)\ (16)^2(0.384)\ (1.2)$ in this case (ft.-lb.)

Similarly across T_2 we find in this case

$M = (0.0118)\ (w)\ (20)\ (25)^2(1.57)$ (ft.-lb.)

and across T_3 we find

$M = (0.0118)\ (w)\ (20)\ (16)^2(-0.09)$ (ft.-lb.)

Note particularly that the coefficient C_6 is negative and that in consequence we have a negative moment at the inner and outer sections across T_3.

The Theorem of Three Moments assumes knife-edge supports at the columns. While this is not strictly correct the assumption will give slightly higher moments in the slab on the side of the column adjoining the short span than actually occur. The bending moment occurring in the column will be taken up later.

We previously found numerical values for M_1, M_2, etc. for the negative moments, considering one-way construction for the arrangement of spans shown. By solving equations (1) to (5) we may also obtain numerical value for these moments for a series of equal spans. For these negative moments we will write P_1, P_2, etc. Then by dividing M_1 by P_1, M_2 by P_2, etc., we will obtain coefficients, C_1, C_3, etc., which are measures of the influence of the unequal spans upon the negative moments.

In obtaining the numerical values of M_2 and P_2, it is immaterial whether we use the span length of T_1 or T_2, provided in our calculations for the moments occurring in the construction, we use the same value for l_2 in the equation $M = 0.0336wl_1l_2^2$.

The best method is to use in all calculations a span equal to the mean of the spans adjoining the column at which the negative moment is being calculated.

The moment M_1 is not affected by the unequal span arrangement and in consequence C_1 is unity.

Solving equations (1) to (5) for a series of spans of $\frac{T_1 + T_2}{2}$ for P_1, $\frac{T_2 + T_3}{2}$ for P_2, etc., we have

$$P_1 = -0.0168wT_1^2 = (-0.0168)(w)(16)^2 = -4.3w$$

$$P_2 = -0.101w\left(\frac{T_1 + T_2}{2}\right)^2 = (-0.101)(w)(20.5)^2 = -42.5w$$

$$P_3 = -0.079w\left(\frac{T_2 + T_3}{2}\right)^2 = (-0.079)(w)(20.5)^2 = -33.3w$$

$$P_4 = -0.085w\left(\frac{T_3 + T_4}{2}\right)^2 = (-0.085)(w)(18)^2 = -27.5w$$

$$P_5 = -0.083w\left(\frac{T_4 + T_5}{2}\right)^2 = -(0.083(w)(18)^2 = -26.9w$$

$$C_1 = \frac{M_1}{P_1} = \frac{-4.3w}{-4.3w} = 1 \qquad C_5 = 1.23$$

$$C_3 = \frac{M_2}{P_2} = \frac{-46.8w}{-42.5w} = 1.1 \qquad C_7 = 0.9$$

$$C_9 = 1.06$$

By means of these coefficients we may determine the correct moments at the column head and mid-sections across the series of spans T_1 T_2, etc. Take the column head section between spans T_1 and T_2. By referring to Fig. 54 we find that on this section

$M = 0.0336wl_1l_2^2$ which in this case

$= (0.0336)(w)(X)\left(\frac{T_1 + T_2}{2}\right)^2 \times 1.2$ for an exterior panel with spans $\left(\frac{T_1 + T_2}{2}\right)$ adjoining

$= (0.0336)(w)(X)\left(\frac{T_2 + T_2}{2}\right)^2 \times 1.2 \times C_3$ for unequal spans

$= (0.0336)(w)(20)(20.5)^2(1.2)(1.1)$ in this case (ft.-lb.)

Similarly at column head section between span T_2 and T_3, we have in this case

$$M = (0.0336)\,(w)\,(20)\,(20.5)^2(1.23) \qquad \text{(ft.-lb.)}$$

Proceeding as above, the moments occurring in the slab at all sections across the span T_1, T_2, T_3, etc., may be determined. The moments at right angles to these will be entirely unaffected by the inequality of the spans T_1, T_2, etc. and may be obtained in the usual manner. The design may then be treated in the usual way. For the sake of uniformity in the construction, the maximum slab and drop thickness should be determined for the worst cases of bending moment and panel size, and these thicknesses allowed to govern in all cases. The dimensions of the drops will be laid out from the column center lines in each direction and the projection from these center lines made the same proportion of the span in which each part of the drop occurs.

In this analysis it will probably be found that the moment at the inner section across the spans T_1, T_2, etc., is not the same as that found across the span X. In two-way construction, then, the steel in these two sections will vary. In four-way construction the steel used in each diagonal band in a rectangular panel should be the same. The designer will, therefore, take the mean of the two bending moments obtained across the inner sections in calculating the steel required in each diagonal band.

By following the methods of examples given above, all of the moments across the spans T_1, T_2. etc., can be found without doubt arising in the designer's mind. For the sake of entire clearness, a few examples of the method of obtaining the moments across the spans X will be given. Take the moment S_1 as indicated in Fig. 56. This moment is made up of the moments in two half outer sections, in one case the panel width being 26 ft. and in the other 16 ft.—the span in both cases being 20 ft. Assuming the same column capital proportion shown in Fig. 54, we have for drop construction M for half outer section $= 0.059wl_1l_2^2$. In this case

$$\begin{aligned} S_1 &= (0.059)(w)(T_2)(X)^2 + (0.059)(w)(T_3)(X)^2 \\ &= 0.059wX^2(T_2 + T_3) \\ &= 0.118wX^2\left(\frac{T_2 + T_3}{2}\right) \end{aligned}$$

Similarly $S_2 = 0.118wX^2\left(\frac{T_3 + T_4}{2}\right)$, $S_3 = 0.129wT_2X^2$, and $S_4 = 0.129wT_3X^2$.

The negative moments at the column head and mid-sections may be found in the same manner.

The analysis given above assumes knife-edge supports as stated previously. This means that if we have uniform loading throughout the structure, there will be no bending in the columns. This is not strictly true, but the departure of the moments obtained under this method of analysis from the precise moments is not sufficient to warrant the application of the extremely laborious calculation necessary if the method of slopes and deflections were to be applied. The bending occurring in any interior column, then, will be that due to any entire panel being unloaded while the adjacent panel is loaded fully. For equal adjoining spans the A. C. I. recommends the use of the formula

$$M = 0.022w_1l_1(l_2 - qc)^2$$

where w_1 is the live load per square foot. Where unequal adjoining panels occur, the dead load moments at the column do not cancel each other. In this case, therefore, the moment in the column between spans T_1 and T_2 would become

$$M = 0.022wX\ (T_2 - qc)^2 - 0.022DX(T'_1 - qc)^2t$$

where D is the dead load per square foot of the structure.

The moment in the exterior column will be found in the usual way.

79. Exterior Panels.—The design of interior panels in flat slab construction is usually a relatively simple matter using any given formula or code, but in the

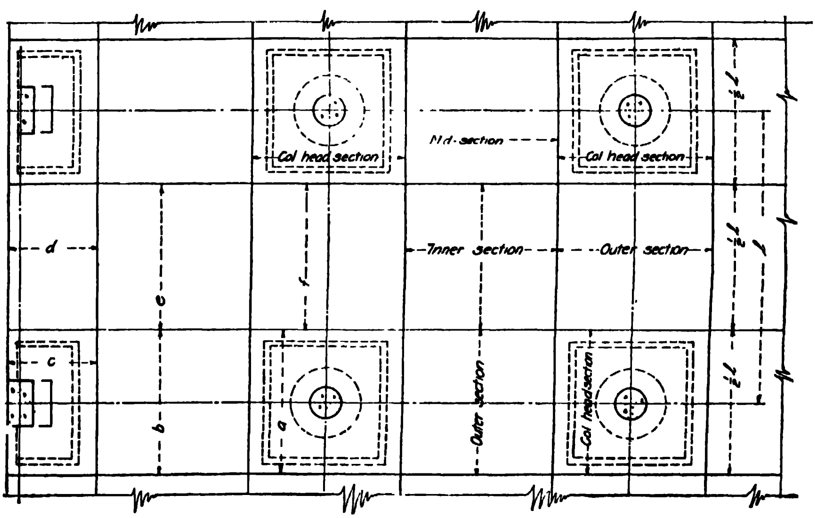


FIG. 57.—Diagram showing different moment sections in flat slab floors.

design of exterior panels with different conditions of spandrel and column capital construction the judgment of the designer is a very important factor. For this reason a rather detailed study of the various code requirements is given herewith.

The Joint Committee requirement for wall and other irregular panels is as follows:

In wall panels and other panels in which the slab is discontinuous at the edge of the panel, the maximum negative moment one panel length away from the discontinuous edge at *a* and *f* (Fig. 57) and the maximum positive moment between *b* and *e* shall be taken as follows:

(*a*) Column strip perpendicular to the wall or discontinuous edge, 15 per cent greater than that given for interior panels.

(*b*) Middle strip perpendicular to wall or discontinuous edge, 30 per cent greater than that given for interior panels.

In these strips the bars used for positive moments perpendicular to the discontinuous edge shall extend to the exterior edge of the panel at which the slab is discontinuous.

The Concrete Institute specifies for exterior panels the following:

The negative moments at the first exterior row of columns (*a* and *f*) and the positive moments at the center of the exterior panel on sections parallel to the wall (*b* and *e*), shall be increased 20 per cent over those specified above for interior panels. If girders are not provided along the column line, the reinforcement parallel to the wall for negative moment in the column-head section and for positive moment in the outer section adjacent to the wall, shall be altered in accordance with the change in the value of *c* (column-head diameter). The negative moment on sections at the wall and parallel thereto should be determined by the conditions of restraint, but must never be taken less than 50 per cent of those for interior panels.

The revised Chicago ruling required that:

(Sec. 27), where wall panels with standard drops and capitals are carried by columns and girders built in walls, as in skeleton construction, the same coefficients shall be used as for an interior panel, except as follows: The positive bending moments on strips *A* and *B* midway between wall and first line of columns shall be increased 25 per cent (at *b* and *e*, Fig. 57); and (Sec. 28), where wall panels are carried on new brick walls these shall be laid in Portland cement mortar, and shall be stiffened with pilasters as follows: If a 16-in. wall is used, it shall have a 4-in. pilaster; if a 12-in. wall is used, it shall have an 8-in. pilaster. The length of pilasters shall be not less than the diameter of the column, nor less than ⅛ of the distance between pilasters. The pilasters shall be located opposite the columns as nearly as practicable and shall be corbeled out 4 in. at the top, starting at the level of the base of the column capital. Not less than 8-in. bearing shall be provided for the slab, the full length of the wall.

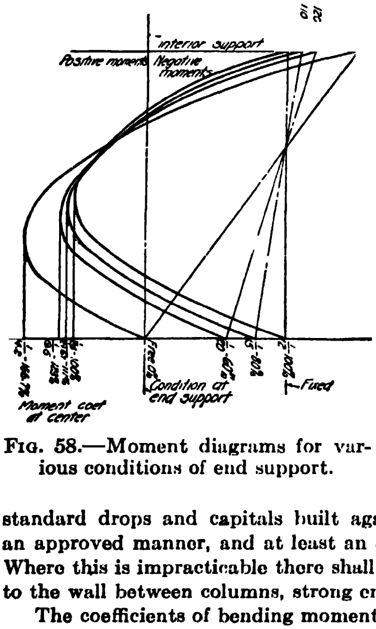


FIG. 58.—Moment diagrams for various conditions of end support.

The coefficients of bending moments required for these panels shall be the same as those for the interior panels except as provided herewith: The positive bending moments on strips *A* and *B* midway between the wall and the first line of columns shall be increased 50 per cent.

(Sec. 29), Where wall panels are supported on old brick walls there shall be columns with standard drops and capitals built against the wall, which shall be tied to the same in an approved manner, and at least an 8-in. bearing provided for the slab, the full length. Where this is impracticable there shall be built a beam on the underside of slab, adjacent to the wall between columns, strong enough to carry 25 per cent of the panel load.

The coefficients of bending moments for the two cases of slab support herein described shall be the same as those specified in Sec. 27 and Sec. 28 for skeleton and wall bearing condition respectively.

The Akme Standards of the Condron Company specify that for exterior panels without cantilever overhang, where wall columns with flaring heads or brackets are used, and for other spans not continuous over both supports, the positive and negative bending moment coefficients in strips perpendicular to wall or opening shall be increased 20 per cent (at *b*, *e*, *a* and *f*).

The increase in moments in exterior panels is of course dependent upon the stiffness and strength of the exterior columns and account must be taken of this fact in fixing the percentage of increase in moments. Referring to Fig. 58 it will be noted that when the slab is considered as fixed at the exterior column or put in another way, when the column is designed to take the entire moment to be developed by the slab, the theoretical negative moment is $\frac{WS}{12}$ and the positive $\frac{WS}{24}$. On the other hand, if the slab is simply supported by the exterior column, the moment at that point will be zero; near the center of span the positive moment will be $\frac{WS}{12}$ and at the first interior row of columns the moment will be $\frac{WS}{8}$. Thus it will be seen that for other conditions of support at exterior columns varying from free support to fixity, the moments must vary between the limits mentioned, and the diagram therefore is of special value to designers in the determination of moments for varying conditions of end support.

If 80 per cent of the slab moment over the interior column head is assumed to be taken by the exterior columns, then the moment at the exterior column becomes $\frac{WS}{15}$, while that at the first interior column becomes $\frac{WS}{10.9}$, and that at or near center of span between columns $\frac{WS}{21.67}$ or an increase of 10 per cent in negative moment over first interior column, and about 11 per cent in the positive moment at or near the center of span between columns.

On the other hand, if the exterior column be designed to take 60 per cent of the bending moment developed over an interior column head, the moment at the exterior column becomes $\frac{WS}{20}$, that over first interior column $\frac{WS}{10}$, and that at middle of span between columns $\frac{WS}{19.5}$, an increase of 20 and 23 per cent respectively for the last mentioned moments as compared with interior panels.

The Akme Standards require that the exterior columns shall resist 80 per cent of the moment developed over the interior column and that the positive moments at the center and the negative moments over the first interior column be increased 20 per cent. This requirement provides for nearly twice the theoretical increase in moments for the assumed condition of end support, but the excess is justified when it is considered that a slight yielding of the exterior column or failure of same to take the assumed moment will materially increase the moments at the points in question. This recommendation in the opinion of the writer is the most logical one to use in design of ordinary exterior panels, but the judgment of the designer must be used for special cases.

80. Spandrel Design.—On the matter of spandrel beam design, a considerable variation in methods exists as confirmed by the following quotations from different rulings:

The Chicago ruling requires that:

The spandrel beams or girders shall in addition to their own weight and the weight of the spandrel wall, be assumed to carry 20 per cent of the wall panel load uniformly distributed upon them.

As previously stated, the Concrete Institute requires that:

If girders are not provided along the column line, the reinforcement parallel to the wall for negative moment in the column-head section and for positive moment in the outer section adjacent to wall shall be altered in accordance with the changed value of *c*.

The Joint Committee Tentative Specifications of 1921 recommend that:

In panels having a marginal beam on one edge or on each of two adjacent edges, the beam shall be designed to carry the load superimposed directly upon it. If the beam has a depth greater than the thickness of the dropped panel into which it frames, the beam shall be designed to carry, in addition to the load superimposed upon it, at least ¼ of the distributed load for which the adjacent panel or panels are designed, and each column strip adjacent to and parallel with the beam shall be designed to resist a moment at least ½ as great as that specified for a column strip. If the beam used has a depth less than the thickness of the dropped panel into which it frames, each column strip adjacent to and parallel with the beam shall be designed to resist the moments specified in Table 17 for a column strip (see *Appendix F*). Where there are beams on two opposite edges of the panel, the slab and the beam shall be designed as though all the load was carried to the beam.

The negative moments on sections at and parallel to the wall, or discontinuous edge of an interior panel, shall be determined by the conditions of restraint.

When exposed rectangular columns are used on the exterior of a building as is now the case with many industrial buildings, the outer strip of reinforcement between exterior columns does not always provide sufficient strength to carry the brick load of the spandrel wall in addition to the floor load, and it then becomes necessary to provide a beam directly under the brick spandrel, either raised above or dropped below the bottom of the slab, or to deepen the slab a small amount for the width of the drop plate between columns and the drop plate dropped below this a sufficient distance to give the required resisting moment at the column-head section. If this method of strengthening the spandrel to carry the load coming upon it is used, the combined action of the spandrel and rest of the slab is more nearly like that of a slab of the same thickness throughout than if a deep beam raised above or dropped below the floor line is used, and this former method of strengthening spandrel strips is recommended and will doubtless become standard, rather than the use of narrow beams. In the latter case the slab adjacent to the beam is not as effective in carrying load owing to the relatively greater stiffness of the beam which must naturally relieve the adjacent slab of its stress, since the beam resists the tendency of the adjacent slab strip to deflect owing to its relatively greater stiffness.

Just how the combination of the deep beam and the adjacent flat slab strip may act is difficult to say, but computations made to determine the amount of load carried by spandrel beams based on the deflection of slab and the deflection of beams will give an idea of the amount. The line in the slab where the deflection due to the load it carries and that of the beam due to the loads acting on it are equal, marks the limit of the load tributary to the spandrel beam. Computations for deflections with various depths of spandrel beam show that on this basis the amount of floor load carried by spandrel beam varies from 0 to a maximum of 10 per cent for a beam four times as deep as the slab, which is unusual (a maximum of three times the slab thickness being the usual one). It would therefore seem logical to design the beam to carry the weight of spandrel

plus a portion of the tributary floor load, say ¼ of that tributary to the spandrel strip (6¼ per cent of panel load), and the flat slab portion sufficiently strong to carry the entire floor load tributary thereto. This duplication of loading will insure sufficient strength in both portions either acting together or independently as would be the case if the beam and slab were separated by cracks caused by difference in respective deflections under load. In placing the reinforcement in the slab portion of the spandrel strip the bars nearest the deep portion should be kept at a distance of, say ½ the depth of the beam away from the inside face. This will insure that the bars are stressed more uniformly and that all carry more nearly their proper portion of the stress.

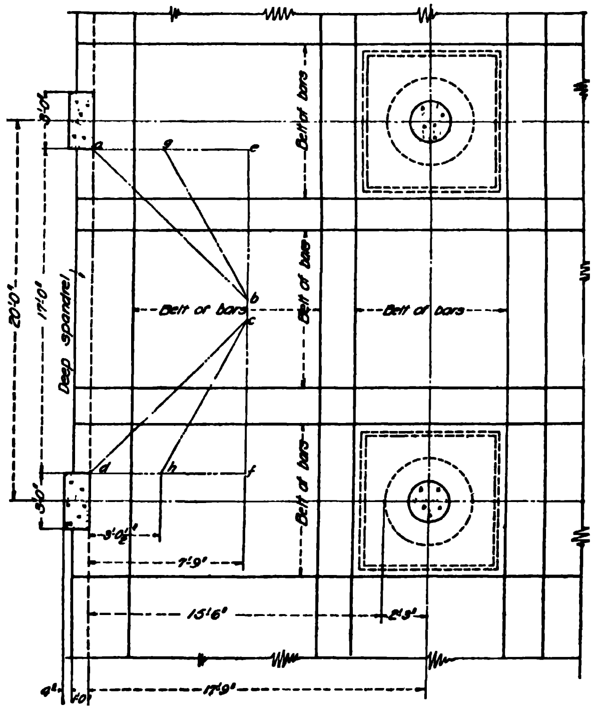


Fig. 59.—Distribution of load for deep beam spandrel.

If the exterior columns have relatively narrow capitals, or simply brackets, as compared with the interior columns, the condition is changed, and the spandrel beam or girder should be designed to carry a greater portion of the panel load; the spandrel strip of the slab, and those perpendicular thereto, should be reinforced so that the load will be transferred to the beam instead of the spandrel slab strip. This may mean that considerably more reinforcement may be required in the strips perpendicular to the spandrel, owing to the increased span.

The worst condition of loading on a spandrel girder would occur when the exterior columns have no capitals or brackets, the outer edge of the slab being carried entirely by the spandrel girder, as is the case with the floor shown in Fig.

59. This approximates the condition of a floor slab supported by brick-bearing walls, although the use of a concrete spandrel girder on concrete columns tends to give greater fixity to the slab at the outer edge.

In designing such panels it seems logical to consider the inside edge of the spandrel girder as corresponding to the edge of the column capital; or stated otherwise, to consider the span of slab as the distance from the center line of the first interior row of columns to a line ½ the width of the interior column capital, beyond the inside face of the girder (L in Fig. 59). In such design the spandrel girder takes the place of the spandrel belt of reinforcement, and other belts, both main and mid, are placed and designed the same as if interior column with capitals were used and the spandrel girder omitted.

By using this assumed span (L) or S, clear span, in Akme Standards with the moment coefficients increased as for ordinary exterior panels, entirely safe and satisfactory results will be obtained. With the belts of reinforcing arranged, as shown in Fig. 59, the amount of floor load producing moment on the spandrel girder will be somewhat greater than indicated by the trapezoid *abcd*, owing to the fact that the main bands are wider than the exterior columns, and hence transfer some load to the spandrel girder which carries it back to the column. Then also, the girder will tend to carry more load than the spandrel strip it replaces owing to its greater stiffness. If one-way reinforcement were used in the exterior panel the spandrel girder would carry the load on the area *aefd*, half the area between spandrel girder and the line of outside edge of interior column heads. Now if two-way reinforcement is used as in Fig. 59, the load carried by the spandrel girder must be less than the rectangle *aefd*, and somewhat greater than the trapezoid *abcd* for the reasons given; the average of these two cases represented by the polygon *agbchd* would seem to represent more than the actual loaded area to be carried by the spandrel girder and hence give safe results.

Now the total area of the panel (Fig. 59) 20 × 17 ft. 9 in. is 355 sq. ft.; the area of the rectangle *aefd* is 132 sq. ft.; the area of the trapezoid *abcd* is 72 sq. ft.; and the area of the polygon *agbchd* is 102 sq. ft. The latter then represents 29 per cent of the panel area, which is the load to be carried by the spandrel girder, which in the case cited is the most severe one to be met with in flat slab construction—that is, no exterior column capitals are used to support the slab. If uniform one-way reinforcement were used in this outside panel the percentage of load carried by the spandrel girder would be 37 per cent.

From the foregoing it will be noted that the load carried by the spandrel strip and the spandrel girder, if one is used, depends entirely upon the shape and size of the column capital and the depth of the spandrel girders, and no rule involving center to center of column dimensions that will meet all conditions is possible of formulation.

The following typical calculations for four types of spandrels in flat slab construction are based on the methods employed in the Akme design standards—that is, on the clear span between column capitals or brackets, or columns in Case 4—which seems the only logical general formula which will fit all cases of support.[1] The unit stresses used are f_s = 18,000 lb. per sq. in., and f_c = 750 lb. per sq. in. at column-head sections and 650 at mid spans. The computations are for:

[1] Effective depths same as for typical panel calculations just given.

Case 1.—Ordinary flat slab spandrel strip of same depth as remainder of floor (Fig. 53).

Case 2.—A spandrel formed by deepening slightly the spandrel strip of the floor (Fig. 60).

Case 3.—A spandrel involving the combination of a narrow beam and the adjacent flat slab strip (Fig. 61).

Case 4.—A deep spandrel girder supported by columns without capitals (Fig. 62).

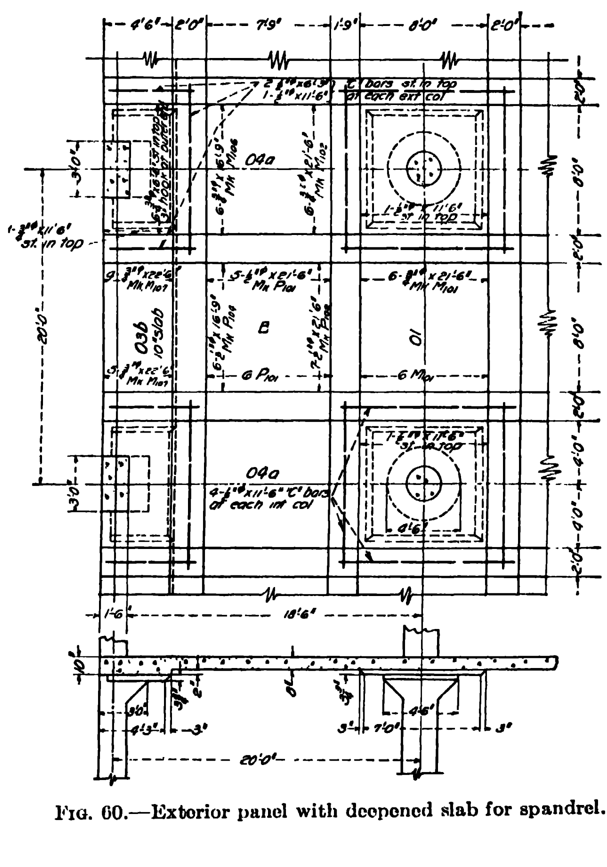


FIG. 60.—Exterior panel with deepened slab for spandrel.

80a. Flat Slab Spandrel (Fig. 53).

Brick 3 ft. high, 9 in. thick at 90 lb. = 270 lb. with sash, say 300 lb. $S = 20 - 4.5 = 15.5$ ft.

Brick load = (15.5)(300) = 4,650 lb.

$$\text{Tributary area} = (15.5)(2.25) + \left[\left(\frac{15.5 + 1}{2}\right)(7.25)\right]$$

$$= 35 + 59 = 94 \text{ sq. ft.}$$

Floor load = (94)(300) = 28,200 lb.

Total 32,850 lb.

$M = (32{,}850)(15.5)(\frac{12}{12}) = 509{,}200$ in.-lb.

$M_c = (509{,}200)(\frac{12}{20}) = 306{,}000$ in.-lb.

Compression width at head $= 3.5 + 7 = 49$ in.
Compression width at center $= (1 + 0.2)(18 + 2t) = 74$ in.

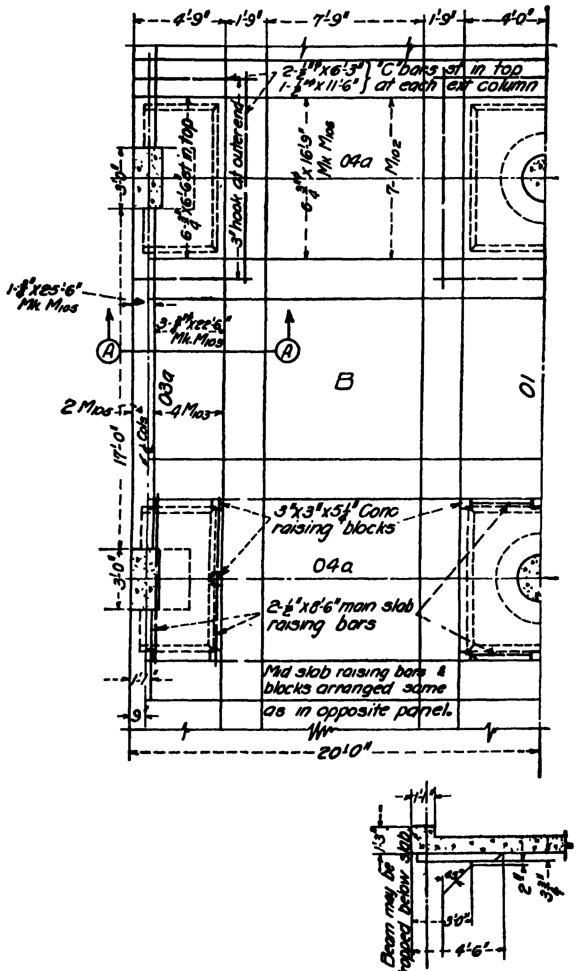


FIG. 61.—Exterior panel with combined raised beam and slab spandrel.

K center $= \frac{\quad}{(74)(6\frac{7}{8})^2}$ 88

K at column $= \frac{509{,}200}{(49)(9\frac{7}{8})^2} = 107$

$p = 0.55$

$A_s = (74)(6\frac{7}{8})(0.005) = 2.8$ sq in.

7—¾ in.ϕ = 3.07 sq. in

$p = 0.675$

$A_s = (49)(9\frac{7}{8})(0.00675) = 3.26$ sq. in.

7—¾ in.ϕ + 1—½ in.ϕ = 3.26 sq. in.

80b. Spandrel with Deepened Slab and Brackets on Exterior Columns (Fig. 60).

$S = 20 - 3 = 7$ ft. 0 in.

Warehouse with brick spandrel walls 8 ft. high.

Brick load 960 lb. per ft. Slab 125 lb. per ft.

Sash 15 lb. per ft. – Total 1,100 lb. per ft.

Spandrel load = (1,100)(17) = 18,700 lb.

$$\text{Tributary floor load}\begin{cases}(17)(3.5) = 59.5 \text{ sq. ft. at } 325 \text{ lb.} = 19{,}350 \text{ lb.}\\ \left(\dfrac{14+1.5}{2}\right)(6) = 46.5 \text{ sq. ft. at } 300 \text{ lb.} = 13{,}950 \text{ lb.}\end{cases}$$

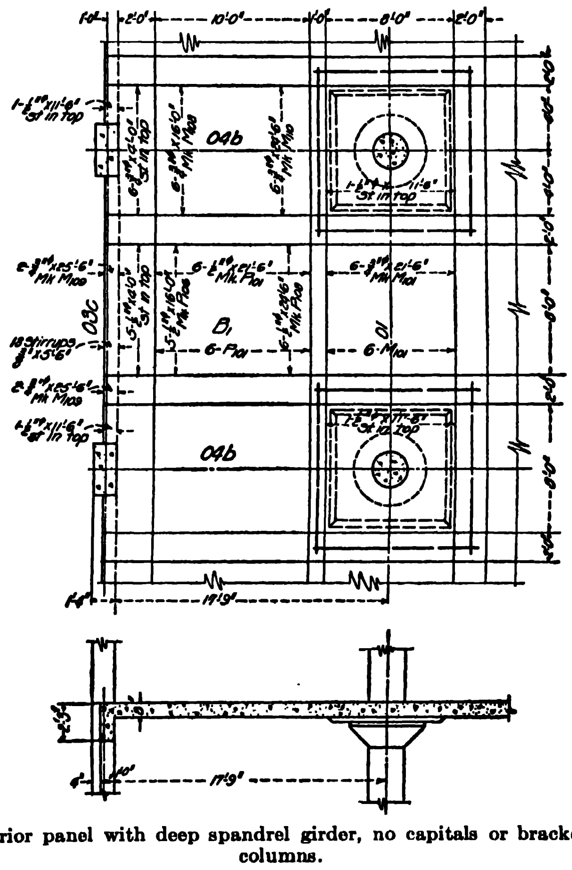


FIG. 62.—Exterior panel with deep spandrel girder, no capitals or brackets on exterior columns.

$M = (52{,}000)(17)\left(\frac{12}{12}\right) = 884{,}000$ in.-lb.

$M_c = (M)(0.6) = 530{,}400$ in.-lb.

Comp. width at center $= [(2)(20) + 2t + 9] = 48 + 16 + 9 = 73$ in.

Comp. width at column = 4.5 ft. = 54 in.

d at center $= 10 - \left(\frac{3}{4} + \frac{3}{8}\right) = 8\frac{7}{8}$ in.

d at column (low bars) $13 - \frac{1}{4} - \left(\frac{3}{4} + \frac{3}{8} + \frac{1}{4}\right) = 11\frac{3}{8}$ in.

K at center $= \dfrac{530{,}400}{(73)(8\frac{7}{8})^2} = 93$ (0.58 per cent)

K at column $= \dfrac{884{,}000}{(54)(11\frac{7}{8})^2} = 116$ (0.73 per cent)

A_s at center $= (73)(8\frac{7}{8})(0.0058) = 3.76$ sq. in. 9—$\frac{3}{4}$ sq. in.$^\phi$ = 3.98 sq. in.

A_s at column $= (54)(11\frac{7}{8})(0.0073) = 4.67$ sq. in. 10—$\frac{3}{4}$ sq. in. and 2—$\frac{1}{2}$ sq. in.$^\phi$ = $4.42 + 0.39 = 4.81$ sq. in.

80c. Spandrel with Beam (Fig. 61). Design with Brackets on Exterior Columns.

$S = 20 - 3 = 17$ ft. 0 in.

Spandrel wall and beam 13 in. wide. Total height = 4 ft. 0 in.

Weight per foot $(120)(4) = 480 + 20$ for sash = 500 lb.

Spandrel load $= (500)(17) = 8{,}500$ lb.

Tributary floor area $= (17)(20) + \left[\left(\dfrac{(17)(15)}{2}\right)(8)\right] = 108$ sq. ft.

Spandrel beam designed to carry weight spandrel + $\frac{1}{4}$ tributary floor area = 8,500 + [(27)(300)] = 166,000 lb.

Beam 1 ft. 1 in. × 1 ft. 3 in. deep.

Beam:

Bending moment $= (16{,}600)(17)(\frac{12}{12}) = 282{,}200$ in.-lb.

$K = \dfrac{282{,}200}{(13)(13\frac{1}{2})^2} = 119$

$p = 0.75$

$A_s = (13)(13\frac{1}{2})(0.0075) = 1.32$ sq. in.

3—$\frac{3}{4}$ sq. in. = 1.32 sq. in.

Shear $= \dfrac{8{,}300}{(13)(13\frac{1}{2})(87)} = 54$ lb. per sq. in.

Use bent bars—no stirrups.

Slab Portion:

Load $= (108)(300) = 32{,}400$ lb.

$M = (32{,}400)(17)(\frac{12}{12}) = 550{,}800$ in.-lb.

$M_c = (M)(0.6) = 330{,}500$ in.-lb.

Compression width at center $(0.2)(19.25) + 2t - 4 = 4$ ft. 10 in.

Compression width at column 4.5 − 1 ft. 1 in. = 3 ft. 5 in.

K center $= \dfrac{330{,}500}{(58)(6\frac{7}{8})^2} = 121$

$p = 0.765$

$A_s = (58)(6\frac{7}{8})(0.00765) = 3.04$ sq. in.

7—$\frac{3}{4}$ in. = 3.07 sq. in.

$K = \dfrac{550{,}800}{(41)(9\frac{7}{8})^2} = 137$ $p = 1.05$—Too high for unit stresses,

therefore, make spandrel belt high bars increasing effective depth.

$K = \dfrac{550{,}800}{(41)(10\frac{5}{8})^2} = 119$ $v = 0.75$

$A_s = (41)(10\frac{5}{8})(0.0075) = 3.26$ sq. in.

7—$\frac{3}{4}$ in. = 3.26 sq. in.

80d. Deep Spandrel Beam. No Brackets or Heads on Columns (Fig. 62).

$S = 20 - 3 = 17$ ft. 0 in.

Floor load to be carried as represented by the polygon *agbchd* in Fig. 59 = 102 sq. ft.

Floor load (102)(300) =	30,600 lb.
Spandrel beam (2)(150)(17) =	5,100 lb.
Spandrel wall (4)(17)(120) =	8,200 lb.
Sash =	700 lb.
Total	44,600 lb.

$$M = (44{,}600)(17)(^{12}\!/_{12}) = 758{,}200 \text{ in.-lb.}$$

$$K = \frac{758{,}200}{(12)(24\tfrac{1}{2})^2} = 107$$

$$p = 0.675$$

$$A_s = (12)(24\tfrac{1}{2})(0.00675) = 1.98 \text{ sq. in.}$$

Use 4—¾ in. bent = 1.76 in. and 1—½ in. short bar in top over columns only, totaling 1.95 sq. in. The 4—¾ in. will be more than sufficient in center of beam.

End shear = 22,300 lb. Unit shear $\dfrac{22{,}300}{(12)(24\tfrac{1}{2})(\tfrac{7}{8})} = 89$ lb. per sq. in.

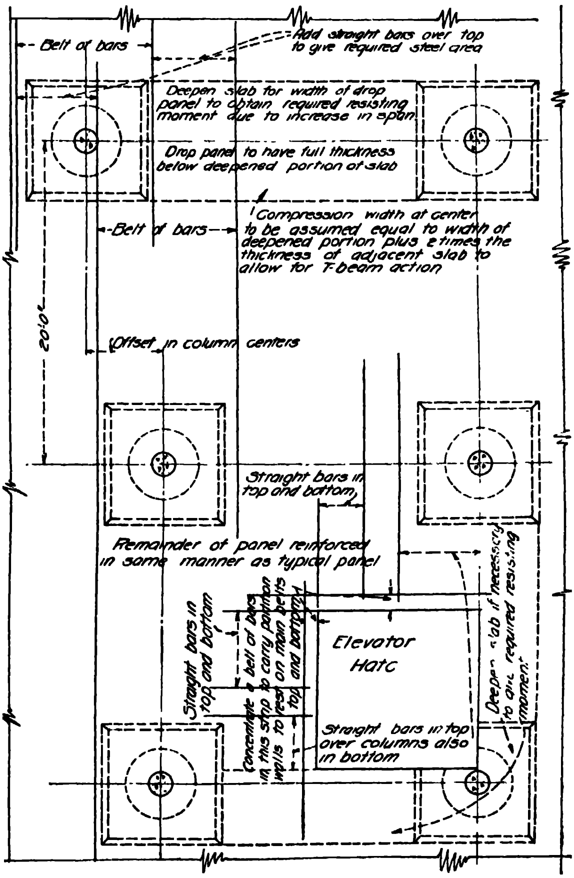


FIG. 63.—Floor framing at openings and special panels.

Bent bars and concrete will take care of 60 lb. per sq. in. shear, leaving 29 lb. per sq. in to be cared for by stirrups. This requires ⅜—in. U-stirrups spaced 9-in. centers for 3 ft. at each end, first one 6 in. from column face; then 12-in. centers for 3 ft., and 16-in. centers for center portion.

Wherever possible, it is, in the writer's opinion, best to use spandrels of the types shown in Cases 1 and 2, since the action in such construction is pure flat slab action in Case 1 and only slightly modified in Case 2, since the moments of inertia of the different slab stripe are the same, or only slightly modified, in the case of the deepened spandrel strip.

The use of a deep-beam spandrel either in conjunction with or without flaring column heads or brackets results in a combination of beam action and flat slab action, which is difficult of solution, and what is more, does not result in the true economy claimed for flat slab construction. Then also the type shown in Figs. 61 and 62 does not give a good lighting effect in the building on account of the drop beam below ceiling level, and if raised above slab, is subjected to considerable torsion stress. Such spandrel construction is also more costly, involving as it does the placing of many stirrups and other reinforcement in a narrow beam. By proper arrangement of columns and proportion of exterior panels, it will very seldom be found necessary to use deep beam spandrels in connection with flat slab floors.

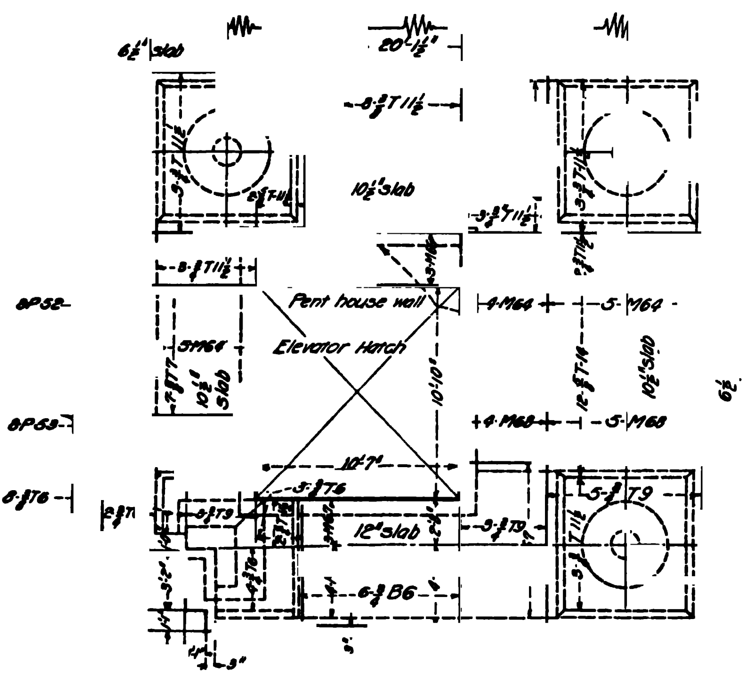


Fig. 64.—Roof framing to carry elevator pent house.

81. Special Framing.—In Fig. 63 is given data for the design of special floor panels with offset panels and elevator hatch in another. This data applies to the Akme System of two-way construction which is more adaptable to such framing than the four-way system.

In Fig. 64 is shown a typical arrangement of reinforcement for a roof panel carrying an elevator pent house.

Where the roof slab as required for interior panels is not quite thick enough to carry the parapet in addition to the roof load, the slab can often be thickened along the outer edge by raising above the level of main roof slab as shown in Fig. 65. The flat ceiling is thus preserved. Spandrels of floor slabs can be raised or thickened an inch or so in the same manner, if necessary, where wood floors are used.

82. Economy of Systems.—When flat slab floors employing different systems of reinforcement are designed in accordance with a given code, in which too much latitude in the placing of values of moments at the various sections is not allowed the designer, the amount of steel and concrete required for the different systems will be very nearly the same. Under the Chicago ruling the two-way system shows some economy in steel over the four-way system. This is because of the requirements for negative moment over the center of direct bands between columns. Since the diagonal bands cannot furnish this negative reinforcement, short bars must be placed in the top of slab between columns. When this is done, the two-way system is more economical in use of steel.

83. Patents.—Practically all the previously described systems of flat slab construction have been patented. The Courts held that the "Norcross Patent"

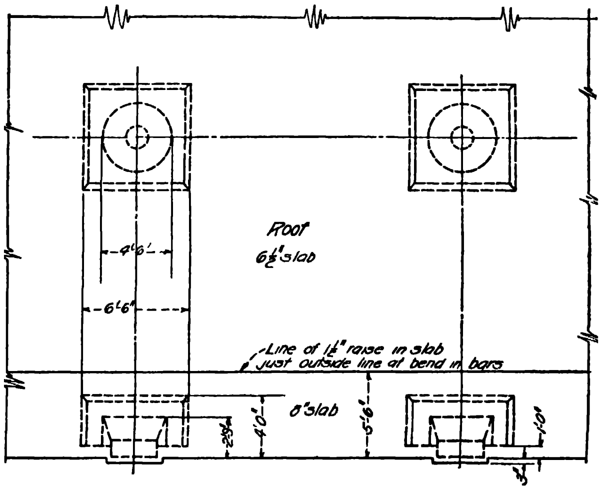


Fig. 65.—Suggested detail roof spandrel.

No. 698542 covering a very primitive four-way system of reinforcement and granted April 29, 1902, was the basic flat slab patent. Patents issued later for similar arrangement of steel could only be considered as improvement patents or as infringements. The Norcross patent did not cover a practical type of construction, no buildings using the system described in the patent ever having been built.

The Courts consistently held that the Turner Mushroom and Spiral Mushroom Systems were infringements of the Norcross patent and rendered an injunction against further infringement.

No suits for infringement were ever brought against the two-way systems by the owners of the Norcross patent before its expiration in 1919 and hence there is no record of the Court's attitude toward these systems in relation to the Norcross patent. A later decision by the United States Circuit Court, Chicago District, declaring the Akme System Patent (Sinks patent, owned by Condron Company) on two-way reinforced floors invalid, would seem to indicate that none of the now existing flat slab patents would be upheld by the Courts since most of

them were granted later than the Sinks (Akme) patent. This does not mean, however, that the owners of these still existing patents cannot start suit and cause an engineer and owner considerable annoyance and expenditure of money defending a suit for infringement should they see fit to start one. This, however, is extremely unlikely, since the owners of patents which have not been held to be infringements or invalid simply because they have not been tested in the Courts will be very slow to start a suit which is very likely to result in their patent being held invalid. The foregoing cannot be said of the S-M-I system which departs quite radically from the general scheme of four-way and two-way flat slab construction since there is no precedent as a guide as in the case of the latter systems.

SPECIAL TYPES OF REINFORCED CONCRETE ROOFS

By Albert M. Wolf

The varied requirements of public and industrial buildings of all kinds have given rise to several special types of reinforced concrete roof construction in addition to ordinary slab, beam and girder construction or flat slab construction. The purpose of these chapters is to discuss the design of such special types of roof construction as the arch, truss, sawtooth and dome.

84. Arched Roof Construction.—While arched roof construction of reinforced concrete rigidly framed to the concrete columns has been used in many structures

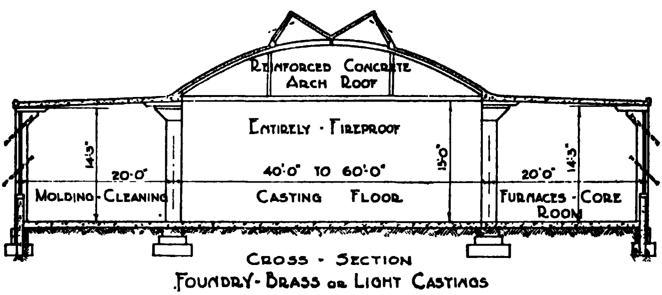


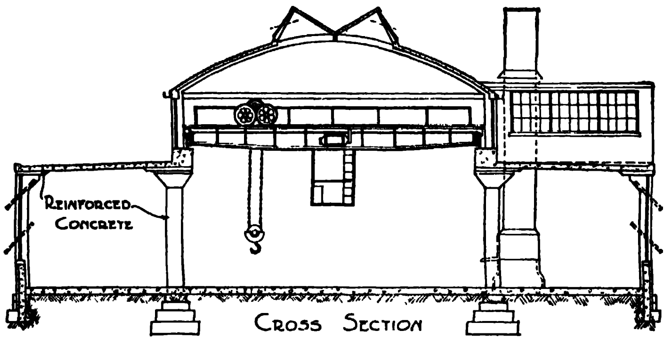
Fig. 66a.—Foundries of combination flat slab and arch roof construction.

in Europe, its use in America has been limited. However, the demand for fire-resisting construction for garages, hangars and factories has been responsible for the use of reinforced concrete arches in a number of structures. Aeroplane hangars when built of steel or wood, are special fire hazards, while if they are built of concrete the losses in case of fire could be limited to contents only. For foundries, machine shops, printing plants and garages where large open floor spaces free from columns are desired, this type of construction with butterfly monitors at the crown to provide light and positive ventilation will be found economical and otherwise most satisfactory (see Figs. 66*a* and 66*b*).

To reduce the dead load as much as possible the roof decking between arch ribs should be of tile and concrete joist construction using preferably a gypsum tile or metal box for the filler between joists because of their light weight. The backs of the butterfly monitors should be of the same construction being sloped at an angle of 30 deg. with the horizontal and the front or window face at right angles thereto. To give proper light and ventilation the sash openings in the

monitors should be about 5 ft. for 40-ft. span arches and 7 ft. 6 in. for 60-ft. span. The roof slabs of monitors should be supported by solid concrete walls built up on the arch ribs.

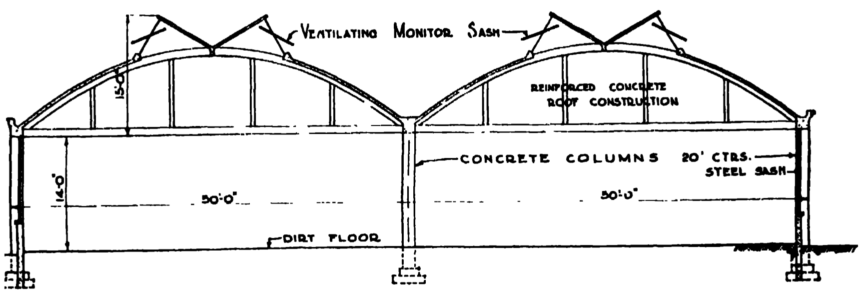
Having designed the roof decking and monitors the loads coming upon the arch ribs at the various points can be readily computed and the arch ribs analyzed. The arch rib ends should be rigidly connected with horizontal tie rods (encased in



Foundry - Heavy Castings

Fig. 66b.—Foundries of combination flat slab and arch roof construction.

concrete) to take up the arch thrust and thereby relieve the bending in columns. The tie rod member should be supported at two, three or four points (depending on the span) from the arch rib by encased hangers in order to prevent its sagging from dead load of concrete encasement, or to transfer the load of light cranes or shafting hung on the tie rod members as is desirable in foundries and machine shops.



Cross - Section.

Fig. 67.—Double span arch roof construction for foundry, machine shop or printing plant.

The arch ribs can be spaced at intervals of from 10 to 20 ft. depending on the requirements and shop layouts. The arch ribs can be analyzed by the ordinary methods used for fixed end arches, the thrust being taken up by the tie member. The curve of the arch ribs should, for economy, be a parabola or closely approximate one. The most effective cross-section for the arch rib is the T-beam section making the wings of the T-beam of solid concrete equal to the total depth of roof deck construction.

In Figs. 66*a* and 66*b* are shown cross-sections of two types of foundry buildings, one for light casting work without a service crane and the other for heavy work with crane service. The exterior panels of standard 18 to 20-ft. size are of flat slab construction while the arch spans can be varied from 40 to 60 ft. as desired.

In Fig. 67 is shown the cross-section of a double span arch roof building suitable for a large foundry not requiring heavy crane service, a machine shop or printing plant. With the increasing use of large size presses this type of building makes an ideal layout with all the advantages of fire-resisting construction.

85. Reinforced Concrete Roof Trusses.

85*a*. General Advantages.—When concrete materials were relatively cheap, as compared with present-day prices, the aim of the designer was to simplify the design as much as possible to cut the labor cost to a minimum, rather than to save on materials at an increase in labor cost. With the existing high prices of concrete materials engineers are beginning to appreciate that cost of construction must be reduced by saving materials in design and this has been effected in long span construction by the use of reinforced concrete trusses. Trusses of this type are especially adapted for long-span roof construction, for framing to carry skylights over wide courts for railway station work, and in fact any place where a fire-resisting construction of the best type is desired or required. In a properly designed reinforced concrete truss the materials are so placed and disposed as to give the greatest carrying capacity with a minimum amount of dead load.

In general it can be said that reinforced concrete roof trusses of other than the most simple types are uneconomical as compared with structural steel or wood, but if a fireproof construction is required it will be found that for ordinary spans reinforced concrete trusses are cheaper than structural steel trusses encased in concrete. This is true because of the more economical arrangement of steel permitted by the use of bar reinforcement, the difficulty of constructing forms around structural steel members, and the greater amount of concrete required to encase structural steel tension members as compared with tension rods in reinforced concrete.

85*b*. Types of Trusses.—The above, of course, would not be true if a complete truss of the same type as would be most economical for structural steel were used. Concrete is especially adapted for compression members, and if properly reinforced, a truss of a smaller number of panels can be used for the reinforced concrete truss. This cuts down on the amount of formwork and hence the cost. The use of reinforced concrete trusses with more than four to six panels, except for Vierendeel or Visintini trusses, should not be encouraged, for with an increase in number of panels the weight increases very rapidly. The ordinary two-panel king-post (Figs. 68 and 69), three-panel queen-rod (Fig. 70), and the simple Pratt, Fan and Fink trusses (Fig. 71), and parallel chord Pratt and Warren trusses are best adapted for reinforced concrete construction.

Warren Trusses.—A Warren truss with 60-deg. diagonals (see Fig. 72), is especially adapted to reinforced concrete construction since (1) the sum of the tensile web stresses in one-half of the truss equals the maximum tensile chord stress, and (2) the difference between the tensile stresses in two adjacent panels of the bottom chord equals the tension in the intervening diagonal. The design then amounts to the determination of the required amount of steel for the middle

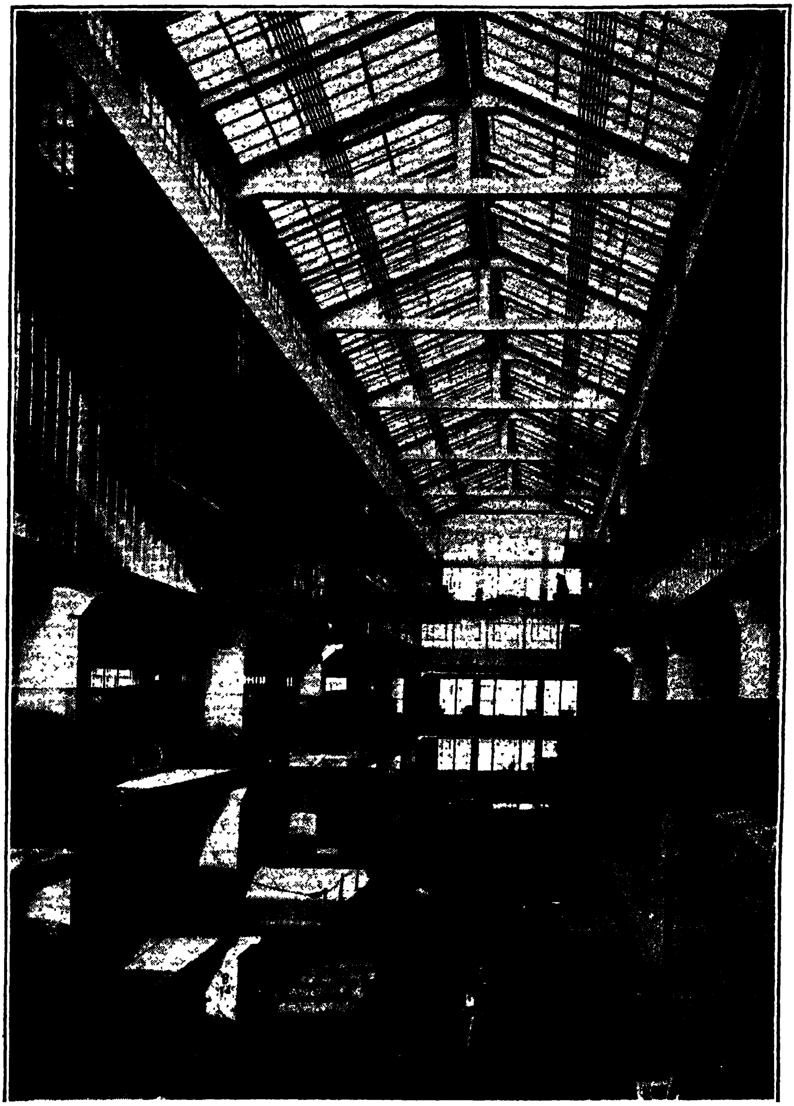


FIG. 68.—Concrete King post trusses—Ford Motor Co. building, Chicago.

panel of the lower chord and bending up at each tension diagonal a sufficient amount of steel and carrying it to the column in the top chord for anchorage. By computing the amount of steel required in each tension diagonal and bending the same so as to extend in the top chord from the column to the diagonal and thence down into the bottom chord, the proper amount of steel will be provided in the lower chord to take care of the tension and in the upper chord to take the local bending.

In transferring stresses from the lower chord to the diagonals three equal stresses must be provided for at the panel point: The tension in the diagonal; the difference in the chord stresses at either side of the panel point; and the bearing of the compression diagonal at the panel point. The bars do not offer sufficient resistance to this bearing and a steel bearing plate or saddle should be provided,

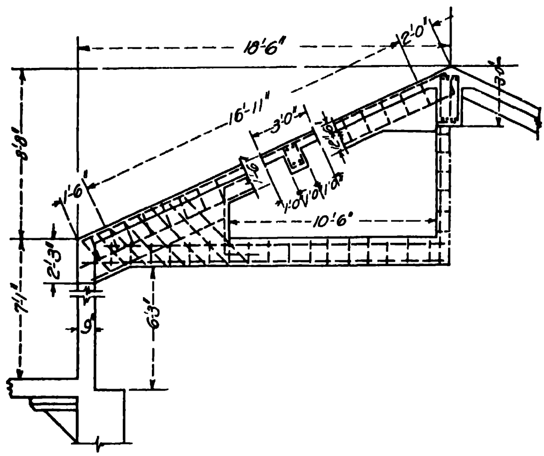


FIG. 69.—Details—Ford Motor Co. building, Chicago.

serving as a spacer and support as well as a bearing plate. All the bars are of the same length and if properly tied into the columns no dependence need be put in concrete for shear along the face of the column, thus eliminating the use of specially built units. The diagonals in special units are nearly always the same sectional area for each diagonal or set of diagonals and do not bear any proportion to the stresses coming upon them. It usually takes nearly all of the length of the diagonal to develop the full strength; therefore full value is not developed for the total length of diagonal, while for this type of construction the steel is in proportion to the load it takes and there is the length from the top of each diagonal to the column to develop bond.

Vierendeel Trusses.—Vierendeel trusses (so named from the inventor, Professor Vierendeel) are used considerably in Europe and are generally built with parallel chords and the members of the same cross-section forming square panels, although in some cases broken top or bottom chords have been used. These trusses are statically indeterminate structures and the analysis is made on the basis of the elastic behavior of the truss.

Owing to the fact that large shearing stresses are developed in the chords and verticals, and that diagonal cracks will form in the members unless large

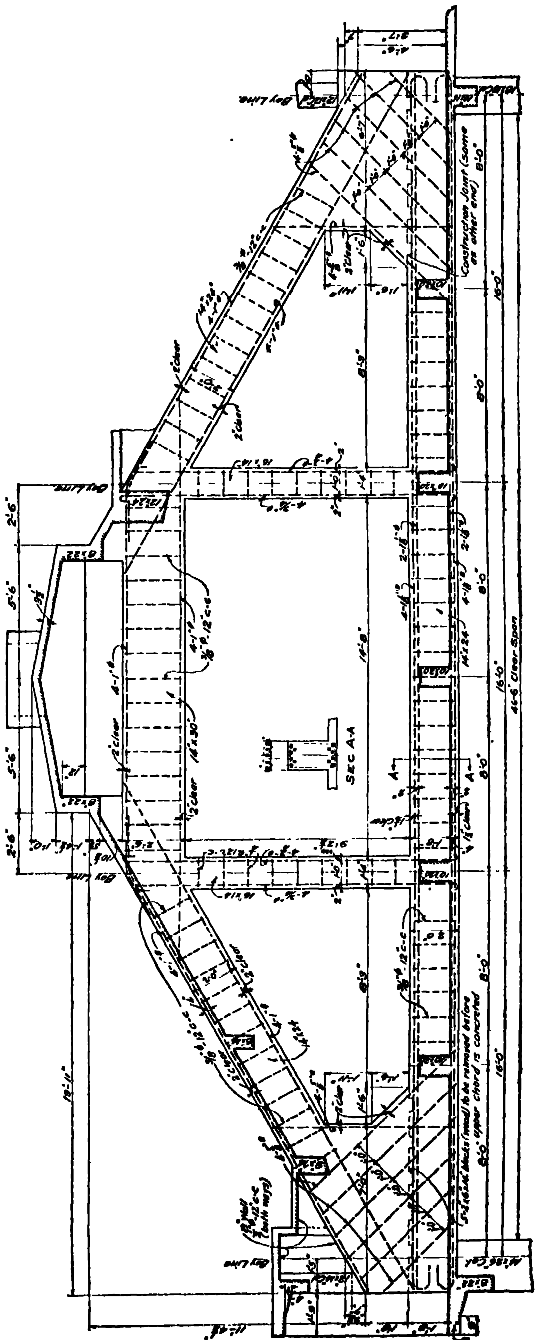
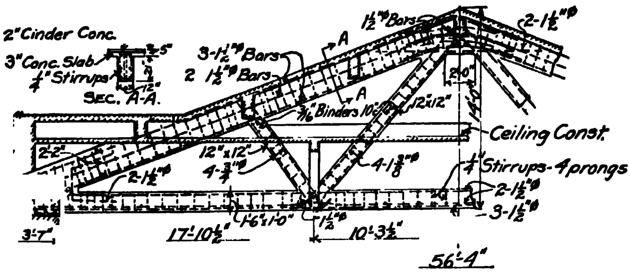


FIG. 70.—Detail of Queen rod trusses, Fulton Co. Court House, Atlanta, Ga.

(*Designed by Wm. C. Spiker, C. E., Atlanta, Ga.*)

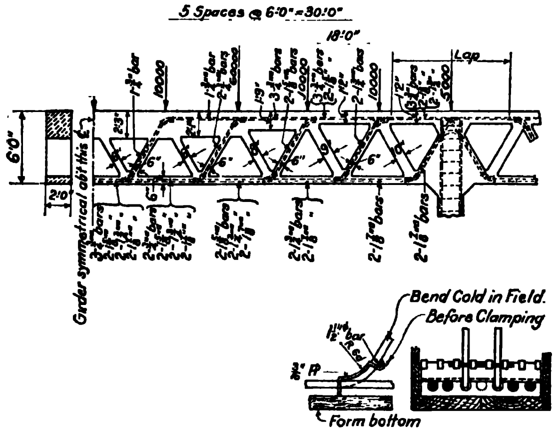
amounts of reinforcing steel are used, the Vierendeel truss proves uneconomical as compared with other types. This condition is further aggravated by the fact that the uncertainty of computation makes it imperative to use an excess of



(*Designed by Wm. C. Spiker, C. E., Atlanta, Ga*

FIG. 71.—Reinforced concrete Fink truss, Atlanta Union Station.

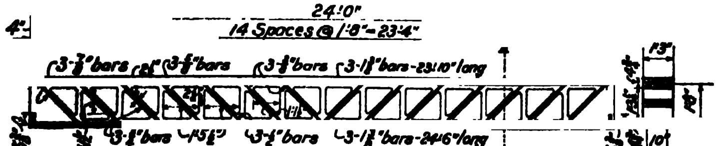
material to insure satisfactory results. In addition to this, the absence of diagonal members allows considerable deformation in the truss, since not only axial



(*Designed by R. T. Bagby*).

FIG. 72.—Warren truss of reinforced concrete.

stresses but bending moments of considerable amount are developed. These facts rather preclude the extensive use of the Vierendeel truss in America.



DETAIL OF TYPICAL VISINTINI TRUSS FLOOR GIRDER. SECTION A-A.
TEXTILE MACHINE WORKS.

FIG. 73.

Visintini Trusses.—Another type of reinforced concrete truss used quite extensively in Europe is the Visintini type (Fig. 73). These are trusses with

parallel chords divided into panels by vertical members connected by single diagonals for long spans, or with the verticals omitted for comparatively short span trusses. The stresses in these trusses are computed in the same manner as for the ordinary parallel chord Pratt or Warren trusses.

85c. Methods of Truss Roof Framing.—The T-beam with the stem reinforced as a column is the most suitable shape to employ for top chord members and especially so where the trusses form a monitor skylight, this shape giving more lateral stiffness to the truss. Where king-post trusses are used they should be tied together by a ridge girder into which intermediate lines of T-beam rafters frame. With three-panel and four-panel trusses, longitudinal roof girders or

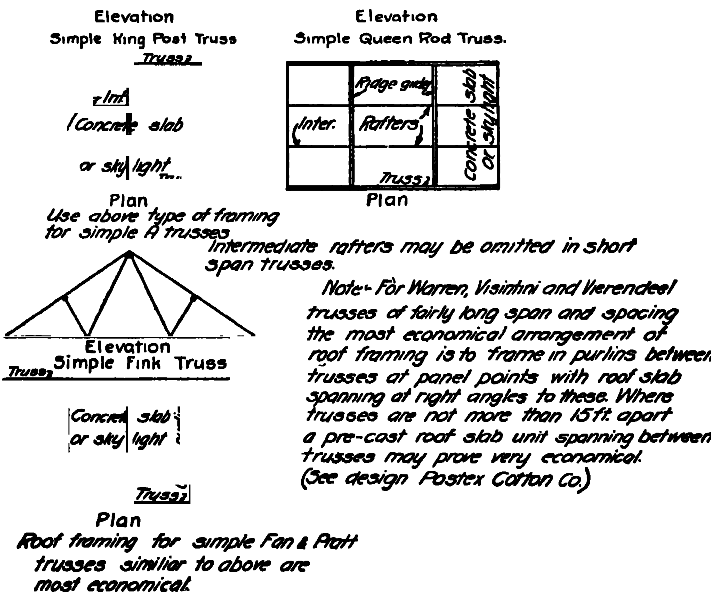


FIG. 74.—Roof framing for various types of concrete trusses.

purlins should be used to support the roof slab directly or, if such slab spans are very long, rafter beams carrying the slab reinforced at right angles to the purlins should be used (Fig. 74).

Where openings occur in the roof a type of framing should be used which will give a minimum amount of concrete for the portion between trusses. Then also the roof deck forms should be as simple as possible, so as to reduce to a minimum the number of supports required, since if the trusses are located at a considerable height above the floor directly below, as in a craneway, the cost of construction will be greatly influenced by the amount of centering required. Under such conditions if the spans of trusses are considerable, the most economical construction may be to concrete the trusses in place (if too heavy to handle as units), and erect the rafters or purlins and slabs as pre-cast units, thus confining the centering to a very small portion of the work.

85*d*. Determination of Stresses.—The loads to be considered in design and the methods of determining the stresses in the various members of the different types of trusses (excepting the Vierendeel truss) are the same as for steel trusses.

85*e*. Details of Design.—Computing the stresses due to truss action is by no means the most important feature of reinforced concrete truss design, this being in most cases simple as compared with the work of detailing the truss. The details to be especially considered are: (1) Construction at intersections of members to the full strength of all intersecting members; (2) the end shear, horizontal and vertical, at the junction of truss with supports and at panel points should also be carefully looked into; (3) the various members should be investigated to ascertain the stresses due to flexure (local bending) due to concentrations, uniform load or dead weight; (4) the methods of proportioning and reinforcing the different members have an important bearing on the economy of the construction.

85*f*. Detailing at Panel Points.—As to point 1, it may be said that in tension members the steel alone is depended upon to take the direct stress and it should therefore be securely connected, in the case of web members, to the steel in the chords so as to insure the full development of the bars. This can ordinarily be done by hooking or looping the reinforcement of web members over the chord reinforcement, the hooks being made of good length with fairly easy bends. For large trusses it may be necessary and advantageous to thread the ends of the reinforcing bars in the web members and anchor them with nuts to steel plates bearing on the chord reinforcement. The additional expense of this detail is very seldom warranted for web members in small trusses (up to 50-ft. span) since positive anchorage can be obtained as previously described. To anchor the reinforcement in the lower or tension chord, the ends of bars can be threaded and run through anchor plates at the ends, or they may be carried well back to the end of the truss and bent up on easy curves into the top chord for a sufficient distance to insure anchorage. The former, however, is to be preferred when such detail is used, for the reinforcement, when placed near a re-entrant corner, will tend to crack out when under stress. It should, therefore, be anchored back into the concrete by loop ties fastened to the bar. Good substantial fillets should be formed at the intersections of all members especially where tension and compression members connect, since the secondary stresses arising at the connections due to their stiffness are likely to overstress the members at such points. The secondary stresses, occurring at panel points, become excessive in long-span trusses unless properly provided for, and therefore such points merit careful investigation. The secondary and not the direct stresses are the ones which in reality limit the spans for which concrete trusses are practicable.

85*g*. End Connections.—The end connection of the upper and the lower chords is one that requires the most careful consideration. At this point sufficient area of concrete and shear reinforcement must be provided at the line of intersection of top chord and upper side of bottom chord to resist the horizontal shearing force represented by the tension in the latter which is the same as the horizontal component of the compression in the upper chord. Diagonal stirrups tying the reinforcement of the intersecting members together are very effective in resisting this stress. Sufficient sectional area should be provided in a vertical

plane at the edge of the support to keep the vertical shear within reasonable limits, 80 to 100 lb. per sq. in. for the maximum load. Filleted or bracketed connections are usually made with the supports to insure a sufficient bearing and shear area.

85*h*. Local Bending.—The bending stresses induced in the members by the external loads and the dead weight of the members themselves in addition to the direct stresses should be ascertained and the design should be such that the combined stresses do not exceed the allowable at any point. In general, good design implies the use of a truss with panel points so arranged as to relieve the chords of local bending caused by loads acting between panel points. This is not always possible owing to other requirements, nor is it possible to avoid bending stresses due to the dead weights of the horizontal or inclined members themselves between panel points. In the compression members such bending stresses increase the compression in the concrete and the steel at the upper surface, and decrease that in the steel reinforcement and the concrete at the lower side. In tension members the tensile stresses in the lower steel are increased and that in the upper reduced by local bending, while the concrete in the upper portion is subjected to some compression. After analyzing the truss and proportioning the member for the stresses thus found, the bending moment due to loads on the various members should be ascertained by considering each member as a simple beam (or partially continuous if the truss has several panels) between panel points. The combined stresses can then be found by the formulas for flexure and direct stresses. The stress in steel should be limited to 16,000 lb. per sq. in. and the maximum allowable compression in concrete due to bending and direct stress should be 650 lb. per sq. in. The bond stress on bars should be low, that is, 50 to 60 lb. per sq. in. to give an added factor of safety.

85*i*. Proportioning and Reinforcing Members.—In the design of concrete roof trusses, the one objectionable feature is the dead load, and it is therefore a cardinal principle of detailing that the concrete used to encase tension members be made as limited in cross-section as is consistent with the general appearance of the truss and the amount of steel in the member. The appearance must be considered, since to use extremely light sections of concrete in tension members and very much heavier ones for compression is not consistent with the idea of massiveness and stability symbolized by concrete. Owing to the methods of anchoring bars of tension members at the panel points, it is not good practice to use a small number of large sized bars in such members as would be done if the desire to keep the sectional area of concrete as small as possible were placed before all other considerations. A greater number of smaller bars will require more concrete for proper encasement, but in so doing, the amount of dependence placed on each individual bar is a relatively small percentage of the whole, and if one should for some reason not act as assumed, the overstress in the others would be slight, while with a few bars it might be dangerous. Although not absolutely essential so far as theory is concerned, the reinforcement of tension members is usually tied together with hoops or ties to facilitate placing of steel to guard against misplacement and dislodgement while concreting.

Web members in compression should be designed as columns for the direct stress with reinforcement well anchored in the chords. The concrete section used should be ample to keep the average unit compressive stress on the effective

cross-section within from 500 to 550 lb. per sq. in. and the same designed as longitudinally reinforced columns with ¼ in. (or larger) hoops or ties spaced from 6- to 9-in. centers. The maximum amount of longitudinal steel used should be about 2½ per cent and in no case should less than ½ of 1 per cent be used in any compression member. The allowable loads on such member can be used by the formula $P = 400(A_c + nA_s)$ where $A_c =$ area of concrete, $A_s =$ area steel, and n the ratio of moduli of elasticity of concrete and steel = 15 (for 1:2:4 concrete). The stresses in the steel should be low since the amount of embedment of bars at the ends where stresses are transferred will in general (except at end connections) be limited.

The design of compression chord members should be made in the manner just described for web members, but it will in some cases be found that a T-section instead of a rectangular section can be used to the best advantage for the former, owing to the greater effectiveness of such a section in resisting the local bending, which is likely to be a large factor in the total stress. In some cases, where the size of top chord members is limited by conditions other than stress, it may be advantageous to use continuous spiral reinforcement around the longitudinal bars, thus allowing the use of higher unit stresses. From a construction standpoint, however, this is to be discouraged, as spirally reinforced horizontal or inclined members interfere with the proper concreting of the web members below and necessitate the use of very fine aggregate in order to fill properly the core of the spiral.

86. Sawtooth Roofs.—One of the best means of lighting and ventilating one-story buildings covering a large ground area is by the use of sawtooth roof construction with movable steel sash in the sawtooth facing to the north, and sloping at an angle of from 25 to 30 deg. with the vertical, depending on the latitude, the steeper pitch being used in the southern states. The back of the sawtooth is usually sloped so that its intersection with the face forms a right angle and the height is usually made about one-fourth of the span.

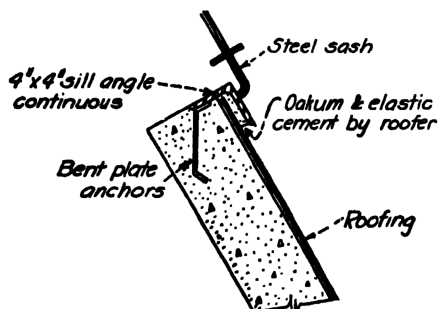


FIG. 75.—Curb wall detail for sawtooth skylight.

For printing establishments, weaving sheds and for machine shops doing fine machine work, abundance of light, freedom from shadows, and direct sunlight are necessary, and these can be supplied most satisfactorily by the use of sawtooth skylights. The main objection to such roofs is their additional cost as compared with flat roof construction, but for close work the extra cost will be more than offset by the greater quantity and better quality of work turned out.

The northern light roof was originally introduced in the south and for a long time was used extensively only in such localities where little or no snow fell, since it was found that the snow banked up between the teeth, blocking the windows, and when melting, leaked in around the window muntins and sash. With the advent of the steel sash, however, this objection was eliminated. The

melting of snow by heat radiating from the roof and from sunlight tends to fill the gutters with saturated snow which quickly freezes in the shadows formed in front of the sawteeth, thus clogging the gutters and downspouts and flooding the roof, causing leaks when the next thaw comes, since the gutters melt out last. This objection can be set aside by giving adequate slopes to gutters, having the curb walls under the sash fairly high, say 18 in. or 2 ft., providing

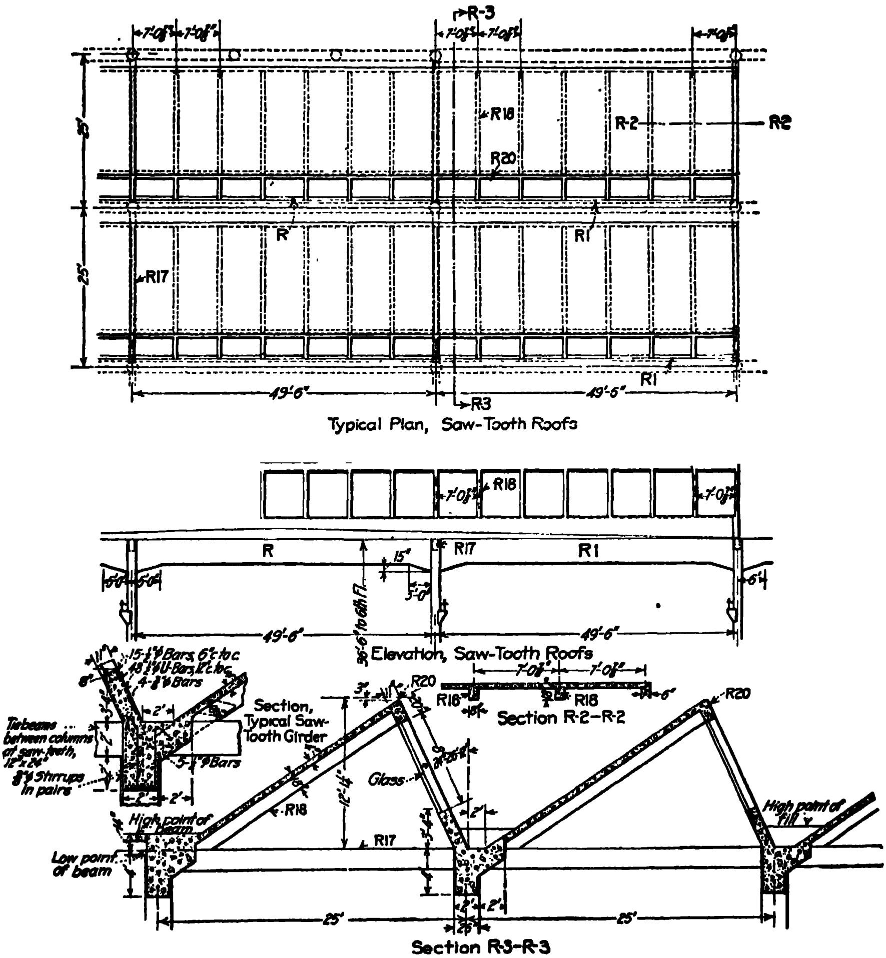


FIG. 76.—Typical sawtooth roof of beam construction.

numerous downspouts and placing heating coils under the gutters so that the radiating heat will keep them free from ice.

Condensation tends to form on the sash of sawtooth skylights unless they are double glazed or sufficient heating coils or radiators are provided along the sill under the sash. Then also condensation gutters should be provided at bottom of sash to carry the condensation. Metal ventilators should be placed in the peaks of a sawtooth skylight to provide ventilation in addition to the

movable sash. It is of the utmost importance that the top of curb walls be detailed so as to provide a tight sash joint and a means of flashing the roofing into same. A detail which fulfills these requirements is shown in Fig. 75.

A typical example of a sawtooth roof of ordinary beam construction is shown in Fig. 76, while in Fig. 77 is shown a sawtooth roof in which the girders supporting the back or roof of the sawtooth are wide flat beams supported on columns

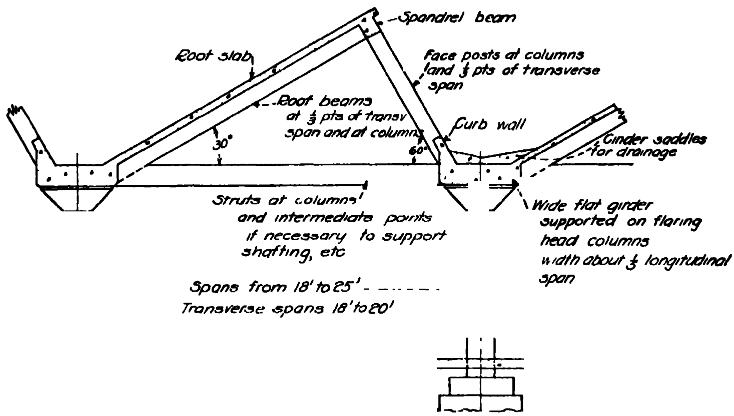


Fig. 77. Sawtooth roof of modified flat slab construction.

with flaring heads as used in flat slab construction, and designed in the same manner as the main belts in flat slab floors. This latter construction possesses the usual advantages of flat slab over beam and girder construction and in addition provides a wider gutter with consequent decrease in the danger of flooding the sash.

The roof slab should be designed as a continuous slab and the beams supporting the roof slab should be designed as partially fixed using a moment of $\frac{wl^2}{10}$

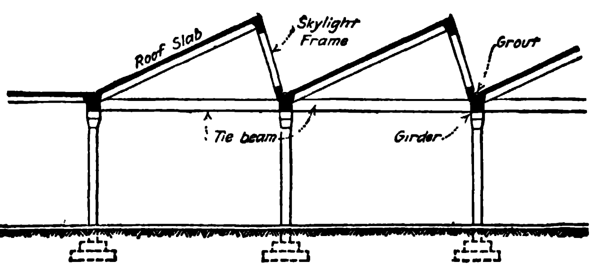


Fig. 78.—Cross-section showing typical arrangement of units in sawtooth construction, *Unit-bilt* system.

at the center and providing for a negative moment of $\frac{wl^2}{12}$ at the supports.

The concrete posts supporting the spandrel beam over sash are usually placed at column centers and if any intermediate support is required by the sash, a light structural steel member is used to which the sash can be readily fastened. The horizontal tie beams between columns take up the thrust from the roof, tie the entire building together securely, and in addition provide shafting supports.

A type of sawtooth roof construction which possesses advantages of construction economy is the "Unit-Bilt" sawtooth roof of separately molded members shown in Fig. 78. The saving in form and falsework which is not required in this type of construction is the important item, all members being poured in unit forms on the ground, cured and then hoisted into place.

In this system the roof slab rests on the skylight frame at the top and in a ledge on the main supporting girders at the lower end. The lower end of skylight frame also rests on a ledge in the girder, and horizontal unit-beams tie the tops of columns together to form a rigid construction.

87. Domes.—The solid dome of reinforced concrete is a very economical structure for roofing over areas of considerable space and with circular periphery, because of the fact that in such structures when properly designed the stresses can be readily taken care of. The use of domes is limited mainly to the roofing over of tanks and reservoirs and of monumental buildings or towers.

Spherical domes, those generated by revolving an arc of a circle about a vertical line, are most commonly used because of their pleasing appearance. However, where utility is the main consideration, a conical dome or a frustrum thereof, formed by revolving an inclined line about a vertical axis, can be used to good advantage.

87*a*. Loads.—In framed domes it is necessary to make separate calculations for stresses due to wind load and snow loads in order that the various joints and members may be properly proportioned and detailed; but with a solid dome such as a monolithic concrete dome, where the thickness of the shell is governed more by practical construction considerations than stresses, the design can be simplified by considering the live load as a combination of the wind load and snow load, acting vertically, and equal to 20 to 30 lb. per sq. ft. of roof surface.

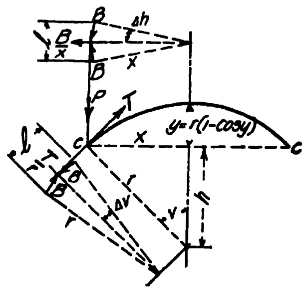


FIG. 79.—Forces acting spherical dome shell.

The dead load for design purposes should be taken as the weight of the concrete shell, the thickness of which should be about 1/150 to 1/200 of the span of the dome, but for practical construction reasons, should be a minimum of 3 in. Some domes are made of uniform thickness throughout, but in order to reduce the dead load as much as possible, it would seem better practice to increase the thickness from crown to base where the stresses are greatest.

Then also, by making the dome thinner in the upper part, the tendency to push out the lower portion is decreased and the dome made more stable.

87*b*. General Theory. *Spherical Domes.*—Let p represent the total live and dead load per square foot of surface of the dome. Now the surface area of a spherical segment of height y equals $2\pi ry$, where r is the radius of the spherical segment.

Then the load on the segment equals $2\pi ryp$. In Fig. 79 $x = r \sin v$, $y = r(1 - \cos v)$, T the meridian or tangential stress per unit length of circumference equals

$$\frac{py}{\sin^2 v} = \frac{pr(1 - \cos v)}{1 - \cos^2 v} = \frac{pr}{1 + \cos v}$$

At the crown of dome $\cos v = 1$ and $T = \frac{pr}{2}$, and at the equator of the sphere $\cos v = 0$ and $T = pr$.

Now the tangential stress T has a horizontal component H which gives rise to another stress commonly called the belt or ring stress B. From Fig. 79

$$H = 2B \sin \Delta h = \frac{B}{\;}$$

and

$$2T \sin \Delta v = \frac{T}{\;}$$

Now B, R and P at any point c must be in equilibrium or in other words, their components in any direction must equal 0. Then in direction r

$$\frac{B}{x} \sin v + \frac{T}{r} \quad p \cos v = 0$$

$$B = pr\left(\cos v - \frac{1}{1 + \cos v}\right)$$

At the crown $\qquad \cos v = 1 \quad$ and $\quad B = \frac{pr}{2}$

At the equator $\qquad \cos v = 0 \quad$ and $\quad B = -rp$ (tension)

From Fig. 80 which illustrates the variation in stress values it will be seen that the belt or ring stress B equals 0 where $v = 51$ deg. 50 min. and $h = 0.618r$. This plane is called the principal joint of rupture since above it (in a full dome) the stresses are all compressive and below it the tangential stress T is compressive and the belt stress B tension with maximum values at the equator.

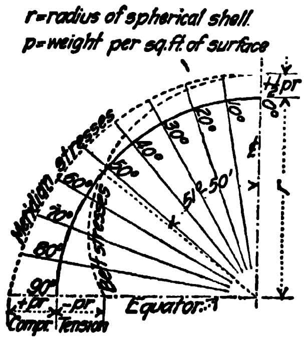


FIG. 80.—Stress values for solid domes.

If the dome is a segment of a sphere cut at any point above the plane of rupture, no tension reinforcement is required in the shell and the dome will be stable provided the abutment is strong enough to resist the horizontal component H of the tangential stress T, or a ring or hoop of steel is provided to resist it.

In a dome whose shell extends below the plane of rupture tension reinforcement must be provided to take care of the tensile stress B.

In domes with lanterns at top or open at the top, special joints of rupture exist and occur at points below the principal joint of rupture when the base is below this plane.

Conical Domes.—In a conical dome the tangential stress $T = +\frac{s^2}{2a}p$ and the belt stress $B \quad -\frac{s^2}{2a}p + \frac{r^2}{a}p$.

Substituting for s^2 its value $a^2 + r^2 \quad B = -p\left(\frac{a^2 - r^2}{2a}\right)$.

Now if $a = r$ (Fig. 81), the cone has a slope of 45 deg. and B becomes zero—that is, none of the courses are in tension, each course simply transmitting the tangential thrust T downward.

With a greater than r the inclination of the sides of the cone is more than 45 deg. with the horizontal and B will have negative values—that is, the stress will be tensile.

If the inclination is less than 45 deg. the courses will be subjected to compression in the direction of parallels of altitude as well as in the direction of the sloping sides.

For determining the tensile stress at the bottom of the main course use the first formula given for the value of B. The first term relates to the effect of the upper part and the second to the lower part of the dome in reference to a particular altitude, hence at the base of any dome the second term vanishes and T and B become equal in amount but opposite in sign.

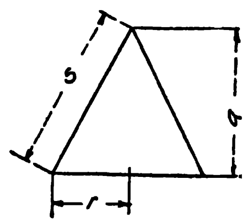


FIG. 81.—Diagram for conical dome.

In a full conical dome no joints of rupture exist, the ring stresses being tension, zero, or compression from top to bottom depending upon the inclination of the sides. In truncated conical domes however, a joint of rupture does exist.

DRAINAGE AND INSULATION OF CONCRETE ROOFS

By ALBERT M. WOLF

The most economical method of providing for drainage on the roof of a concrete building is by pitching the roof slab in one or more directions toward the drains. This type of roof construction, however, does not make a very pleasing appearance in the upper story, neither does it provide the roof insulation necessary in certain kinds of buildings to prevent condensation on the underside of the slab. For one or both of these reasons the roofs of the majority of concrete buildings are built flat (the same as the floors) and the necessary slopes for drainage and the insulating material placed on the top of the roof slab. Various methods have been evolved for draining and insulating concrete roofs, and these will be described herein.

88. Insulation.—When air, laden with moisture in the form of vapor, comes in contact with a cold body or surface, the air becomes chilled and the vapor forms in drops of water which collect on the cold surface and form what is known as condensation or dew. Condensation only occurs when the temperature inside the building is considerably higher than that outside, or when the air becomes so heavily laden with moisture as to reach its dew-point or, in other words, becomes saturated. The amount of moisture which air can hold in vapor form depends upon its temperature and barometric pressure; in general, the warmer the air, the more moisture it will carry.

If the building is of such a character as to require considerable heat during winter and moisture is present, condensation takes place on such surfaces as are directly affected by the cold outer air. Generally speaking, concrete is known as a poor conductor of heat, but when compared with wood, especially in the case of thin construction, such as is ordinarily employed in roof and floor slabs of concrete, it is a relatively good conductor of heat and cold.

One of the main objections to the use of concrete roofs in northern latitudes is that wherever steam-laden or hot moist air is present, there is sure to be

condensation on the concrete ceiling during cold weather, unless some special provision is made to prevent it. This objection has led to the use of various methods, designed to prevent condensation, and has led some engineers and architects to refuse to use concrete roofs. In some cases, wooden roofs have been used on structures otherwise entirely of concrete, just to eliminate the trouble encountered with condensation on cold concrete slabs. This, of course, is a poor policy since one fault is eliminated by introducing another—that is, placing a combustible roof on an otherwise fire-resisting building.

In storage warehouses and buildings of a similar character, where no artificial heating is required, condensation can be eliminated by proper ventilation. Concrete buildings of this type require little or no roofi nsulation. Power houses, paper mills, roundhouses and concrete structures of a similar occupancy, where much steam and moist air is present, require very good insulation and ventilation to prevent condensation and subsequent dripping of water of condensation on machinery and employees. Manufacturing and industrial buildings form a class which, as far as insulation of the roof is concerned, lie between the two above extremes, since they will require a more or less positive roof insulation, depending upon the character of the occupancy and the locality.

From the foregoing it can be seen that it would not be wise or consistent with economy to use the same kind of insulation for all types of concrete buildings. Practically every building forms a separate case, which demands a careful investigation into conditions of air in the building at different times, the means of ventilation, and the climate of the locality. In one case a certain method of insulation may meet all requirements and be entirely satisfactory, while in another, owing to different conditions, the results may be entirely unsatisfactory.

88a. Clay Tile Insulation.—Hollow clay tile laid directly on the concrete roof slab and covered with a mortar finish upon which to lay the roofing, form a good insulating medium for sloping roofs. Partition tile, 3 or 4 in. thick, with scratched or keyed surfaces to furnish a mechanical bond for the cement mortar coating, are suitable for this work. They should be laid end to end to form continuous air spaces, and tar or asphalt expansion joints so arranged as to prevent the fill from running into and between the tile, should be placed at all walls. The mortar coating is necessary to render a smooth surface upon which to lay the roofing, owing to the irregularities of the tile. It should be from ¾ to 1 in. thick, depending upon the uniformity of the tile, and be composed of a 1:3 mixture of cement and sand.

This type of insulation is a good one for all ordinary buildings with sloping roofs, since it combines the advantages of fairly light weight (25 to 30 lb. per sq. ft. not including roofing, depending on the thickness of tile), comparatively low cost and a positive insulation. It can be constructed very rapidly and is suitable for use on all types of buildings except where an unusual amount of moist air or steam is present.

88b. Gypsum Tile Insulation.—An insulation similar to that just described, but possessing a lighter weight and even better insulating qualities, is a concrete slab covered with gypsum roof tile. Gypsum tile is a much more efficient non-conductor than clay tile, and, owing to its uniformity, does not have to be covered with a mortar coat to form a surface for the application of

the roofing, which materially lessens the cost of construction. Gypsum roof tile are made of gypsum and a small amount of wood fiber in the standard size 12 × 30 in., with a 3-in. thickness. After drying out they are especially treated and waterproofed.

When placing an insulation of this type the sloping roof slab should be perfectly dry, then mopped with hot asphalt or pitch and the gypsum roofing tile laid immediately end to end and breaking joints. After all the tile are placed, the tops should be mopped with asphalt or pitch and a tar and gravel roof placed the same as when applied directly on a concrete slab.

88c. Suspended Ceilings.—One of the early types of construction used to prevent condensation was the suspended ceiling, and it is still being used to considerable extent. In beam and girder roofs the metal lath can be fastened directly to the bottoms of the beams with wires or expansion bolts, while for flat slab roofs the metal lath is fastened to ¼ × 1½-in. plate or 1¼-in. channels, supported by No. 7 wire hangers or ¼ × 1-in. flats hung from the roof slab. For spans under 2 ft. ordinary metal lath can be used, but for longer spans a stiffened lath is required. These ceilings should be plastered with a plaster composed of 1 part hydrated lime, 5 parts Portland cement, and 12 parts sand and some long cow-hair thoroughly mixed before adding the water.

The dead air space formed by this construction is quite effective in preventing condensation, and this accounts for the use of suspended ceilings in nearly all classes of buildings, including mills, power houses, etc., where they have in general given good service. The weight of the suspended ceiling is only about 12 or 14 lb. per sq. ft., which is a point in its favor. The cost is considerably higher than that of the types of insulation just described. However, where a plastered ceiling is desired, the suspended ceiling becomes the more economical.

The disadvantages of suspended ceilings are the tendency of the lath to rust out where moisture is present, and the need of providing slopes on the roof for drainage, which some of the other types furnish along with insulation. Then again, experience has shown that suspended ceilings will break down in hot fires.

88d. Cork Tile Insulation.—Slab cork insulation can be used either on the top of the slab or attached to the underside or ceiling, where a very positive insulation is required, but the cost thereof makes its use uneconomical except in cold-storage plants where the problem of maintaining low inside temperatures is foremost.

89. Insulation and Drainage of Flat Roofs.—A great majority of reinforced concrete buildings are built with the top of the roof slab level inasmuch as this construction is cheaper as regards form work, and gives a more pleasing appearance to the ceiling. It is, therefore, necessary to build up slopes on the top of the roof deck in order to provide for the necessary drainage. In such cases the insulation of the roof slab can be obtained with the same material used for making the roof slopes or in combination with other insulators placed below the material used to form the slopes. The most common type of construction for this purpose is the ordinary cinder fill with a top finish of cement, upon which the roofing is laid. In order to provide a more positive insulation, hollow clay tile or gypsum tile can be placed directly on the roof slab and the cinder filling placed thereon. A cinder concrete filling has been used quite extensively, but has objections which make it undesirable. These will be pointed out later.

89*a*. Cinder Fill.—For flat roofs the cinder fill insulator is probably the most used, since it performs two functions: (1) Forming drainage slopes; (2) insulating the roof slab at a comparatively low cost. The use of a cinder fill necessitates a heavier roof construction, but this is offset to some extent by the lesser cost of building forms for a level roof slab as compared with a sloping or warped surface, the extra cost of forms for a sloped roof amounting to from ½ to 1 ct. per sq. ft. and for a warped roof from 3 to 4 cts. per sq. ft.

The cinder fill, with a minimum thickness of 3 or 4 in. (at downspouts) composed of a porous grade of steam boiler cinders free from refuse or slag, should be wet down, then placed on the roof properly sloped and tamped to an even surface. It is essential that the cinders be wet down before hoisting to the roof, for if this is done after placing on the roof the excess water will stand on the slab and probably cause trouble by seeping through the ceiling or freezing in cold weather and disrupting the fill. If the cinders are not wet down before placing they do not tamp well, and when the surface finish of cement mortar is applied they will take up water from the mortar and destroy its value. Before the cinders have dried out after placing on the roof a 1:3 cement mortar coat, mixed quite wet, should be placed on the cinders for a depth of 1 in. and given a smooth float finish. This provides a smooth, unyielding surface upon which to lay the tar and gravel roofing, and also acts as a seal for the dead air space formed by the fill, which gives it its insulating value.

It is important to see that expansion joints are provided in the top portion of the fill around all pent-houses and parapet walls to allow for the expansion of the top surface. The top portion of the fill and the mortar finish should, therefore, be kept 1 in. or so from all walls and the joints filled with bituminous or asphalt paving pitch (see Fig. 82).

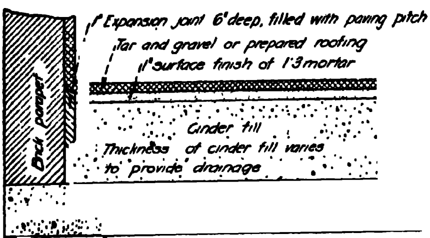


Fig. 82.—Cinder fill for drainage and insulation.

A cinder fill placed in accordance with the above instructions will weigh from 65 to 75 lb. per cu. ft. (not including roofing) and on account of this excessive weight it is highly important that the downspouts be arranged so as to keep the average depth of fill to a minimum, say 12 in It will be found more economical to provide additional downspouts than to overload the slab with additional cinders to care for the drainage and thus increase the cost of construction. In designing concrete roofs carrying cinder fills the weight of fill and roofing should be taken into account in the calculations in addition to the live load. Many designers ignore the weight of the cinder fill in design, assuming that the live load, which seldom acts, will take care of this extra load. This is not the case and the practice can only be condemned.

A cinder fill insulation as described is solid enough to bear all ordinary weights to which it may be subjected, gives no trouble from expansion, and can readily be used on concrete slabs designed as future floors, since, when the future stories are added the fill can be torn up and practically all of it re-used. It gives good service on heated warehouses, factories and light manufacturing buildings, and is

very satisfactory for any type of building except those where much steam or moist air is present.

89b. Cinder Concrete.—Cinder concrete fills have been used to some extent for providing drainage on flat concrete roofs and as an insulator against condensation. Such a fill is made of a rather dry, porous, cinder concrete, mixed in the proportions of 1 part, by volume, of cement to 8 or 10 parts of porous, screened steam boiler cinders, placed in the same manner as a cinder fill and finished with a mortar coat on which to lay the roofing. Expansion joints should be provided at all walls for the entire depth of the fill, for on hot days it expands considerably and exerts much pressure. Many parapet walls have been pushed out of place where expansion joints have not been placed in the cinder concrete fill.

The cinder concrete fill is by no means as efficient an insulator as a cinder fill and inasmuch as it weighs considerably more than cinder filling and gives trouble due to excessive expansion in hot weather, it is not to be recommended as material to be used for insulating or providing drainage slopes.

89c. Combination Hollow Clay Tile and Cinder Fill Insulation.—On flat concrete roofs, where some sort of fill must be applied to take care of drainage, the combination of the hollow clay tile and cinder fill insulators forms an excellent construction, since it combines and augments the advantages of each method considered separately (Fig. 83).

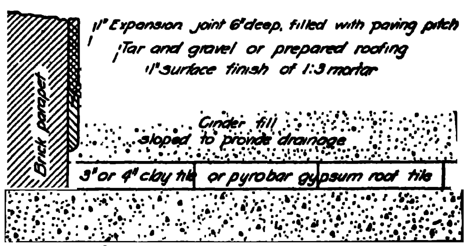


FIG. 83.—Combined cinder fill and tile insulation.

This insulation is constructed in the same manner as the cinder fill except that the clay tile are placed end to end on the roof slab before placing the fill. The weight for an average total depth of 12 in. amounts to from 70 to 75 lb. per sq. ft.

The dead air space directly over the roof slab formed by the tile and the protecting covering of cinders (forming the roof slopes) keeps the temperature of the air in the dead air space at about normal, and allows little chance of condensation except when the ventilation is poor. For this reason the insulation is as nearly perfect as can be made (with exception of the type next described) without the use of expensive cork insulation combined with tile, etc. It is entirely satisfactory for use on power houses, paper mills, roundhouses and structures of a similar nature, and with proper ventilation no trouble should be experienced from condensation.

The principal objection to this type of insulation is the excessive weight, but where a positive insulator is required the advantages cited overcome the disadvantage and extra cost of roof construction caused by the additional weight to be carried.

89d. Combination Gypsum Roof Tile and Cinder Fill Insulation.—Perhaps the most positive insulation which can be constructed is the combination Pyrobar gypsum roof tile and cinder fill. The better insulating qualities are obtained by the use of gypsum roof tile instead of clay tile. The weight of

this construction is less than the type just described and the cost is about the same, so that, all other conditions being equal, it should be given preference when insulating the roofs of roundhouses, power houses, paper mills and similar buildings. It is especially adapted to flat roofs, since the cinder fill provides for the drainage as well as abetting the insulating qualities of the gypsum tile.

In constructing this insulator the gypsum roof tile are laid on the concrete slab in the manner previously described, except that the coating of pitch on the top of tile is omitted, and the cinder fill is placed and finished as heretofore noted. This construction weighs about 55 lb. per sq. ft. for an average total depth of 12 in., and the cost will be about 15 to 16 cts. per sq. ft. not including the roofing.

A great many conditions and factors enter into the selection of the proper type of insulation for a concrete roof slab, and a very careful study should, therefore, be made of the case at hand so as not to cause a costly insulation to be used where a cheaper one would have prevented condensation just as effectually. Hard and fast rules are difficult of application, and individual judgment and the past experience of others in the same locality with the insulation of buildings against condensation should in all cases be given proper weight in making the selection.

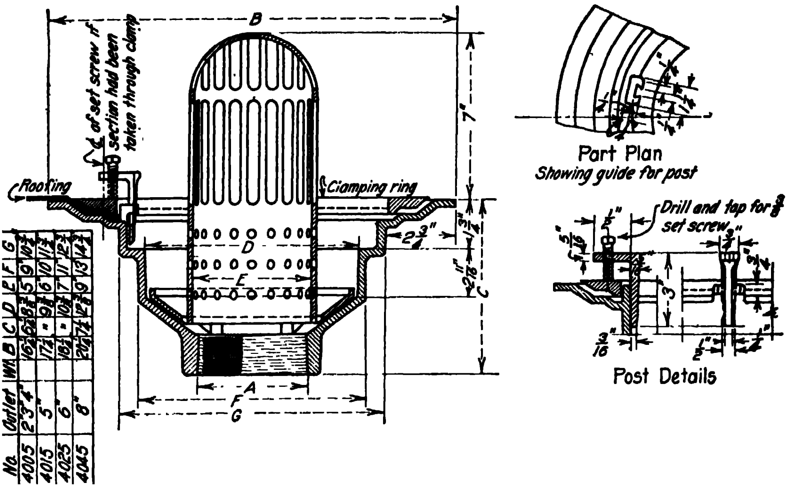
90. Conductor Heads for Roof Drainage.—In order that downspouts provided to carry off the drainage from roofs may function efficiently it is essential that the conductor head, or inlet, be so constructed as to preclude the possibility of its being stopped up by accumulations of gravel, leaves, dirt, etc. Then again some consideration as to the type and pitch of the roof should be given when detailing the conductor head and fixing the size of the conductor. It is apparent that the same conductor head detail and the same size of conductor should not be used for equal areas of flat and pitched roof.

90*a*. Size of Downspout.—The common practice in determining the proper size of downspout is to provide 1 sq. in. of conductor area for each 100 sq. ft. of area of what are commonly known as flat roofs (slope ¼ to ¾ in. per ft.) in the Central States and 1 sq. in. of conductor for each 75 sq. ft. in localities where the rains are more severe. For roofs of greater pitch the areas to be cared for by 1 sq. in. of conductor should be decreased in the following ratios: 15 per cent for ½ pitch and 25 per cent for ¾ pitch. In no case should a downspout be less than 2 in. in diameter and preferably not less than 4 in.

The conductor head should be of such size and arrangement as to present about twice or three times as great a width or diameter as the conductor, so that even if a portion of the screen should become clogged or stopped up with debris, there will still be sufficient inlet area to carry away the water without flooding the roof.

The downspout arrangements for sawtooth roofs demand special attention in order that satisfactory results may be obtained. On such roofs it is highly essential that downspouts be placed in each line of valleys and that they be designed to serve a much smaller area than for ordinary roofs of the same pitch. In winter there is always a tendency for snow to drift into the valleys; and in mild weather, when alternate thawing and freezing take place, difficulties of getting the water to the conductors are sure to be experienced unless they are spaced fairly close together and the screens or strainers carried up some distance above the roof line.

Wherever practicable, the downspouts should be placed on the inside of the building, to prevent freezing and bursting in the winter. It will be found that wrought-iron pipe gives better satisfaction for interior conductors than cast-iron pipe, since it can be obtained in longer lengths, is not as heavy,



No.	Outlet	Wt.	B	C	D	E	F	G
4005	2" 3" 4"		16½	6⅝	8¾	5	9	10¾
4015	5"		17¼	"	9⅛	6	10	11¾
4025	6"		18½	"	10⅛	7	11	12¾
4045	8"		20¼	7¼	12⅜	9	13	14¾

FIG. 84. Typical Josam cast-iron conductor head for ordinary roofs.

requires no calking (threaded sleeve connections being made between sections) and does not take up as much space as hub and spigot cast-iron pipe.

90b. Construction of Conductor Head.—To be durable and at the same time relatively inexpensive requires that conductor heads be constructed of cast iron. Until a few years ago, there were no standardized designs of

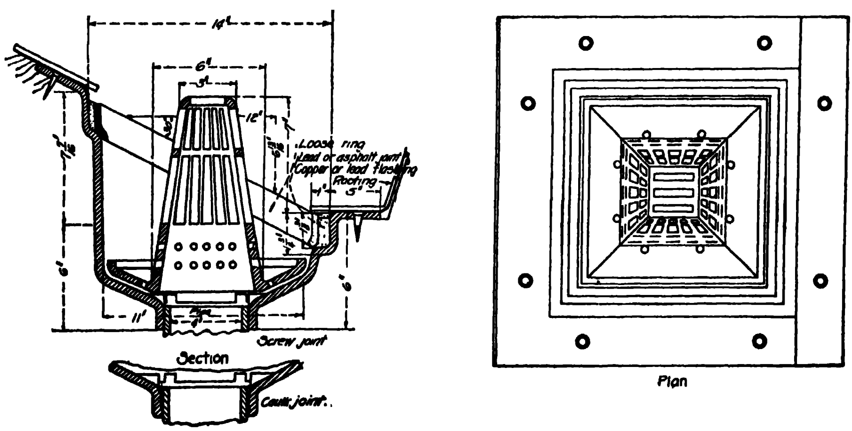


FIG. 85.—Josam cast-iron conductor head for sawtooth roof construction.

conductor heads which properly fulfilled requirements, and the making of special castings made the use of cast iron rather expensive. This fact lead to the quite extensive use of copper wire baskets set in conductor boxes connected to the downspout.

At the present time, however, standardized designs for conductor heads to meet various requirements as patented and manufactured by the Josam Manufacturing Company of Michigan City, Indiana, can be obtained at a very reasonable cost and are being extensively used.

In Figs. 84 and 85 are shown details of these conductor heads for ordinary concrete roofs and for sawtooth roofs. It will be noted that special precautions are taken against leakage around the casting embedded in the slab and that a removable sediment cup is provided.

ROOF PARAPET WALLS

By Albert M. Wolf

Parapet walls for reinforced concrete buildings are of two general types, brick or reinforced concrete or a combination of both, the type used depending upon the architectural treatment desired. To prevent flooding of a roof enclosed with parapet walls in case the downspouts should become clogged, holes should be left in the parapets at frequent intervals. These openings should be formed of sheet metal or short pieces of pipe.

91. Brick Parapets.—Brick parapet walls have been extensively used, but unless properly built and waterproofed on the inner side are very apt to give trouble from leakage at the junction of roofing and parapet. This has given rise to the quite extensive use of reinforced concrete parapet walls. If a brick parapet wall is used on a reinforced concrete building, the inner side of the parapet should be of hard burned vitrified brick (not ordinary soft-burned common brick) laid up in cement mortar and coated with high melting point roofing tar or asphalt.

Where cinder or cinder concrete fills are used on roofs to provide for drainage, the tendency of the fill to expand and push out the parapet must be counteracted by placing expansion joints 1 or 2 in. wide filled with tar or asphalt paving pitch of relative low melting point between the fill and the parapet. Then also it is good practice on large roofs to anchor the brick parapet to the roof slab as an added safeguard. No rules for exact size or spacing of anchors can be given, but in general ½ in. diameter bars placed in the roof slab at intervals of 18 in. to 2 ft. and bent up into the parapet will do the work, if the wall is built up solid around them and laid in cement mortar.

92. Flashing Details.—It is of the utmost importance that the proper means of fastening the roof flashing and counterflashing at the parapet walls are provided. A very satisfactory detail for flashing of built-up roofings at brick parapets is shown in Fig. 86. This detail recommended as good practice by the American Railway Engineering Association makes use of a 2×4-in. timber with one edge beveled, laid continuous in the parapet at the proper height in place of a stretcher course of brick. This serves as a nailing strip for a light wood strip, holding down the flashing and counterflashing. After placing the flashing the slot should be completely closed with cement grout or plastic roofing cement.

A recommended flashing detail for concrete parapets is shown in Fig. 87. A 2×4 timber is ripped on the diagonal and both pieces are then placed in the forms at the desired height, the upper strip being securely nailed to the forms so

as to insure its removal when forms are taken down, while the lower piece is just tacked to the forms and has wires or nails driven into it as shown to anchor it to the concrete. The flashing and counterflashing are then placed in the same manner as for brick walls.

93. Concrete Parapets.—As generally designed, concrete parapets in addition to retaining or masking the drainage slopes, carry a portion of the roof load as beams, but owing to the fact that they are generally much deeper than required simply to carry the load, other considerations beside the load must be taken into account. That is, enough reinforcing must be provided and distributed in a manner which will prevent the formation of expansion and contraction cracks resulting from the excessive changes of temperature to which side walls are subjected. Ordinarily parapets are at least 3 ft. deep overall, and usually this

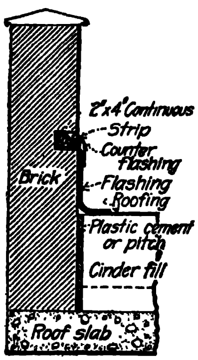


Fig. 86.

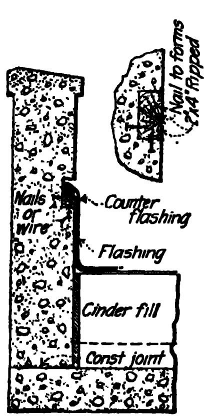


Fig. 87.

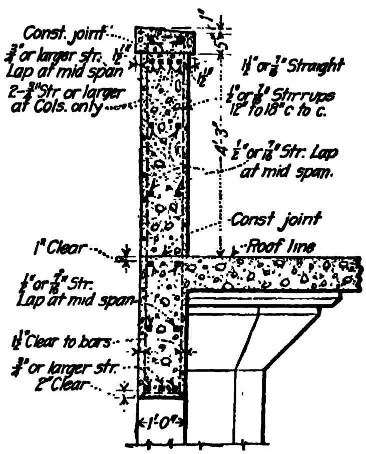


Fig. 88.

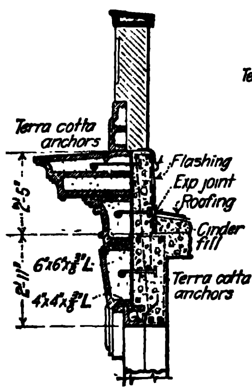
depth is much more than is required to resist the bending moments induced therein, especially in flat slab construction where the portion of the flat slab adjacent to the parapet carries most of the roof panel load. This means that a very low percentage of steel will be required to resist the tensile stresses produced by bending. In fact, it may be such a small amount as to be incapable of resisting the stresses set up by temperature changes. For this reason it is always well to so detail the reinforcement of the portion of the parapet above the roof as to have not less than 0.25 to 0.30 of 1 per cent of longitudinal reinforcement arranged somewhat as shown in Fig. 88 with plenty of vertical reinforcement in the form of stirrups to tie together the portions of parapet poured at different times (portion below and above roof line).

Concrete parapets should always be designed and reinforced as fully or partially continuous beams, depending upon the location thereof, and the condition of end supports, for unless this is done, unsightly cracks are sure to appear near the supports. If the parapet walls are of such form as to require pouring in two operations as in Fig. 88 they will, of course, not be so strong as if poured in one operation, and therefore, if the total depth is to be considered effective, a bending moment coefficient somewhat lower than used in the formula for fully continuous

beams, namely, $M = \frac{wl^2}{12}$, should be used. In the writer's opinion, this should be for beams of the type in question, $M = \frac{wl^2}{10}$ for interior spans and $M = \frac{wl^2}{9}$ for end spans at support and mid-span.

Parapets seldom require very much diagonal tension reinforcement owing to the depth of same and the relatively light loads to be carried, and the use of bent bars is therefore seldom warranted, since the stirrups used to tie the portions of the wall together can be made of sufficient number and size to care for all diagonal tension stresses in excess of that which the concrete alone will resist. At corners extra horizontal bars should be provided bent around the corner so as to lap with the main bars, for unless this is done, cracks are sure to develop, owing to expansive and contractive forces acting at right angles to each other. In large buildings it will be found advisable to provide expansion and contraction joints in parapets about every 200 ft. over columns, the spans adjacent to such joints being designed and reinforced same as end spans; or better still to provide expansion and contraction joints for the full height of building (see Art. **99**, p. 358).

94. Concrete Parapets to Support Brick or Terra Cotta.—In commercial, monumental and some industrial buildings, the architectural treatment requires



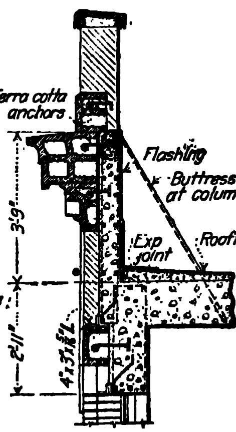


FIG. 89. FIG. 90.

concrete parapets designed to support the cornices and brick, terra cotta, or stone facing of the parapet. Such concrete beams must be designed to resist the twisting action produced by the weight of the material hung from the side of the beam as in Figs. 89 and 90.

When the concrete portion of the parapet supporting the projecting cornice and brick parapet above is not very high, as in Fig. 89, the bending stresses produced can be taken care of by placing vertical bars or stirrups so as to reinforce the wall as a cantilever with sufficient longitudinal reinforcement to resist the tensile stresses due to beam action in a vertical plane.

Where the concrete parapet beam is relatively high as in Fig. 90, and the ornamental stone or terra cotta projects a considerable distance beyond the face

of the wall, or in other words, if the center of gravity of the entire parapet lies outside the face of the concrete wall, buttresses should be used at the columns and the wall reinforced as a counterfort wall between the buttresses. The main reinforcement in the buttresses (at the back) should be well anchored into the floor construction or into the column below, and after extending up the back of the buttress it should be bent down in front of the longitudinal wall bars, thus giving a positive support for the continuous wall slab. Buttressed parapet walls should also be reinforced to take care of the ordinary beam action which may develop due to the weight of the parapet. The same bending moment coefficients are recommended as for ordinary roof parapets previously described.

If the covering of brick or terra cotta is only a few inches thick it can be supported by spandrel angles anchored to the concrete, and placed wherever offsets occur in the veneer. In addition to these angles, corrugated metal brick ties set in the concrete should be built into the brick joints to aid in holding the brick in place. Terra cotta or stonework which projects 9 in. or more beyond the face of concrete should be anchored to the parapet by bolts passing through the webs of the blocks of terra cotta or by bent plate anchors securely fastened into the stone courses. These anchors should be used in addition to the spandrel angle supports previously mentioned (see Figs. 89 and 90). When a well-designed concrete parapet is used, there is no danger of failure of the main support of the cornice, and if the spandrel angles and anchors are properly placed and well covered with mortar when placing the veneer, practically all danger of part of the cornice falling is removed.

MISCELLANEOUS STRUCTURES

By Albert M. Wolf

95. Reinforced Concrete Stairs.

95a. General Types.—Generally speaking, there are three types of concrete stairs: (1) A simple inclined slab reinforced longitudinally between floors (or floor and landing) with the steps cast integral with the slab; (2) inclined stringer beams at sides between supports carrying a slab reinforced transversely between the beams and with the steps cast on the slab; and (3) separate pre-cast units for each step reinforced as individual cantilevers anchored in the wall of the stair well or as simple beams between walls. Some examples of spiral or helical stairs cast monolithically or as pre-cast units also exist.

95b. Requirements as to Width, etc.—Actual counts made by reliable authorities show that with freely moving crowds going in one direction, an average of thirteen people per foot of width per minute will pass down a stairway. In general, an allowance of two minutes per floor is made for complete egress. Thus, by dividing the number of occupants per floor by twenty-six (2×13), the combined width of stairs necessary to furnish proper exit for that floor is given. Since the occupants from all floors would not pass a given point in the two minutes specified, it would seem justifiable to compute the necessary width of stairs based on the number of people on the two most densely occupied floors.

The minimum clear width of stairs between handrails for warehouse buildings subjected to light traffic should be 3 ft. In office or factory where stairs are used considerably, a minimum width of 4 ft. should be used, and a width of 5 ft.

between handrails is recommended as best practice. It is generally assumed that a 3-ft. stair will accommodate two persons abreast, and a 4-ft. stair, three persons. In any case, no matter how few persons occupy a floor, two stairs should be used except for small buildings (those under 5,000 sq. ft. floor areas) in order that the distance to a stair be not too great.

The following requirements as to width and other details of stairs of fireproof construction, taken from the Building Code recommended by the National Board of Fire Underwriters, are representative of good practice, and can be followed except in large cities where building codes exist which must be complied with if more stringent than these.

Stairways used as required means of exit shall be at least 44 in. wide between faces of walls, or 40 in. wide between face of wall and an open balustrade, or between two open balustrades. All such widths shall be clear of all obstructions except that handrails attached to walls may project not more than 3½ in. within them. If newels project above tops of rails, a clear width of at least 44 in. shall be provided between the faces of the newel and the face of the wall or newel opposite. All stairs shall have walls or well-secured balustrades or guards on both sides, and except in dwellings, shall have handrails on both sides. A stairway 7 ft. or more in width shall be provided with a continuous intermediate handrail substantially supported. All stairs shall have treads and risers of uniform width and height throughout each flight; the rise shall be not more than 7¾ in., and the tread, exclusive of the nosing, not less than 9½ in. Stairways exceeding 12 in. in height shall have an intermediate landing at least 3 in. in length.

All stairways that serve as required means of exit for one or more of the upper four stories of every building shall be continued their full width to the roof and shall lead by a direct line of travel to the first story, and open directly on the street, or to an open-air or fireproof passage leading to the street, or to a yard or court connected with the street. Such fireproof passage shall be not less than 7 ft. in height.

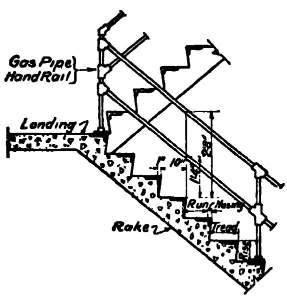


Fig. 91.—Diagram showing rise, run, tread, etc. of stair.

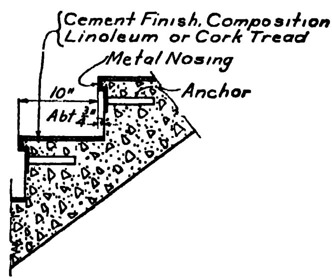


Fig. 92.—Stair with projecting nosing.

To make stairs safe for traffic under all conditions it is essential, also, that risers and treads be properly proportioned and bear a certain relation to each other and that where changes in direction are made they shall be effected by the use of landings and platforms instead of winders.

The run of a stair is the horizontal distance from the face of one riser to the next (see Fig. 91). The horizontal portions are called the treads, and the faces of steps the risers. The tread should be distinguished from the run since the former is usually from ¾ to 1¾ in. wider than the run because of the nosing which is usually added, either by sloping the riser forward or by projecting the tread as in Figs. 91 and 92. In concrete buildings the type of step shown in Fig. 91 is used most because of the relative simplicity of the formwork.

Rules for proportioning of risers and treads which have become recognized as giving best results are:

(*a*) The sum of two risers and a tread should not be less than 24 in. nor more than 25 in.

(*b*) The product of the rise and run shall not be less than 70 nor more than 75. From these it will be seen that the width of run should be determined by the rise—that is, the less the rise, the greater should be the run, and vice versa.

The relative proportions of rise to run should be the same throughout the entire run of stairs from one floor to another, for in walking up or down a stair a person instinctively makes the same step each time and if there is any change in the rise or run a mis-step is likely to occur which may prove very serious, especially in case of fire or panic. Statistics and records show that large percentage of the accidents on stairs are due to slipping or tripping on the treads, and it is therefore important that the steps be provided with nosings or treads which are not slippery or will not wear smooth and give a false feeling of security which may prove disastrous.

95*c*. Designing Loads and Unit Stresses.—Concrete stairs for warehouses, shops, stores and factories, are generally designed for a live load of 100 lb. per horizontal sq. ft. of stairs and landings, although this is considerably more than ever comes on most stairs. In general the loads for which stairs must be designed are fixed by city ordinance, but if no ordinance governs, a live load of 100 lb. per sq. ft. may be safely used for almost any concrete stair, except where subjected to additional loads carried up or down. Reliable experiments seem to indicate that a dense crowd of people may weigh as much as 140 to 150 lb. per sq. ft. on a level surface, but such a crowd could hardly be conceived as massed on a stairway, except perhaps in case of a panic, and then only for a short time, and the factor of safety is ample to care for the overload. In steel stairway construction this is not true, since the connections of strings are usually weak and any vibration caused by a rapidly moving crowd might be disastrous if the stairs were designed for a light live load.

The National Board of Fire Underwriters recommends:

All stairs, platforms, landings, balconies, and stair hallways shall be of sufficient strength to sustain safely a live load of not less than 100 lb. per sq. ft. for interior construction, and 150 lb. per sq. ft. for exterior construction, with a factor of safety of 4 in each case.

For unit stresses to govern the design of stairs, the following are recommended:

Compression in cross-bending on extreme fiber of 1:2:4 concrete 700 lb. per sq. in.

Tension in steel, medium grade 16,000 lb. per sq. in.; high carbon grade 18,000 lb. per sq. in.

Bond stress 80 lb. per sq. in. for plain bars and 100 lb. per sq. in. for approved deformed bars (not square twisted).

95*d*. Methods of Moment Calculation.—The available literature on the action of forces on inclined bent beams is rather meager and in fact most works on mechanics and statics neglect the subject entirely. To obtain a correct idea of the action of stair slabs under load, a knowledge of the action of inclined and bent beams is necessary.

Rankine in his treatise on Mechanics says:

A sloping beam (a simply supported beam is here referred to) is treated like a horizontal beam so far as the bending stress produced by that component of the load which is normal to the beam is concerned. The component of the load which acts along the beam is to be considered as producing a direct thrust along the beam, which is to be combined with the stress due to the bending moment of the load.

In stair construction, however, the condition of slabs is usually that of a beam fixed at one end and approximately simply supported at the other, or a beam fixed (or partially so) at both ends. For beams of this character, uniformly loaded, Merriman demonstrates in Mechanics of Materials:

Where the lower end of an inclined beam is supported and the upper end fixed, and the difference in levels is considerable (as in stairs), the reaction on the supported (lower) end is zero and the beam is a cantilever fixed at the upper end; while for a fixed end beam the moments are negative at both ends, the beam being in the condition of a pair of constrained cantilevers uniformly loaded and having a concentrated load at the free end. The maximum moment occurs at the upper end and its theoretical value is

$$M = \frac{1}{12} wl^2 - \left(\frac{6EI}{l^2}\right) h$$

where w = the uniform load per unit of length, l = the horizontal distance between supports, and h = the vertical distance between supports.

Since absolutely fixed ends may not always be obtained on account of construction joints at top and bottom of flights, and since if stub bars are provided at such joints the condition of ends is that of partial fixity, it would seem logical and on the side of safety to use a moment of

$$M = \frac{Wl}{10}$$

for both positive and negative moments, in which W is the combined total dead weight of the inclined slab and landings, if included in the span, and the total live load measured at 100 lb. per sq. ft. of horizontal projection, and l is the horizontal span between supports.

95e. Slab Thickness and Reinforcement.—Using the basis of design recommended, the following table of slab thicknesses and reinforcement for various horizontal span distances between supports has been computed. These slab thicknesses will also apply to slab span, including horizontal landings as well as the inclined stair slab, such as the design shown in Fig. 93.

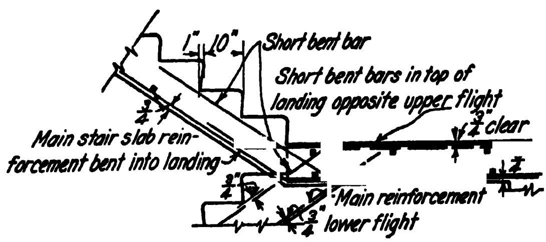


FIG. 93.—Typical reinforced concrete stair detail at landing

TABLE OF SLAB THICKNESSES AND REINFORCEMENT FOR STAIRS

HORIZONTAL SPAN	SLAB THICKNESS	MAIN REINFORCEMENT
7 feet 0 inches	4¼ inches	⅜ inches sq. 5½ inches o.c.
8 feet 0 inches	4¾ inches	⅜ inches sq. 5 inches o.c.
9 feet 0 inches	5½ inches	⅜ inches sq. 4½ inches o.c.
10 feet 0 inches	6 inches	½ inches sq. 6½ inches o.c.
11 feet 0 inches	6½ inches	½ inches sq. 6 inches o.c.
12 feet 0 inches	7¼ inches	½ inches sq. 5½ inches o.c.
13 feet 0 inches	7¾ inches	½ inches sq. 5¼ inches o.c.
14 feet 0 inches	8½ inches	½ inches sq. 4½ inches o.c.

In addition to the main reinforcement as given in the table, which is the amount required to resist negative as well as positive moments, ⅜-in. distributing and spacing bars should be placed above the main reinforcement, as shown in Fig. 94. The main bars should be of such lengths as adequately to reinforce all possible regions of tension. This means that the negative reinforcement should extend about one-fourth the distance between supports from each support, and where landings are included in the span, the negative reinforcement should be carried well into the inclined slab, since the stiffness of the landing slab connected to the two inclined flights may cause negative moments in the flights for some distance out from the landings.

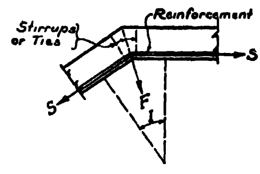


Fig. 94.—Stresses at stair landing.

With stairs of the type shown in Figs. 95 to 97, it is highly essential that the arrangement of reinforcement at points where horizontal and inclined slabs join be given very careful study. As indicated in the large scale detail of Fig. 93 the bars at such junction points should be bent so that when tending to straighten

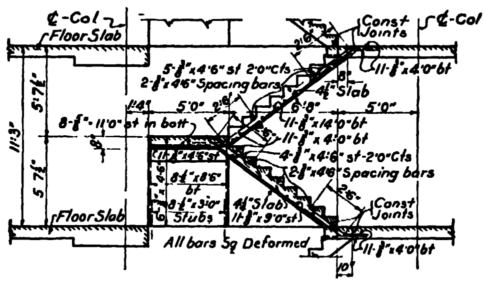


Fig. 95.—Two-flight stair.

out under the influence of tensile stresses nearly the full thickness of slab will be available to resist such a tendency—in other words, the bending of bars in any particular plane of a slab should not be such as to follow directly along the tension face of both sides of the "break" in the slab, unless stirrups are placed around such

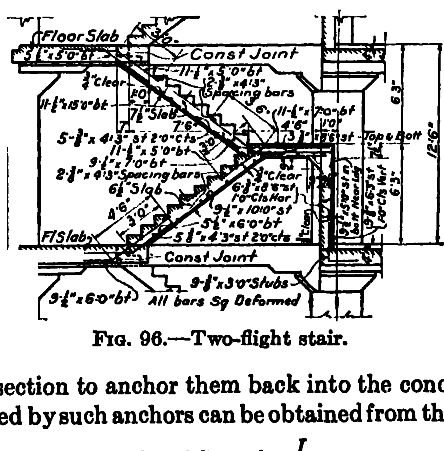


Fig. 96.—Two-flight stair.

bars at the critical section to anchor them back into the concrete. Some idea of the force to be resisted by such anchors can be obtained from the following formula:

$$F = 2S - \sin \frac{I}{2}$$

where S = stress in the bar, and I = the angle made by the intersection of perpendiculars to the horizontal and inclined slab sections (see Fig. 94).

The use of relatively small sized bars at fairly close spacing is to be recommended in stair construction, because they may be easily bent in place. Where the stairs are poured after the floors are completed, as is often the case, the reinforcement should be so arranged that relatively short stub bars can be placed in the floor at the time of pouring to lap with the main bars in bottom of slab and to provide negative reinforcement in top of inclined slabs. Construction joints should be indicated in Figs. 95 and 96.

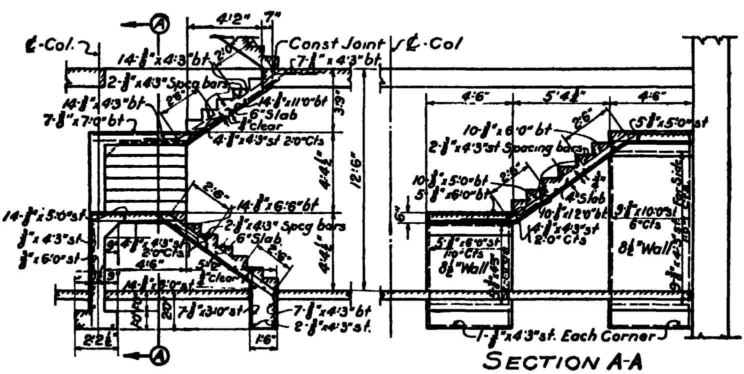


FIG. 97.—Three-flight stair.

95*f*. Examples of Factory and Warehouse Stairs.—The most common type of concrete stairs used in reinforced concrete factory, warehouse and mercantile buildings is the inclined slab type, wherein the slab supports the steps (with which they are integrally cast) and is reinforced longitudinally between the floors and landings which the flight connects. The simplest form is that of a single inclined slab or flight connecting two floors, where the story heights are relatively low. When the story heights are more than 9 or 10 ft. it is better practice to use at least two flights with an intermediate landing, as shown in Figs. 95 and 96, thus decreasing the slab spans and thickness required. The flights of this stair were designed as inclined slabs between the floors and the landing, which was reinforced as a slab between the concrete side walls to carry the reactions from flights and its own dead and live load. Similar stairs are often built with the landing slab supported on the brick enclosing walls of the stair well. The stairs shown in Fig. 96 were designed as inclined and bent slabs spanning from floor to the concrete wall at the back of the landing. Where heavy brick walls enclose the stair well, as is often the case, one of the walls may be used to support the back of the landing, instead of the concrete wall; or if the landings are wide, they can be supported on transverse beams, one of which is designed to take the stair flight reactions in addition to the landing slab reaction. However, in modern concrete building construction, it is the usual and best practice to make the stairs self-supporting, avoiding the necessity of laying brick while concrete work is going on, or of building the stairs after the floors are finished.

The three-flight concrete stair shown in Fig. 97 was designed on the same basis as the stair shown in Fig. 96, the upper and lower flights and landings being designed as bent slabs, strong enough to take the reaction of the intermediate flight carried by them. The stair illustrated in Fig. 98 is the opposite form, the upper and lower flights being supported at one end by the slab forming the intermediate flight and landings. It will be noted that one end of this bent or

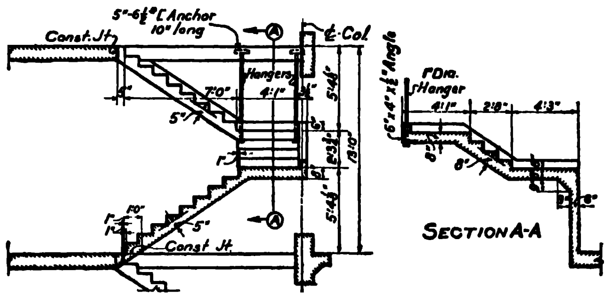


FIG. 98.—Three-flight stair with hangers.

"broken back" slab is supported by a concrete wall resting on the lower floor, while the other end is suspended from the floor above by rod hangers. Hangers can be often used advantageously in stair construction to support landings or ends of flights where concrete walls would be expensive on account of their height or where it is desired to omit the enclosing stair walls. The hanger rods should preferably be encased in concrete to protect them in case of fire, which

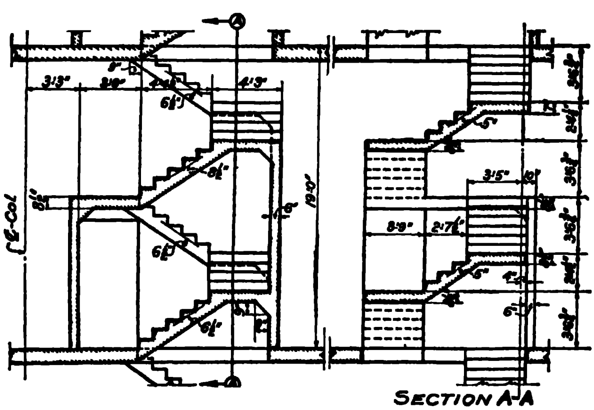


FIG. 99.—Six-flight stair.

might prove disastrous to the stair construction if they are left unprotected. The curb shown on one side of the upper flights is not a structural feature, being added as part of the architectural treatment and to prevent small articles accidently dropped on the stairs breaking the glass in the steel sash and partitions enclosing them.

A reinforced concrete stair of rather unusual construction is shown in Fig. 99. The six flights can be divided into an upper and lower group, designed in the

same manner as the three-flight stair shown in Fig. 97. Such construction is necessarily rather costly and is only justifiable where the story heights are great (19 ft. in this particular case) and the stair must be kept within a minimum amount of space.

In warehouse buildings and structures of a similar character where the stairs are used but seldom and then by only a few persons, long single flights of stairs are permissible, and are often used. These can be built most economically by reinforcing the slab carrying the risers, transversely between inclined concrete beams spanning from floor to floor. Such slabs can be made relatively thin (3 or 4 in. for stairs 3 to 5 ft. wide) since when the risers are cast integral with the slab, each step can be considered to form a trapezoidal beam, the effective depth of which is considerably more than that of the slab considered alone.

96. Framed Towers.—In large buildings good practice requires the use of water tanks above the roof to insure water supply for house purposes and fire protection. The tendency during the past few years has been to house these tanks in a tower which may or may not be treated as an architectural feature. Usually such towers consist only of one panel of the building carried up to the desired height.

In addition to designing the columns, girders, beams and slabs composing the tower for the direct loads coming upon them, the wind load stresses under different conditions must be investigated and combined with the direct load stresses.

Since the tower usually occupies a relatively small portion of the building area, the tower can be considered as a separate structure, with its columns fixed at the roof line if the reinforcement is continuous at the connection. Due to this rigid connection, the tendency of the wind pressure to overturn the tower as a unit is overcome and the other tendency of wind pressure—namely, to collapse the tower—must be taken care of in the design of the individual members. The tower should, however, be carefully investigated as to overturning with tank empty and sufficient anchorage provided to prevent it.

Inasmuch as towers are usually exposed to the full effects of the wind, it is advisable to use a wind pressure of at least 30 lb. per sq. ft. of vertical surface in the design of the members. As to unit stresses for members under combined wind and gravity loads it is well-established practice to allow a 50 per cent increase in unit stresses for wind alone or combined with gravity stresses; the section of the member however, must not be less than required for the gravity loads.

The architectural design of a tower, if enclosed with brick walls, usually precludes the use of a diagonal bracing in a vertical plane and hence the resistance to collapse under wind pressure must be gained by the use of rectangular or portal framing instead of triangular framing. In resisting the horizontal force of wind, a rectangular frame tends to distort and hence develop shearing and bending stresses. These must be ascertained and combined algebraically with the gravity stresses and the members designed accordingly, but in no case should the members be decreased in strength below that required by the gravity loads.

In Fig. 100 assume hinges at the points of contraflexure of the members as indicated by the moment diagram, Fig. 101. For a four-column tower the wind load W acting on one bent of two columns will be *one-half* of the total wind load on the exposed surface. The bending moments due to this load at a, b, c and d

in the vertical members and at a and b in the horizontal members will all be equal and have a value of $\frac{1}{4}WH$. Besides these bending stresses a direct compressive stress of $\frac{1}{2}W$ will develop in ab, and a compressive stress $V = \frac{1}{2}W\frac{H}{L}$ in member bd, and a tensile stress of the same amount in member ac. A shear of $\frac{1}{2}W$ must be resisted at the base of each vertical member.

If the tower is more than one story high, each story can be investigated for wind stresses in the manner just described.

In detailing the reinforced concrete members, substantial fillets at beam connections with columns should be used and reinforcing bars placed in these fillets to tie the members rigidly together. Since the beams under wind load act as compression struts, the top and bottom beam reinforcement should be tied together as in rodded columns by the use of closed loop stirrups.

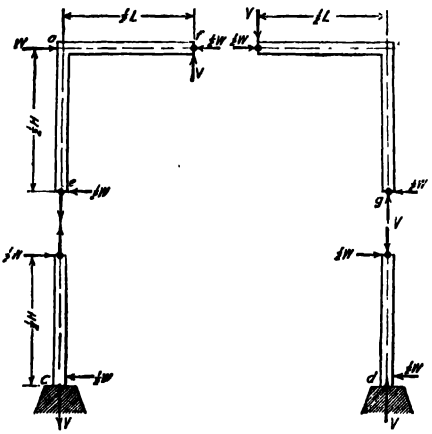


Fig. 100.—Illustrating wind load and reactions on a stiff bent.

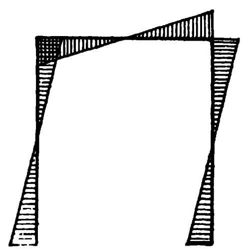


Fig. 101.—Bending moments in framed bent.

The layout of beams for the tank floor will, of course, depend upon the spacing of tower columns in each direction and the size of tank. Generally speaking, however, the tower will be approximately square and the tank of such size as to cover the major area of the tower floor (see Fig. 101a). Under such conditions, the greatest economy will result in using two floor beams in each direction, framing into the girders at the third-points. If these floor beams are all of the same section and same span, each will carry one-fourth of the tank load regardless of the intersection of beams at the third-points of the panel. The floor beams can be designed as T-beams while the girders should be designed as rectangular beams.

97. Elevator Pits and Elevator Machinery Supports.

97a. Elevator Pits.—The fundamental considerations in the design of elevator pits are: (1) Depth of pit, sufficient to provide for buffer installation and for travel of doors; (2) waterproofing and drainage; and (3) reinforcement of pit walls to prevent cracks and resultant seepage.

Depth of Pit.—For the type of elevators generally used in factories and warehouses of reinforced concrete, the depth of pit required below the lowest floor served, is governed not so much by the buffer requirements, as by the type of door used. Ordinarily a depth of $3\frac{1}{2}$ ft. is sufficient for the proper installation of buffers, but if the Meeker type or Peelle counterbalanced doors are used at open-

ings, the pit must have sufficient depth to allow the lower half of the door to travel into the pit. The depth usually required is from 6 in. to 1 ft. more than

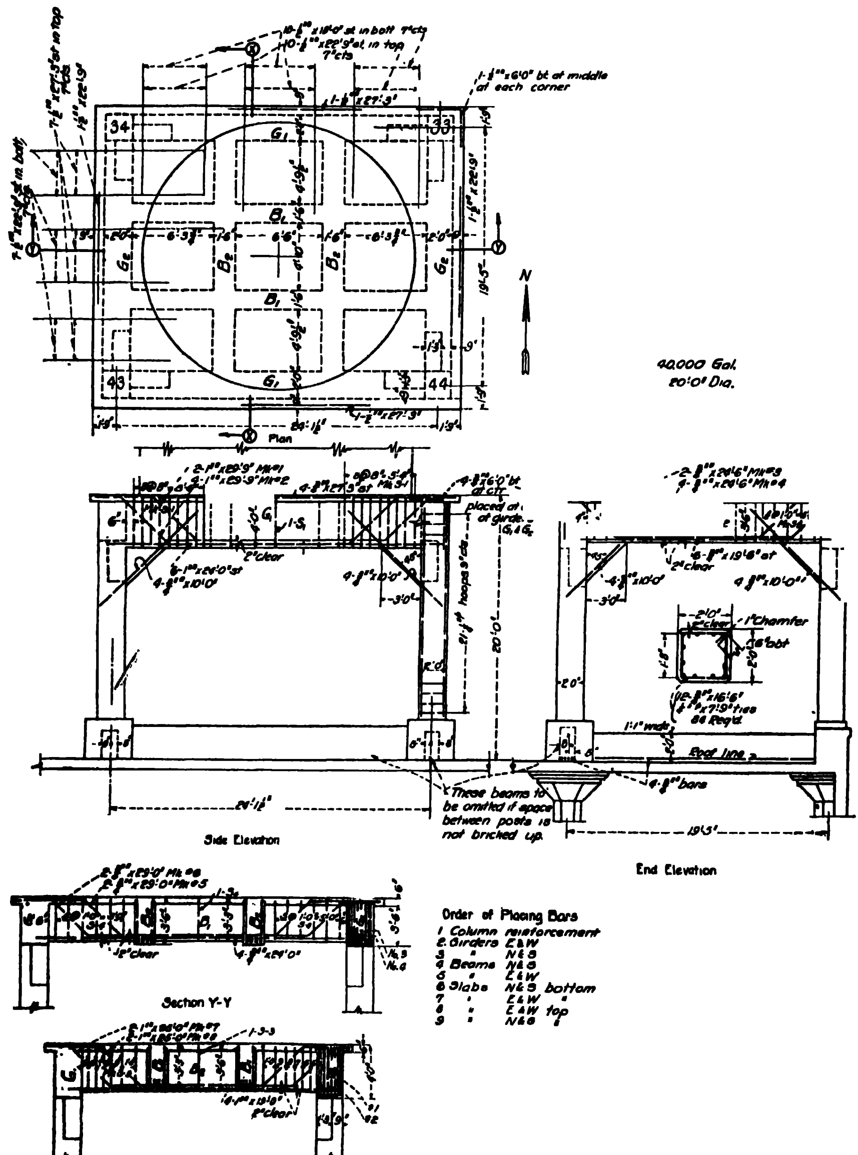


Fig. 101a.—Typical framed tower of reinforced concrete.

half the height of the door opening at the lower landing depending on the make of door used, the Peelle type requiring more than the Meeker. Since elevator doors are generally made 7 or 8 ft. high, the depth of pits necessary will vary from 4 to

5 ft. The elevator contractor and the door manufacturer should, of course, be consulted before designing pits for special elevators or those involving the use of unusual types of doors.

Waterproofing and Drainage of Pits.—Since in most factories and warehouses the elevators run to the basement, the bottom of the pits will be below the sewer and drain lines. This means that special attention must be given to waterproofing, or ejectors provided to pump out the seepage collecting in the pits. If the pit is to be made waterproof, all column or wall footings adjoining the elevator shaft must be carried down so that the portions under the pit are at least 6 in. and preferably 1 ft. below the finished floor line of the pit. This will allow the construction of the pit floor as a monolithic slab with reinforcement extending both the length and width of the pit, and bent up into the side walls near the outer face to act as cantilever reinforcement. A key should be formed in the top of the slab directly under the walls, and the walls poured as soon after the base as possible so as to obtain a watertight joint. A rich and very dense concrete (with an integral waterproofing compound or hydrate of lime added if the work is to resist any considerable head of water) should be used in elevator pit construction, and the concrete should be very thoroughly tamped and spaded to insure slabs free from voids and stone pockets. To aid in preventing the formation of shrinkage cracks in the top of walls, at least two ½- in. diameter bars should be placed near the top, bent to lap at the corners with bars in walls at right angles.

Reinforcement of Pit Walls.—Except for the smallest size pits, it is poor economy to use less than 8- to 12-in. walls and floors owing to the difficulty of waterproofing thinner slabs. The sidewalls of the pit are supported at the bottom by the pit floor and at the ends by the walls at right angles. This means that they can be reinforced as cantilever walls to resist the earth pressure, or, when the pits are very deep, the top portion of the side wall for a width of a foot or so can be reinforced as a beam between the walls at right angles to it and the slab between this and the pit floor reinforced as a vertical slab, or the wall may be considered as a restrained slab between end walls. The bottom of the pit should be reinforced between side walls as a two-way slab resisting the upward pressure of water (if the pit is under hydrostatic pressure) and the bars can be bent up into the side walls to act either as cantilever reinforcement or as vertical slab reinforcement on the inside of slab between bottom and the assumed beam at the top. In deep pits the corners should be tied together by short bars bent at right angles and placed at intervals in height of walls near outer face to act as negative moment reinforcement.

Sometimes the tops of side walls are anchored to the adjacent floor slab in lieu of reinforcing the top portion as a beam, but this action is rather uncertain and any settlement of floor or pit will cause unsightly cracks in the floor. The best practice seems to indicate that the pit walls should be built up to the level of the top of the floor slab and the latter separated from the pit walls by an expansion joint filled with asphaltic cement, thus allowing the floor to be placed at any convenient time after the pit is completed.

97b. Elevator Machinery Supports.—The machines for operating elevators are generally placed on supports spanning the shaft at some distance (usually 3 to 4 ft.) above the roof line of the building. In the great majority of

cases structural beams are used as supports for the elevator machine and a plank floor laid on them to furnish a platform for workmen inspecting or repairing the machinery. Sometimes the plank floor is omitted and a protective screen of wire mesh placed below the beams.

In fireproof buildings of reinforced concrete it would seem more logical to construct the elevator machinery floors of a fire-resisting material such as concrete, or structural steel encased in concrete, and thus avoid the danger of the elevator machines being dropped into the basement by a small fire starting in or about the pent-house while the remainder of the structure might be unharmed.

Just which type of floor will prove the most economical for any given case depends upon the size, speed and capacity of elevator, the arrangement of the machinery and time of placing the floor with respect to the completion of the remainder of the building.

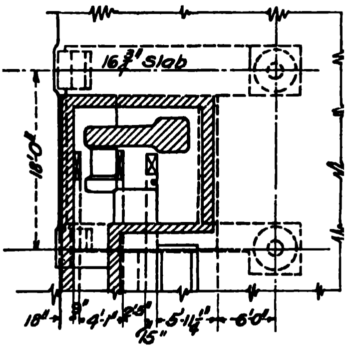


FIG. 102.—Concrete elevator machinery floor built as part of main roof.

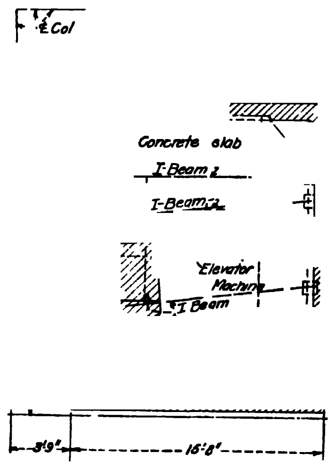


FIG. 103.—Elevator machinery floor of concrete supported on steel beams.

Reinforced Concrete Supports.—For comparatively small elevators in buildings with high story heights which will allow placing the elevator machinery floor at the same level as the roof, reinforced concrete may be used to good advantage, unless the arrangement of holes in the floor is such as to make the main support members very narrow and deep. An elevator machinery floor of this type is shown in Fig. 102. The roof construction is of the flat slab type (two way reinforced) with the portion of the slab supporting the elevator machinery and the pent-house walls deepened sufficiently to make up the resisting moment required to support the additional loads imposed. The slab being built as a monolithic portion of the main roof, considerable advantage is gained due to continuous beam action, which is not the case when the machinery floor is raised some distance above the roof, which makes all beams and slabs simply supported.

Steel and Concrete Supports.—For complicated machinery layouts requiring skewed setting of beams similar to that shown in Fig. 103, reinforced concrete is not economically adapted since the loads are usually concentrated at points where the floor is pierced with various openings, making impossible the economi-

cal design of concrete beams to carry the loads. The larger the elevator and the higher the speed at which it operates, the greater are the loads (and hence impact) to be carried by the machinery floor and also the difficulties attendant with the design of concrete beam and slab floors. Where there is only a width of a few inches between openings with heavy concentrations on the slab nearby, structural steel beams present practically the only satisfactory solution of the problem of design of main supports of the elevator machine, for very narrow and deep concrete beams are impractical, and they also materially cut down the head room or necessitate a raise in the level of the floor if used. (The depth of floor construction shown in Fig. 104, a concrete floor, should be compared with that in Fig. 105). Then, also, if elevator service is desired as soon as possible after the floors of the building have been completed, the machines should rest on supports which can be placed and used immediately. Concrete beams must cure and harden for a few weeks before being subjected to suddenly applied loads producing much impact, and, therefore, where speedy completion of the elevator plant is required, reinforced concrete is out of the question, at least for the main members of the floor.

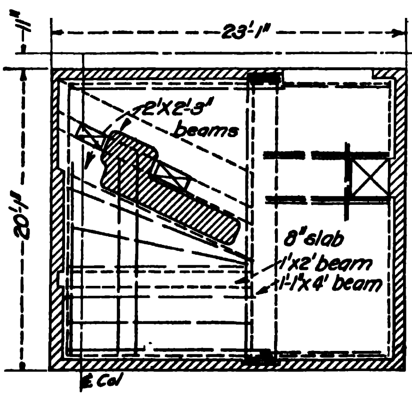


FIG. 104.—Elevator machinery floor of reinforced concrete.

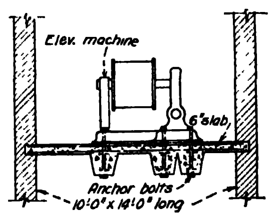


FIG. 105.—Section of elevator machinery floor of concrete on steel beams encased in concrete.

Structural steel beams can be quickly set in place, the elevator machine blocked up on them, and the reinforced concrete floor slab and the encasement of the beams poured at any time thereafter in forms suspended directly from the steel beams without other support, except for very long spans between beams. Herein lies one point of economy over the floor with reinforced concrete beams, which when poured must be supported from below by costly centering and formwork spanning the elevator shaft. Of course this economy will not always be apparent in the bids on a large job, for this item is a very small percentage of the total cost of the building. Nevertheless, the writer has found by careful comparative estimates made on machinery floors with layouts of machines and openings giving the reinforced concrete floor its most favorable arrangement, that a concrete floor on structural steel I-beams or channels costs the same or even less, to say nothing of the advantages gained in speed of erection, saving in headroom in the elevator shaft and ease of erecting the elevator machinery on steel beams before placing the floor as compared with placing it afterward as is the case where reinforced concrete supports and floors are used.

Design of Elevator Machinery Floors.—A typical elevator machinery floor detail is shown in Fig. 105. The main supporting beams consist of pairs of channels set back to back with space enough between to allow placing the anchor bolts for the machine with washers on the underside. This allows very accurate setting and leveling of the machine and a more convenient layout of openings in the floor for cables, counterweight guides, etc. because of the relatively small width taken up by the structural beams as compared with those of concrete. The concrete slab should be reinforced transversely between the supporting beams and the side walls on a corbel or better still a 6- or 8-in. bearing on the pent-house walls. On account of the heavy concentrations and the fact that the slab will tend to act as a continuous beam over the steel beams, it is essential that reinforcing bars as required by moment $\left(M = \frac{WL}{12}\right)$ should be placed in the top of the slab at such points. The slabs should not be designed to carry the elevator machinery, the load from which goes directly to the steel beams on account of their location directly under the same, but a live load of from 150 to 200 lb. per sq. ft. of surrounding floor, depending upon the weight of machine parts which may be handled thereon. Owing to the fact that the loading may be non-uniform and concentrated, it is well to provide reinforcement in the bottom of the slab to resist a moment, $M = \frac{WL}{8}$. For this same reason the slab should be made somewhat thicker than actually required for the most economical percentage of steel. It should be remembered that in special structures of this type it does not pay to save a few pounds of steel or cubic feet of concrete by making assumptions which do not provide for unusual, but plausible conditions of loading. At least 0.3 per cent of the distributing steel should be placed in the bottom of the slab parallel to the beams.

The steel beams should be designed to carry the loads indicated on the elevator manufacturer's layout, doubled to provide for impact, and the proper proportion of the dead and live load of slab, with bearing plates at ends to distribute the load over a portion of the pent-house walls. Before encasing the beams they should be wrapped with a wire mesh or netting to give a bond for the fireproofing. The top of the beams if placed at a level 1 in. above the bottom of the floor slab, furnish a support for the lower slab bars, and insure the proper location of steel in the slab.

98. Ramps.—The great increase in the use of automobiles and automobile trucks during the past few years has resulted in the construction of numerous multi-storied garages of reinforced concrete. In such buildings, housing a large number of cars, the use of elevators alone to take up and bring down cars is not economical or rapid; hence, various forms of ramps, or inclined slabs have been designed upon which the cars can be driven from floor to floor.

In designing a building with ramps it is very important that they be so located as to give a uniform layout for each floor and that they be as straight as possible, thus occupying the minimum amount of space. This requires that the ramp system be located as a flight of stairs and using the aisles to connect the ramp ends. Typical layouts for ramps in a building 100 ft. square and 100 by 150 ft. are shown in Figs. 106 and 107 respectively.[1]

[1] From an article in the *Architectural Forum*, Nov., 1921, by Harold F. Blanchard.

These plans show that while a straight ramp cannot be laid out in a building 100 ft. long, a length of 150 ft. will allow such a design.

The steeper the ramp, the less will be the space occupied, but since a 20 per cent grade is about as steep as the average car will ascend in second speed, the grade should be something less than that, but not necessarily less than 15 per cent.

Actual experience shows that for garages housing up to 250 to 300 cars, single-track ramps are sufficient—that is, the same ramp is used for both up

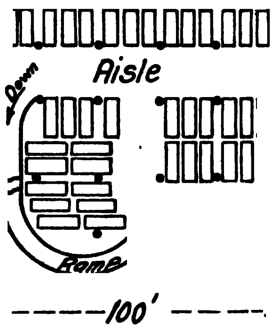


FIG. 106.—Floor plan garage 100 ft. square with curved ramps. Capacity 46 cars.

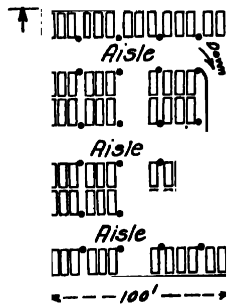


FIG. 107.—Floor plan garage 100 ft. by 150 ft. with straight ramp.

and down traffic. The ramp proper should be from 8 to 9 ft. wide with the outside curb 12 in. wide and the inner curb 9 in. wide and 9 in. high. The ramp should be designed on the same basis as a stair slab, or if advantage is taken of the curb construction these can be designed as inclined beams supporting the ramp slab.

The economy of operation in ramp garages doubtless has been the cause for the patenting of several special ramp designs. In one of these the entire floor of the garage slopes, being helical in form, with modifications to fit a square building. This design is very economical of space but the sloping floor is a disadvantage both from the construction and operating standpoint. If the floor is of the flat slab type, however, the disadvantage from the construction standpoint is minimized. Such floors are best designed as ordinary flab slab construction, making due allowance for the side thrust on the columns due to the pitch of the spiralled floor. The greater the pitch of the floor, the greater will be the component of the load which produces a direct thrust along the slab.

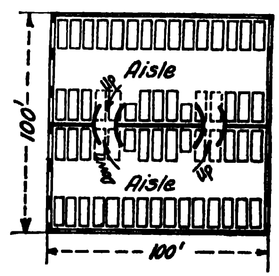


FIG. 108.—Floor plan garage 100 ft. square with patent ramp design and staggered floors. Capacity 52 cars.

Another patented type of ramp design has the building divided into two parts, the floors in one part being offset one-half the story height from those in the other half. The floors in the two sections are connected by the ramps near the ends of the building, each rising one-half a story height at a time (see Fig. 108). This type of ramp takes up little more space than would be occupied by one

elevator. These ramps have the advantage of being open, and for fire protection automatic fire doors can be placed at the ramp opening in the dividing wall.

99. Expansion and Contraction in Buildings.—Although the influence of temperature changes on the expansion and contraction of concrete may be considered a secondary one, it is easy to see that in some cases these changes combined with the effects of the hardening of the concrete mass may produce stresses greatly in excess of those produced by hardening alone. The Boonton Dam experiments and others show conclusively that heat is generated in large masses of concrete due to the chemical action taking place when the concrete hardens. Now when this chemical action ceases and the heat dissipates, additional contraction due to fall in temperature will result, besides that which takes place during the hardening process of concrete cured in air. In mass concrete poured in the late fall there is the combination of contraction due to fall of temperature of the concrete mass, to fall of atmospheric temperature and that contraction which takes place when concrete hardens in air regardless of temperature changes. For example, let us suppose that the drop in temperature of the concrete mass for a period of six months due to atmospheric change and the dissipation of the heat generated by the chemical action is 100 deg. F.; then the contraction due to the atmospheric change would amount to 0.00055 in. per inch[1] of length and that due to the drying out of the mass to 0.00068 in.[1] or a total of 0.00123 in. per inch of length. If the concrete structure was 50 ft. long, the total contraction would amount to nearly ¾ in. and if rigidly held at the ends this contraction would manifest itself in one or more cracks in the concrete.

With the above information at hand it can be said that in most cases it is contraction joints and not expansion joints that must be provided in long concrete structures in order not to have unsightly ragged cracks formed. This, however, does not hold good for a wall with recesses, sharp bends or angles. In such cases the expansion due to increase of pressure is sure to rupture the wall at an angle due to the fact that the expansive forces are acting against each other. Numerous cracks at the corners of concrete reservoir walls and at points in concrete retaining walls in track elevation work, where walls have been built at the same time as abutments with which they connect at right angles, are examples of this disruptive force.

Shrinkage cracks are sure to manifest themselves at points where large masses or thick walls join relatively smaller or thin walls unless provision is made in design to care for the shrinkage stresses. This can be accomplished in a way by the use of fillets, thereby reducing gradually from the larger dimensions to the smaller. Shrin. age cracks are very likely to occur at points where new concrete is joined to that which has already set and for this reason it is desirable to have construction points made on horizontal or vertical lines if for no other reason than for the sake of appearance.

In straight walls contraction joints properly placed will care for all expansion which takes place after hardening, due to moisture or rise of temperature. These joints which only need to be a fraction of an inch in thickness so as to insure free movement between the two portions, should be placed so that when the walls contract, the shortening will take place at the joints and not at intermediate

[1] Figures derived from tests by the Office of Public Roads.

points. That is to say, they shall be so placed as not to allow fixed end conditions to arise due to friction between the wall and fill and due to weight of wall which if present would be sure to cause contraction cracks between contraction joints. The desired result can usually be accomplished with joints every 25 to 40 ft. The joints should be so arranged and constructed as to prevent them from being filled with earth, gravel, etc. which would not allow proper expansion and result in excessive compressive stresses being set up in concrete.

In reinforced concrete walls contraction joints can be placed farther apart, for if properly reinforced, the steel tends to distribute the contraction stresses and to make the cracks very small, in fact practically invisible, but never entirely eliminating them.

Assuming that reinforced concrete will shrink the same as plain concrete, and that the value of 0.0003 in. per in. obtained for a reinforced concrete specimen mentioned in the tests cited above indicates the net contraction and that there is a high compression in the steel which tends to resist the shrinkage, by using the value of 0.00068 as the unit of shrinkage we can obtain the stresses in steel and concrete.

The net contraction as measured by the concrete will be $C = (f_c - E_c)$ where C = the coefficient of contraction of plain concrete, f_c = unit tensile stress in concrete and E_c = the modulus of elasticity of concrete. The contraction as measured by the steel will be $f_s - E_s$, where f_s = the compressive stress in steel and E_s = the modulus of elasticity of steel. These two values must needs be equal and for equilibrium $f_c = pf_s$ where p = the steel ratio.

The value of f_c can be found as follows:

$$f_c = CE_c \frac{np}{1 + np}$$

Now if $C = 0.00068$, $E_c = 2{,}000{,}000$, $n = 15$ and $p = 1$ per cent, $f_c = 177$ lb. per sq. in. tension and $f_s = 17{,}700$ lb. per sq. in. compression. The actual stresses are probably not as large as indicated by the formulas because of a probable adjustment in the concrete developing.

The figures, nevertheless, indicate that stresses due to skrinkage of concrete are very important and in many cases should be taken into account in designing structures in order to prevent cracks or failures. Then also the repetition of the stresses caused by alternate expansion and contraction may subject concrete to a fatigue which will ultimately cause failure.

The changes in length in concrete buildings due to changes in temperature are relatively small as compared with those taking place in bridges, walls and other exposed structures. This, it is at once apparent, is due to the fact that the inside temperature of a building is fairly uniform and nullifies to a large extent the effects of changes in outside temperature except of course, for the roof. The amount of reinforcement required in a concrete roof to carry the relatively light roof load is comparatively small, because the roof slab thickness is usually fixed by ordinance, as for example, a minimum of 6 in. for flat slab construction which is greater than required for the most economical percentage of reinforcement. This means that the reinforcement necessary to carry properly the applied load may not be sufficient to care for temperature stresses developed. For large exposed roofs and parapet walls the usually accepted 0.3 of 1 per cent

reinforcing is not sufficient and a minimum of 0.5 of 1 per cent of reinforcing steel is therefore recommended.

At the present time there is no uniform practice as to the provision of expansion and contraction joints in reinforced concrete buildings. Cases can be found where buildings 400 ft. long and over without expansion and contraction joints show no ill effects from the force of expansion and contraction exerted upon them, while other buildings of considerably less length can be found which show clearly evidences of damage due to these forces. Undoubtedly the former structures have resisted the stress set up by expansion and contraction mainly

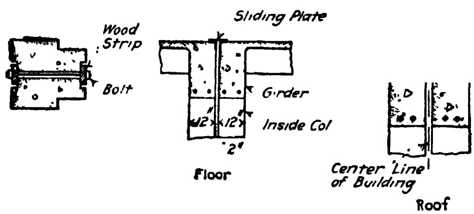


FIG. 109.—Typical expansion and contraction joints in concrete building.

because the method of reinforcing the floors was given proper consideration.

While no definite rules can be given to suit all cases since the height and location of the building are factors which must also be considered, in general it will be found good practice to place expansion and contraction joints in buildings over 3 stories high and over 300 ft. long. For example, buildings of 400 to 500 ft. long should be divided into two sections; buildings 600 to 800 ft. long into three sections by joints, and so on.

It is essential that these joints separate the sections of the building entirely so that the different units will be free to move independently of one another.

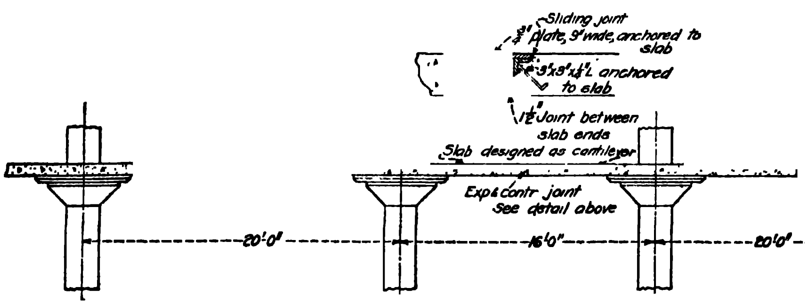


FIG. 110.—Longitudinal section of long building showing expansion and contraction panel.

This is usually accomplished by constructing two independent sets of columns and beams at the expansion and contraction joint separated by a space of 1 in. or more (Fig. 109). The columns usually rest upon the same footing since the movement of the building is practically nil below ground, except possibly in the case of one- or two-story structures.

To prevent weather and dirt from entering the space between outside columns and the roof sections a metal diaphragm of a "V" shape made of copper or sheet lead should be inserted in the beams and columns adjacent to the opening.

Such a diaphragm will allow free movement of the different units and still keep the space closed tightly.

On the interior of the building it is essential to the proper working of the joints that dirt be prevented from filling them and that the space between columns be closed from the standpoint of fire protection. The floor joint can be readily taken care of by sliding plates anchored to one of the slabs. The joint between columns can best be closed by sheet metal strips anchored to one column and slightly bent so as to spring tightly against the side of the other.

A more logical type of expansion and contraction joint especially adapted to flat slab buildings is shown in the accompanying detail (Fig. 110). Instead of using two separate lines of columns placed close together at the joint, the bay in which the expansion and contraction joint is placed is made of lesser width than the typical, that is, from 12 to 14 ft. depending on the width of the typical panels and has a joint down the middle so that the slab on either side cantilevers over from the row of columns adjacent. Such a joint has the advantage of making the design of floor panel in adjacent bays the same as for a typical panel owing to the continuous effect obtained by the cantilevered overhang. The joint should be so constructed as to allow free movement of the slabs and prevent dirt from entering. Further advantages are: The shape of columns can be readily made the same throughout; and the bending stresses in columns alongside the joint can be made independent of each other.

SECTION 6

RETAINING WALL DESIGN

By C. A. Willson

1. Retaining Walls in General.—Retaining walls are walls designed to withstand the lateral pressure of earth or similar material. They may be constructed of plain masonry such as brick, stone, or concrete, or they may be constructed of reinforced concrete. In a plain masonry wall, the weight of the wall itself must be sufficient to prevent overturning. A reinforced concrete wall is proportioned so that the weight of a part of the material retained may be utilized in stabilizing the wall. Due to the fact that a plain masonry wall depends upon its weight for stability, the term gravity is frequently used to denote this type of wall.

The pressure which a given material exerts upon a wall is dependent upon the character of the material, its compactness, cohesion, moisture content, coefficient of internal friction, and the coefficient of friction between the material and the wall. Seldom, if ever, are all of these factors known in a practical problem. Because of the lack of adequate experimental data it would be extremely difficult to determine the pressure in a given case even if these factors were known. However, the accumulated experiences of a large number of engineers extended over a period of years have shown that walls analyzed according to certain theories produce satisfactory results. Therefore this section will be restricted to the use of the two theories most commonly used in practice. Emphasis will be placed upon the methods of design and their application to practical problems.

2. Rankine's Theory and the Equivalent Fluid Pressure Theory.—Most retaining walls of both the gravity and the reinforced concrete types are designed according to either Rankine's theory or the equivalent fluid pressure theory. From the standpoint of design, the chief difference between the two theories lies in the fact that the effect of a sloping earth fill back of the wall may be determined by Rankine's theory whereas in the equivalent fluid pressure theory this difference is not usually taken into account, although it has been considered in this discussion as brought out later.

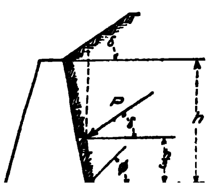


Fig. 1.

Since a straight-line variation in pressure is assumed in both theories, it is possible to reduce the pressure formula for a known set of conditions given by either theory to the following form:

$$P = C\frac{wh^2}{2} \quad \text{(See Fig. 1)}$$

in which P = total earth pressure in pounds.

w = weight of material retained in pounds per cubic foot.

h = height of wall in feet.

$$\cos\delta\,\frac{\cos\delta-\sqrt{\cos^2\delta-\cos^2\phi}}{\cos\delta+\sqrt{\cos^2\delta-\cos^2\phi}}\ \text{for Rankine's theory.}^1$$

δ = angle which surface of earth makes with horizontal.

ϕ = angle of internal friction of earth fill.

Since the earth thrust P is assumed to act parallel to the surface of the earth fill for all positive values of δ according to Rankine's theory, the horizontal component of the earth thrust P_H is

$$P_H = C\cos\delta\,\frac{wh^2}{2} \tag{1}$$

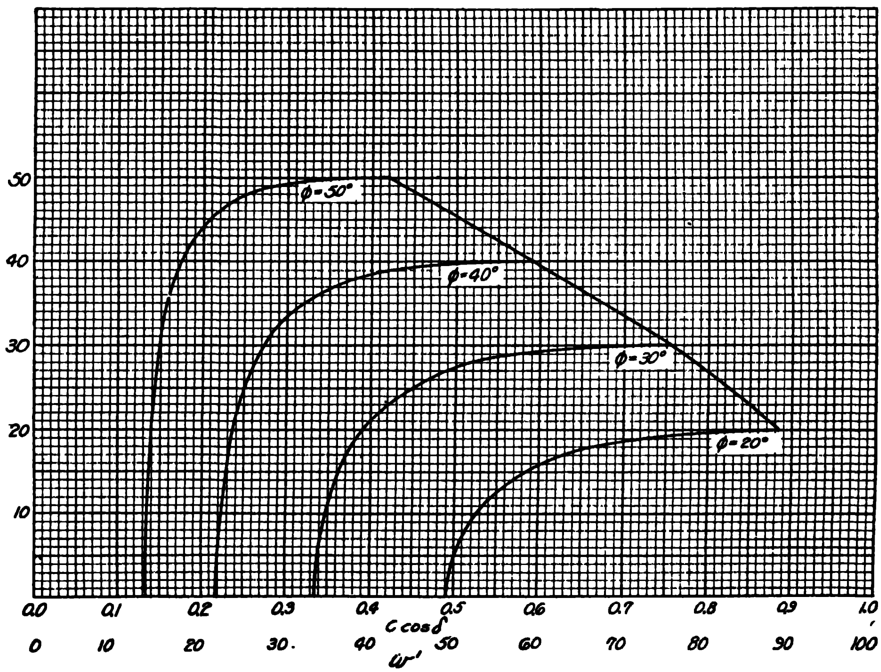


FIG. 2.

In determining the stability of a retaining wall the vertical component of the earth thrust P_V is usually neglected because the analysis is simplified thereby, while the results differ from those given by more refined methods by relatively small amounts on the side of safety. The variation of $C\cos\delta$ with δ and ϕ is shown in Fig. 2.

In the equivalent fluid pressure theory $C = 1$, so that

$$P = \frac{w'h^2}{2} \tag{2}$$

in which w' = weight of equivalent fluid in pounds per cubic foot.

[1] For development, see Ketchum's "Walls, Bins, and Grain Elevators."

3. Stability of a Retaining Wall.—A gravity wall must be heavy enough and of sufficient proportions to be able to safely withstand the tendency to slide and the tendency to overturn. Experience has shown that, in general, when a gravity wall will safely withstand the tendency to overturn, it will also safely withstand the tendency to slide.

4. Foundation Pressures.—In order that a wall may be safe against overturning the resultant of the earth pressure and the weight of the wall and that part of the earth above the base of the wall must not only fall within the base, but the safe bearing pressure of the material beneath the wall must not be exceeded. Furthermore, since there can be no tension between the base of the wall and its foundation, the resistance to overturning must be provided wholly by compressive forces.

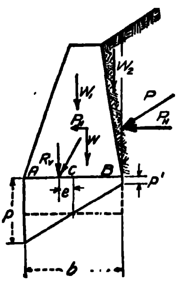


Fig. 3.

From an inspection of Fig. 3 it is evident that the distribution of pressure on the base of a retaining wall is similar to the distribution of fiber stress on a structural member subjected to bending and compression. The extreme fiber stresses in this latter case are given by the formulas

$$f = \frac{P}{A} + \frac{Mc}{I}$$

and

$$f' = \frac{P}{A} - \frac{Mc}{I}$$

In Fig. 3, f and f' are replaced by p and p' respectively, $P = R_V = W$, $A = b$ for a section 1 ft. long, $M = R_V e = We$, $c = \frac{b}{2}$, and $I = \frac{b^3}{12}$ or $\frac{c}{I} = \frac{6}{b^2}$.

Then

$$p = \frac{W}{b} + \frac{6We}{b^2}$$

or

$$p = \frac{W}{b}\left(1 + \frac{6e}{b}\right) \tag{3}$$

and

$$p' = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$$

If no tension is to exist at point B, the heel of the wall, $\frac{e}{b}$ must not exceed $\frac{b}{6}$. Hence the familiar rule that the resultant should not fall outside the middle third of the base.

DESIGN OF GRAVITY RETAINING WALLS

5. Proportions of Wall.

5a. Level Fill.—According to the discussion given in the preceding article the resultant of the earth pressure and the weight of the wall and earth should cut the edge of the middle third of the base or should fall within the middle third. If the resultant cuts the edge of the middle third and passes through point A of the wall shown in Fig. 4, then the algebraic sum of the moments about A must equal zero, or

$$Wa = P_H z \tag{4}$$

In this equation W is the total weight of the wall and earth whose length is one foot and whose cross-section is $BCDE$. The term Wa may be found readily by noting that

$$Wa = W\left(\frac{b}{3} + a\right) - W\frac{b}{3}$$

The term $W\left(\frac{b}{3} + a\right)$ equals the summation of moments of the various elements of the wall and earth section about the toe, point B. Assuming that concrete weighs 150 lb. per cu. ft. and that earth weighs 100 lb. per cu. ft.

$$W\left(\frac{b}{3} + a\right) = 150bh\frac{b}{2} - 50\frac{xh}{2}\left(b - \frac{x}{2}\right) - 150\frac{yh}{2}\left(\frac{y}{3}\right)$$

$$= 150h\left(\frac{b^2}{2} - \frac{bx}{6} + \frac{x^2}{18} - \frac{y^2}{6}\right)$$

$$W\frac{b}{3} = \frac{b}{3}\left(150bh - 50\frac{xh}{2} - 150\frac{yh}{2}\right)$$

$$= 150h\left(\frac{b^2}{3} - \frac{bx}{18} - \frac{by}{6}\right)$$

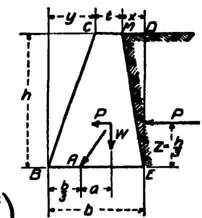


FIG. 4.

$$W\left(\frac{b}{3} + a\right) - W\frac{b}{3} = 150h\left(\frac{b^2}{6} - \frac{bx}{9} + \frac{x^2}{18} + \frac{by}{6} - \frac{y^2}{6}\right)$$

$$= 25hb^2\left[1 - \frac{1}{3}\cdot\frac{x}{b}\left(2 - \frac{x}{b}\right) + \frac{y}{b}\left(1 - \frac{y}{b}\right)\right]$$

$$= Wa = P_Hz = C\cos\delta\frac{wh^2}{2}\left(\frac{h}{3}\right)$$

With $\delta = 0$ and $w = 100$ lb. per cu. ft.

$$C\frac{100h^3}{6} = 25hb^2\left[1 - \frac{1}{3}\cdot\frac{x}{b}\left(2 - \frac{x}{b}\right) + \frac{y}{b}\left(1 - \frac{y}{b}\right)\right]$$

$$\frac{b^2}{h^2} = \frac{2}{3}\,\frac{C}{1 - \frac{1}{3}\cdot\frac{x}{b}\left(2 - \frac{x}{b}\right) + \frac{y}{b}\left(1 - \frac{y}{b}\right)}$$

$$\text{Let } G = \frac{1}{1 - \frac{1}{3}\cdot\frac{x}{b}\left(2 - \frac{x}{b}\right) + \frac{y}{b}\left(1 - \frac{y}{b}\right)}$$

$$\text{Then } \frac{b}{h} = 0.816\,G\sqrt{C} \tag{5}$$

The variation of the value of G with different values of $\frac{y}{b}$ and $\frac{t}{b}$ is shown in Fig. 5. Although $\frac{t}{b}$ does not appear in the formula for G, it is evident from Fig. 4 that $\frac{t}{b} = 1 - \left(\frac{y}{b} + \frac{x}{b}\right)$. In order to show the variation of $\frac{b}{h}$ with $\frac{y}{b}$ and $\frac{t}{b}$ the value of C must be determined. This factor depends upon δ and ϕ. Two commonly used values of ϕ are 33° 42′ and 45° 00′. These angles correspond to slopes of 1½:1 and 1:1 respectively. These values of the angle of internal friction are not necessarily the best ones, but they are values which are quite commonly assumed and they may be used here for the purpose of illustration. With $\delta = 0°\ 00'$ and $\phi = 33°\ 42'$, $C = 0.288$ and $0.816\sqrt{C} = 0.44$. With

$\delta = 0° \ 00'$ and $\phi = 45° \ 00'$, $C = 0.172$ and $0.816 \sqrt{C} = 0.34$. Since $P_H = C\frac{wh^2}{2}$ (for level fill), the results produced by this method when $\phi = 33° \ 42'$ are equal to those given be the equivalent fluid pressure theory using a fluid weighing 28.8 lb. per cu. ft. Similarly, results produced by this method when

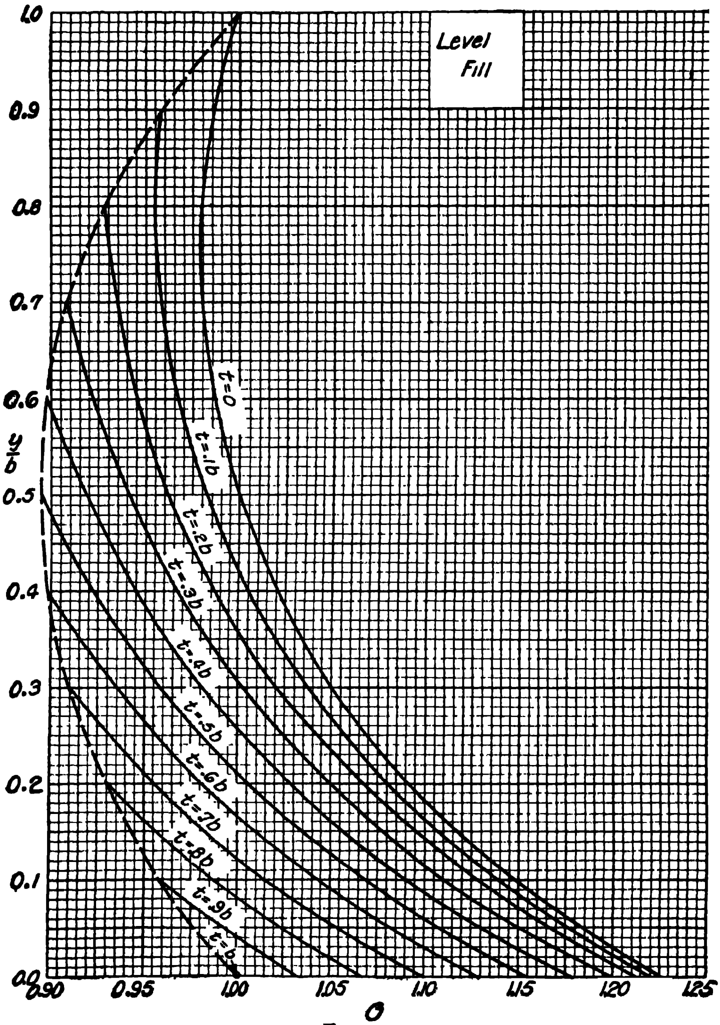


Fig. 5.

$\phi = 45° \ 00'$ are equal to those given by the equivalent fluid pressure theory using a fluid weighing 17.2 lb. per cu. ft.

From the above discussion or from eq. (5) it is evident that $\frac{b}{h}$ is directly proportional to $\sqrt{C}$. If Rankine's theory is used values of C may be taken from Fig. 2. When C is determined, the variation of $\frac{b}{h}$ with $\frac{y}{b}$ and $\frac{t}{b}$ may be found

by multiplying values of G taken from Fig. 5 by $0.816\sqrt{C}$. It should be borne in mind that Fig. 5 can be used only in the case of a level back-fill. If the equivalent fluid pressure theory is used C equals the weight of the fluid divided by 100, since the weight of earth has been assumed to be 100 lb. per cu. ft. Then

$$\frac{b}{h} = 0.0816\,G\sqrt{w'} \tag{6}$$

Formulas will now be developed for the cases in which the earth fill slopes upward from the top of the wall. Since cases in which earth slopes downward from the top of the wall are of rather infrequent occurrence and are not of a great deal of practical importance, these cases will not be considered here.

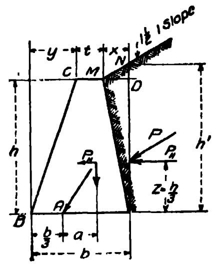


FIG. 6.

5b. Sloping Fill.—Two common values of earth slope will be considered: Namely, $1\frac{1}{2}:1$ and $1:1$. The conditions for the first case are shown in Fig. 6. If the resultant cuts the base at the edge of the middle third and passes through point A of the wall shown in Fig. 6, then the algebraic sum of the moments about A must equal zero as in Fig. 4, or $Wa = P_Hz$. However, the value of W in Fig. 6 is greater than that in Fig. 4 by a weight of earth whose cross-section is DMN. Also P_Hz in Fig. 6 is proportional to the cube of the height EN while in Fig. 4 it is proportional to the cube of the height ED.

Since the cross-section shown in Fig. 6 differs from that shown in Fig. 4 by the area DMN it is necessary to find the value of $W\left(\frac{b}{3} + a\right) - W\frac{b}{3}$ for that area only and modify the value of G given in the preceding article accordingly.

$$W\left(\frac{b}{3} + a\right) - W\frac{b}{3} = 100\frac{x^2}{3}\left(b - \frac{x}{3}\right) - 100\frac{x^2}{3}\frac{b}{3}$$
$$= 100\frac{x^2}{9}(2b - x) = 25\left(\frac{4}{9}\right)x^2\left(2 - \frac{x}{b}\right)b$$
$$= 25\,hb^2\left[\frac{4}{9}\frac{x^2}{b^2}\left(2 - \frac{x}{b}\right)\frac{b}{h}\right]$$

The values of this quantity and $\frac{b}{h}$ depend upon each other and therefore one cannot be determined accurately without the other being known. However, since the value of $\frac{b}{h}$ usually ranges from 0.3 to 0.7 in any practical design, it seems reasonable to substitute a value of 0.5 for $\frac{b}{h}$ in this last equation. Then

$$W\left(\frac{b}{3} + a\right) - W\frac{b}{3} = 25\,hb^2\left[\frac{2}{9}\frac{x^2}{b^2}\left(2 - \frac{x}{b}\right)\right]$$

Evidently a factor $\frac{2}{9}\frac{x^2}{b^2}\left(2 - \frac{x}{b}\right)$ must be incorporated in the formula for G. Let this value of G, modified to fit the case of a $1\frac{1}{2}:1$ earth slope be denoted by G_1. Then

$$G_1 = \frac{\qquad}{\sqrt{1 - \frac{1}{3}\cdot\frac{x}{b}\left(2 - \frac{x}{b}\right) + \frac{y}{b}\left(1 - \frac{y}{b}\right) + \frac{2}{9}\cdot\frac{x^2}{b^2}\left(2 - \frac{x}{b}\right)}}$$

or

$$G_1 = \frac{1}{-\frac{1}{9}\cdot\frac{x}{b}\left(3 - 2\frac{x}{b}\right)\left(2 - \frac{x}{b}\right) + \frac{y}{b}\left(1 - \frac{y}{b}\right)}$$

When the earth slope is 1 : 1, the factor $\frac{2}{9}\cdot\frac{x^2}{b^2}\left(2 - \frac{x}{b}\right)$ is increased by 50 per cent and is equal to $\frac{1}{3}\cdot\frac{x^2}{b^2}\left(2 - \frac{x}{b}\right)$. Let the value of G, modified to fit the case of a 1 : 1 earth slope be denoted by G_2. Then

$$G_2 = \frac{1}{1 - \frac{1}{3}\cdot\frac{x}{b}\left(2 - \frac{x}{b}\right) + \frac{y}{b}\left(1 - \frac{y}{b}\right) + \frac{1}{3}\cdot\frac{x^2}{b^2}\left(2 - \frac{x}{b}\right)}$$

or

$$G_2 = \sqrt{\frac{1}{1 - \frac{1}{3}\cdot\frac{x}{b}\left(1 - \frac{x}{b}\right)\left(2 - \frac{x}{b}\right) + \frac{y}{b}\left(1 - \frac{y}{b}\right)}}$$

It was found that the values of G_1 and G_2 for all values of $\frac{t}{b}$ would lie within the area bounded by the solid curves shown in Fig. 7. Therefore it was thought advisable to use average values shown by the dotted curve and denoted G' since the final results would be practically the same and since the work which follows would be greatly simplified thereby.

If

$$G' = \sqrt{\frac{1}{g'}}$$

Then $Wa = 25hb^2g' = P_Hz = C \cos \delta \frac{wh_1^2}{2}\left(\frac{h_1}{3}\right)$, in which h_1 = vertical distance from heel of wall to sloping surface of fill. If w = 100 lb. per cu. ft.

$$\frac{b^2}{h_1^2} = \frac{2}{3}\frac{C \cos \delta}{g'}\frac{h_1}{h}$$

or

$$\frac{b}{h_1} = \sqrt{\frac{2}{3}\frac{C \cos \delta}{g'}\frac{h_1}{h}}$$

When the slope of the earth surface is 1½ : 1,

$$h_1 = h + \frac{2}{3}x = h\left(1 + \frac{2}{3}\frac{x}{h}\right) = h\left(1 + \frac{2}{3}\frac{x}{b}\frac{b}{h}\right)$$

Then

$$\frac{b}{h} = 0.816\sqrt{C \cos \delta}\ G'\left(1 + \frac{2}{3}\frac{x}{b}\frac{b}{h}\right)^{3/2} \tag{7}$$

When the slope of the earth surface is 1 : 1

$$\frac{b}{h} = 0.816\sqrt{C \cos \delta}\ G'\left(1 + \frac{x}{b}\frac{b}{h}\right)^{3/2} \tag{8}$$

The value of $\sqrt{C \cos \delta}$ may be determined from Fig. 2. With known or assumed values of two of the variables $\frac{y}{b}$, $\frac{t}{b}$, and $\frac{x}{b}$, the third may be found, since

the sum of the three is unity. Then with these factors known, $\frac{v}{h}$ must be determined by assuming various values and testing them by eqs. (7) or (8) until the true value is found.

In order to illustrate the method assume that $\delta = \phi = 33° 42'$, $\frac{y}{b} = 0.7$, $\frac{t}{b} =$

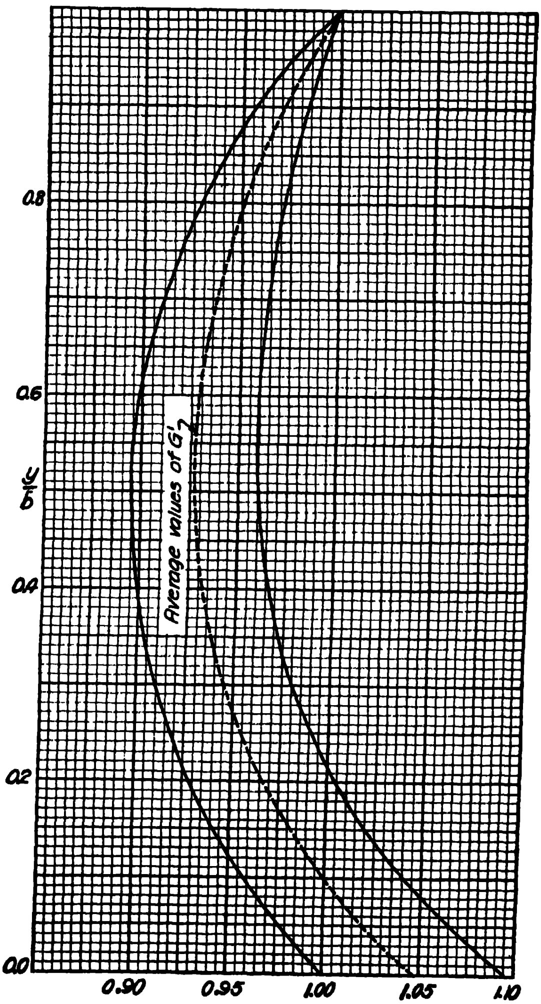


Fig. 7.

0.2, and $\frac{x}{b} = 0.1$. From Fig. 2, $\sqrt{C \cos \delta} = 0.833$, and from Fig. 7, $G' = 0.938$. Assume that $\frac{b}{h} = 0.665$. Then

$$\frac{b}{h} = 0.816(0.833)(0.938)\left[1 + \frac{2}{3}(0.1)(0.665)\right]^{2/3} = 0.677$$

Since the assumed value was too small, try $\frac{b}{h} = 0.68$. Then

$$\frac{b}{h} = 0.816(0.833)(0.938)\left[1 + \frac{2}{3}(0.1)(0.68)\right]^{3/2} = 0.68$$

This is the true value.

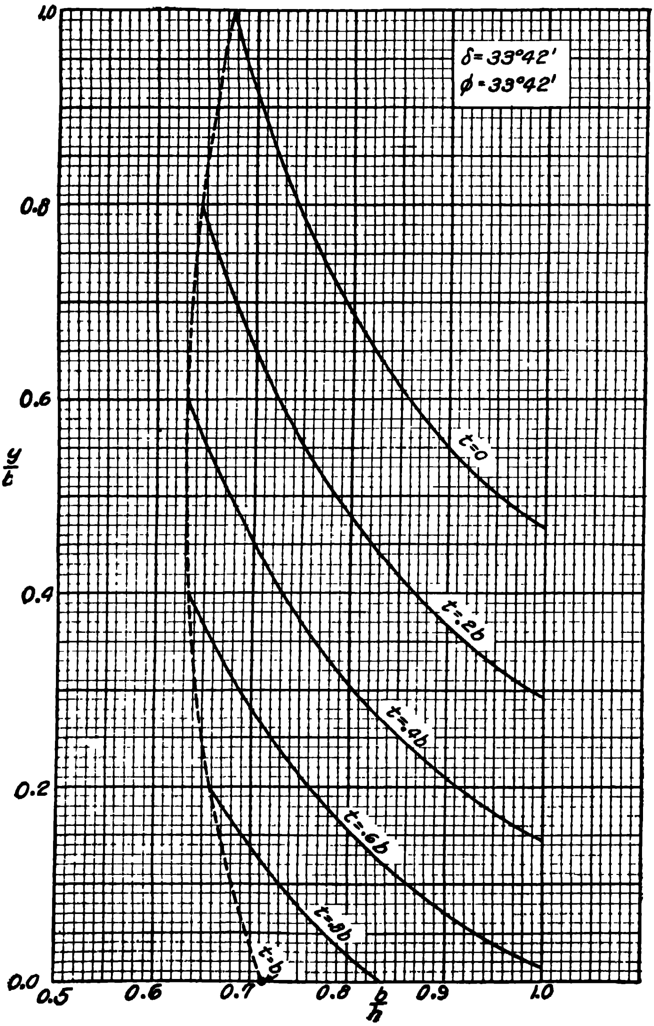


Fig. 8.

In this same manner the variation of $\frac{b}{h}$ with $\frac{y}{b}$ and $\frac{t}{b}$ was determined for three different cases, as follows: $\phi = 33° 42'$, $\delta = 33° 42'$; $\phi = 45° 00'$, $\delta = 33° 42'$; and $\phi = 45° 00'$, $\delta = 45° 00'$. The results are shown in Figs. 8, 9 and 10, respectively. These three cases give values of $\frac{b}{h}$ equal to those produced by the

equivalent fluid pressure theory using fluids which weigh 69.2, 21.4, and 50 lb. per cu. ft. respectively (see Fig. 2).

5c. Approximate Method for Sloping Fill.—It is evident that the work of calculation of values for the construction of diagrams similar to Figs. 8, 9, and 10 constitutes a long and tedious process. Therefore, in order to show

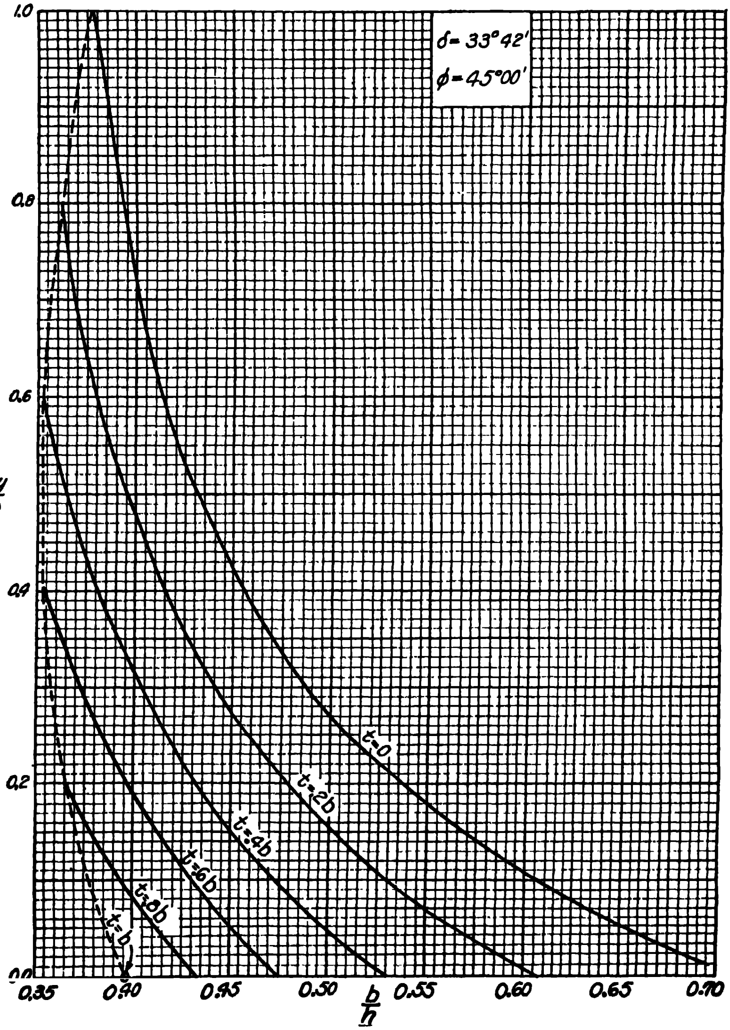


Fig. 9.

the influence of a variation in the value of $C \cos \delta$ upon the value of $\frac{y}{h}$ an approximate method has been developed. In the approximate method it is assumed that G' is a constant and is equal to unity. From an inspection of Fig. 7 and eqs. (7) and (8) it may be noted that the ratio $\frac{b}{h}$ obtained by this method will be slightly in

excess of that obtained by the more exact method in the majority of cases. Also it may be noted that while the sum of $\frac{y}{b}$ and $\frac{t}{b}$ is taken into account by means of $\frac{x}{b}$, that except for this condition the values of $\frac{y}{b}$ and $\frac{t}{b}$ may vary widely without

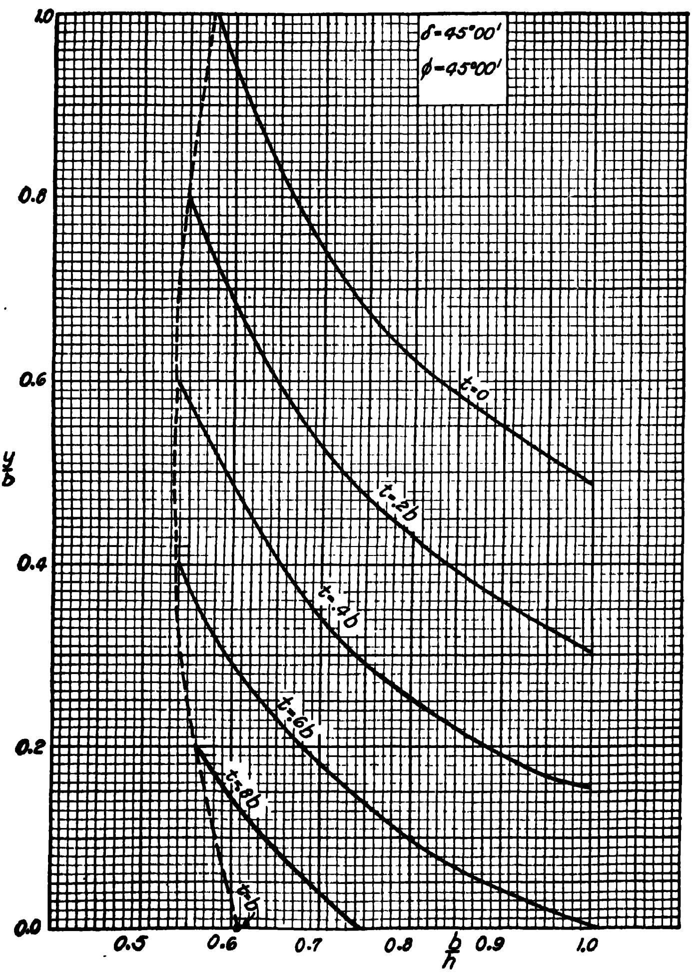


Fig. 10.

having any influence on this approximate value of $\frac{o}{h}$. In other words, with a certain value of $\frac{x}{b}$, the sum of $\frac{y}{b}$ and $\frac{t}{b}$ is $1 - \frac{x}{b}$, but either $\frac{y}{b}$ or $\frac{t}{b}$ may vary from 0 to $1 - \frac{x}{b}$.

Figures 11 (*a*), 11 (*b*), and 11 (*c*) have been prepared for conditions the same as those for Figs. 8, 9, and 10. In Fig. 11 (*a*) the solid line curve on the right is a graph of the maximum values of $\frac{b}{h}$ for various values of $\frac{x}{b}$ while the solid line curve on the left is a graph of the minimum values of $\frac{b}{h}$ for various values of $\frac{x}{b}$. These maximum and minimum values of $\frac{b}{h}$ for different values of $\frac{x}{b}$ were taken from Fig. 8.

The solid line curves for Figs. 11 (*b*) and 11 (*c*) were obtained from Figs. 9 and 10. The dotted line curves are graphs of eqs. (7) and (8). The values

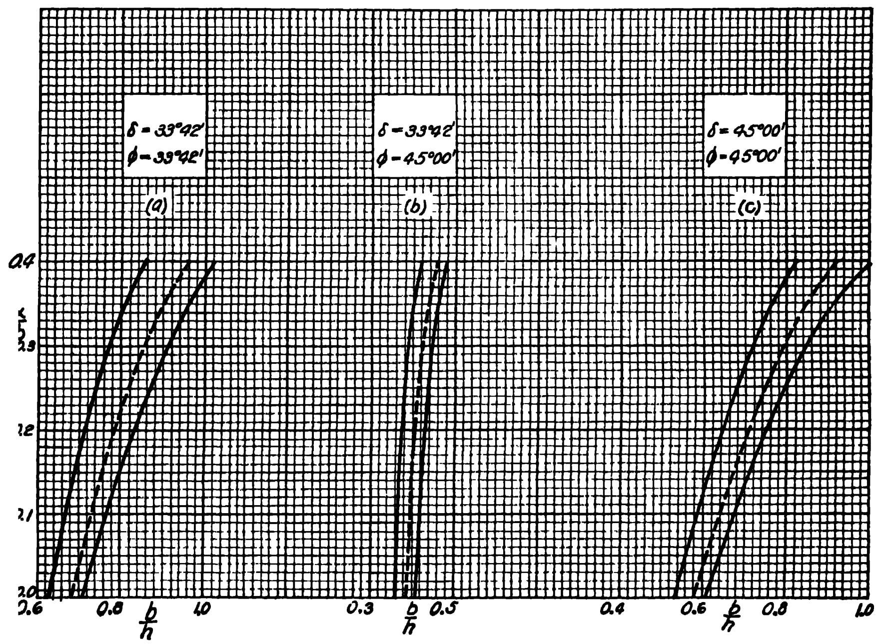


FIG. 11.

of $\frac{o}{h}$ which would satisfy these equations were determined by a cut and try process.

From a study of Figs. 11 (*a*), 11 (*b*), and 11 (*c*) it is evident that accuracy will be sacrificed by the use of this approximate method but on the other hand it is also evident that a fairly good notion of the influence of $C \cos \delta$ upon the quantity $\frac{b}{h}$ may be obtained in this way.

Figure 12 shows the variation of $\frac{b}{h}$ with $\frac{x}{b}$ for different values of $C \cos \delta$ when $G = 1$, and when the earth slope is $1\frac{1}{2} : 1$. Figure 13 is the same except that the earth slope is $1 : 1$.

Two factors will usually determine the choice of the proportions of a wall for a given set of conditions. One requirement is that the chosen wall shall be the

most economical, and the other requirement is that the unit pressure of the wall on the soil at the toe shall not exceed the allowable unit bearing pressure for the soil in question.

6. Relation of Economy to Proportions of Wall.—If it is assumed that the cost of a gravity wall varies directly with the cross-section, then for a given height cost

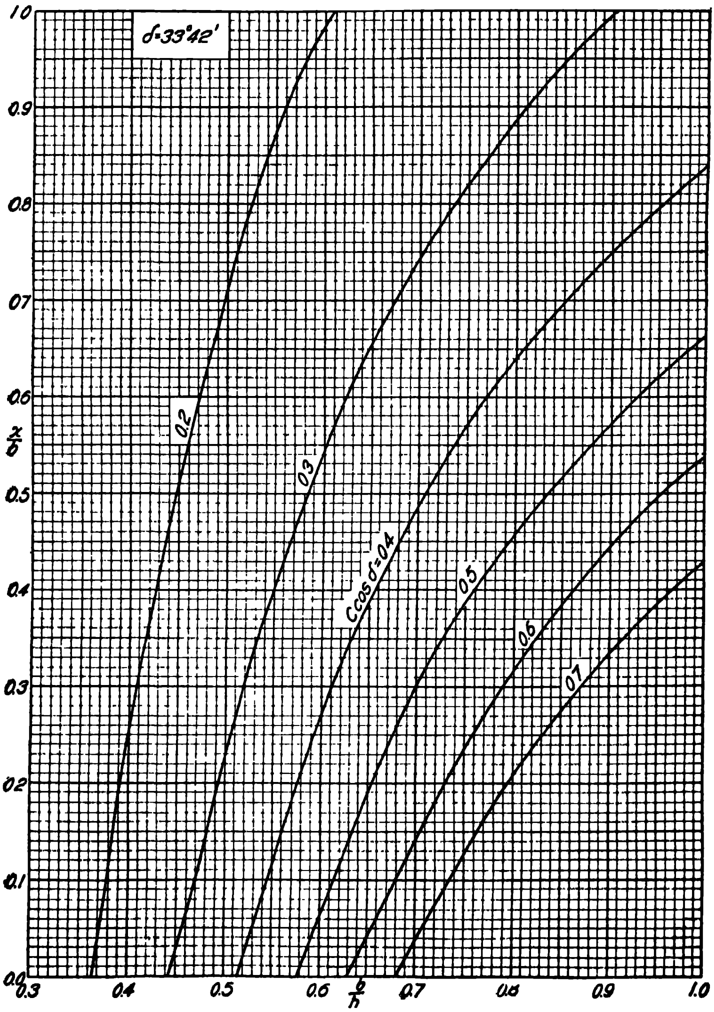


FIG. 2.

varies directly with the quantity $\frac{b}{h}\left(1 + \frac{t}{b}\right)$. Using the value of this quantity when $\frac{y}{b} = 1$ as the basis of comparison Figs. 14, 15, 16 and 17 have been made. These diagrams are for the same conditions assumed in making Figs. 5, 8, 9 and 10 respectively and were constructed by multiplying values of $\frac{b}{h}$ taken from those

diagrams by $\left(1 + \frac{t}{b}\right)$ and plotting the resulting values. In Fig. 5, G corresponds to $\frac{b}{h}$.

From an inspection of Figs. 14, 15, 16 and 17, it is evident that, in general, $\frac{y}{b}$ should be made as large as possible for maximum economy. Also, it may be noted

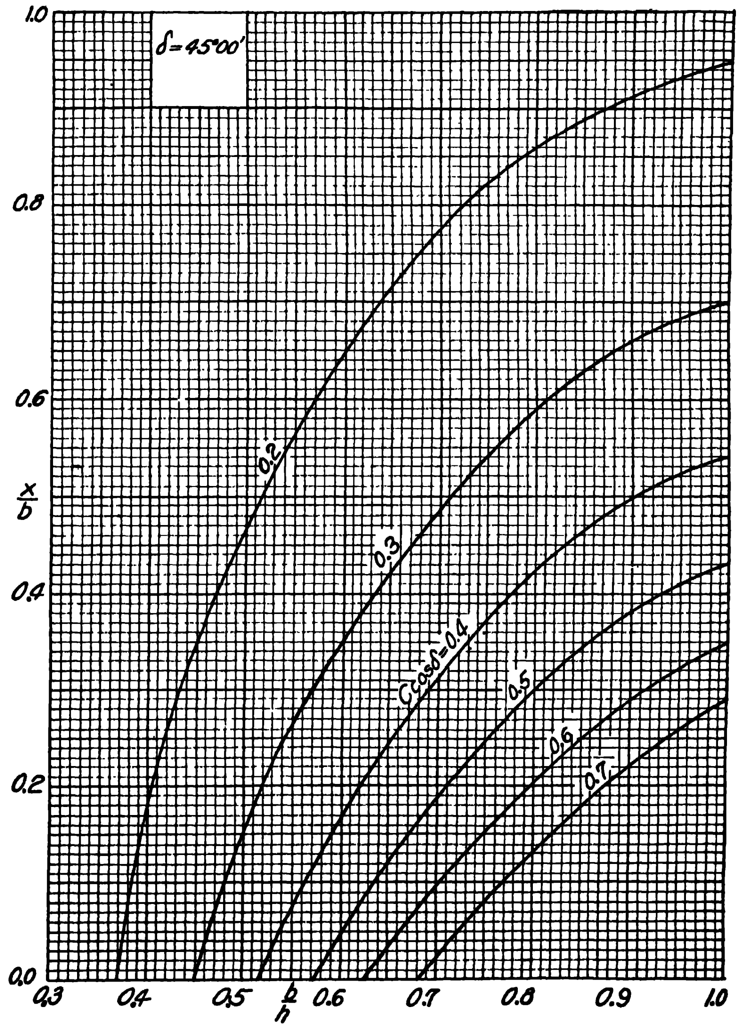


FIG. 13.

that for the conditions shown in Figs. 14 and 16 and for a given value of $\frac{y}{b}$ the most economical wall will usually result from the use of a minimum value of $\frac{t}{b}$. For the

conditions shown in Figs. 15 and 17 and for a given value of $\frac{y}{b}$ the most economical wall will usually result from the use of a maximum value of $\frac{t}{b}$.

7. Relation of Toe Unit Pressure to Proportions of Wall.—In order to determine whether or not the unit pressure at the toe of the wall exceeds the

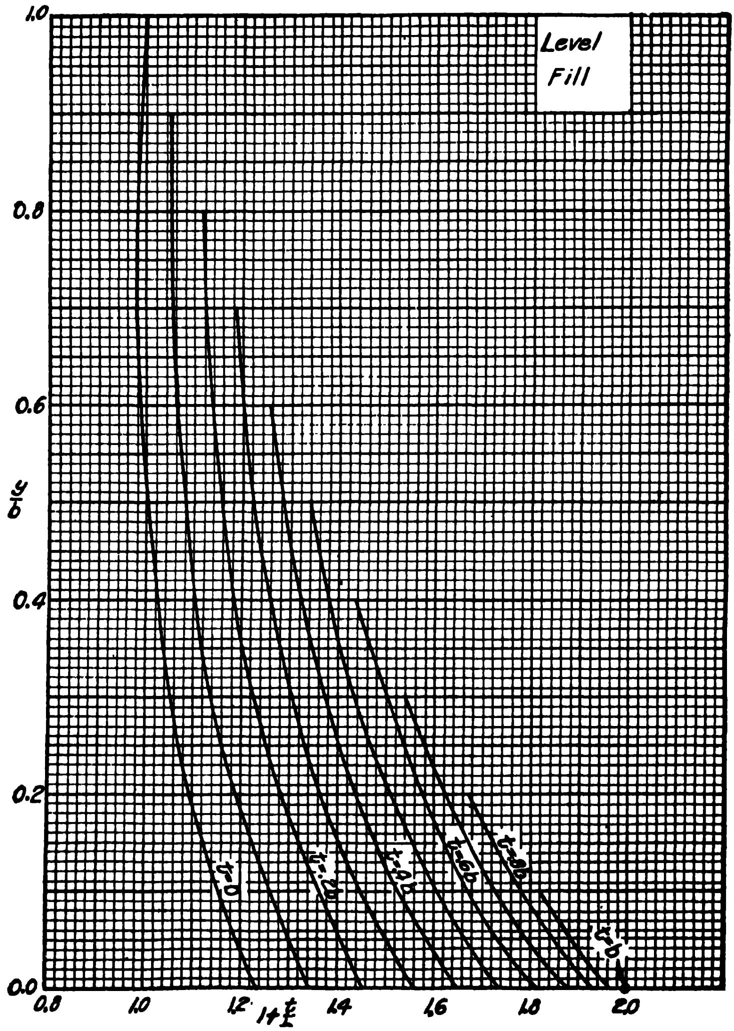


FIG. 14.

allowable, it seems desirable to establish a relation between toe unit pressure and proportions of wall. Since it has been assumed thus far in the discussion that the resultant cuts the base at the edge of the middle third, then the toe unit pressure is twice the average unit pressure for then $\frac{6e}{b} = 1$. The average unit pressure

equals the weight of the wall and earth divided by the width of the base. Therefore, for level earth fill, toe unit pressure

$$= \frac{2}{b}\left[150hb\left(1 + \frac{t}{b}\right)\frac{1}{2} + 100hb\left(\frac{x}{b}\right)\frac{1}{2}\right]$$

$$= 100h\left(1.5 + 1.5\frac{t}{b} + \frac{x}{b}\right)$$

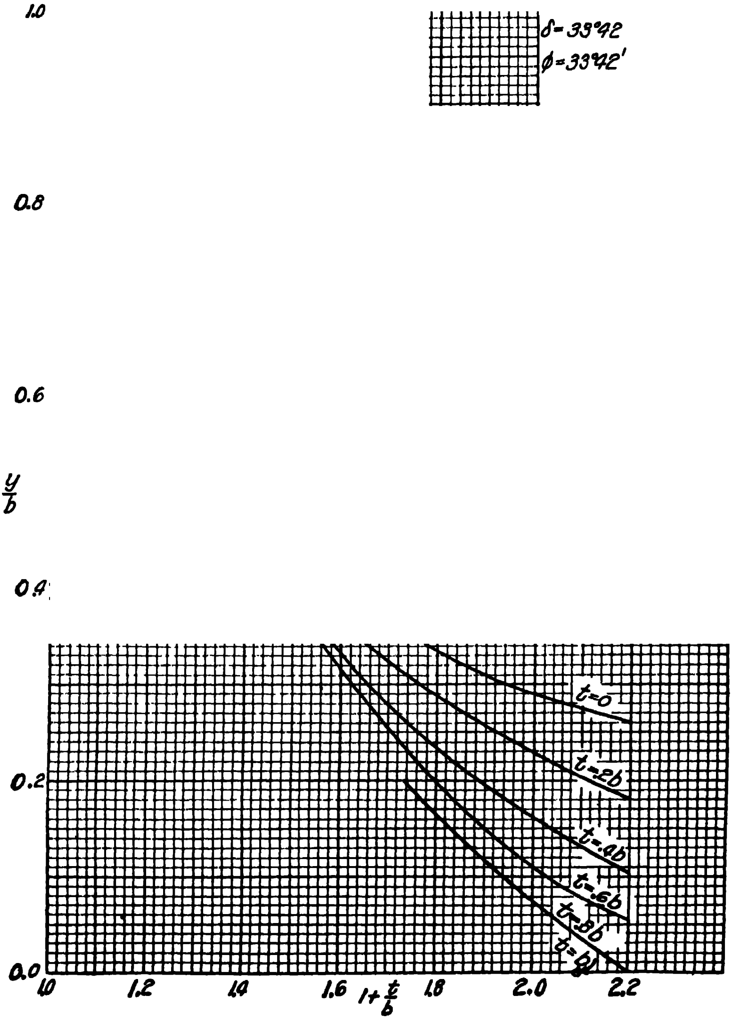


Fig. 15.

$$= 100h\left(2.5 - \frac{y}{b} + 0.5\frac{t}{b}\right)$$

The effect of sloping earth fill upon the toe unit pressure is slight. After a study of the relation between toe unit pressure and wall proportions had been made for all ordinary values of ϕ and δ it was found that the value of the toe unit

pressure for both level and sloping earth fills would generally fall within a region bounded by two straight lines. The equations of these two lines are as follows:

$$\text{Toe unit pressure} = 100h\left(2.5 - \frac{y}{b}\right)$$

$$\text{Toe unit pressure} = 150h\left(2.0 - \frac{y}{b}\right)$$

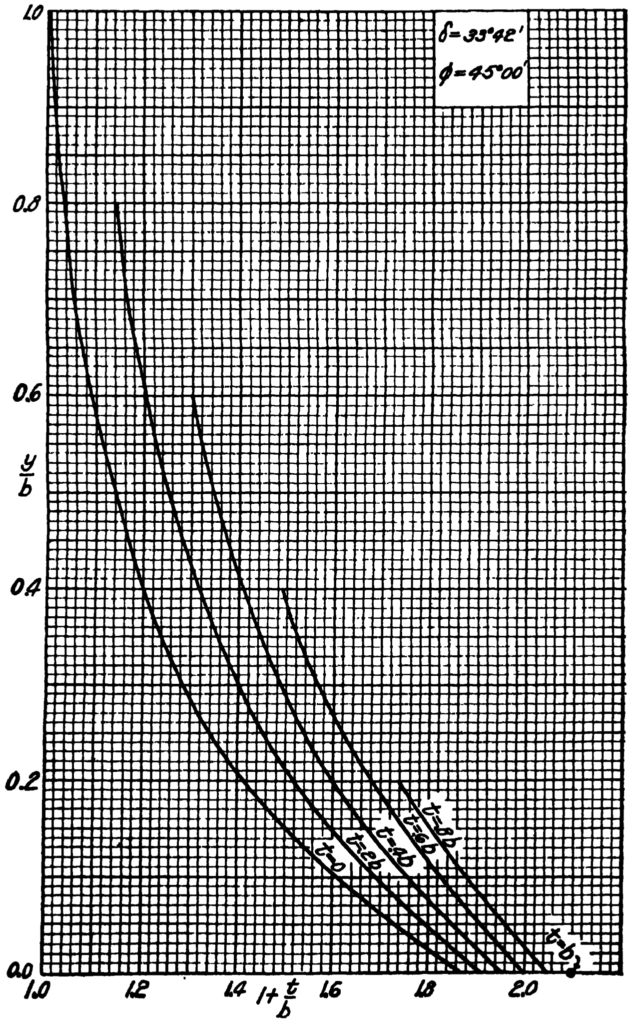


Fig. 16.

In a great many cases, values given by the first equation will be too small, while values given by the second equation will be too large in other cases. Since no equation in between these two seems to be entirely satisfactory it is recom-

mended that the second equation be used even though it may give values slightly in excess of the true ones under certain conditions. Then

$$\text{Toe unit pressure} = 150\,h\left(2 - \frac{y}{b}\right) \tag{9}$$

8. Relation of Proportions of Wall to Assumed Position of Resultant.—Thus far in the discussion it has been assumed that the resultant cuts the base at the

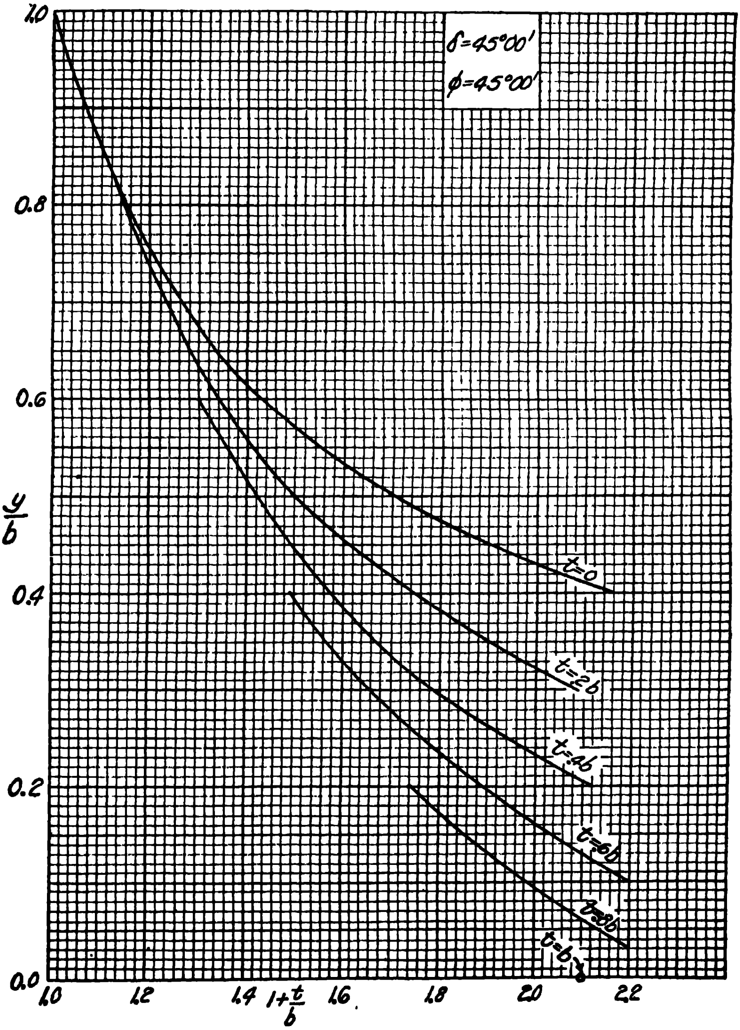


Fig. 17.

edge of the middle third. In order to show the relation between $\frac{b}{h}$ and the position of the resultant, Fig. 18 has been prepared. The shaded area in Fig. 18 (*a*) is a reproduction of Fig. 5. The left-hand boundary line of this area is a graph of the minimum values of $\frac{b}{h}$ for different values of $\frac{y}{b}$, while the right-hand

boundary line is a graph of the maximum values of $\frac{y}{h}$. Figure 18(b) gives the same data for the case in which the resultant cuts the base at a point $\frac{2}{3}b$ from the toe to the wall. Figure 18(c) is for the case in which the resultant cuts the base at the middle.

It is evident from a study of these figures that the most economical wall will result when the wall is porportioned so that the resultant will cut the base at or near the edge of the middle third of the base.

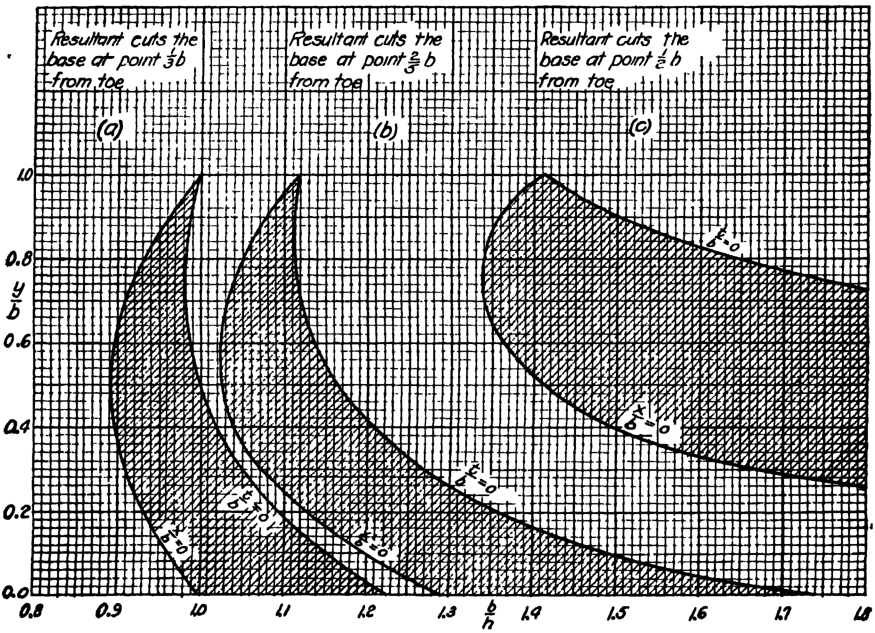


FIG. 18.

9. Schedule for the Design of Gravity Retaining Walls.

Given: Values of h, ϕ, δ, minimum value of t, and maximum allowable soil pressure.

Find: Proportions of wall such that the resultant cuts the base at the edge of the middle third.

Solution: The value of δ will generally equal 0° 00′, 33° 42′, or 45° 00′. The value of ϕ may equal 33° 42′, 45° 00′, or some other angle. From Figs. 14, 15, 16 or 17 choose a trial value of $\frac{t}{b}$.

(a) With $\delta = 0° \; 00'$

Get C from Fig. 2, G from Fig. 5, and $\frac{b}{h}$ from eq. (5).

(b) With $\delta = 33° \; 42'$ or $45° \; 00'$

If ϕ equals 33° 42′ or 45° 00′ get $\frac{b}{h}$ from Figs. 8, 9 or 10. If ϕ does not equal 33° 42′ or 45° 00′ get $C \cos \delta$ from Fig. 2, and get $\frac{b}{h}$ from Figs. 12 or 13.

Find b by multiplying this ratio by h. Compare the resulting value of $\frac{t}{b}$ with the assumed value and modify the design if necessary. Determine the unit pressure at the toe by means of eq. (9). Compare this value with the allowable value and modify the design if necessary.

Given: Values of h, w', minimum value of t, and maximum allowable soil pressure.

Find: Proportions of wall such that the resultant cuts the base at the edge of the middle third.

Solution: From Fig. 14 choose a trial value of $\frac{t}{b}$.

Get C from Fig. 5, and $\frac{b}{h}$ from eq. (6).

Find b by multiplying this ratio by h. Compare the resulting value of $\frac{t}{b}$ with the assumed value and modify the design if necessary. Determine the unit pressure at the toe by means of eq. (9). Compare this value with the allowable value and modify the design if necessary.

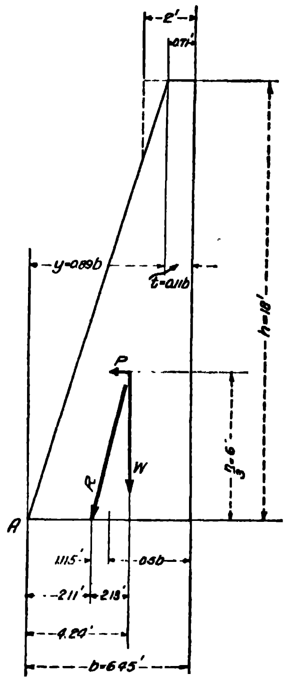


FIG. 19.

Illustrative Problem.—*Given:* Height = 18 ft., minimum top width = 2 ft., ϕ = 40° 00′, δ = 0° 00′, and the maximum allowable unit pressure at the toe is 3,000 lb. per sq. ft.

Find: Proportions of wall.

Solution: In order to get the most economical proportions of wall for a given value of t it is necessary to assume a trial value of From Fig. 14 it is evident that $\frac{t}{b}$ should be made as small as possible in order to get the most economical wall.

For a first trial assume $\frac{t}{b} = 0.2$ and $\frac{y}{b} = 0.8$. Then $G = 0.93$ (Fig. 5). From Fig. 2, $C = 0.214$ and from eq. (5), $\frac{b}{h} = 0.816\ (0.93)\ \sqrt{0.214} = 0.351$. With $h = 18$ ft., $b = 0.351\ (18) = 6.32$ ft., and $\frac{t}{b} = 0.316$.

Assume $\frac{t}{b} = 0.3$ and $\frac{y}{b}$ 0.7. Then $G = 0.91$, $b = 0.816\ (0.91)\ \sqrt{0.214}(18) = 6.19$ ft., and $\frac{t}{b}$ 0.324.

With $\frac{t}{b} - 0.33$ and $\frac{y}{b} = 0.67$, $G = 0.905$ and $b = 0.816\ (0.905)\ \sqrt{0.214}(18) = 6.15$ ft.

The unit pressure at the toe is given by eq. (9). Substituting in this equation $p = 150\ (18)\ (2 - 0.67) = 3{,}600$ lb. per sq. ft. To reduce the toe unit pressure to 3,000 lb. per sq. ft., substitute this value for p and solve for $\frac{y}{b}$. Doing this $\frac{y}{b} = 2 - \frac{3{,}000}{150(18)} = 0.89$. Then $\frac{t}{b} = 0.11$, $G = 0.95$ and $b = 0.816(0.95)\sqrt{0.214}(18) = 6.45$ ft. With these conditions $t = 0.11(6.45) = 0.71$ ft. The required top width may be secured by the means indicated in Fig. 19. This modification will have practically no effect upon the position of the resultant nor the unit pressure at the toe.

Check on the solution: (See Fig. 19.) Position of center of gravity, $c = \frac{M_A}{W} =$

$$\left[\frac{\frac{0.89}{2}\left(\frac{2}{3}\right)(0.89) + 0.11(0.945)}{\frac{1.11}{2}}\right]6.45 = 4.24 \text{ ft. from toe of wall.}$$

From eq. (4) $Wa = P_H z$, or,

$$a = \frac{\frac{1}{2}(100)(18)^2(0.214)(6)}{\frac{1.11}{2}(18)(150)(6.45)} = 2.13 \text{ ft. from center of gravity, or } 4.24 - 2.13 = 2.11 \text{ ft. from toe,}$$

true location of point where resultant cuts the base. It was assumed that the resultant cuts the base at the edge of the middle third or $\frac{6.45}{3} = 2.15$ ft. from toe of wall. The resultant actually cuts the base at a distance $\frac{6.45}{2} - 2.11 = 1.115$ ft. from the center. Then the unit pressure at the toe may be found readily from eq. (3),

$$p = \frac{W}{b}\left(1 + \frac{6e}{b}\right) = \frac{\frac{1.11}{2}(18)(150)(6.45)}{6.45}\left[1 + \frac{6(1.115)}{6.45}\right] = 3{,}080 \text{ lb. per sq. ft.}$$

A value of 3,000 was obtained originally.

Illustrative Problem.—*Given:* Height = 24 ft., minimum top width = 2 ft., $\phi = 45° 00'$, $\delta = 33° 42'$, and the maximum allowable unit pressure at the toe is 6,000 lb. per sq. ft.

Find: Proportions of wall.

Solution: (*a*) It is evident from Fig. 16 that an economical wall will be secured when $\frac{t}{b} = 0.2$ and $\frac{y}{b} = 0.8$. Then from Fig. 9, $\frac{b}{h} = 0.36$ and $b = 0.36(24) = 8.65$ ft., and $\frac{t}{b} = 0.232$. With $\frac{t}{b} = 0.24$, $\frac{y}{b} = 0.76$, $\frac{b}{h} = 0.36$. Toe unit pressure $p = 150(24)(2 - 0.76) = 4{,}460$ lb. per sq. ft.

(*b*) Assume as an added requirement that the front face shall be as nearly vertical as possible. For maximum economy make $\frac{t}{b}$ as small as possible (Fig. 16). From Fig. 9 with $\frac{t}{b} = 0.2$ and $\frac{y}{b} = 0$, $b = 0.606(24) = 14.58$ ft., and $\frac{t}{b} = \frac{2}{14.58} = 0.137$. With $\frac{t}{b} = 0.13$, $b = 0.64(24) = 15.37$ ft., and $\frac{t}{b} = \frac{2}{15.37} = 0.13$. The toe unit pressure $p = 150(24)(2 - 0) = 7{,}200$ lb. per sq. ft. To reduce this value to 6,000, $\frac{y}{b} = 2 - \frac{6{,}000}{150(24)} = 0.33$. From Fig. 9 with $\frac{y}{b} = 0.33$ and $\frac{t}{b} = 0.2$, $b = 0.433(24) = 10.40$ ft., and $\frac{t}{b} = 0.192$. This is close enough to the assumed value to be satisfactory.

Check on the solution (*a*): (See Fig. 20.) Position of the center of gravity,

$$c = \frac{M_A}{W} = \left[\frac{\frac{0.76}{2}\left(\frac{2}{3}\right)(0.76) + 0.24(0.88)}{\frac{1.24}{2}}\right]8.65 = 5.62 \text{ ft.}$$

from toe of wall. The distance from the point where the resultant cuts the base to the center of gravity is

$$a = \frac{P_H z}{W} = \frac{\frac{1}{2}(100)(24)^2(0.214)(8)}{\frac{1.24}{2}(24)(150)(8.65)} = 2.56 \text{ ft. or } 5.62 - 2.56 = 3.06 \text{ ft.}$$

from toe of wall. It was assumed that the resultant cuts the base at the edge of the middle third or $\frac{8.65}{3} = 2.88$ ft. from toe of wall. The resultant actually cuts the base at a distance $\frac{8.65}{2} - 3.06 = 1.265$ ft. from the center. Then the unit pressure at the toe is

$$p = \frac{W}{b}\left(1 + \frac{6e}{b}\right) = \frac{\frac{1.24}{2}(24)(150)(8.65)}{8.65}\left[1 + \frac{6(1.265)}{8.65}\right] = 4{,}200 \text{ lb. per sq. ft.}$$

A value of 4,460 was obtained originally.

Check on the solution (*b*): (See Fig. 21.) Position of the center of gravity, $c = \frac{M_A}{W}$

$$= \left[\frac{150(24)\left\{\frac{0.33}{2}(0.22) + 0.20(0.43) + \frac{0.47}{2}(0.68)\right\} + \frac{0.47}{2}(27.26)(100)(0.84)}{24\left(\frac{1.20}{2}\right)150 + \frac{0.47}{2}(27.26)\ 100}\right]10.40$$

$$= 5.77 \text{ ft.}$$

from toe of wall. The distance from the point where the resultant cuts the base to the center of gravity is

$$a = \frac{P_H z}{W} = \frac{\frac{1}{2}(100)(27.26)^2(0.214)(9.09)}{\left[24\left(\frac{1.20}{2}\right)(150) + \frac{0.47}{2}(27.26)(100)\right]10.40} = 2.48 \text{ ft.} = 5.77 - 2.48 = 3.29 \text{ ft.}$$

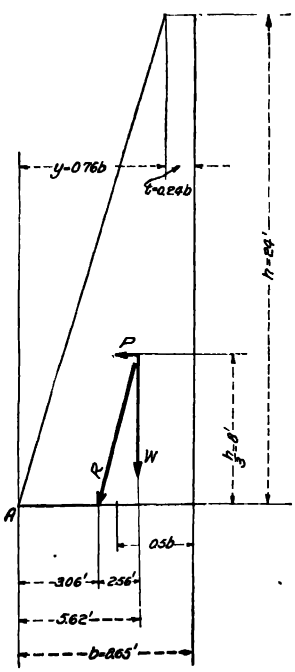


Fig. 20.

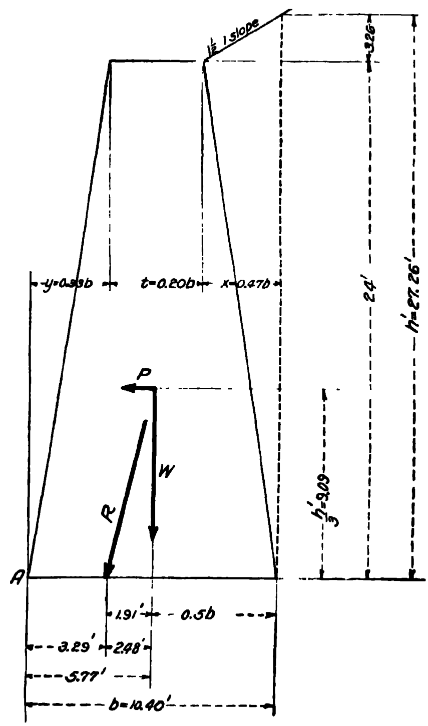


Fig. 21.

from toe of wall. It was assumed that the resultant cuts the base at the edge of the middle third or $\frac{10.40}{3} = 3.47$ ft. from toe of wall. The resultant actually cuts the base at a distance $\frac{10.40}{2} - 3.29 = 1.91$ ft. from the center. Then the unit pressure at the toe is

$$p = \frac{W}{b}\left(1 + \frac{6e}{b}\right) = \frac{\left[24\left(\frac{1.20}{2}\right)(150) + \frac{0.47}{2}(27.26)(100)\right]10.40}{10.40}\left[1 + \frac{6(1.91)}{10.40}\right]$$

$$= 5{,}9\ 00 \text{lb. per sq. ft.}$$

A value of 6,000 was obtained originally.

Illustrative Problem.—*Given:* Height = 15 ft., minimum top width = 1 ft., $\phi = 40°\ 00'$, $\delta = 33°\ 42'$, and the maximum allowable pressure at the toe is 3,000 lb. per sq. ft.

Find: Proportions of wall.

Solution: From Figs. 15 and 16 it is evident that for maximum economy, $\frac{t}{b}$ should be made as small as possible and $\frac{y}{b}$ should be made as large as possible. Try $\frac{t}{b} = 0.15$ and $\frac{y}{b} = 0.85$. From Fig. 2, $C \cos \delta = 0.300$: From Fig. 12, with $\frac{x}{b} = 0$, $b = 0.45(15) = 6.75$ ft. Then $\frac{t}{b} = \frac{1}{6.75} = 0.148$ which checks the value assumed. The toe unit pressure $= 150h\left(2 - \frac{y}{b}\right) = 150(15)(2 - 0.85) = 2{,}585$ lb. per sq. ft. This value is less than the maximum allowable, 3,000, and therefore is satisfactory.

Check on the solution: (See Fig. 22.) Position of the center of gravity

$$c = \frac{M_A}{W} = \left[\frac{\frac{0.85}{2}\left(\frac{2}{3}\right)(0.85) + 0.15(0.925)}{\frac{1.15}{2}}\right] 6.75 = 4.38 \text{ ft. from toe of wall.}$$

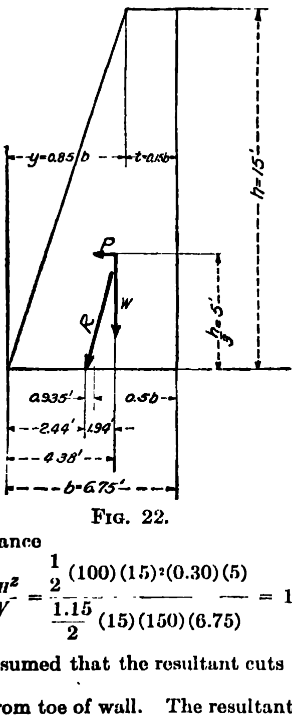


FIG. 22.

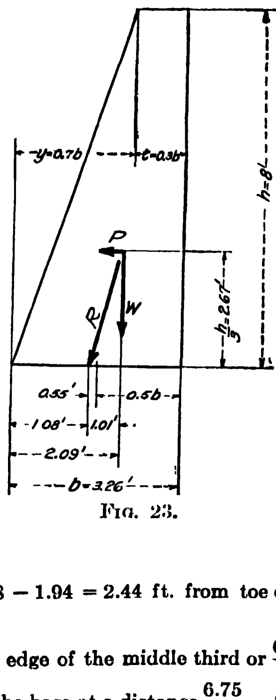


FIG. 23.

The distance

$$a = \frac{P_H z}{W} = \frac{\frac{1}{2}(100)(15)^2(0.30)(5)}{\frac{1.15}{2}(15)(150)(6.75)} = 1.94 \text{ ft. or } 4.38 - 1.94 = 2.44 \text{ ft. from toe of wall.}$$

It was assumed that the resultant cuts the base at the edge of the middle third or $\frac{6.75}{3} = 2.25$ ft. from toe of wall. The resultant actually cuts the base at a distance $\frac{6.75}{2} - 2.44 = 0.935$ ft. from the center. Then the unit pressure at the toe is $p = \frac{W}{b}\left(1 + \frac{6e}{b}\right) =$

$$\frac{\frac{1.15}{2}(15)(150)(6.75)}{6.75}\left[1 + \frac{6(0.935)}{6.75}\right] = 2{,}380 \text{ lb. per sq. ft.}$$

A value of 2,585 was obtained originally.

Illustrative Problem.—*Given:* **Height = 8 ft., minimum top width = 1 ft., equivalent fluid pressure = 30 lb. per sq. ft., and the maximum allowable pressure at the toe is 3,000 lb. per sq. ft.**

Find: **Proportions of wall.**

Solution: It is evident from Fig. 14 that $\frac{t}{b}$ should be small and $\frac{y}{b}$ should be large to secure an economical wall. Try $\frac{t}{b} = 0.3$ and $\frac{y}{b} = 0.7$. Then from Fig. 5, $G = 0.91$ and

from eq. (6), $b = 0.0816\, Gh\sqrt{w'} = 0.0816(0.91)(8)\sqrt{30} = 3.26$ ft. Then $\frac{t}{b} = \frac{1}{3.26} = 0.306$ which checks the value assumed. The unit pressure at the toe is $150h\left(2 - \frac{y}{b}\right) = 150(8)(2 - 0.7) = 1{,}560$ lb. per sq. ft. This value is satisfactory.

Check on the solution: (See Fig. 23.) Position of the center of gravity

$$\frac{M_A}{W} \quad \frac{\frac{0.70}{2}\left(\frac{2}{3}\right)(0.70) + 0.3(0.85)}{\frac{1.30}{2}} \quad 3.26 = 2.09 \text{ ft. from toe of wall.}$$

The distance $a = \frac{P_H z}{W} = \frac{\frac{1}{2}(100)(8)^2(0.30)(2.67)}{\frac{1.30}{2}(8)(150)(3.26)} = 1.01$ ft. or $2.09 - 1.01 = 1.08$ ft. from toe of wall. It was assumed that the resultant cuts the base at the edge of the middle third or $\frac{3.26}{3} = 1.09$ ft. from toe of wall. The resultant actually cuts the base at a distance $\frac{3.26}{2} - 1.08 = 0.55$ ft. from the center. Then the unit pressure at the toe is

$$p = \frac{W}{b}\left(1 + \frac{6e}{b}\right) = \frac{\frac{1.30}{2}(8)(150)(3.26)}{3.26}\left[1 + \frac{6(0.55)}{3.26}\right] = 1{,}570 \text{ lb. per sq. ft.}$$

A value of 1,560 was obtained originally.

DESIGN OF REINFORCED CONCRETE WALLS

10. Proportions of Wall.—A study of reinforced concrete walls was made similar to that explained in detail in Art. **5** for gravity walls. Using the same notation as given in preceding articles the variation of the factor G with different values of $\frac{y}{b}$ and $\frac{t}{b}$ was found to be as shown in Fig. 24. These results are based on the assumption of level earth fill back of the wall and a thickness of base equal to one-tenth of the total height of wall. It was found that the relative thickness of the base had very little effect upon the results and so this value was chosen because it was convenient and would produce results sufficiently accurate for all practical purposes. Figure 24 gives the same information for reinforced concrete walls as Fig. 5 does for gravity walls. Figures 25, 26 and 27 are similar to Figs. 8, 9 and 10 respectively.

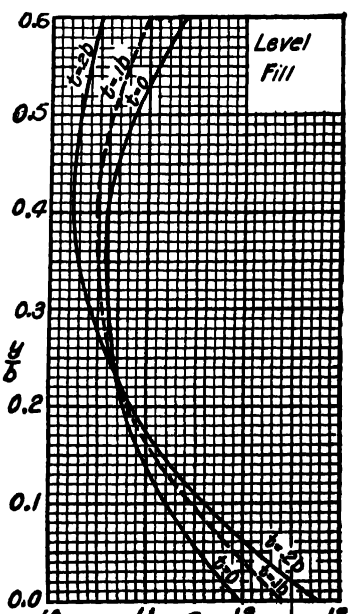
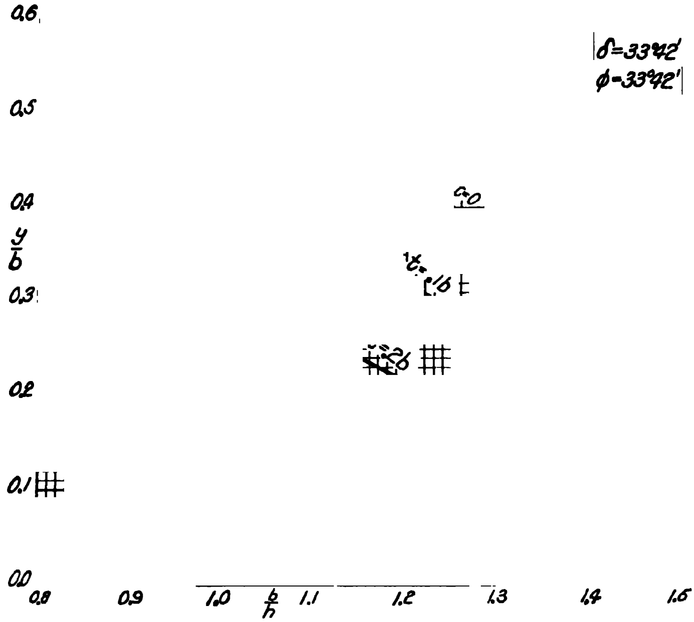


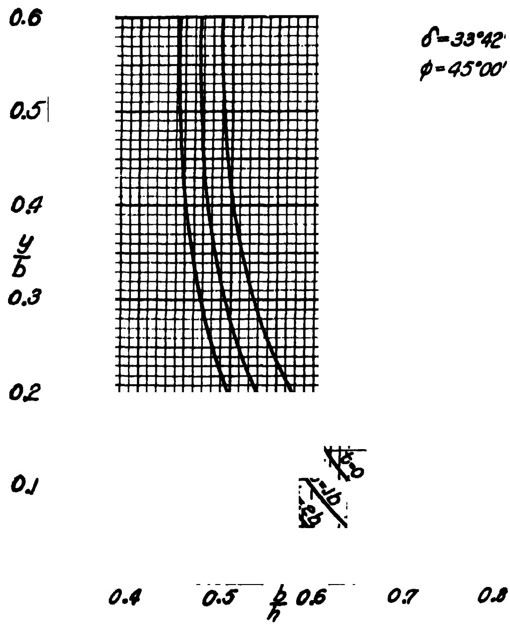
Fig. 24.

Since many engineers favor the use of the equivalent fluid pressure theory in the design of reinforced concrete walls, emphasis will be placed on the design of reinforced concrete walls according to this theory just as emphasis was placed on Rankine's theory in the design of gravity walls. However, diagrams have been specially prepared for the design of both types of walls according to both methods.



0.6
0.5
0.4
y/b
0.3
0.2
0.1
0.0
0.8
0.9
1.0
b/h
1.1
1.2
1.3
1.4
1.5
δ=33°42'
φ=33°42'

Fig. 25.



0.6
0.5
0.4
y/b
0.3
0.2
0.1
δ=33°42'
φ=45°00'
0.4
0.5
b/h
0.6
0.7
0.8

Fig. 26.

In Figs. 28, 29 and 30, w' has been used as one of the variables instead of ϕ, the angle of internal friction. In these three figures data for the curves labeled $\delta = 0° \; 00'$ were computed from the formula

$$\frac{b}{h} = 0.0816G\sqrt{w'}$$

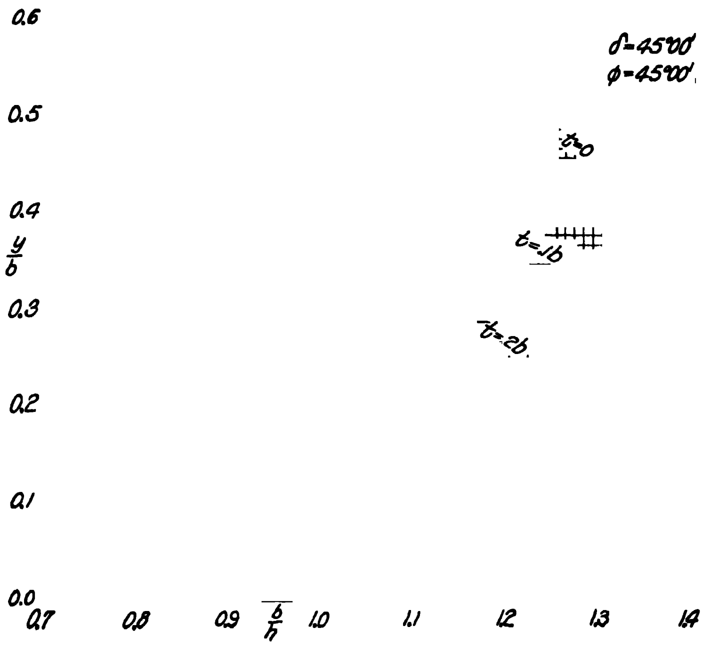


Fig. 27.

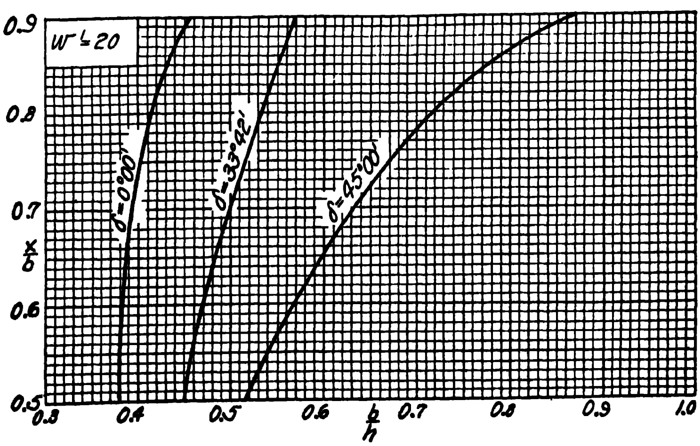


Fig. 28.

in which values of the factor G were taken from the curve labeled $t = 0.1b$ in Fig. 24. This is the same as eq. (6) of Art. 5*a* but with a different value of G.

Data for the curves labeled $\delta = 33° 42'$ and $\delta = 45° 00'$ were computed respectively from the following formulas:

$$\frac{b}{h} = 0.0816\sqrt{w'}\left(1 + \frac{2}{3}\frac{x}{b}\frac{b}{h}\right)^{3/2}$$

$$\frac{b}{h} = 0.0816\sqrt{w'}\left(1 + \frac{x}{b}\frac{b}{h}\right)^{3/2}$$

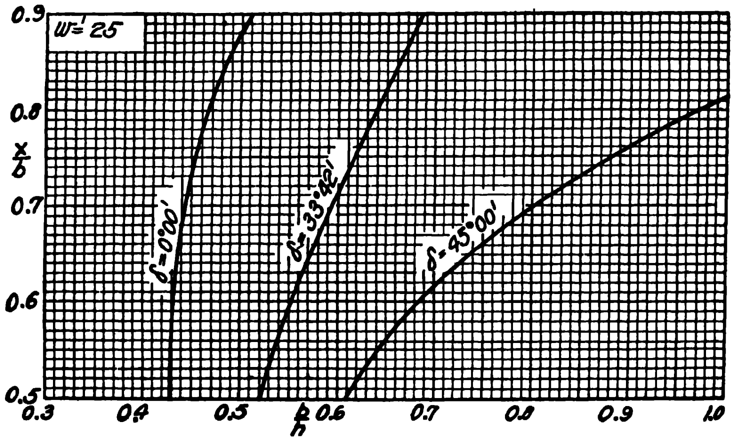


FIG. 29.

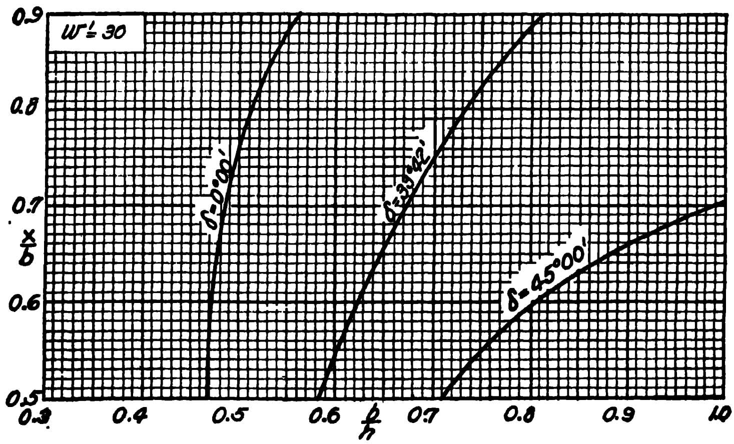


FIG. 30.

These formulas are the same as eqs. (7) and (8) of Art. 5b except that $C \cos \delta$ has been replaced by $\frac{w'}{100}$ and G' has been assumed equal to unity as in the approximate method of Art. 5c.

Rules quite often used in the design of reinforced concrete walls state that the base should be made five-tenths or six-tenths the height of the wall. An inspection of Figs. 28, 29 and 30 will show under what conditions these rules may

be used safely and economically and under what conditions they should be modified.

These diagrams may be used to advantage in formulating simple rules for design of walls under a certain set of conditions. For example: Given $w' = 25$, $\delta = 0° 00'$, and $33° 42'$, $\frac{x}{b} = 0.6$ to 0.7. From Fig. 29 it is evident that for $\delta = 0° 00'$, $\frac{b}{h} = 0.45$ will produce satisfactory results. Likewise for $\delta = 33° 42'$, $\frac{b}{h} = 0.6$ will produce satisfactory results.

A study of Figs. 28, 29 and 30 indicates that for the most economical wall $\frac{x}{b}$ should be made as small as possible. However, it must be borne in mind that these diagrams show the proportions that satisfy one condition, namely that the resultant must cut the base at or near the edge of the middle third. In addition to this condition care must be taken in the design of a reinforced concrete wall to see that enough weight is provided to prevent sliding of the wall bodily. A value of four-tenths is quite often used for the coefficient of friction between the wall and its foundation. In other words, the lateral thrust of the earth against the wall should not exceed four-tenths of the weight of the wall and the earth above the heel of the wall.

11. Relation of Toe Unit Pressure to Proportions of Wall.—In determining the relation between the proportions of a wall and the unit pressure at the toe a study similar to that described in Art. 7 was made. It was assumed that the resultant cut the edge of the middle third of the base and that therefore the unit pressure at the toe was equal to twice the average pressure. It was also assumed that the vertical wall had a thickness equal to one-tenth the base width, and that the base slab had a thickness equal to one-tenth the overall height of wall and base.

The following rules were formulated:

For level earth fill

$$p = 60\left(h + 3h\frac{x}{b}\right) \tag{10}$$

For $1\frac{1}{2}:1$ earth slope

$$p = 60\left(h + 3h\frac{x}{b} + \frac{x^2}{0.9b}\right) \tag{11}$$

For $1:1$ earth slope

$$p = 60\left(h + 3h\frac{x}{b} + \frac{x^2}{0.6b}\right) \tag{12}$$

12. Design of a Reinforced Concrete Retaining Wall, Cantilever Type.—The design of a reinforced concrete wall of the cantilever type comprises the design of three independent cantilever walls—the back part of the base slab, or heel of the wall, the front part of the base slab, or toe of the wall, and the vertical wall. A variation in the position of the vertical wall on the base will cause a variation in all parts of the design, except that of the vertical wall for a level earth fill.

The detailed analysis of a cantilever wall may be explained best by means of an illustrative problem. A complete design is presented in the following problem.

Illustrative Problem.—Given: Height of wall = 16 ft., $\delta = 33° 42'$, $w' = 25$, maximum allowable unit pressure at the toe is 4,000 lb. per sq. ft., coefficient of friction between concrete and foundation material = 0.4, f_s = 16,000 lb. per sq. in., f_c = 650 lb. per sq. in., u = 100 lb per sq. in. (deformed bars), and maximum allowable unit shear taken by concrete = 40 lb. per sq. in.

Assume 0.7 and investigate toe unit pressure and resistance to sliding. From Fig. 29 with $\frac{x}{b} = 0.7$, $\frac{b}{h}$ 0.6 or b = 0.6(16) = 9.6 ft. and x = 0.7(9.6) = 6.72 ft. The unit pressure at the toe is

$$p = 60\left(h + 3h\frac{x}{b} + \frac{x^2}{0.9b}\right) = 60\left[16 + 3(16)(0.7) + \frac{(6.72)^2}{0.9(9.6)}\right] = 3{,}300 \text{ lb. per sq. ft.}$$

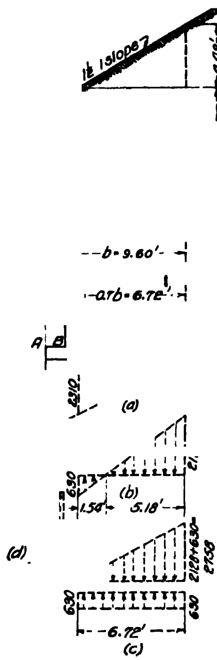


FIG. 31.

This value is less than the allowable and is therefore satisfactory.

Before proceeding further the resistance to sliding must be investigated. The weight of the wall and earth shown in Fig. 31 (*a*) may be determined approximately by assuming the vertical wall to have a thickness equal to one-tenth the base width, and the base to have a thickness equal to one-tenth the total height of the wall. Then $W = [0.1(16)(9.6) + 0.9(16)(0.1)(9.6)]150$

$$+ \left[0.9(16)(6.72) + \left(\frac{4.48}{2}\right)6.72\right]100 = 15{,}580 \text{ lb.}$$

$$P_H = w'h^2 \quad 25(20.28)^2 = 5{,}140 \text{ lb.}$$

Since P_H is less than four-tenths of W, this base width and position of vertical wall on the base is satisfactory.

Since the height of the vertical wall is unknown until the thickness of the base slab is determined the base slab back of the vertical wall will be designed first. To get the downward pressure on this slab assume that the thickness of the base slab is one-tenth the given height of wall. Then the downward pressure at C, Fig. 31(*a*), is 0.9(16)(100) + 0.1(16)(150) = 1,680 lb. per sq. ft. The upward pressure at C is 0.7(3,300) = 2,310 lb. per sq. ft. The resultant upward pressure at C is 2,310 − 1,680 = 630 lb. per sq. ft. The downward pressure at D = 1,680 + 4.48(100) = 2,128 lb. per sq. ft. This is the resultant downward pressure at this point since the upward pressure is zero. The distribution of pressure is shown in Fig. 31(*b*).

The distance from C to the point of zero pressure is

$$\left(\frac{630}{630 + 2{,}128}\right)6.72 = 1.54 \text{ ft.}$$

The moment at C is

$$\frac{5.18(2{,}128)}{2}\left[1.54 + \frac{2}{3}(5.18)\right] - \frac{1.54(630)}{2}\left(\frac{1.54}{3}\right) = 27{,}300 \text{ ft.-lb.}$$

The computation for the location of the point of zero pressure may be eliminated and the moment computation may be simplified by considering the pressures to be distributed as indicated in Fig. 31 (*c*). Then the moment at C is

$$\frac{6.72(2{,}758)}{2}(6.72)\left(\frac{2}{3}\right) - 6.72(630)\left(\frac{6.72}{2}\right) = (6.72)^2\left(\frac{2{,}758}{3} - \frac{630}{2}\right) = 27{,}300 \text{ ft.-lb.}$$

From the formula $M \quad Kbd^2$, $d = \sqrt{\frac{M}{Kb}} \quad \sqrt{\frac{27{,}300(12)}{107(12)}}$ 16 in. or total thickness of slab = 18 in. In the determination of the downward pressure it was assumed that this slab was 1.6 ft. thick. The results are therefore satisfactory. From the formula $V = vbjd$, $d = \frac{V}{vbj} = \frac{V}{40(12)(7/8)} = \frac{V}{420}$. The shear $V = \left(\frac{2{,}758}{2} - 630\right)6.72 = 5{,}040$ lb. Then

the depth required for shear is $\frac{5{,}040}{420} = 12$ in. Moment governs as might be expected. From the formula, $A_s = \frac{M}{f_s jd}$,

$$A_s = \frac{27{,}300(12)}{16{,}000(\frac{7}{8})16} = 1.46 \text{ sq. in.}$$

This area may be provided with $\frac{3}{4}$-in. round rods, spaced $3\frac{1}{2}$ in. apart. Investigating the unit bond stress by considering a strip of wall $3\frac{1}{2}$ in. wide; it is found that $u = \frac{V}{\Sigma_0 jd} = \frac{\left(\frac{3.5}{12}\right)5{,}040}{(\frac{7}{8})(16)2.356} = 45$ lb. per sq. in. This value is less than the allowable. The rods should be extended from C (Fig. 31 (a)) toward A a distance equal to 40 diameters.

The height of earth causing pressure on the vertical wall is $16.00 + 4.48 - 1.50 = 18.98$ ft.

$$P = 25(18.98)^2 = 4{,}500 \text{ lb.}$$

and

$$M = \frac{4{,}500(18.98)}{3} = 28{,}500 \text{ ft.-lb.}$$

The depth required for moment is

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{28{,}500(12)}{107(12)}} = 16.3 \text{ in.}$$

The depth required for shear is

$$d = \frac{V}{vbj} = \frac{4{,}500}{40(12)(\frac{7}{8})} = 10.7 \text{ in.}$$

The effective depth of the wall at the base will be made 16.5 in. and the thickness will be made 18 in. The effective depth of the wall at the top will be made 10.5 in. and the thickness will be made 12 in. In order that the size and spacing of the reinforcing rods may be selected, the area of steel required at the foot of each of four equal divisions of the vertical wall will be determined. The computations are shown in the following table:

Point	Height to top of wall	$\frac{\text{Height}}{14.5}$	$\left(\frac{\text{Height}}{14.5}\right)^3$	Moment (ft.-lb.)	Effective depth (in.)
1	3.625	0.25	0 0156	0.0156(28,500) = 450	12.0
2	7.250	0.50	0.1250	0.1250(28,500) = 3,560	13.5
3	10.875	0.75	0 4219	0.4219(28,500) = 12,020	15.0
4	14.500	1.00	1.0000	1.0000(28,500) = 28,500	16.5

In finding the effective depths at various heights a uniform variation from 10.5 in. at the top to 16.5 in. at the bottom was assumed. The area of steel required at each of the four points is as follows:

$$A_s = \frac{M}{f_s jd}$$

$$A_{s_1} = \frac{450(12)}{16{,}000(\frac{7}{8})(12.0)} = 0.032 \text{ sq. in.}$$

$$A_{s_2} = \frac{3{,}560(12)}{16{,}000(\frac{7}{8})(13.5)} = 0.226 \text{ sq. in.}$$

$$A_{s_3} = \frac{12{,}020(12)}{16{,}000(\frac{7}{8})(15.0)} = 0.688 \text{ sq. in.}$$

$$A_{s_4} = \frac{28{,}500(12)}{16{,}000(\frac{7}{8})(165.)} = 1.480 \text{ sq. in.}$$

The variation in area of steel required is shown in Fig. 32. The size and arrangement of the reinforcing rods will not be determined until after the portion AB of the base slab has been designed.

The length of this toe $AB = 9.60 - (6.72 + 1.50) = 1.38$ ft. The upward pressure at the outside edge of the vertical wall $= \left(\frac{9.60 - 1.38}{9.60}\right)3{,}300 = 2{,}830$ lb. per sq. ft. It will be assumed that the slab AB is 18 in. thick. Then the downward pressure is $\frac{18}{12}(150) = 225$ lb. per sq. ft. The resultant upward pressure at $A = 3{,}300 - 225 = 3{,}075$ lb. per sq. ft. and at $B = 2{,}830 - 225 = 2{,}605$ lb. per sq. ft. The distribution of pressure is shown in Fig. 31 (d).

$$\text{The moment} = \frac{2{,}605(1.38)^2}{2} + \frac{(3{,}075 - 2{,}605)(1.38)^2}{3} = 3{,}030 \text{ ft.-lb.}$$

The depth required for moment is

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{3{,}030(12)}{107(12)}} = 5.32 \text{ in.}$$

The depth required for shear is

$$d = \frac{V}{vbj} = \frac{\left(\frac{3{,}075 + 2{,}605}{2}\right)1.38}{40(12)\left(\frac{7}{8}\right)} = 9.32 \text{ in.}$$

Since the thickness is 18 in. and the effective depth is 16 in. it is evident that satisfactory results will be obtained by using the same reinforcement in the toe as at the bottom of the vertical slab. These rods must be embedded 40 diameters in the base slab anyway in order that the bond requirements may be satisfied. Therefore they will be carried along the lower surface of the slab AB to the front edge of the base slab. The area of steel required at the foot of the vertical wall may be provided by ¾-in. round rods spaced 3½ in. apart. Some of these rods may be cut off where no longer needed as indicated in Fig. 32. The bars should be continued about 30 diameters beyond the theoretical cut-off points shown in Fig. 32 in order to develop some resistance in bond.

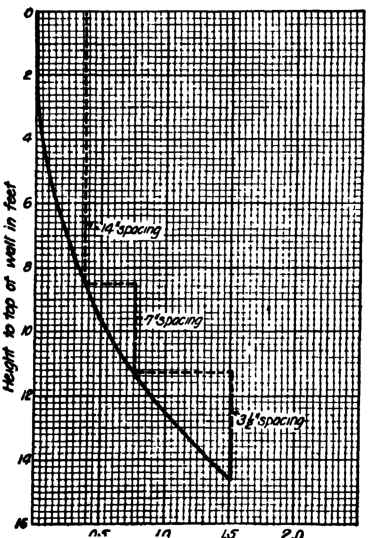


Fig. 32.

Temperature reinforcement may be provided by means of ½-in. round rods, 2 ft. apart, placed horizontally along the front face of the wall.

The completed design is shown in Fig. 33.

13. Design of a Reinforced Concrete Retaining Wall, Counterforted Type.—The analysis and design of a reinforced concrete counterforted retaining wall presents one of the most difficult problems which the structural engineer must solve. Due to the introduction of counterforts at frequent intervals, the simple cantilever action of an ordinary cantilever retaining wall is changed, because the vertical wall and the back part of the base slab are divided into panels which are supported along three edges.

Since these two slabs are rigidly connected it seems reasonable to assume that each acts as a restraining influence upon the other, causing a small amount of simple cantilever action near their junction. However, since the amount of the restraint and the extent of the cantilever action is unknown, it is common practice to disregard the connection between the vertical wall and the base.

Then both are designed as continuous slabs supported on counterforts. Because of the condition of continuity a bending moment coefficient of $\frac{1}{12}$ is generally

Whether shear or moment will govern the thickness of these slabs will depend largely upon the spacing of the counterforts. Until the engineer has had con-largelyle experience in counterforted wall design both should be investigated. The size or spacing of rods is frequently controlled by the bond stress so that this point should be investigated also.

The spacing of the counterforts generally varies from a minimum of 6 or 8 ft. to a maximum of 12 or 15 ft. The thickness of the counterfort is governed by the amount and arrangement of the steel which must be accommodated, and not by theoretical considerations.

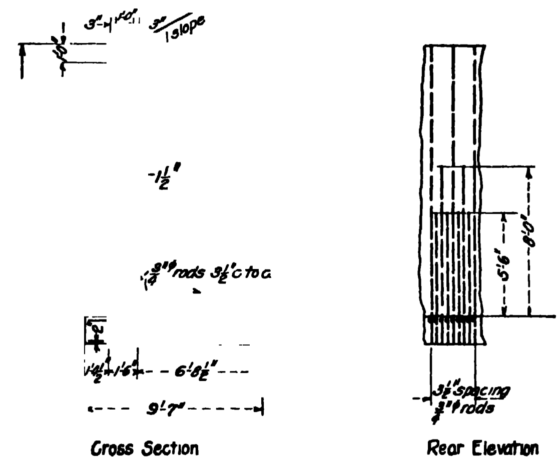


FIG. 33.

The design of the toe slab is the same as in the case of the cantilever wall except when counterforts are placed in front of the vertical wall. Usually the toe slab is relatively narrow and is designed as a cantilever.

The detailed analysis of a counterforted wall may be explained best by means of an illustrative problem. A complete design is presented in connection with the following problem.

Illustrative Problem.—Given: Height of wall = 24 ft., spacing of counterforts = 8 ft., δ = 33° 42′, w' = 25, maximum allowable unit pressure at the toe = 6,000 lb. per sq. ft., coefficient of friction between concrete and foundation material = 0.4, f_s = 16,000 lb. per sq. in., f_c = 650 lb. per sq. in., u = 100 lb. per sq. in. (deformed bars), and allowable unit shear taken by concrete = 40 lb. per sq. in.

Assume $\frac{x}{b} = 0.7$ and investigate toe unit pressure and resistance to sliding. From Fig. 29, with $\frac{x}{b} = 0.7$, $\frac{b}{h} = 0.6$, or $b = 0.6(24) = 14.4$ ft., and $x = 0.7(14.4) = 10.08$ ft. The unit pressure at the toe is

$$p = 60\left(h + 3h\frac{x}{b} + \frac{x^2}{0.9b}\right) = 60\left[24 + 3(24)(0.7) + \frac{(10.08)^2}{0.9(14.4)}\right] = 4{,}940 \text{ lb. per sq. ft.}$$

This value is less than the allowable and is therefore satisfactory.

For the purpose of investigating the resistance to sliding it may be assumed that the thickness of the vertical wall is one-tenth the width of the base and that the thickness of the base is one-tenth the total height of the wall. The difference between the weight of a counterfort and an equivalent volume of earth is small and may be neglected without appreciable error. Then

$$W = \Big[0.1(24)(14.4) + 0.9(24)(0.1)(14.4)\Big]150 + \left[0.9(24)(10.08) + \left(\frac{6.72}{2}\right)(10.08)\right]100$$
$$= 35{,}050 \text{ lb.}$$

$$P_H = \frac{w'h^2}{2} = \frac{25(30.72)^2}{2} = 11{,}800 \text{ lb.}$$

Since P_H is less than four-tenths of W, this base width and position of vertical wall is satisfactory.

The rear portion of the base slab will be designed first as in the case of the cantilever wall. To get the downward pressure on this slab assume that the thickness of the base slab is one-tenth the total height of wall. Then the downward pressure at C (Fig. 34 (*a*)) is 0.9(24)100 + 0.1(24)150 = 2,520 lb. per sq. ft. The upward pressure at C is 0.7(4,940) = 3,458 lb. per sq. ft. The resultant upward pressure is 3,458 − 2,520 = 938 lb. per sq. ft. The downward pressure at D is 2,520 + (6.72)100 = 3,192 lb. per sq. ft. Since the upward pressure at this point is zero, this value is the resultant downward pressure at this point. The pressure distribution diagram is shown in Fig. 34 (*b*).

The portion of the slab CD subjected to the greatest bending moment will be a strip 1 ft. wide adjacent to point D. If it is assumed that the slab is continuous under the counterforts and that therefore a moment coefficient of $\frac{1}{12}$ may be used then the moment is

$$M = \frac{1}{12}wl^2\ 12\text{ in.-lb.} = \left(\frac{2{,}780 + 3{,}192}{2}\right)(8)^2 = 191{,}000\text{ in.-lb.}$$

From the formula $M = Kbd^2$, $d = \sqrt{\dfrac{M}{Kb}} = \sqrt{\dfrac{191{,}000}{107(12)}} = 12.2$ in. The shear $V = \dfrac{wl}{2} = \dfrac{(2{,}780 + 3{,}192)8}{4} = 11{,}944$ lb. For shear $d = \dfrac{V}{bjv} = \dfrac{11{,}944}{12(\frac{7}{8})40} = 28$ in. A slab thickness of 2.5 ft. will be satisfactory. Substituting in the formula $A_s = \dfrac{M}{f_sjd}$, the area of steel required at D for a strip of wall 1 ft. wide is

$$A_s = \frac{191{,}000}{16{,}000(\frac{7}{8})28} = 0.49\text{ sq. in.}$$

Since the pressure at E is zero, the area of steel required at this point is also zero. The bending moment produced in the 1-ft. strip of slab adjacent to C by the upward pressure is

$$M = \frac{1}{12}wl^2 12\text{ in.-lb.} = \left(\frac{938 + 528}{2}\right)(8)^2 = 46{,}900\text{ in.-lb.}$$

From the formula

$$A_s = \frac{M}{f_sjd} = \frac{46{,}900}{16{,}000(\frac{7}{8})28} = 0.12\text{ sq. in.}$$

The area of steel required at E is zero, while at a point midway between E and D it is half of that required at D or 0.25 sq. in. So for the 4 ft. of slab adjacent to D, ½-in. square rods 6 in. c. to c. will be used, and in the remainder of the slab, ½-in. square rods 12 in. c. to c. will be used (see Fig. 35). The bond stress in the bars near D is $u = \dfrac{vb}{\Sigma_0} = \dfrac{40(6)}{2} = 120$ lb. per sq. in. This value is slightly over the allowable but the above spacing will be used regardless.

Since the slab CD is 2.5 ft. thick the clear height of the vertical wall is 24.00 − 2.50 = 21.50 ft. The height of earth acting upon the vertical wall is 21.50 + 6.72 = 28.22 ft. Therefore the pressure at the base of the wall is (28.22)25 = 705 lb. per sq. ft. Since the vertical wall is continuous across counterforts $M = \frac{1}{12}wl^2$ 12 in.-lb. = 705(8)² = 45,000 in.-lb.

The depth required for moment is

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{45{,}000}{107(12)}} = 5.9\text{ in.}$$

The shear $V = \dfrac{wl}{2} = \dfrac{705(8)}{2} = 2{,}820$ lb. The depth required for shear is $d = \dfrac{V}{bjv} = \dfrac{2{,}820}{12(\frac{7}{8})40} = 6.7$ in. Use a thickness of 9 in. and an effective depth of 7½ in.

The area of steel required at the base of the wall is

$$A_s = \frac{M}{f_sjd} = \frac{44{,}600}{16{,}000(\frac{7}{8})7.5} = 0.43\text{ sq. in.}$$

Assuming ½-in. square bars 6 in. c. to c. $u = \dfrac{vb}{\Sigma_0} = \dfrac{40(6)}{2} = 120$ lb. per sq. in. Since this size and spacing of rods is convenient and since this high bond stress exists over only a

small region near the base of the wall, this arrangement of rods will be adopted for the lower part of the wall. For the central part ½-in. square rods, 8 in. c. to c. will be used, while for the upper part of the wall ½ in. square rods, 12 in. c. to c. will be used. In the former case $A_s = 0.37$ sq. in. while in the latter case $A_s = 0.25$ sq. in. At the base of the wall, 21.5 ft. from the top, area required = 0.43 sq. in. The 8-in. spacing may be started $\frac{0.37}{0.43}(21.5) = 18.5$ ft. below the top of the wall, and the 12-in. spacing may be started $\frac{0.25}{0.43}(21.5) = 12.5$ ft. below the top of the wall. The adopted spacing of reinforcement is shown in Fig. 35.

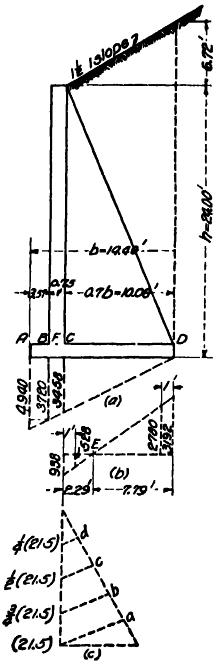


Fig. 34.

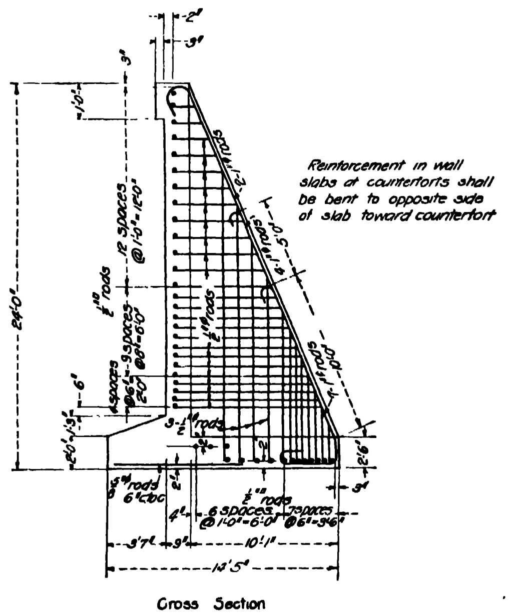


Fig. 35.—Shows position of reinforcement in counterforts and in wall slabs between counterforts.

The length of the toe AB is $14.40 - (10.08 + 0.75) = 3.57$ ft. The moment at B is

$$\frac{3{,}720(3.57)^2}{2} + \frac{(4{,}940 - 3{,}720)(3.57)^2}{3} = 28{,}900 \text{ ft.-lb.}$$

The depth required for moment is $d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{28{,}900(12)}{107(12)}} = 16.43$ in.

The shear at B is $\left(\frac{4{,}940 + 3{,}720}{2}\right)3.57 = 15{,}470$ lb. The depth required for shear is $d = \frac{V}{bjv} = \frac{15{,}470}{12(\frac{7}{8})40} = 36.8$ in. or say 37 in. The toe will be made 3.25 ft. thick at B and 2 ft. thick at A. The area of steel required is $A_s = \frac{28{,}900(12)}{16{,}000(\frac{7}{8})(37)} = 0.67$ sq. in. This area can be provided by ⅝-in. round rods, 5½ in. c. to c. Then the bond stress is

$$u = \frac{V}{\Sigma_0 jd} = \frac{15{,}470}{1.96(\frac{7}{8})(37)\left(\frac{12}{5\frac{1}{2}}\right)} = 112 \text{ lb. per sq. in.}$$

This value is slightly over the allowable. The area of steel provided by ⅝-in. round rods, 5 in. c. to c. is 0.74 sq. in. Then the bond stress is 102 lb. per sq. in. which is satisfactory.

The horizontal steel in the counterfort must be so designed as to be able safely to withstand the horizontal pressure against the vertical wall. As given in the design of the vertical wall, the horizontal pressure on the lowest 1-ft. strip of vertical wall is 705 lb. per sq. ft.

Since the spacing of the counterforts is 8 ft. the total horizontal force is 8(705) = 5,640 lb., and the area of steel required is $\frac{5,640}{16,000} = 0.35$ sq. in. This area may be provided by ½-in. round rods, 7 in. c. to c. The center of the 1-ft. strip where the pressure is 705 lb. per sq. ft. is 21 ft. below the top of the wall. The area provided by ½-in. round rods, 12 in. c. to c. is 0.20 sq. in. This spacing may be started at a point $\left(\frac{0.20}{0.35}\right)21 = 12.08$ ft. below the top of the wall. Since this spacing of rods is practically the same as that of the horizontal rods in the front vertical slab, the same spacing will be adopted for both (see Fig. 35).

The total downward force to be resisted in the outermost 1-ft. section of the counterfort is $8\left(\frac{2,780 + 3,192}{2}\right) = 23,888$ lb. To carry this an area of vertical steel equal to $\frac{23,888}{16,000} = 1.49$ sq. in. must be provided. This area can be furnished by three ½-in. square rods, 6 in. c. to c. This spacing will be adopted for the 4 ft. of counterfort adjacent to D, and will be changed to 12 in. c. to c. for the next 4 ft. From E to C no vertical rods are needed since the resultant pressure is upward.

Both the horizontal and the vertical rods in the counterforts should be hooked around the horizontal rods in the vertical slab and the back part of the base slab.

The amount of steel required along the inclined edge of the counterfort may be determined by taking moments about point F The perpendicular distance from the inclined edge of the counterfort to this point is about 10 ft. Allowing 3 in. of protective covering for the steel the effective depth is 9 ft. 9 in. or 117 in.

The height of earth acting upon the vertical wall is 28.22 ft. as determined in the design of this wall. Then the bending moment is $M = \frac{w'h^2}{2}\left(\frac{h}{3}\right)12 = \frac{25(28.22)^3 12}{6} = 1,125,000$ in.-lb. per ft. of wall, or 8 (1,125,000) = 9,000,000 in.-lb. per counterfort. The area of steel required is

$$A_s = \frac{M}{f_s jd} = \frac{9,000,000}{16,000(\frac{7}{8})117} = 5.50 \text{ sq. in.}$$

The area of steel required at various points is directly proportional to the cube of the height, but is inversely proportional to the effective depth. Since the effective depth is directly proportional to the height, the result is to make the area of steel required directly proportional to the square of the height.

If 5.50 sq. in. of steel is needed at point a, Fig. 34c, then the steel needed at points b, c, and d respectively is as follows:

$$(\tfrac{3}{4})^2\, 5.50 = 3.10 \text{ sq. in.}$$
$$(\tfrac{1}{2})^2\, 5.50 = 1.38 \text{ sq. in.}$$
$$(\tfrac{1}{4})^2\, 5.50 = 0.34 \text{ sq. in.}$$

These requirements can be fulfilled by using 1-in. round rods, 7 at point a, 4 at point b, and 2 for the remainder of the distance. The rods should be extended beyond the theoretical points of cut-off a short distance in order to satisfy the bond stress. Additional strength may be gained by providing hooks at the ends of these rods as indicated in the completed design shown in Fig. 35.

SECTION 7

SLAB AND GIRDER BRIDGES

By Walter S. Todd

The simple slab is used for spans from 12 to 30 ft., the through girder for spans from 30 to 55 ft. and the deck girder for spans from 40 to 60 ft. The deck girder usually proves more economical than the through girder whenever there is sufficient headroom to permit the use of girders below the floor.

SLAB BRIDGES

1. Design of Floor.—The method to be followed in designing the floor of a reinforced concrete slab bridge is similar to the general methods employed in the design of slabs. It is customary to consider the floor as being a series of simple beams 1 ft. in width, reinforced in the direction of the center line of the road, and with span length equal to the distance between centers of supports, but not to exceed the clear span plus depth of slab.

2. Loads.—It is obviously impossible to specify the live load applicable to all locations. It is easy to arrive at the dead load, but the designer must have a knowledge of the traffic to which the structure is to be subjected before he can make a reasonable assumption concerning the live load. The structure should be designed for the maximum live load which may reasonably be expected within its lifetime. In the design given in this chapter a uniform live load of 125 lb. per sq. ft. of floor surface, or an engine or truck load of 24 tons distributed as shown in Fig. 1, is assumed.

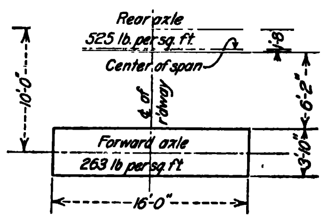


Fig. 1.—Assumed distribution of 24-ton engine load.

3. Impact.—The amount of impact caused by the various types of vehicles traveling over highway bridges is not definitely known, and as yet no standard practice has been established, although a number of tests have been made. In the following articles it is assumed that the live loads mentioned include impact.

4. Allowable Stresses in Superstructure.—When there is no provision made for expansion or contraction in the superstructure due to temperature changes, provision must be made for temperature stresses in the design. For a drop in temperature of 40 deg. from the normal, a tensile stress of 8,000 lb. per sq. in. is produced in the longitudinal reinforcing steel, if the abutments remain fixed, and there is no relative movement between abutments and superstructure. Tests, however, indicate that this temperature stress rarely exceeds 6,000 lb. per sq. in. As the maximum highway loads, such as tractors, seldom cross the bridge during the coldest weather, it is not probable that the maximum stress due to temperature will occur at the same time with the maximum stress due to loads.

In the design which follows a stress in steel of 12,000 lb. per sq. in. is used, which allows 4,000 lb. for stresses due to temperature fall before the allowable working stress in steel is reached. The allowable unit stress in concrete is 800 lb. per sq. in. The proportions used are 1 : 2½ : 4 with a maximum size of coarse aggregate of 1½ in.

5. Design of Superstructure.—Assume a clear span of 24 ft., the dead load to be the weight of the floor slab itself plus a wearing surface weighing 50 lb. per sq. ft., and the live loads as previously stated.

Assume $n = 15$; $f_s = 12{,}000$ lb. per sq. in.; $f_c = 800$ lb. per sq. in.; $v = 40$ lb. per sq. in.; $u = 80$ lb. per sq. in.; span to be used in design = 25 ft.; weight of concrete (including reinforcement) = 144 lb. per cu. ft.

L.L. moment due to engine is

$$(3.83)(525)(14.17) + (3.83)(263)(4.17)\left(\frac{10.83}{25.0}\right) - \frac{(1{,}005)(3.83)}{4} = 13{,}200 \text{ ft.-lb.}$$

(Figure 1 shows the 24-ton load placed in its relative position to the center of span so as to give the maximum bending moment.)

Uniform *L.L.* moment is

$$(\tfrac{1}{8})(125)(25)(25) = 9{,}770 \text{ ft.-lb.}$$

The moment due to the engine is the greater and will govern the design.

D.L. moment, assuming a slab having a total thickness of 15¾ in., is

$$\begin{aligned} \text{Concrete} &= (12)\ (15.75) = 189 \\ \text{Wearing surface} &= 50 \\ \hline \text{Total} &= 239 \text{ lb.} \end{aligned}$$

$$M = (\tfrac{1}{8})\ (239)(25)(25) = 18{,}670 \text{ ft.-lb.}$$

Total moment *L.L.* + *D.L.* = 13,200 + 18,670 = 31,870 ft.-lb.

Deducting 1¾ in. from the total depth of the slab as protective covering for the steel, the effective depth is 14 in.

From the flexure formulas for working loads, and based on the straight line theory, $M_s = pf_sjbd^2$ and $M_c = \frac{1}{2}f_ckjbd^2$, K is a coefficient which is equal to pf_sj in the formula for M_s and is equal to $\frac{1}{2} f_ckj$ in the formula for M_c. K should not exceed 168 for stresses of 12,000 lb. per sq. in. in steel and 800 lb. per sq. in. in concrete.

$$M = (31{,}870)(12) = (K)(12)(14)(14)$$

or

$$K = 162.6$$

The values of p and j may be determined directly from a curve giving the coefficients of resistance of beams. In this case $p = 0.0162$, and $j = 0.83$.

A_s required = (0.0162)(12)(14) = 2.72

$$\begin{aligned} \text{Dead load shear} &= (239)(12) &&= 2{,}868 \\ \text{Live load shear} &= \frac{(2{,}010)(23.08)}{25} + \frac{(1{,}005)(13.08)}{25} &&= 2{,}381 \\ \hline &&\text{Total } V &= 5{,}249 \text{ lb.} \end{aligned}$$

$$v = \frac{5{,}249}{(12)\ (0.83)(14)} = 37.6 \text{ lb. per sq. in.}$$

$$\Sigma_0 = \frac{5249}{(80)(0.83)(14)} = 5.65 \text{ sq. in.}$$

Use 1-in. square bars at 4½-in. centers. A_s provided = 2.67 sq. in. and Σ_0 = 10.66 in.

The details of the slab design are shown in Fig. 2.

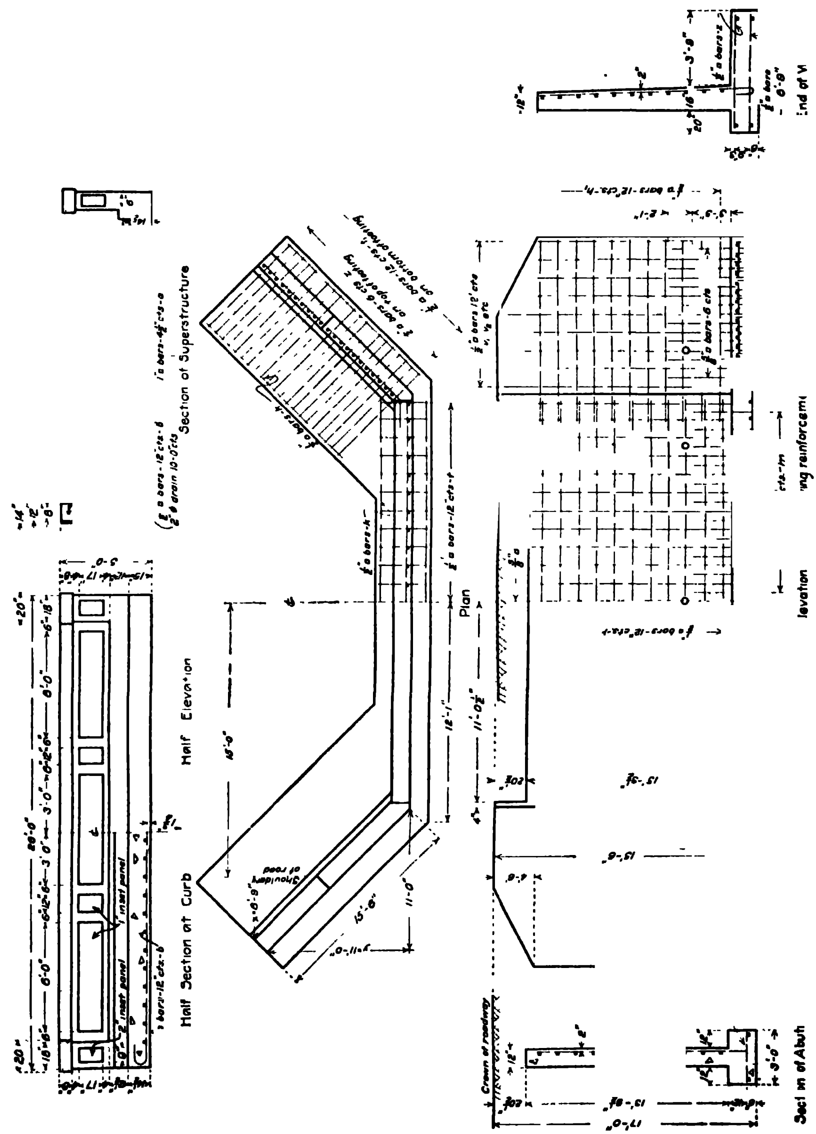


FIG De b bridg Sp igh

6. Abutments for Slab Bridge.—The abutment wall supporting a slab superstructure is usually designed as a vertical beam simply supported at the top by the slab and at the bottom by the footing, and carrying a uniformly varying load due to the horizontal component of the earth pressure back of the wall.

The amount of this pressure will vary depending upon the kind of soil, per cent of moisture in the soil, and the method by which the back fill is made. For the purpose of illustration it will be assumed in this design that the horizontal component of the pressure will be equal to that of a fluid weighing 21 lb. per cu. ft.

For convenience in design, most highway bridge engineers assume that the horizontal thrust due to earth retained by walls, varies uniformly with the depth of fill. The equivalent fluid pressures in pounds per cubic foot by some of the State Highway Departments are as follows:

Missouri—25; Illinois—21; North Carolina—21; Texas—25; Wisconsin—25; California—30 lb. per cu. ft. for usual backfills and 36 lb. per cu. ft. for sand fill.

The footing is designed to carry half the entire superstructure load, plus the weight of the wall itself and the earth fill directly over the footing. In designing the toe of the footing it is on the side of safety to neglect the weight of the earth on the front of the wall.

The wing walls are designed as independent cantilever retaining walls.

The following computations give the design for a substructure supporting a slab bridge with clear span of 24 ft. roadway of 20 ft. and a height overall of 17 ft. Detailed dimensions for this design are shown in Fig. 2.

The assumptions made in this case are given as follows: $f_s = 16{,}000$ lb. per sq. in.; $f_c = 650$ lb. per sq. in.; $n = 15$; $v = 40$ lb. per sq. in.; $u = 80$ lb. per sq. in.; weight of earth $= 100$ lb. per cu. ft.; equivalent fluid pressure $= 21$ lb. per cu. ft.; maximum soil pressure 6,000 lb. per sq. ft.; and average soil pressure $= 4{,}000$ lb. per sq. ft.

6a. Design of the Main Wall.—Denoting the intensity of pressure at the top of the wall by w_1 and at the top of the footing by $(w + w_1)$ it may be shown that the shear is equal to zero when

$$\frac{1}{6}wl - \frac{(w)(2)}{2l} + w_1\frac{L}{2} - w_1x = 0$$

and that the

$$\text{Max. } M = \frac{1}{6}wLx - \frac{wx^3}{6L} + \frac{w_1Lx}{2} - \frac{w_1x^2}{2}$$

where x is measured down from the top of the wall and L is the distance from top of footing to bridge seat.

Solving

$$\text{Max. } M = 4{,}380 \text{ ft.-lb.}$$

For the conditions ordinarily met with, it is very nearly exact to assume that $x = 0.56\,L$.

With dimensions shown

$M = (4{,}380)\ (12) = (K)\ (12)\ (10)^2$
$K = 43.8 \qquad p = 0.003 \qquad j = 0.92$
A_s required $= (0.003)\ (12)\ (10) = 0.36$ sq. in. per ft.
Use ⅝-in. square bars at 12-in. centers A_s provided $= 0.39$ sq. in.

The shear and bond stresses on the abutment wall are low and need not be investigated.

6b. Design of Main Wall Footing.—The superstructure loads are as follows:

$$\begin{aligned}
\text{Super. concrete} &= (33)(27)(144)(\tfrac{1}{2}) &&= 64{,}200\\
\text{Wearing surface} &= (50)(19)(26)(\tfrac{1}{2}) &&= 12{,}350\\
\text{Engine loading} &= 32{,}000 + \frac{(16{,}000)(15)}{25} &&= 41{,}600\\
& & \text{Total} &= 118{,}150 \text{ lb.}
\end{aligned}$$

A uniform live load of 125 lb. per sq. ft. of roadway surface would impose a load of only 30,875 lb. so the engine load will govern the design.

$$\frac{118{,}150}{22.08} = 5{,}350 \text{ lb. per lin. ft. of wall}$$

The loads on the footing are as follows:

$$\begin{aligned}
\text{Main wall} &= (13.79)(1)(144) = && 1{,}985 \text{ lb.}\\
\text{Footing} &= (3)(1.5)(144) = && 650\\
\text{Earth} &= (15.5)(1)(100) = && 1{,}550\\
\text{Super load} &= && 5{,}350\\
& \qquad\qquad \text{Total} = && 9{,}535 \text{ lb.}
\end{aligned}$$

$$\text{Average soil pressure} = \frac{9{,}535}{3} = 3{,}175 \text{ lb. per sq. ft.}$$

Moment about base of wall neglecting small downward moment of footing slab is

$$\frac{3{,}175}{2} = 1{,}590 \text{ ft.-lb.} \qquad K = \frac{1{,}590}{(12)(12)} = 11.0 \qquad p = 0.0007$$

A_s required $= 0.101$ sq. in.

Use $\frac{1}{2}$-in. square bars at 12-in. centers. A_s provided $= 0.25$ sq. in.

No shear or bond need be figured on the footing since a line drawn downward at 45 deg. from the juncture of the wall and the top of the footing will intersect the reinforcing steel at the outer edge of the footing.[1]

In the design of the abutment wall it was assumed that support was furnished the wall at the top and bottom. In order to obtain this condition, the frictional resistance between the slab and the top of the wall must equal or exceed the horizontal force at the top caused by the earth pressure. In the case assumed, this force

$$P_H = (\tfrac{1}{6})(21)(13.79)(13.79) + (\tfrac{1}{2})(1.71)(21)(13.79) = 913 \text{ lb. per ft. of bridge seat}$$

This force is resisted by the weight of the superstructure (neglecting the wearing surface) times a coefficient of friction suitable to the existing conditions. For concrete on concrete, it is safe to assume a value of 0.6 which would give a factor of safety equal to $\left(\frac{64{,}20}{22.08}\right)(0.6) + 913 = 1.91$.

Experience with slab bridges shows that there is no relative movement between slab and abutment, even when a tar-paper joint is provided between them.

6c. Wing Walls.—Moment of earth about base of wall is

$$M = (\tfrac{1}{6})(21)(15.5)^3 = 13{,}050 \text{ ft.-lb.} \quad K = \frac{13{,}050}{(14)^2} = 66.5$$

[1] See Proposed Specifications of New Joint Committee, *Appendix G.*

$p = 0.0047 \quad A_s$ required $= 0.79$ sq. in. $\quad j \quad 0.90$

$V = (\tfrac{1}{2})(21)(15.5)^2 = 2{,}520$ lb.

$$\frac{2{,}520}{(12)(0.90)(14)} = 16.7 \text{ lb. per sq. in.} \quad \Sigma_0 = \frac{2{,}520}{(80)(0.90)(14)} = 2.5 \text{ sq. in.}$$

Use ⅝-in. square bars at 6-in. centers $\begin{cases} A_s = 0.78 \text{ sq. in.} \\ \Sigma_0 = 5.00 \text{ in.} \end{cases}$

At a point 5 ft. 6 in. above top of footing

$d = 12.58$ in. and $M = (\tfrac{1}{6})(21)(10)^3 = 3{,}500$ ft.-lb.

$$K = \frac{3{,}500}{(12.58)^2} = 22.1$$

$p = 0.0015 \quad A_s$ required $= 0.227$ sq. in.

Use ½-in. square bars at 12-in. centers $\quad A_s = 0.25$ sq. in.

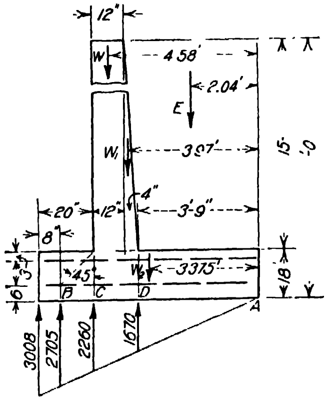


FIG. 2a.—Section of wing wall.

6d. Footing Design of Wing Walls.—The footing is designed as two cantilevers, the front or toe being reinforced to withstand resultant stresses upward, and the rear portion to withstand downward stresses.

The width of base is taken as 0.4 of the overall height—in this case 6 ft. 9 in. (see Fig. 2a).

Taking moments about point A:

$W = (15.5)(1)(144)$	$= (2{,}233)(4.58)$	$= 10{,}220$ ft.-lb.
$W_1 = (15.5)(\tfrac{1}{2})(\tfrac{1}{3})(44)$[1]	$= (114)(3.97)$	$= 453$
$W_2 = (6.75)(1.5)(144)$	$= (1{,}458)(3.375)$	$= 4{,}922$
$E = (15.5)(4.08)(100)$	$= (6{,}325)(2.0\ )$	$= 12{,}905$
Total vertical force	$= 10{,}130$ lb.	
Moment due to earth fill	$= (\tfrac{1}{6})(21)(17)^3$	$= 17{,}200$
	Total M	$= 45{,}700$ ft.-lb.

$$\text{Eccentricity} = \frac{45{,}700}{10{,}130} - 3.375 = 1.135$$

$$P = \frac{4w}{3(b - 2e)} = \frac{(4)(10{,}130)}{3[6.75 - (2)(1.13)]} = 3{,}008 \text{ lb. per sq. ft.}$$

$$\text{Pressure equals zero at point, } b' = \frac{(2)(10{,}130)}{3{,}008} = 6.74 \text{ ft. from toe.}$$

[1] Forty-four is the difference in weight between earth and concrete.

It will be seen that this point corresponds very nearly with the heel of the footing

$$V_B = \left(\frac{3{,}008 + 2{,}705}{2}\right)(0.67) - (0.67)(144)(1.5) = 1{,}760 \text{ lb.}$$

$$v = \frac{1{,}760}{(12)(0.92)(12)} = 13.27 \text{ lb. per sq. in. } \Sigma_0 = \frac{1{,}760}{(80)(0.92)(12)} = 1.99 \text{ in.}$$

$$M_C = \frac{2{,}260(1.67)^2}{2} + \frac{(748)(1.67)}{2}\left(\frac{2}{3}\right)(1.67) - \frac{(144)(1.5)(1.67)^2}{2} = 3{,}545 \text{ ft.-lb.}$$

$$K = \frac{3{,}545}{(12)^2} = 24.7 \qquad p = 0.0017 \qquad j = 0.92$$

$$A_s = (0.0017)(12)(12) = 0.245 \text{ sq. in.}$$

½-in. sq. bars at 12-in. centers = 0.25 sq. in. $\Sigma_0 = 2.00$ in.

$$M_D = (1{,}670)\left(\frac{3.75}{2}\right)\left(\frac{3.75}{3}\right) - \frac{(15.5)(100)(3.75)^2}{2} - \frac{(144)(1.5)(3.75)^2}{2} = -8{,}500 \text{ ft.-lb.}$$

$$K = \frac{8{,}500}{(15)^2} = 37.8 \qquad A_s = (12)(0.0026)(15) = 0.468 \text{ sq. in.}$$

$$p = 0.0026 \qquad j = 0.92$$

$$V_D = (15.5)(100)(3.75) + (3.75)(144)(1.5) - \frac{(1{,}670)(3.75)}{2} = 3{,}490 \text{ lb.}$$

$$v = \frac{3{,}490}{(12)(0.92)(15)} = 21.1 \text{ lb. per sq. in. } \Sigma_0 = \frac{3{,}490}{(0.92)(80)(15)} = 3.17 \text{ in.}$$

Use ½-in. square bars at 6-in. centers, $A_s = 0.50$ sq. in. $\Sigma_0 = 4.00$ in.

6e. Stability of the Wall Against Overturning.—The factor of safety of the wall against overturning is equal to the moment of all the vertical loads about the toe of the footings, divided by the horizontal moment of the earth about the same point. When the resultant of all the forces acting passes through the outside edge of the middle third as it does in the problem above cited, the factor of safety is 2.

6f. Stability Against Sliding.—The force tending to move the wall forward is equal to the horizontal thrust of the earth or

$$(½)(21)(17)^2 = 3{,}035 \text{ lb.}$$

This force is resisted by the vertical loads multiplied by a coefficient of friction. With the 17-ft. wall, this coefficient must equal at least

$$\frac{3{,}035}{10{,}130} = 0.3$$

in order to prevent the wall from slipping along the base. With firm, unsaturated subsoil, a coefficient of friction of about 0.57 may safely be assumed. This gives a factor of safety of 1.9 against sliding, which is large enough in view of the fact that the tendency to slip is materially resisted by the backfill placed against the front of the footing and the wall.

6g. Length of Wing and Height of Wing at End.—For figuring the length of wing, and height of wall at the end, it is necessary to know the difference in elevation between the shoulder and the stream bed on line with the abutment wall, and to assume the slope of the earth in place. Referring to the elevation of the wing wall shown in the drawing (see Fig. 2), assume that the shoulder is 12 ft. 6 in. above the stream bed. If the earth assumes a slope of 1½ horizontal to 1 vertical, the distance $(X + 1 + y)$ must equal (1.5) (12.5 ft.) = 18.75 ft. to prevent the earth from reaching the stream when the lowest

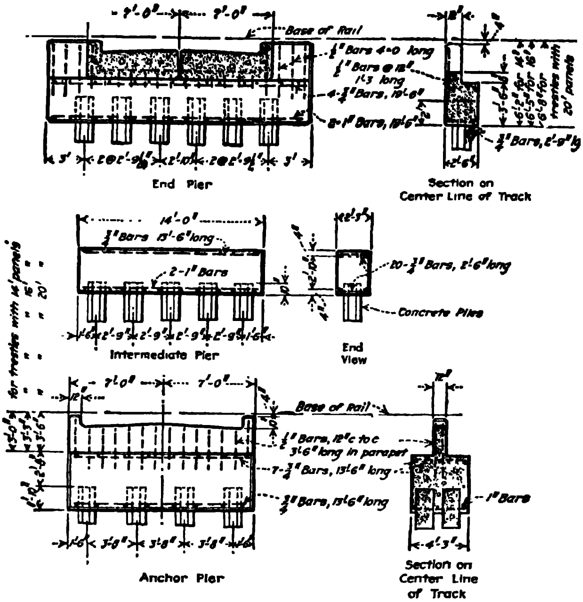


FIG. 3.—Details of substructure, standard concrete pile trestle, Illinois Central R. R.

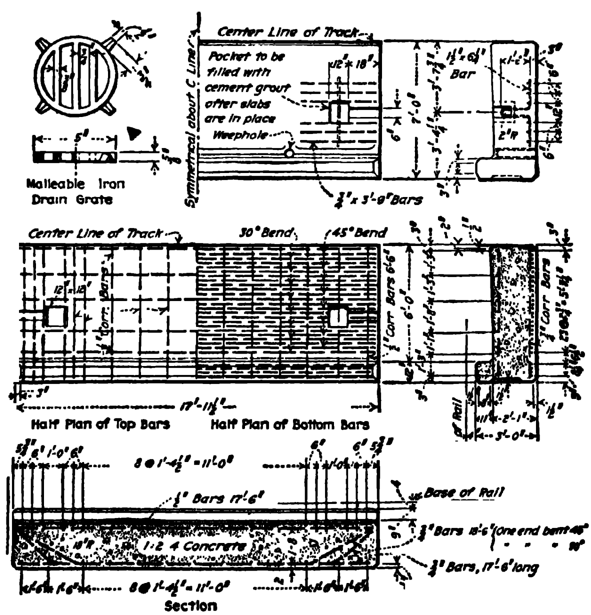


FIG. 4.—Standard slab for clear spans of 16 ft., Illinois Central R. R.

point in the stream is adjacent to the face of the abutment. This gives a wing length of 15 ft. 6 in. The drop at the end of the wing is $\frac{2}{3} X = 4$ ft. 6 in.

7. Slab Bridges of Multiple Spans.—Slab bridges of multiple spans will be treated under the three following headings:

Concrete pile trestles.
Pier trestles.
Trestles with framed bents.

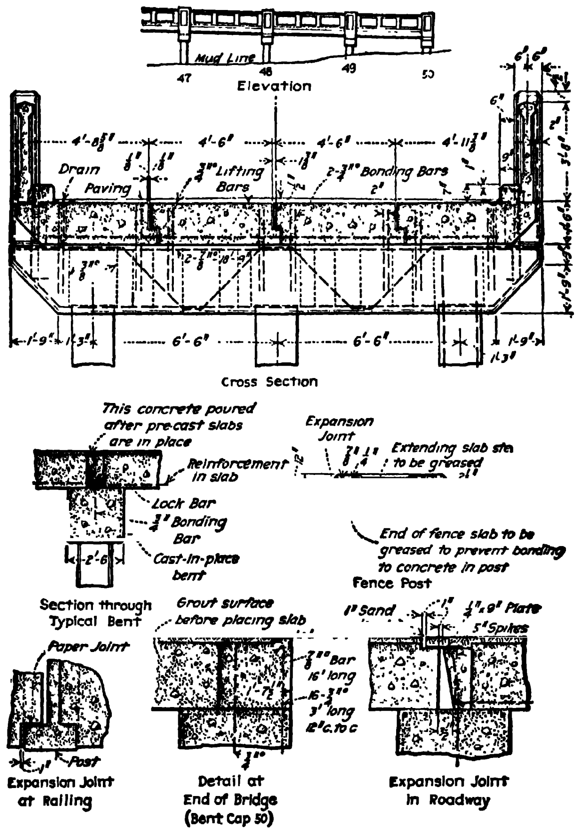


FIG. 5.—Details of pile trestle across the Miles River near Easton, Md.

7a. Concrete Pile Trestles.—Figures 3 and 4 give the essential details of design of the pile trestles built by the Illinois Central Railroad. They can be considered typical of concrete pile trestles in general. These trestles replace similar wooden structures over swamps and shallow streams which may not be filled and where bridges on more permanent supports would be extremely expensive because of their great length. The construction consists of pile bents spaced generally from 16 to 20 ft. c. to c., and with a height above ground not greater than the span. The piles are capped with reinforced concrete girders which support the floor slabs.

The piles and deck slabs are usually cast in a convenient yard, allowed to season from 60 to 90 days, and are then hauled to the bridge site. The lifting stirrups shown permit of the slabs being set in place by a wrecking crane.

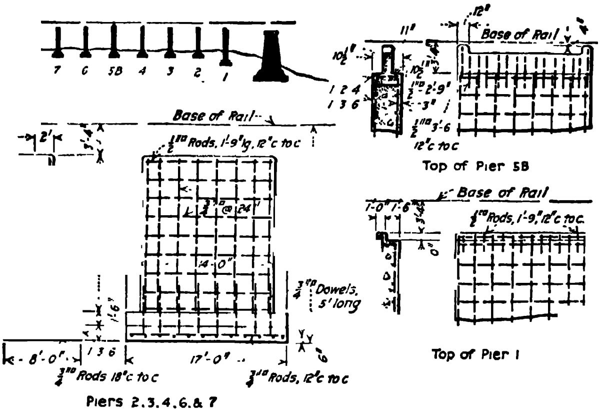


FIG. 6.—Pier details, Illinois Central R. R. trestle over Kaskaskia River near New Athens Illinois.

The ballast and track are laid directly on the slabs after the longitudinal and transverse joints (except at anchor bents) are filled with cement mortar and after

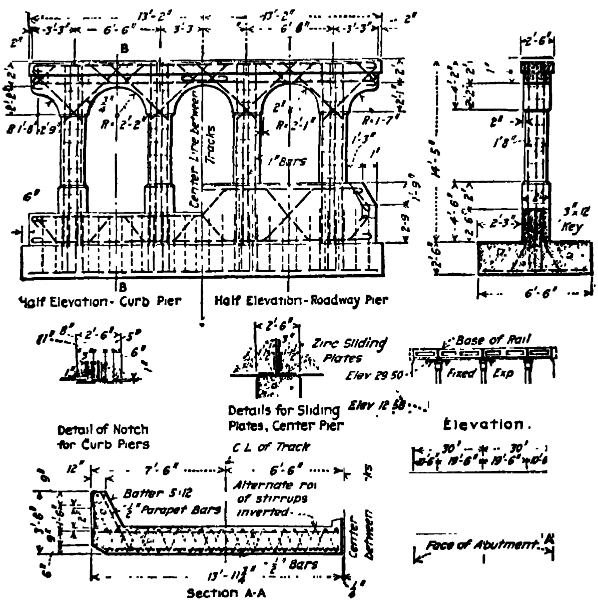


FIG. 7.—Details of Mozart Street subway, Bloomingdale Road track elevation, Chicago, Milwaukee & St. Paul Railway.

the floor surface is thoroughly waterproofed. The slabs are set on a bed of grout on the pile caps. An anchor bent is used at suitable intervals to take up longitudinal stresses due to tractive force and, by means of an expansion

joint, to prevent any great accumulation of movement of the deck due to temperature changes.

A concrete pile trestle for carrying a highway is shown in Fig. 5.[1] It was found economical to cast the piles, deck slabs, and railing slabs at Baltimore, 60 miles away, and transport them to the site on scows. Expansion joints were located in the roadway slabs, curb, and railing slabs at every fifth bent.

7b. **Pier Trestles.**—Thin concrete piers are preferable to pile bents when the height of bridge above the ground line is greater than about 16 ft. Figure 6 shows a typical trestle of the solid bench-wall type built by the Illinois Central Railroad.

7c. **Trestles with Framed Bents.**—Slab bridges with framed bents forming subways are used on at least fifteen railroads in this country. A design which may be considered typical is shown in Fig. 7. The deck slabs may either be cast in place or cast at some central yard and placed in a similar manner to the slabs for pile or pier trestles. In Fig. 7 the design is shown for slabs to be cast in place.

GIRDER BRIDGES

8. Through Girders.—The through girder type of structure (Fig. 8) is adapted to spans of from about 30 to 60 ft., and to locations where the clearance between high water and finished grade is limited. This type is not economical for wide roadways—that is, for roadways of more than 20 ft. in width.

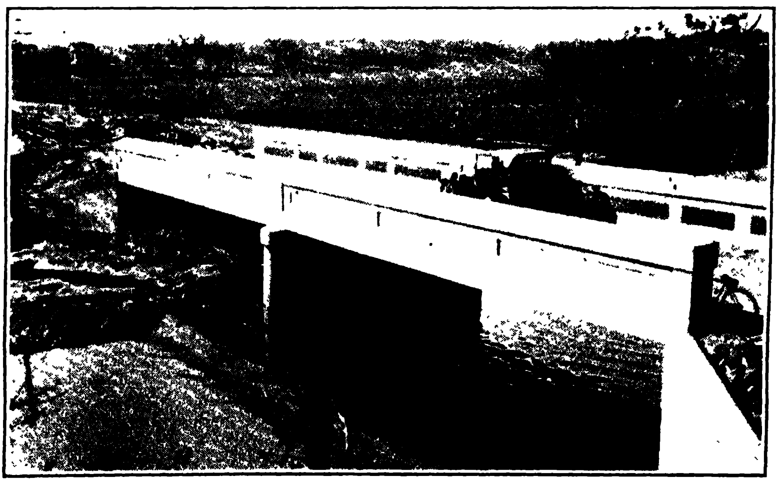


FIG. 8.—Through girder bridge built in 1920 near Springfield, Illinois—spans 50 ft. each—roadway 20 ft.

In a through girder structure the loads from the roadway surface are carried to the girders through the floor slab; and the girders in turn carry the loads to the abutments. In the longer spans the girders are quite large and present a rather unpleasant appearance due to their size.

8a. **Allowable Stresses and Loads.**—By the use of an expansion device at one support of the superstructure, the temperature stresses may be

[1] See also *Eng. News*, Feb. 5, 1914.

eliminated, and a much higher working stress may be used in the design. For the following example, the superstructure will be designed for a maximum compressive stress in the concrete of 1,000 lb. per sq. in.; tension in reinforcing steel, 16,000 lb. per sq. in.; bond stress, 80 lb. per sq. in. of steel surface area; shear (when no reinforcement is provided), 40 lb. per sq. in.; shear (when shear reinforcement is provided), 120 lb. per sq. in.

The loads will be assumed the same as for the slab bridges—that is, concrete, 144 lb. per cu. ft.; wearing surface, 50 lb. per sq. ft.; and earth, 100 lb. per cu. ft. The live loads will be either the engine load indicated in Fig. 1 or a uniform load of 125 lb. per sq. ft. of roadway surface.

8b. Design of Floor Slab.—The design of a through girder with a span of 40 ft. and a clear roadway of 18 ft. will be given in detail. Assume a crown in floor slab of 2½ in., an effective depth of 12 in. at the center of floor slab span, a total depth at this point of 14½ in., and a total depth of slab at the curb of 12 in. (see Fig. 9).

$$\begin{aligned}
&\text{Weight of floor slab} = \frac{12 + 14.5}{(2)(12)}(144) &&= 159 \\
&\text{Wearing surface} &&= \underline{\ \ 50\ } \\
&\qquad\qquad\qquad\text{Total} &&= 209 \text{ lb. per sq. ft.}
\end{aligned}$$

$$\begin{aligned}
\text{Dead load moment} &= (\tfrac{1}{8})(209)(17)^2 = 7{,}550 \\
\text{Live load moment} &= (8)(525)(8.5 - 4) = 18{,}900 \\
\text{Total moment} &= 26{,}450 \text{ ft.-lb.}
\end{aligned}$$

It will be noted that the span length is taken as the distance between hub guards rather than the clear width of roadway as the detail of the connection of the floor slab to the girders provides a partially fixed beam.

For $f_c = 1{,}000$ and $f_s = 16{,}000$, the maximum value for the coefficient K in the formula $M = Kbd^2$, is 202.5. In this case

$$K = \frac{(26{,}450)(12)}{(12)(12)(12)} = 183.7$$

From the curve

$$p = 0.0137 \text{ and}$$
$$j = 0.84$$

A_s required $= (12)(12)(0.0137) = 1.973$ sq. in. per ft. width of slab.

By using 1-in. square bars spaced 6-in. centers, the area of steel provided will be 2 sq. in.

8c. Shear in Floor Slab.—For this width of roadway it is reasonable to assume that the engine moves over the bridge along the center of the roadway so that equal loads are transmitted to each girder at all times.

$$\begin{aligned}
&\text{At the hub guard the shears, } V = (8\tfrac{1}{2})(209) + (8)(525) &&= 5{,}975 \\
&\text{Minus shear carried by concrete} = 40bjd = (40)(12)(0.84)(9.5) &&= \underline{-3{,}830} \\
&\text{Reinforcing steel should be provided for the difference} &&\quad 2{,}145 \text{ lb.}
\end{aligned}$$

A_s required per foot $= \dfrac{(2{,}145)(12)}{(9.5)(0.84)(16{,}000)} = 0.199$ sq. in. or $(0.199)(0.707) = 0.141$ sq. in. inclined at 45 deg. By using one ⅜-in. sq. bar on alternate floor slab bars, the area of shear steel provided will be 0.1875 sq. in.

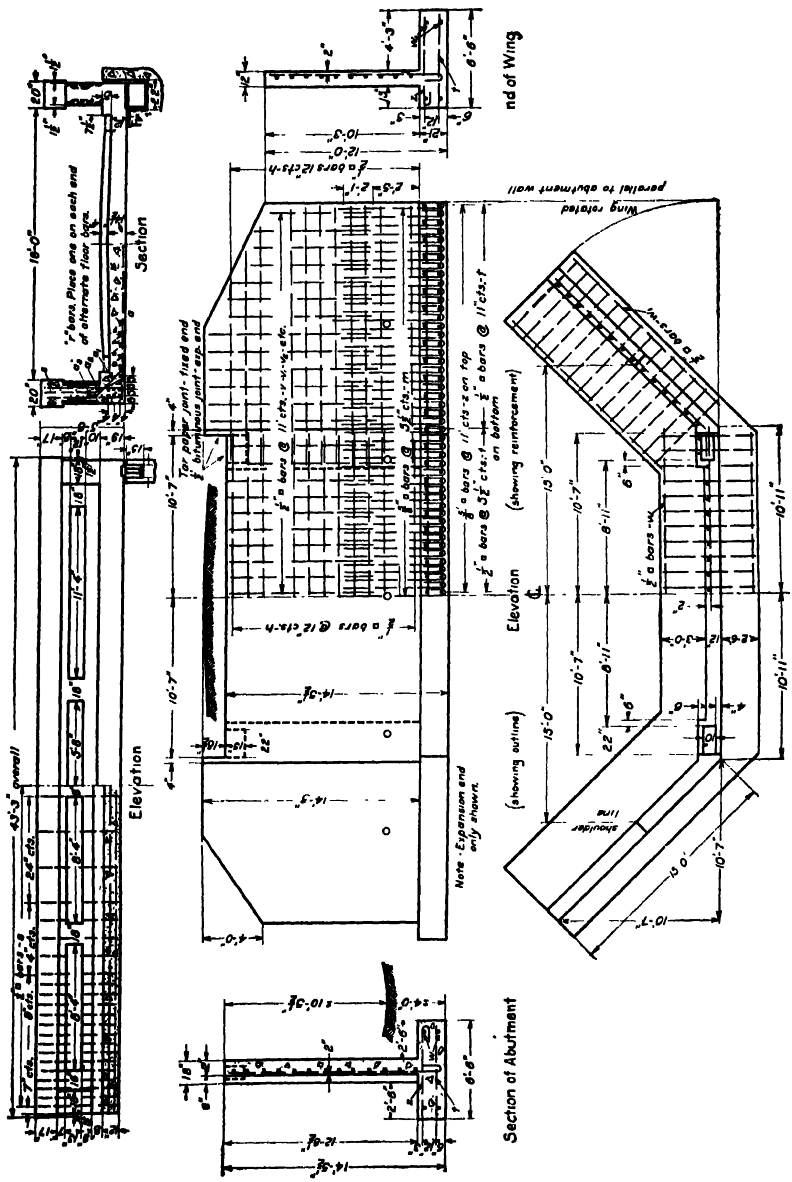


FIG. 9.—Typical design reinforced concrete through girder bridge—span 40 ft.—roadway 18 ft.—height 16 ft.

The unit end shear is

$$\frac{5,975}{(12)(0.84)(9.5)} = 62.1 \text{ lb. per sq. in.}$$

while the maximum allowed is 120 lb. per sq. in. The shear 2 ft. 10 in. from the hub guard will be carried entirely by the concrete and the small shear bars, r may be discontinued at this point.

8*d*. Hanger Bars.—If we assume that the hanger bars, a_2, carry the entire end reaction of the floor slab, then the area of steel in these hanger bars per lineal foot of girder would be

$$\frac{5,977}{16,000} = 0.373 \text{ sq. in.}$$

while these bars spaced 24 in. apart provide 0.5 sq. in. The length of embedment of these bars should be not less than

$$\frac{(2)(5,977)}{(4)(80)} = 37.3 \text{ in.}$$

8*e*. Design of Main Girders.—The weight of one girder and half of the floor slab per lineal foot of bridge is 2,586 lb., and the wearing surface weighs 425 lb. per lin. ft., making a total of 3,011 lb.

Dead load moment $= (\frac{1}{8})(3,011)(41.5)^2(12) = 7,779,000$ in.-lb.

Engine live load moment =

$$\frac{(16,000)(22.42) + (8,000)(12.42)}{41.5}(19.08)(12) = 2,530,000 \text{ in.-lb.}$$

Uniform live load moment = (125) (9) (41.5) ($\frac{12}{8}$) = 2,906,000 in.-lb. The uniform live load moment will govern the design. The total moment is equal to 10,685,000 in.-lb.

Determination of Center of Steel and Effective Depth of Girders.—Try seven 1¼-in. square bars and two 1⅛-in. square bars arranged as shown in the girders. The area of steel provided by these bars is 13.468 sq. in.

The center of gravity of this steel may be determined as follows:

No. of bars	Area of one bar	Distance above bottom of girder (in.)	Product
2	1.2656	12	30.4
3	1.5625	8	37.5
4	1.5625	4	25.0
Total........	......	..	92.9

The distance from the bottom of girder to center of gravity of the reinforcing steel is

$$\frac{92.9}{13.468} = 6.9 \text{ in.}$$

and the effective depth

$$d = 66 - 6.9 = 59.1$$

Since the coping is quite deep the results will be only slightly in error if the width of beam is taken as the width of coping, or $b = 20$ in.

$$K = \frac{10,685,000}{(20)(59.1)(59.1)} = 153.0 \quad p = 0.0112 \quad j = 0.86$$

$$A_s \text{ required} = (0.0112)\ (20)\ (59.1) = 13.24 \text{ sq. in.}$$

The amount of steel assumed for the girders is sufficient and it will be arranged as shown to provide clearance for the floor slab reinforcement.

Design of Stirrups.—Vertical stirrups will be provided to carry the shear in excess of the shear carried by the concrete.

The total dead load shear at the support is

$$(3,011)\ (20.75) = 62,500 \text{ lb.}$$

The uniform live load shear is greater than that produced by the engine load, and is

$$(125)\ (9)\ (20.75) = 23,340 \text{ lb.}$$

The maximum end shear is equal to the sum of these two values, or 85,840 lb.

$$\text{Unit shear, } v = \frac{85,840}{(14)(0.86)(59.1)} = 121 \text{ lb. per sq. in.}$$

In this design the shear governs the design of the girder.

$$\text{Unit bond, } u = \frac{85,840}{(44)(0.86)(59.1)} = 38.5 \text{ lb. per sq. in.}$$

The surface area of the main girder steel is equal to 44 sq. in. per lineal inch of girder.

The stirrup spacing is determined as follows:

Maximum end shear	= 85,480
40 bjd = (40) (14) (0.86) (59.1)	= 28,330
Shear to be carried by the stirrups	= 57,510 lb.

If ½-in. square bars are used for the stirrups, the spacing of same at the end of the girder should be

$$\frac{(0.5)(16,000)(59.1)(0.86)}{57,510} = 7 \text{ in.}$$

Similarly, at a point 4 ft. from the support the total shear is 70,190 lb., and the shear to be taken by the stirrups is 41,860 lb., requiring a stirrup spacing of 9.7 in. At a point 8 ft. from the support, the total shear is 55,840 lb., the shear to be carried by stirrups is 27,510 lb., and the stirrup spacing is 14.7 in. At a point 12 ft. from the support, the total shear is 41,480 lb., the shear to be carried by the stirrups is 13,150 lb., and the spacing of the stirrups is 30.8 in. From the point 12 ft. from the support to the center of the span the spacing will be such as not to exceed half the effective depth.

8f. Design of Expansion Rockers.—As stated above, some means must be provided for expansion of the superstructure since no allowance is made for temperature changes in the unit stresses used in the design. Cast-iron rockers are placed in suitable pockets in one abutment, and directly beneath the girders. Sections through a rocker pocket are shown in Fig. 10. The rockers rest upon

steel plates and additional plates are placed above the rockers to receive the superstructure load. The lower plates are accurately set in mortar beds and the rockers are held in place by means of wooden wedges at the ends of the rockers as shown.

The pockets are filled with asphalt. A bituminous felt joint is placed along the bridge seat to prevent the slab from resting directly upon the abutments. Rectangular holes are cut in the bituminous felt to allow direct bearing of rockers, steel plate and superstructure. The top plates are held in place by means of sticks placed vertically, as shown, and stiff mortar is placed around the edges of the plates to prevent, in a measure, possible leakage into the pocket while the superstructure is being poured. This method of providing for expansion has proven quite satisfactory.

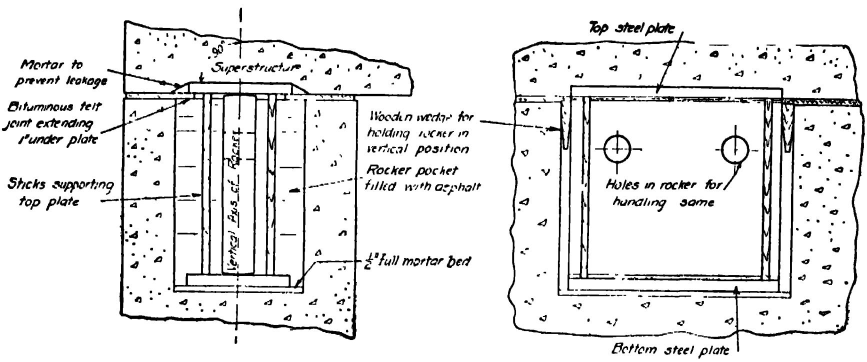


FIG. 10.

The rockers are designed so that the bearing per lineal inch does not exceed 300 D, where D is equal to the diameter of the rocker in inches. In the design shown, if a 20-in. rocker is used, the diameter required is

$$\frac{89{,}000}{(20)(300)} = 14.8 \text{ in.}$$

A 15-in. diameter is used. The rocker should also be checked for column action, if it is made comparatively thin and has a large diameter.

8g. Design of Abutments.—The main wall and the wing walls of the abutment will be designed as cantilever retaining walls. The wing walls will be placed so that they make an angle with the face of the abutment of 45 deg. The wing walls are poured monolithically with the main wall and bonded to it by means of reinforcing steel, but no allowance will be made for this in design. The maximum stresses used will be as follows:

Compression in concrete, f_c = 650 lb. per sq. in.
Tension in steel, f_s = 16,000 lb. per sq. in.
Unit bond stress, u = 80 lb. per sq. in.
Unit shear, v = 40 lb. per sq. in.
Soil pressures, maximum, 6,000 lb. per sq. in.
average, 4,000 lb. per sq. in.

The weights of materials used are given in Art. 8a.

This design will be made for abutments to support a 40-ft. span through girder superstructure with an 18-ft. roadway, and the distance from bottom of footings to crown of roadway will be 16 ft. A section of the wing wall and main wall each 1 ft. in length will be designed to resist the horizontal pressure produced by the back fill and equivalent to that produced by a fluid weighing 21 lb. per cu. ft. The width of footings will be made equal to 0.4 the height.

Quite frequently contracts are let for bridges and grading on a section of highway at the same time and to different contractors. In the case of a multiple span structure, the bridge contractor is required to build the abutments first, so that the grading contractor may complete the earth fill adjacent without delay.

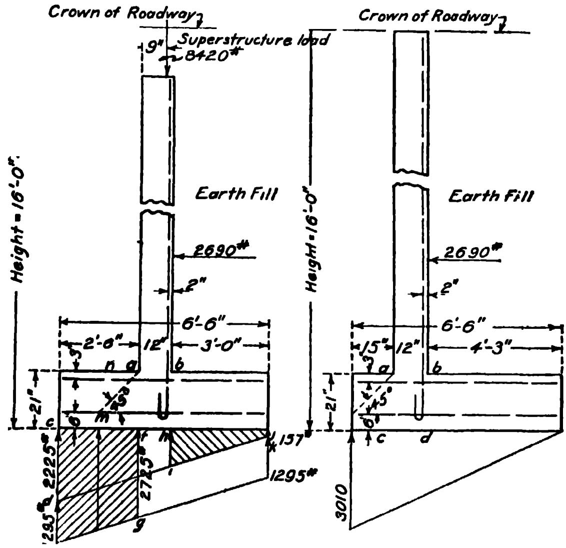


FIG. 11.—Section of abutment. FIG. 12.—Section of wing.

With a slab bridge, where the abutment wall depends upon the superstructure for stability, the back fill, of course, should not be made before the superstructure is poured. However, with an abutment similar to the one shown in Fig. 9 for a through girder bridge, the back fill may be placed before the superstructure is in place. The critical period for an abutment occurs just after the back fill is completed, and the concrete is comparatively new.

In the accompanying design, the abutment footing will be investigated for two cases. The first case will be with the superstructure and back fill completed. This condition will govern the design of the footing on the stream side of the abutment wall and the upward pressure of the soil is represented diagrammatically by the area *cegf* in Fig. 11. The second case which will govern the design is with the back fill completed and with the superstructure not yet constructed. This condition will govern the design of the footing on the earth side of the abutment, and the upward pressure of the soil is represented by the area *hijk*; while the downward forces include the weight of the footing and earth back of the abutment wall. The condition resulting from the superstructure being poured before the back fill is made, will govern the design of the reinforcement

near the bottom of the footing on section *bh*; but the requirements here are provided for if the steel provided at section *af* is sufficient at that point.

Main Wall Design.—The moment on section *ab* is equal to 10,130 ft.-lb. The stress in concrete on the stream face is 630 lb. per sq. in. due to this cantilever moment. The area of steel required is 0.862 sq. in. and ⅝-in. square bars at 5½-in. centers will provide this amount. At a point 4 ft. 6 in. above the top of the footing, ½-in. square bars at 11-in. centers will be sufficient. The shear and bond stresses on the main wall are small and seldom govern the design.

The moment on section *af* is produced by the forces represented by *cefg* (Fig. 11) minus the moment due to the weight of the footing offset. This moment is equal to 9,385 ft.-lb., requiring an area of steel of 0.522 sq. in. One-half-inch square bars at 5½-in. centers will provide the necessary area of steel. The critical section for the footing may be taken at section *mn*. The total shear at this point is equal to 3,835 lb., and the unit shear is equal to 23 lb. per sq. in. The surface area required for bond per lineal inch of bar is 3.5 sq. in. The steel provided is sufficient.

The moment on section *bh* is produced by the forces shown by the diagram *hijk* minus the moment due to the weight of the footing offset and the earth fill. This moment is equal to 5,410 ft.-lb. and the total shear is equal to 3,130 lb., requiring an area of steel of 0.26 sq. in. and a surface area for bond of 2.31 sq. in. Five-eighths-inch square bars at 11-in. centers will be used to satisfy these conditions and to make the spacing conform to that of the other bars in the footing and wall.

Wing Wall Design.—In a similar manner the wing wall is designed for the critical sections *ab*, *ac* and *bd* (Fig. 12), for the conditions prevailing with the back fill in place. It will be noted that the critical sections for shear and bond on the toe of the footing would fall at the forward edge of the footing, and therefore in this short offset these factors need not be investigated. The details of the wing walls are shown in Fig. 9. The lengths of wings may be determined in a manner similar to that used for the wing walls on the abutment for a slab bridge.

9. Deck Girders.—With the growing demand for bridges on the public highways with wider roadways to accommodate fast moving traffic, the tendency is to turn to the deck girder (Fig. 13) for spans from 30 to 60 ft. in preference to the through girder. This is due to the fact that the latter type is restricted to roadway widths of about 18 to 20 ft. The rails on the deck girder present a more pleasing appearance than is the case with the through girder, since the rails on the latter are designed to carry loads and are often made quite bulky and out of proportion with the rest of the structure.

The fact that the deck girder on the other hand requires a deeper floor system, resulting in the necessity of raising the grade line, is often a determining factor in choosing the type to be used.

The complete design for the superstructure of a reinforced concrete deck girder bridge with a clear span of 45 ft. and a roadway width of 20 ft. will be given. The structure will be designed for loads and stresses similar to those used in the design of a through girder bridge (Art. 8) with one exception. The maximum bond stress used will be 120 lb. per sq. in. of surface area of plain bars. This high bond stress is considered permissible when at least four of the bars in a beam are bent up, and when the bends are made at two or more points at each end of the beam.

Three girders will be used, so spaced that the maximum moment on all will be equal or nearly so, and the size and amount of steel in each may be the same. The theorem of three moments is used in the design of the floor slab, neglecting the errors due to the varying thickness of the floor slab, the slight difference in elevation of the supports, and any stability against rotating at the girder supports. Fillets are added as an added factor of safety, and are neglected in the design except in computing dead loads on the girders. The width, depth and spacing of girders will be made as shown in Fig. 15.

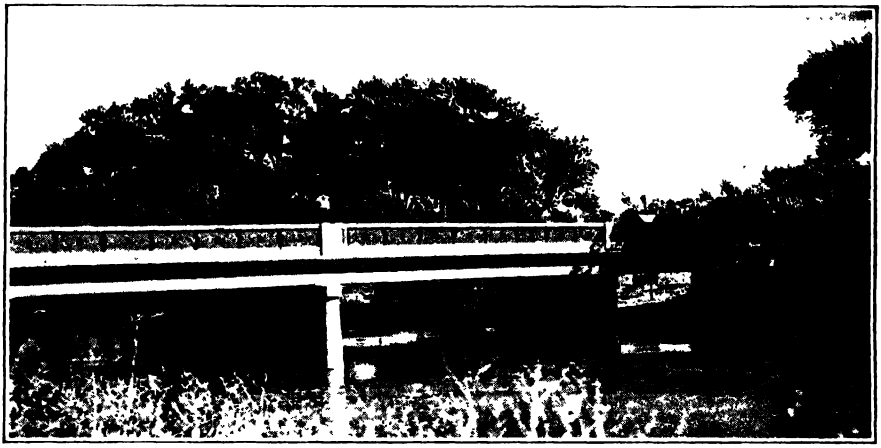


FIG. 13.—Deck girder bridge built in 1921 over Little Calumet River, Cook County, Illinois—spans 45 ft. each—roadway 24 ft.

9a. Design of Slab.—The slab will be designed as a three-span, continuous beam overhanging the supports. The dead load shear at V_1 (Fig. 14) of the cantilever is 590 lb. and the moment at this point is -850 ft.-lb. The average weight of the floor slab between the girders is 140 lb. per sq. ft. The shears and moments at the other critical points in the floor slab are as follows:

$$M_1 + 4M_2 + M_3 = -\tfrac{1}{2}wl^2;\ M_1 = M_3 = -850 \text{ ft.-lb.}$$

$$M_2(D.L.) = \frac{(140)(6.625)^2}{8} + \frac{(2)(850)}{4} = -340 \text{ ft.-lb.}$$

$$V_2(D.L.) = \frac{M_2 - M_1 + \frac{1}{2}wl^2}{l} = \frac{-340 + 850 + (\frac{1}{2})(140)(6.625)^2}{6.625} = 540 \text{ lb.}$$

$$V_3(D.L.) = wl - V_2 = (140)(6.625) - 540 = 390 \text{ lb.}$$

For the maximum V_2 (*L.L.*), assume that the slab is loaded with 525 lb. per sq. ft. as shown in (*a*), Fig. 14, then

$$V_1(L.L.) = (0.75)(525) = 395 \text{ lb.}$$

$$M_1(L.L.) = (-395)\left(\frac{0.75}{2}\right) = -148 \text{ ft.-lb.} \quad M_3 = 0$$

$$M_2(L.L.) = -\frac{1}{16}wl^2 - \frac{M_1}{4} = \frac{-(525)(6.625)^2}{16} + \frac{148}{4} = -1{,}400 \text{ ft.-lb.}$$

$$V_2(L.L.) = \frac{-1{,}400 + 148 + \left(\frac{525}{2}\right)(6.625)^2}{6.625} = 1{,}550 \text{ lb.}$$

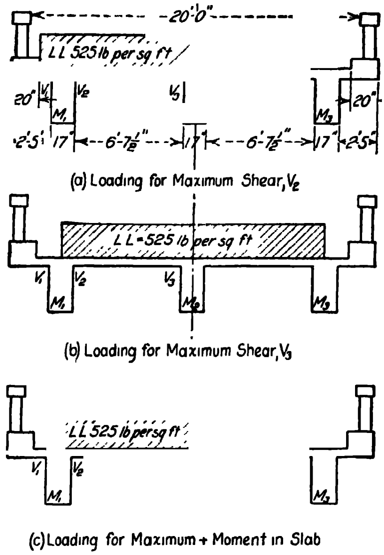


FIG. 14.—Positions of live loads for maximum stresses.

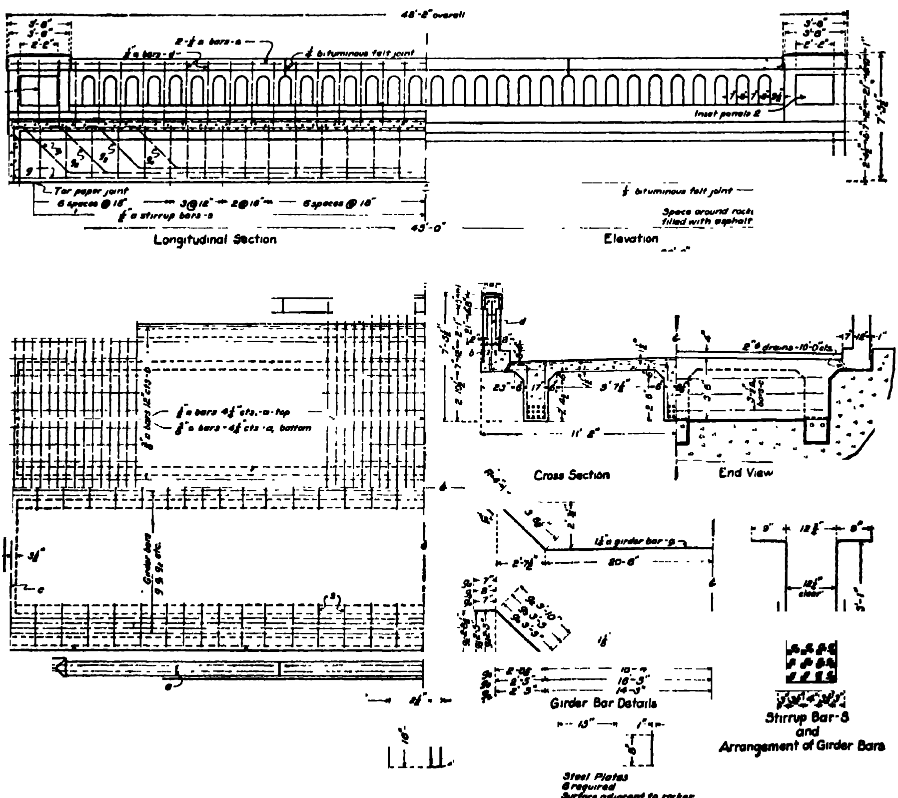


FIG. 15.—Typical design of deck girder superstructure—span 45 ft.—roadway 20 ft.

For the maximum $V_3(L.L.)$, assume that the slab is loaded as indicated in (b), Fig. 14 then

$$M_1(L.L.) = M_3(L.L.) = 0$$

$$M_2(L.L.) = -\tfrac{1}{8}wl^2 = \frac{-(525)(6.625)^2}{8} = -2{,}880 \text{ ft.-lb.}$$

$$V_3(L.L.) = \tfrac{5}{8}wl = \frac{(5)(525)(6.625)}{8} = 2{,}175 \text{ lb.}$$

For the maximum positive moment in the slab, assume that the slab is loaded as indicated in c, Fig. 14, then

$$M_1(L.L.) = M_3(L.L.) = 0$$

$$M_2(L.L.) = -\tfrac{1}{16}wl^2 = \frac{-(525)(6.625)^2}{16} = -1{,}440 \text{ ft.-lb.}$$

$$V_2(L.L.) = \tfrac{7}{16}wl = \frac{(7)(525)(6.625)}{16} = 1{,}520 \text{ lb.}$$

$$V_3(L.L.) = \tfrac{9}{16}wl = \frac{(9)(525)(6.625)}{16} = 1{,}960 \text{ lb.}$$

The total dead load on the slab = 140 + 525 = 665 lb. per sq. ft.

	M_1	V_2	M_2	V_3
D.L. =	−850	540	−340	390
L.L. =	0	1,520	−1,440	1,960
Total	−850 ft.-lb.	2,060 lb.	−1,780 ft.-lb.	2,350 lb.

The moment is a maximum where the shear is equal to zero, or $\frac{2{,}060}{665} = 3.10$ ft. from the outer girder.

The maximum positive moment in the slab is as follows:

$$+M = (2{,}060)(3.10) - 850 - \frac{(3.10)^2}{2}(665) = +2{,}345 \text{ ft.-lb.}$$

The shear at the point of contraflexure is as follows:

$M = 0$ at a point X from the left support or outer girder

$$M = 2{,}060X - 850 - \left(\frac{X^2}{2}\right)(665) = 0 \quad X = 0.44 \text{ or } 5.76 \text{ ft.}$$

$$\text{Shear} = 2{,}060 - (0.44)(665) = 1{,}770 \text{ lb.}$$

The floor slab at V_2 should be designed for a moment of −340 ft.-lb. dead load and −2,880 ft.-lb. live load, making a total of −3,220 ft.-lb. The maximum shear at the same point is 390 lb. dead load plus 2,175 lb. live load, making a total of 2,565 lb. $d = 6.3$ in.

$$K = \frac{3{,}220}{(6.3)(6.3)} = 81.2 \quad p = 0.0058 \quad j = 0.89 \quad f_c = 545 \text{ lb. per sq. in.}$$

$$A_s = (6.3)(12)(0.0058) = 0.44 \text{ sq. in. per ft. required}$$

$$v = \frac{2{,}565}{(12)(0.89)(6.3)} = 38.1 \text{ lb. per sq. in.}$$

$$\Sigma O = \frac{2{,}565}{(80)(0.89)(6.3)} = 5.72 \text{ sq. in. required}$$

Use ½-in. square bars, 4¼-in. centers, 1½ in. from top of slab.

For the steel in the bottom of the slab, the minimum effective depth is 6.1 in., and the maximum positive moment is 2,350 ft.-lb.

$$K = \frac{2{,}350}{(6.1)(6.1)} = 63.0 \quad p = 0.0044 \quad j = 0.90$$

The concrete and shearing stresses are low.

$$A_s = (6.1)(12)(0.0044) = 0.32 \text{ sq. in. per ft. required}$$

$$\Sigma O = \frac{1{,}770}{(80)(0.90)(6.1)} = 4.03 \text{ sq. in. required}$$

Use $\frac{3}{8}$-in. sq. bars, $4\frac{1}{4}$-in. centers, $1\frac{1}{2}$ in. from bottom of slab.

9b. Design of Girders.—The uniform live load distribution to the girders is figured as follows:

Assuming that the uniform live load covers the roadway surface from the center line of roadway to the hub guard, $M_3(L.L.) = 0$

$$V_1(L.L.) = (0.75)(125) = 94 \text{ lb. per ft.}$$

$$M_1(L.L.) = \left(\frac{0.75}{2}\right)(94) = 35 \text{ ft.-lb.}$$

$$4M_2 = \frac{-(125)(6.625)^2}{4} + 35 \quad M_2 = -334 \text{ ft.-lb.}$$

$$V_2 = \frac{-334 + 35 + (\frac{1}{2})(125)(6.625)^2}{6.625} = 370 \text{ lb. per ft.}$$

Assuming that the uniform live load covers the entire roadway surface for maximum load on center girder, $M_1 = M_3 = 0$

$$V_3 = \frac{5}{8}wl = \frac{(5)(125)(6.625)}{8} = 518 \text{ lb. per ft.}$$

The maximum engine live load on the outer girder is equal to $V_1 + V_2 + (1.41)(525) = 395 + 1{,}550 + 740 = 2{,}685$ lb.

The maximum engine live load on the center girder is equal to $2V_3 + (1.41)(525) = 4{,}350 + 740 = 5{,}090$ lb. The maximum rear axle load on the outer girder is $(3.83)(2{,}685) = 10{,}300$ lb. The maximum front axle load on the outer girder is 5,150 lb. The maximum rear axle load on the center girder is $(3.83)(5{,}090) = 19{,}510$ lb., and the front axle load is one-half this amount. The maximum uniform live load on the outer girder is 640 lb. per lin. ft. of girder, and on the center girder the uniform live load is 1,215 lb. per lin. ft. of girder.

Live load shear and moments in outer girder:

$$\text{Uniform } V(L.L.) = \left(\frac{46.5}{2}\right)(640) = 14{,}880 \text{ lb.}$$

$$\text{Engine } V(L.L.) = \frac{(44.58)(10{,}300)}{46.5} + \frac{(34.58)(5{,}150)}{46.5} = 13{,}700 \text{ lb.}$$

$$\text{Uniform } M(L.L.) = \frac{(640)(46.5)^2}{8} = 173{,}000 \text{ ft.-lb.}$$

$$\text{Engine } M(L.L.) = \frac{(24.92)(10{,}300) + (14.92)(5{,}150)}{46.5}(21.58) - \frac{(3.83)^2}{8}(2{,}685) = 150{,}000 \text{ ft.-lb.}$$

Live load shear and moments in center girder:

$$\text{Uniform } V(L.L.) = \left(\frac{46.5}{2}\right)(1{,}215) = 28{,}200 \text{ lb.}$$

$$\text{Engine } V(L.L.) = \frac{(44.58)(19{,}510)}{46.5} + \frac{(34.58)(9{,}755)}{46.5} = 26{,}000 \text{ lb.}$$

$$\text{Uniform } M(L.L.) = (1{,}215)\frac{(46.5)^2}{8} = 328{,}000 \text{ ft.-lb.}$$

$$\text{Engine } M(L.L.) = \left(\frac{5{,}090}{2{,}685}\right)(150{,}000) = 284{,}000 \text{ ft.-lb.}$$

Dead load shears and moments:

	Outer Girder		Center Girder
$V_1(D.L.)$	590	$V_3(D.L.)$	390
$V_2(D.L.)$	540	$V_3(D.L.)$	390
Fillets	36		36
Total per foot	1,166 lb.		816 lb.

Assume a distance of 48 in. from crown of finished roadway to bottom of center girder. This will make the total depth of the outer girder and wearing surface equal to 46 in. Since the size of girder is probably dependent upon the shearing stress rather than the moment, we may assume $j = 0.90$ and $v = 120$ lb. per sq. in. and solve for the depth required; thus, correct the assumed dead load weight if necessary.

Assuming that the girders will be 17 in. wide, the weight of the outer girder is 780 lb. per lin. ft. and the inner girder, 810 lb. per lin. ft.

The total end shear on the outer girder =

$$V(D.L.) + V(L.L.) = 14{,}880 + (\tfrac{1}{2})(46.5)(1{,}166 + 780) = 60{,}080 \text{ lb.}$$

$$d = \frac{60{,}080}{(17)(0.90)(120)} = 33.1 \text{ in. required}$$

The total end shear on center girder =

$$28{,}200 + (\tfrac{1}{2})(46.5)(816 + 810) = 66{,}000 \text{ lb.}$$

$$d = \frac{66{,}000}{(17)(0.90)(120)} = 36.0 \text{ in. required}$$

The total moment on outer girder =

$$M(L.L.) + M(D.L.) = 173{,}000 + (1{,}946)\frac{(46.5)^2}{8} = 699{,}200 \text{ ft.-lb.}$$

$$\text{Approximate } A_s \text{ required} = \frac{(699{,}200)(12)}{(16{,}000)(0.90)(33.1)} = 17.6 \text{ sq. in.}$$

The total moment on center girder =

$$328{,}000 + (1{,}626)\frac{(46.5)^2}{8} = 768{,}000 \text{ ft.-lb.}$$

$$\text{Approximate } A_s \text{ required} = \frac{(768{,}000)(12)}{(16{,}000)(0.90)(36.0)} = 17.8 \text{ sq. in.}$$

Try six $1\tfrac{1}{4}$-in. square bars and six $1\tfrac{1}{8}$-in. square bars.

Determination of center of gravity of steel.

Equation	Area	Distance above Bottom of Girder	Product
(4)(1.5625) =	6.25 ×	3.0 in.	= 18.75
(2)(1.5625) =	3.125 ×	6.5 in.	= 20.31
(2)(1.2656) =	2.531 ×	6.5 in.	= 16.45
(4)(1.2656) =	5.062 ×	10.0 in.	= 50.62
Total A_s	= 16.968 sq. in.	Total	= 106.13

The center of gravity of steel $= \frac{106.13}{16.968} = 6.26$ in. above bottom of girder.

Final Design of Outer Girder.—The effective depth $d = 48 - 4$ (wearing surface) $- 2.1$ (crown in slab) $- 6.26 = 35.64$ in. $t = 7.4$ in. $b = 85.75$ in.

$$\frac{t}{d} = \frac{7.4}{35.64} = 0.207$$

$$p = \frac{16.968}{(85.75)(35.64)} = 0.00555 \quad np = (15)(0.00555) = 0.0833$$

$$k = 0.3605$$

$$j = 0.91$$

$$f_s = \frac{(699{,}200)(12)}{(16.968)(0.91)(35.64)} = 15{,}240 \text{ lb. per sq. in.}$$

$$f_c = \frac{(15{,}240)(0.3605)}{15(1 - 0.3605)} = 575 \text{ lb. per sq. in.}$$

Final Design of Center Girder.—The effective depth $d = 48 - 4 - 0.2 - 6.26 = 37.54$ in. $t = 7.7$. $b = 96.5$ in. $p = 0.00472$. $\frac{t}{d} = 0.205$. $pn = 0.0708$.

$$k = 0.333$$

$$j = 0.91$$

$$f_s = \frac{(768{,}000)(12)}{(16.968)(0.91)(37.54)} = 15{,}900 \text{ lb. per sq. in.}$$

$$f_c = \frac{(15{,}900)(0.333)}{15(1 - 0.333)} = 530 \text{ lb. per sq. in.}$$

The maximum bond stress at the ends of the girders, if all but the four bars in the bottom row are bent up, will be as follows ($\Sigma O = 20$ in.):

$$\text{Outer girder } u = \frac{60{,}080}{(20)(0.91)(35.64)} = 92.6 \text{ lb. per sq. in.}$$

$$\text{Center girder } u = \frac{66{,}000}{(20)(0.91)(37.54)} = 96.6 \text{ lb. per sq. in.}$$

Shear in Girders and Design of Stirrups.

At support—Outer girder.

$$V = \frac{60{,}080}{(17)(0.91)(35.64)} = 110 \text{ lb. per sq. in.}$$

At support—Center girder.

$$V = \frac{66{,}000}{(17)(0.91)(37.54)} = 113.5 \text{ lb. per sq. in.}$$

The center girder has the greater percentage of the live load, so the stirrup spacing will be computed for center girder and used in outer girder also. The spacing of bars bent up at an angle of 45 deg. should have a horizontal spacing of not greater than $\frac{3}{4}d$, or 26 in.

Shear carried by bent up bars in the center girder =

$V - 40bjd = 66{,}000 - (40)(17)(0.91)(37.54) = 42{,}770$ lb.

Bend up two $1\frac{1}{4}$-in. square bars at a point 2 ft. 9 in. from support. These bars must extend to a point which is

$$\frac{(42{,}770)(33)(0.707)}{(10)(80)(37.54)(0.91)} = 36.5 \text{ in.}$$

beyond a point $0.6d$ below top of slab.

Shear 2 ft. 9 in. from the support.

$$V(D.L.) = (20.5)(1{,}626) = 33{,}330 \text{ lb.}$$

$$V(L.L.) = \frac{(43.75)^2(1{,}215)}{(2)(46.5)} = 24{,}970$$

$$\text{Total } V = 58{,}300$$

$$-40bjd = 23{,}230$$

Shear to be carried by the steel $= 35{,}070$ lb.

Extend $1\frac{1}{8}$-in. square bars

$$\frac{(35{,}070)(26)(0.707)}{(9)(80)(37.54)(0.91)} = 26 \text{ in.}$$

beyond a point $0.6d$ below top of slab. Provide stirrups to carry $V - 40bjd$ at a point $2.75 + (2.5)(2.17) = 8.17$ ft. from the support. A maximum stirrup spacing of 18 in. will be used from this point back to the support since bent up bars are used.

Shear 8 ft. 2 in. from support.

$$V(D.L.) = (15.08)(1{,}626) = 24{,}560 \text{ lb.}$$

$$\text{Uniform } V(L.L.) = \frac{(38.33)^2(1{,}215)}{(2)(46.5)} = 19{,}210$$

$$\text{Engine } V(L.L.) = \frac{(33.08)(29{,}265)}{46.5} = 20{,}820$$

$$\text{Total } V(D.L.) + \text{Engine } V(L.L.) = 45{,}380$$

$$-40bjd = 23{,}230$$

Shear to be carried by the steel $= 22{,}150$ lb.

$$A_s \text{ required for stirrups} = \frac{(22{,}150)(12)}{(16{,}000)(0.91)(37.54)} = 0.486 \text{ sq. in. per lin. ft. of girder}$$

Use $\frac{1}{2}$-in. square bars, 12-in. centers. $A_s = 0.50$ sq. in.

Shear 11 ft. 0 in. from support.

$$V(D.L.) = (12.25)(1{,}626) = 19{,}930 \text{ lb.}$$

$$\text{Engine } V(L.L.) = \frac{(30.25)(29{,}265)}{46.5} = 19{,}040$$

$$\text{Total } V = 38{,}970$$

$$-40bjd = 23{,}230$$

Shear to be carried by the steel $= 15{,}740$ lb.

$$A_s \text{ required} = \frac{(15{,}740)(12)}{(16{,}000)(0.91)(37.54)} = 0.346 \text{ sq. in. per lin. ft. of girder}$$

½-in. square bars spaced 16 in. apart will provide 0.37 sq. in.

Shear 15 ft. 0 in. from support.

$$V(D.L.) = (8.25)(1{,}626) \qquad = 13{,}420 \text{ lb.}$$

$$\text{Engine } V(L.L.) = \frac{(26.25)(29{,}265)}{46.5} = 16{,}510$$

$$\text{Total } V = 29{,}930$$

$$-\ 40\ bjd = 23{,}230$$

$$6{,}700 \text{ lb.}$$

Use spacing not to exceed ½d to the center of the span.

9c. Design of Rockers and Plates.

$$\text{Maximum load per rocker} = \frac{(24)(66{,}000)}{23.25} = 68{,}160 \text{ lb.}$$

A rocker with a length 2 in. less than width of girder will be used.

$$\text{Diam. of rocker} = \frac{68{,}160}{(15)(300)} = 15.1 \text{ in.} \quad \text{Diameter used is 16 in.}$$

If the bearing stress allowed on concrete is 600 lb. per sq. in., the size of plate to use is $\frac{68{,}160}{600} = 114$ sq. in. An 8×15-in. plate will provide sufficient area for bearing.

The complete design for the deck girder superstructure is shown in Fig. 15.

10. Estimate of Quantities of Concrete in Abutments.—In comparing bridges of different types in order to determine the most economical one to use for a particular site, it is very convenient to have some means of quickly determining quantities. Most highway departments have standard plans prepared for superstructures, from which the exact quantities may be obtained, but due to the number of variables in the abutments, standard plans are not readily prepared to fit all conditions.

The variables are: (1) Width of bridge seat, (2) height over all, and (3) length of wing walls. In Figs. 16 and 17, curves have been plotted with volume of concrete in two abutments as ordinates, and an equation which embodies the three variables, as abscissæ. These curves may be plotted from actual plans prepared, and give fairly accurate results for any combination of roadway width R, height of abutments H, and length of wings W. These curves apply to abutments similar in design to those shown in Figs. 2 and 9.

11. Camber in a Single Span.—Camber is provided in concrete structures mainly for appearance. A truly horizontal line is rather difficult to obtain in a bridge of this type due to possible settlement of falsework as the superstructure is being poured. If the lines which are intended to be horizontal—such as the bottom of a girder or beam—sag slightly, the effect is quite noticeable and not pleasing to the eye. While on the other hand, if these lines are bowed slightly above the horizontal or grade line, the resulting effect is pleasing.

To prevent the possibility of sag in the superstructure, a slight camber is given to the falsework. The camber for slab and girder bridges should be about one-

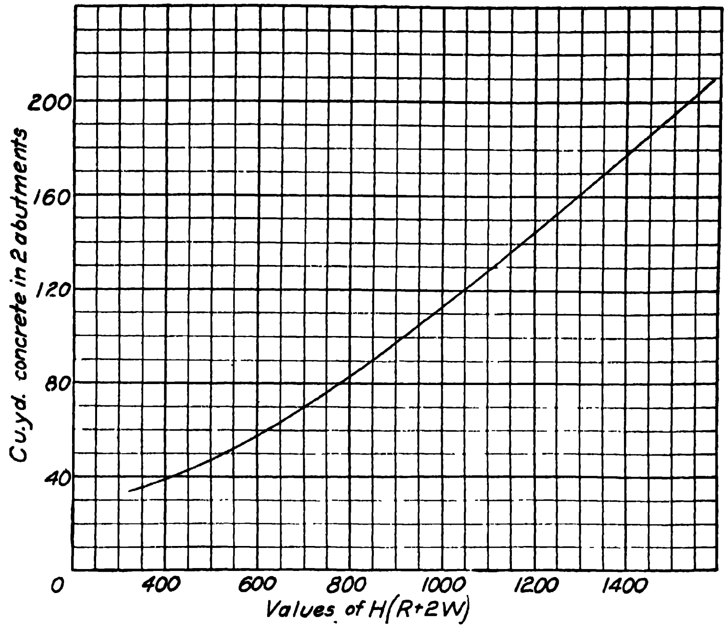


FIG. 16.—Curve showing volume of concrete in abutments for slab bridges.

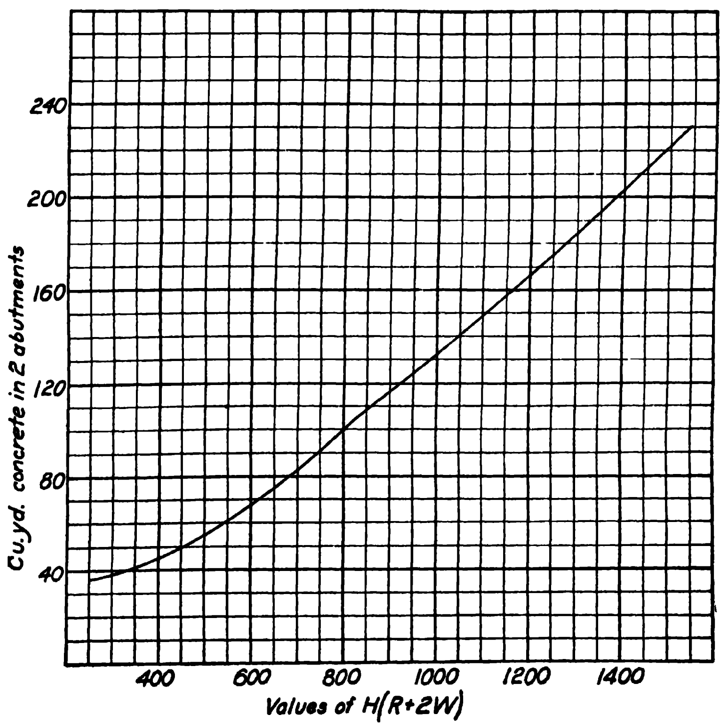


FIG. 17.—Curve showing volume of concrete in abutments for through girder bridges.

twentieth of an inch per foot of span. This would mean ½ in. for a 20-ft. span, or 1 in. for a 40-ft. span.

12. Camber in a Multiple Span.—In addition to the camber in an individual span, the tops of piers in a multiple span structure are raised above the elevations of the bridge seats at the abutments. The purpose of this is also to present a pleasing appearance. The total amount of camber at the center of the structure should be $\frac{1}{1,000}$ to $\frac{1}{1,200}$ of the total length of the bridge. This means that for a bridge 500 ft. long the camber should be from 5 to 6 in. The elevations of piers intermediate between the faces of abutments and center of bridge may be figured as points on a parabolic curve.

The camber in multiple span slab and girder bridges should not be as great as is sometimes provided for in a multiple span arch bridge.

13. Piers.—The design of solid concrete piers for highway bridges, with distance from bottom of footing to top of coping, up to 25 or 30 ft. is not very difficult. A massive pier of this type will usually be found safe against overturning if the batter of the shaft is made ½ in. per ft., and the width of shaft at the top is made great enough to provide ample bearing for the superstructures. The effect of wind, ice and water pressures rarely need to be considered to determine the stability of a plain concrete pier of average height. In northern climates, however, special provision should be made for ice. The upstream nose of the pier may be made of such a shape that it will present a cutting edge to the sheet of ice, and this edge should be armoured and inclined to the vertical so that the pressure will not be normal to the surface of the pier. The offset of the pier footing should be designed as a cantilever with the load on the offset equal to the upward reaction of the piles, or, when bearing piles are not used, equal to the bearing pressure on the foundations. When the offset is not greater than half the depth of footing, it is not necessary to use reinforcing steel.

In a case where a pier is carrying superstructures with widely differing end reactions, the resulting eccentricity of loads upon the pier should be investigated.

The shapes of the nose and tail of the pier are important factors in determining the backwater effect caused by the bridge, and the scouring action of the stream around the piers. In an experiment with miniature pier models of various shapes in a flume at the Argo Dam, Ann Arbor, Michigan,[1] it was found that the loss in head, and consequently a backing up of the water to produce a given velocity, was less for an elliptical or half-round nose and tail, than for any other practical form. It was also found that the loss in head, due to friction of the water as it passes the wetted pier surface, is small and may generally be neglected.

It is quite important to place solid piers with their sides parallel to the current prevailing during flood stages in the stream. Piers placed otherwise present a greater obstruction to flow, by reducing the effective opening, and by creating eddies which may cause considerable scour around the footings. Whenever necessary to maintain a good alignment of the highway, and where the stream crosses at an acute angle, it is preferable to design a skew structure.

Often it is possible to decrease the yardage in piers materially by designing post piers similar to that shown in Fig. 18. This pier has a height of 18 ft. 0 in.,

[1] See "Obstruction of Bridge Piers to the Flow of Water" by FLOYD A. NAGLER, *Trans. Am. Soc. C. E.* (1918), vol. 82, p. 334.

and was designed to carry a 28-ft. clear span slab superstructure, with a clear roadway of 20 ft. The cost of forms, labor and materials is greater per yard of concrete for this type than for a solid pier and the workmanship required is of a higher quality. These facts partially counteract the saving gained due to the decreased volume in the pier. The post pier, being of smaller volume, requires fewer piles to support it. A pier of this type will probably present a greater retarding effect to the flood flow of a stream, than a solid pier, and there is possibly a greater danger of drift lodging at the bridge than would be likely with the latter.

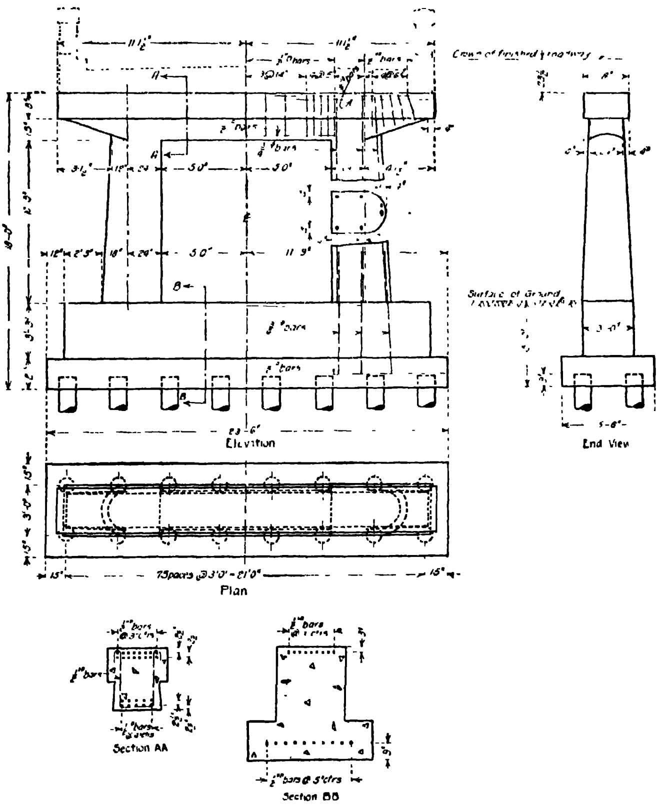


Fig. 18.—Details of pier for bridge over Little Wabash River near Louisville, Illinois.

14. U or "Tied Back" Abutments.—A type of abutment sometimes used is the "U" or "tied-back" abutment, the wings of which are designed to resist the horizontal thrust of the earth upon the front or main wall, they in turn being held together by means of connecting ties. An abutment of this type is shown in Fig. 19.

The main wall is designed as a series of partially fixed horizontal beams extending between the wings, using the same principle of design as for a counterfort wall. The main steel is denoted by *h*. The main wall is tied into the wings by means of this horizontal steel which is bent around the corners sufficiently to develop

its full strength through bond. In practice, it will be found that the requirements of shear and bond rarely govern the design of the main wall except with the combination of great heights of abutments and relatively narrow roadways. Reinforcement for negative moment should be provided at the juncture between main wall and wing.

Between the crown of roadway and the center of the row of ties, the wings are designed as short cantilever walls. The main steel will, therefore, be located near the back of the wall. From the ties down to the footing, the wall is assumed to act as a vertical beam simply supported at the top by a continuous horizontal beam, integral with the wing wall, and which in turn is supported by the ties. At

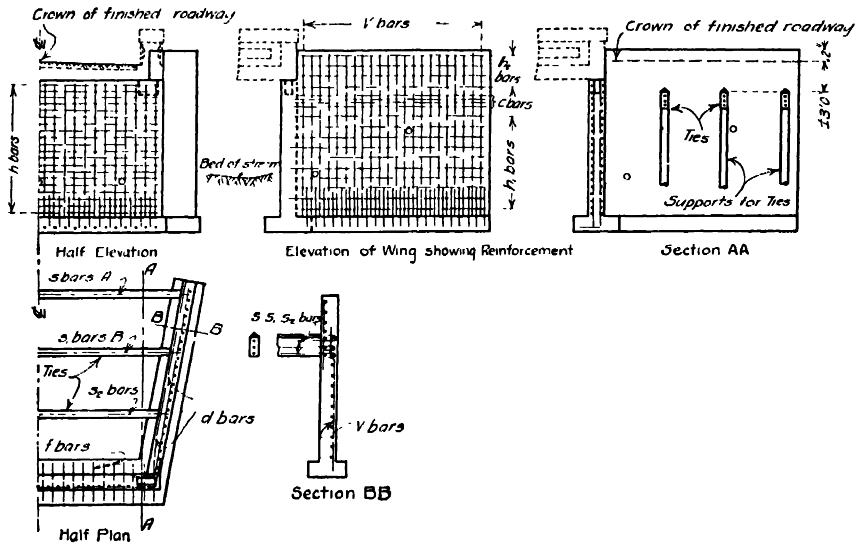


FIG. 19.—"U" or "tied back" abutment.

the bottom, the wall is supported by the footing. Under this assumption the vertical steel will be located near the front of the wall. In order that this assumption may be carried out, it is necessary to introduce a number of horizontal rods, as the c and the d bars shown. These are placed horizontally at the ends of the ties, and are designed to resist the upper reaction of the vertical slab. The wing wall may be considered as a simple vertical beam overhanging one support. The reaction at the bottom of the vertical beam is assumed to be counteracted by the frictional resistance between footing and the supporting earth, and also by the inward pressure from the back fill outside the wall.

The wing walls are tied together as shown in the figure, the tie steel (s, s_1 and s_2 bars) being designed to counteract both direct and flexural stresses—that is, (1) the direct tension caused by the earth pressure normal to the walls, and (2) the bending stresses produced in the tie by its own weight and the weight of the superimposed earth and live loads.

It will often be found economical to cut down the effective length of the ties by supporting them at intermediate points on short, creosoted wood or concrete

posts. The h_1 h_2 bars are nominal in size and are added for the purpose of preventing cracks due to shrinkage and unequal settlement.

In general, this type of abutment shows an appreciable economy of materials, but its construction requires more rigid inspection than the type shown in Fig. 9.

Special care must be observed, when placing the back fill, to keep out water which, if impounded, would cause pressures greater than those for which the structure was designed. Since each portion of this structure depends upon some

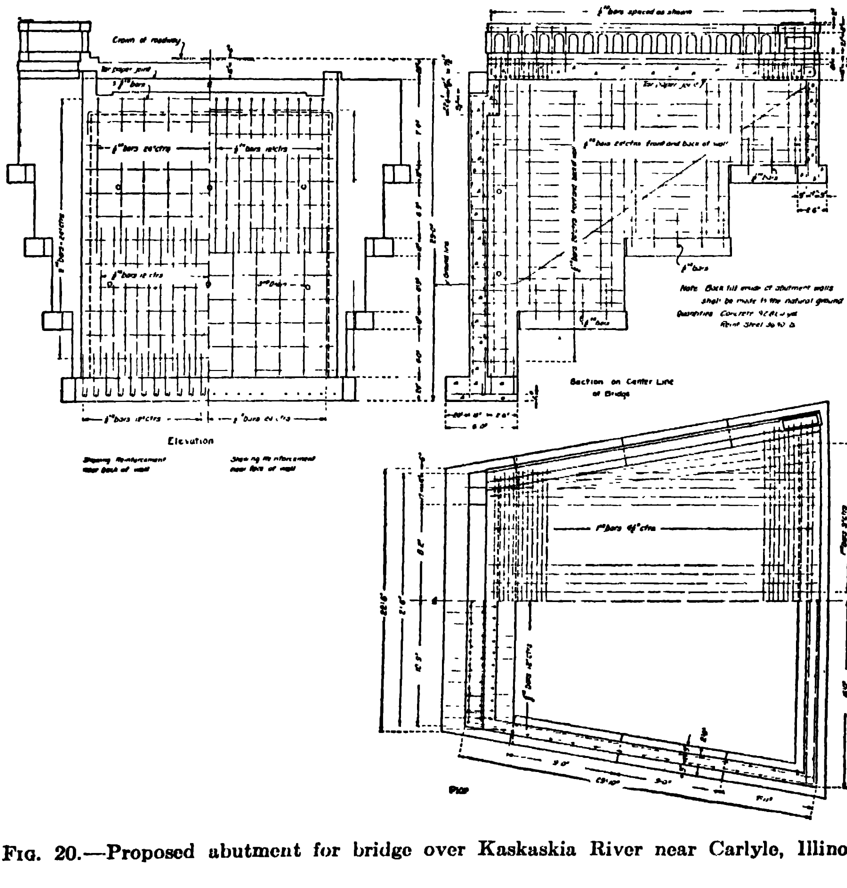


Fig. 20.—Proposed abutment for bridge over Kaskaskia River near Carlyle, Illinois.

other portion for its stability, a failure at one point may result in the failure of the entire abutment.

15. Cellular Abutments.—Another type of abutment is shown in Fig. 20. This abutment has a height over all of 29 ft. and is designed to carry a 40-ft. span reinforced concrete deck girder superstructure. The wing walls are not designed to carry earth pressure, since the back fill inside the abutment is to be made only up to the surface of the ground as it exists outside the abutment, and the pressure will be neutralized. A slab with two-way reinforcement is provided which rests upon the wing walls. A parapet wall at the main wall of the abut-

ment is added, also an additional wall connecting the ends of the wings. Since the abutment is entirely enclosed, the falsework beneath the slab cannot be removed.

16. Cantilever Bridges.—It is possible to construct a bridge resembling a concrete arch structure in appearance, in locations where the foundation conditions would not permit the construction of an arch, due to the large horizontal component of thrust of the arch rib. Figure 21 respesents a structure of this type. The horizontal component of dead and uniform live loads are eliminated since

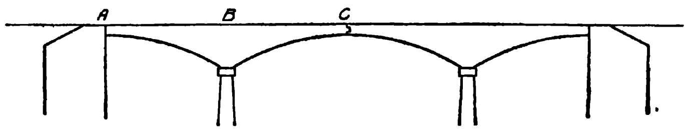


Fig. 21.—Cantilever bridge.

the two arms are the same length, and the unit is balanced over the pier. The reactions due to dead and uniform live loads over the entire structure act vertically. Each cantilever is designed to carry the dead and maximum live loads on the half span from A to B. The piers are designed to carry the maximum eccentric load caused by the entire dead load of the unit from A to C plus the maximum live load on one arm only, and the footings are designed for the same loading.

The joint at C is designed to take live load shear, and it is considered that there is no arch action from one pier to the adjacent one.

(*Designed by California Highway Commission*)
Fig. 22.—Bridge over Atchison, Topeka and Santa Fe tracks at Riverbank, California.

The cantilever bridge has an advantage over a continuous girder bridge, due to the fact that slight settlement of the substructure will not cause serious stresses in the superstructure, while in the latter type, settlement of the substructure may cause considerable damage or possibly a failure of the superstructure. Cantilever reinforced concrete bridges are adaptable for viaducts.

17. Continuous Girder Bridges.—The overhead structure shown in Fig. 22 consists of two end spans of 26 ft. 9 in. each and a middle span of 53 ft. The structure is designed as a continuous girder over three spans. The floor beams

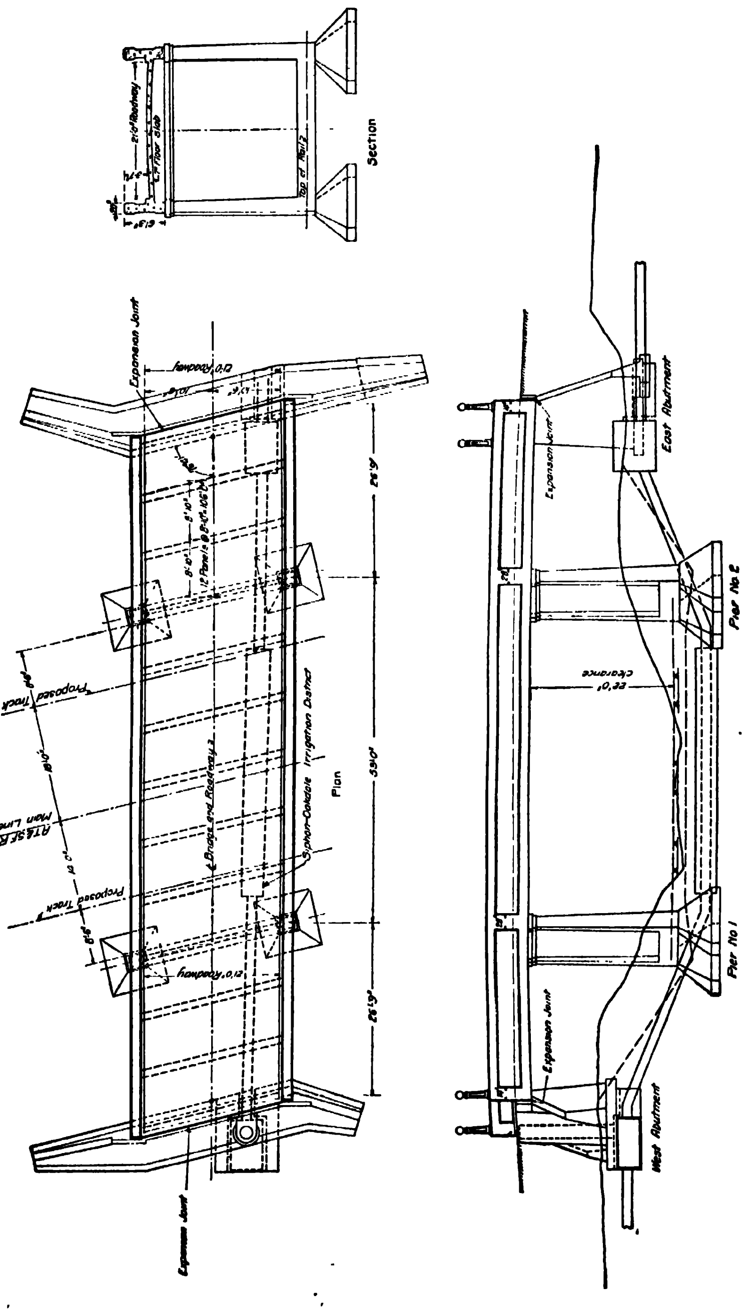
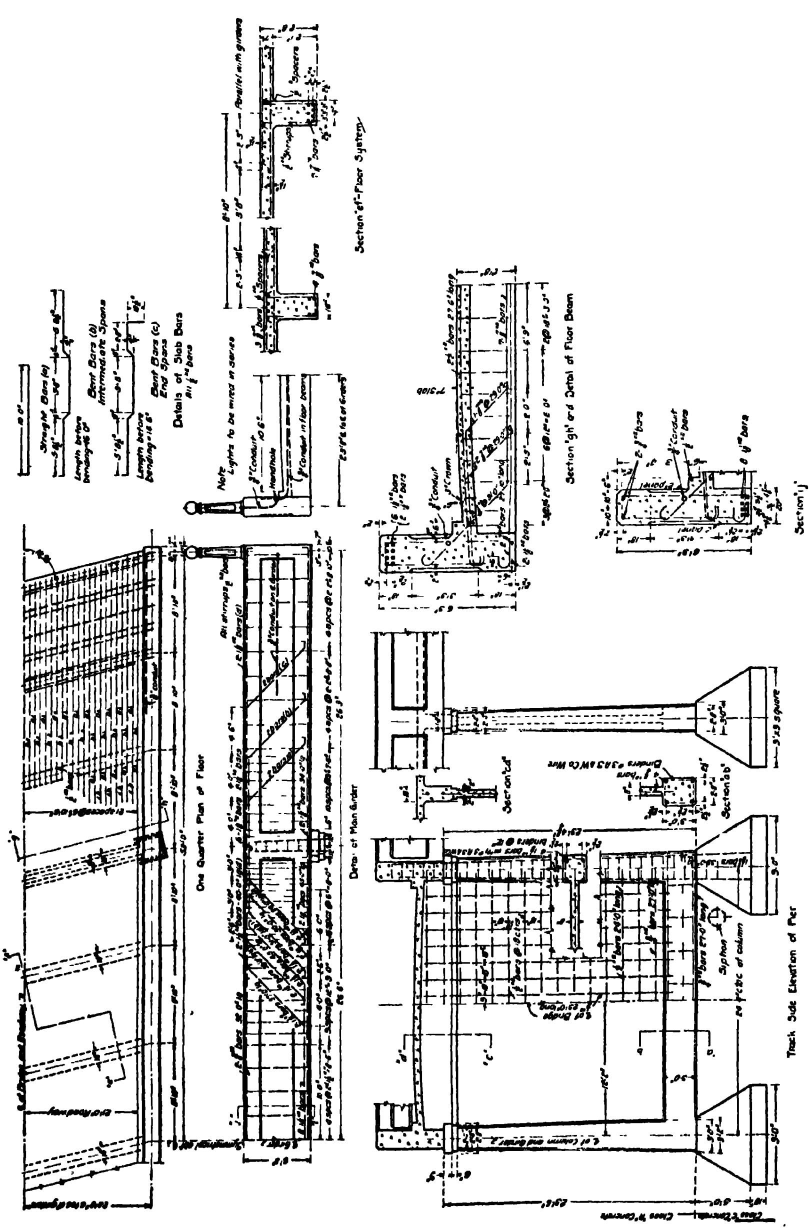
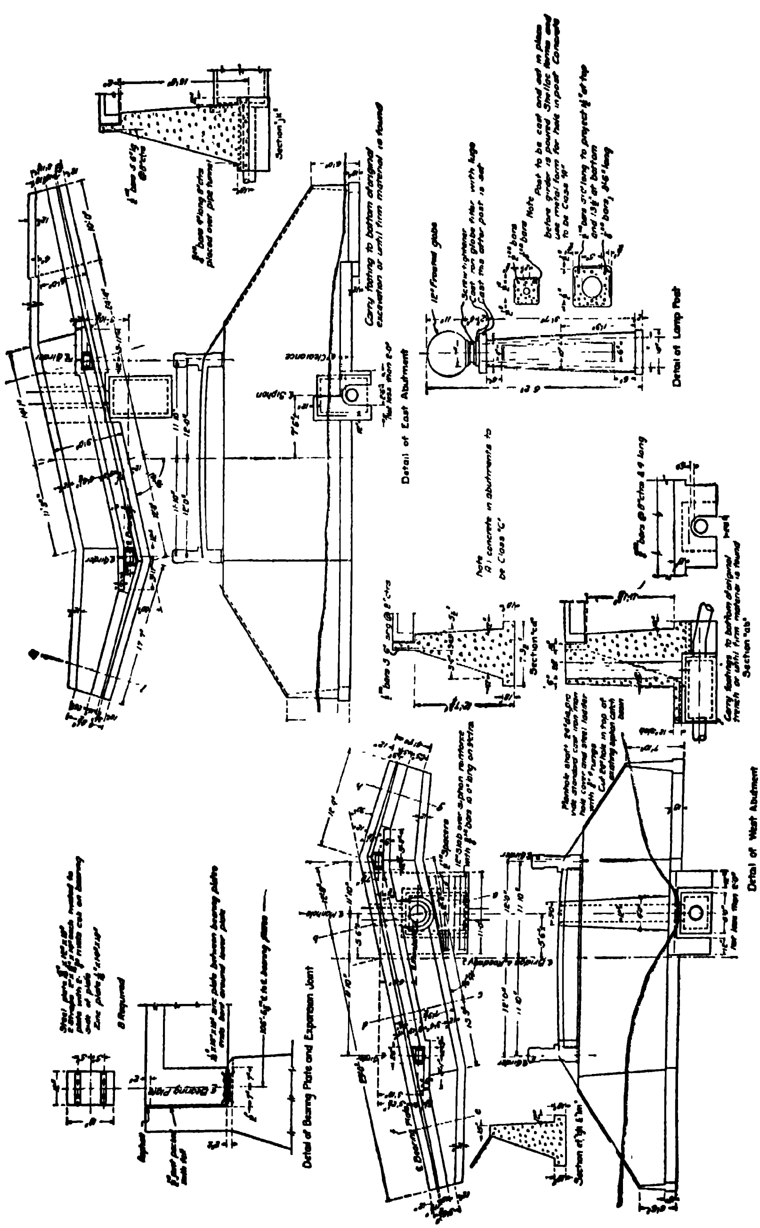


FIG. 23.—Details of bridge over Atchison, Topeka and Santa Fe tracks at Riverbank, California. (*Designed by California Highway Commission*)



(*Designed by California Highway Commission*)

FIG. 23a.—Details of bridge over Atchison, Topeka and Santa Fe tracks at Riverbank, California.



(Designed by California Highway Commission)

FIG. 23b.—Details of bridge over Atchison, Topeka and Santa Fe tracks at Riverbank, California.

are placed parallel to the piers and abutments, and the reinforcing steel in the floor slab is parallel to the center line of roadway. The pier footings are about 7.9 ft. below top of rail and the footings for the abutments are about 6 and 8 ft. above top of rail. The details of the structure are shown in Fig. 23.

A three-span structure of this nature is generally more economical for an overhead crossing, than a one-span structure, with the faces of abutments placed at the clearance line required by railroad traffic. The abutment footings need not be placed at a very great depth, and the size of footings, thickness of abutment walls and lengths of wings are materially decreased.

On the other hand, a continuous girder bridge requires more careful workmanship than a series of simple spans. The load on the pier is greater, requiring larger footings. In general, there is little or no saving by the use of a continuous girder structure, instead of a series of simple spans, and the use of this type is not recommended ordinarily for highway bridges.

SECTION 8

ARCHES

By C. B. McCullough

GENERAL DATA

1. Terms Used in Designating and Dimensioning Masonry Arches.—Following are some of the more commonly used technical terms:

Skewback.—The surface (generally inclined) upon which the arch ring or rib rests. This surface is the assumed dividing line between arch and abutment and is a purely imaginary plane. Either of the sections *a-a*, *b-b*, or *c-c* (Fig. 2) may be assumed as constituting the skewback section without materially affecting the analysis.

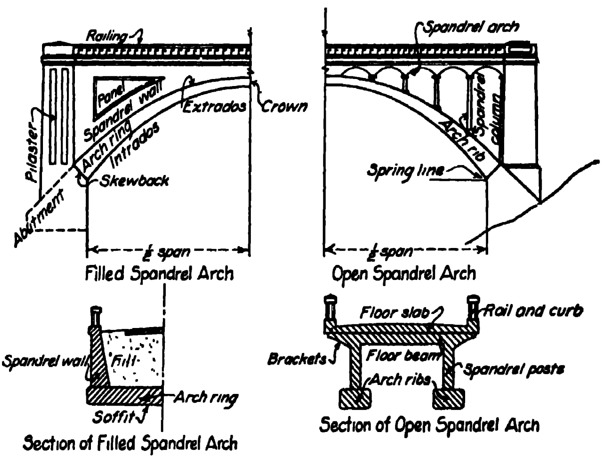


Fig. 1.

Crown.—The highest point on the center line of the arch ring.

Spring Line.—The intersection of skewback and soffit.

Soffit.—The under surface of arch ring or rib.

Intrados.—The curve of intersection of the soffit plane and a vertical plane parallel to the center line of the roadway.

Extrados.—The intersection of the curved back or upper surface of the arch with a vertical plane parallel to the center line of the roadway.

Span.—The clear distance between spring lines measured horizontally and parallel to the center line of the roadway.

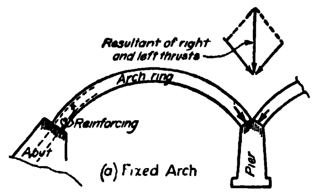
Rise (*Clear*).—The vertical distance between the spring line and the intrados at the crown. For unsymmetrical spans, this definition must be qualified by designating which spring line is meant. The term "center line rise," designating the maximum vertical center line ordinate measured from the intersection of arch center line and skewback, is also frequently used (see Fig. 2)

Spandrel.—That portion of the structure lying above the arch ring.

Other Terms.—As designated by Figs. 1 and 2.

In general, it may be said that many of these terms are used rather loosely —for example, the span of the arch rib is often considered as the distance L' or L'' rather than the clear distance L (see Fig. 2). It is always well, therefore, in discussing questions involving such dimensions or quantities to qualify or restrict the meaning of the terms used so that no confusion will result.

2. Classification of Arches.—In reference to the method in which the stresses are distributed throughout the arch ring, such structures may be classified as:



(1) Fixed or hingeless arches.
(2) Single hinged arches.
(3) Two-hinged arches.
(4) Three-hinged arches.

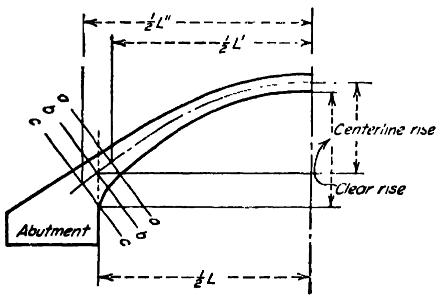


Fig. 2.

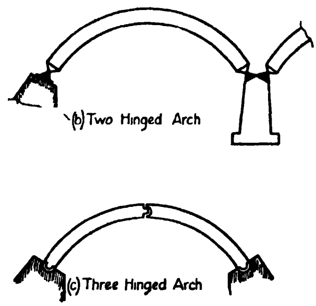


Fig. 3.

Masonry arches with one and two hinges are a very rare occurrence and possess no distinct advantage over the three-hinged type. Arches with three hinges have been built of reinforced concrete masonry in several instances, but by far the most common type of construction in masonry arches is the fixed or hingeless type.

Figure 3 illustrates the difference between the fixed and hinged types of construction. Figure 3*a* is the ordinary type of fixed masonry arch rib. The massive abutment at the left hand support is rigidly connected with the arch ring (by reinforcing bars in reinforced concrete construction) and operates to hold it in a fixed and unyielding position at this point. The pier at the right hand end of the span is more flexible, but ordinarily is so arranged that the dead load pressures from the two arch rings meeting at this point balance or neutralize each other as far as horizontal movement is concerned. This condition so relieves the lateral stress on the pier that to all intents and purposes it may be considered as fixed and immovable, and as holding the arch ring in a fixed and unyielding position at the right skewback. For very high and slender piers, the deflection of the pier is taken into account as discussed in the chapters on arches with elastic supports.

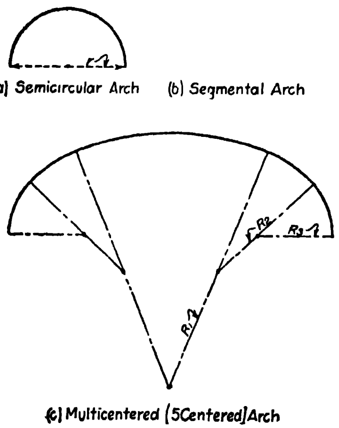
Figure 3*b* illustrates the general nature of a two-hinged arch rib and Fig. 3*c* that of a three-hinged arch rib. As mentioned hereinbefore, the arch rib fixed at

the skewbacks and with a single hinge at the crown is another possible type of construction.

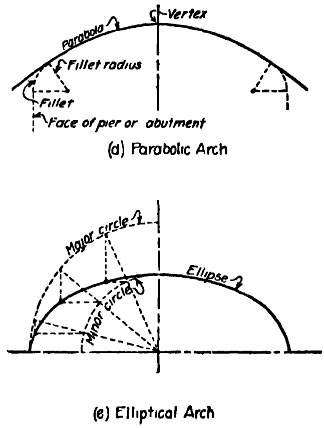
The principle disadvantage in hinged construction with masonry as a material is the fabrication of the hinge detail. Figure 3*b* illustrates the ordinary type of pin hinge, and Fig. 3*c* indicates another type which is sometimes employed.

Classified in accordance with the method in which the deck load is carried by the arch, this type of construction may be divided into: (1) Filled spandrel arches, and (2) open spandrel arches.

In the filled spandrel type (see Fig. 1), the arch ring carries a longitudinal wall at each side of the roadway. This acts as a retaining wall for the spandrel filling material. In the open spandrel arch, the deck load is carried longitudinally by the floor slab to transverse floor beams which are supported by spandrel posts or columns resting upon the arch rib proper. Curved *spandrel arches* (see



FIGS. 4*a*, 4*b* and 4*c*.



FIGS. 4*d* and 4*e*.

Fig. 1) are sometimes employed for architectural effect. These are usually simply curtain walls and have no function in transmitting the stresses from deck to rib.

The principal disadvantage of the filled spandrel arch lies in the excessive amount of dead load from the massive spandrel fill. There is also some question as to just how the dead and live loads are transmitted to the rib, whether the pressure of the earth filling against the rib is vertical or inclined (and if the latter, at what angle of inclination). The open spandrel arch on the other hand distributes its deck loads through a definite path and in a definite way. In the opinion of the writer, this latter type is much to be preferred over the filled spandrel arch except for heavy moving live loads where the cushioning effect of the spandrel fill is a distinct advantage.

Classified in accordance with the curve of the axial line or of the intrados, arch bridges may be grouped into:

(1) Semicircular arches (Fig. 4*a*).
(2) Segmental arches (Fig. 4*b*).
(3) Multicentered arches (Fig. 4*c*).
(4) Parabolic arches (Fig. 4*d*).
(5) Elliptical arches (Fig. 4*e*).
(6) Other curves (not in frequent use).

Generally the above designation is applied to the intradosal curve, although sometimes the term is used to describe the curve of the arch axis. This last is particularly true for a parabolic arch rib the axial line of which is generally made to lie on a parabola.

Where arches are sprung from horizontal beds, as in the case of Figs. 4*a*. 4*c* and 4*e*, the arch is said to be *full centered.*

It is sometimes convenient to employ short radius fillets tangent to both the intradosal arch curve and to the abutment or pier face (as shown in Fig. 4*d*) thus improving the appearance of the design. These fillets, however, should not be regarded as adding to the rib section—being merely added for appearance. It is, of course, true that such fillets do stiffen up the rib somewhat at the skewback, but their effect is uncertain and disregarding them in the analysis simply errs on the side of safety.

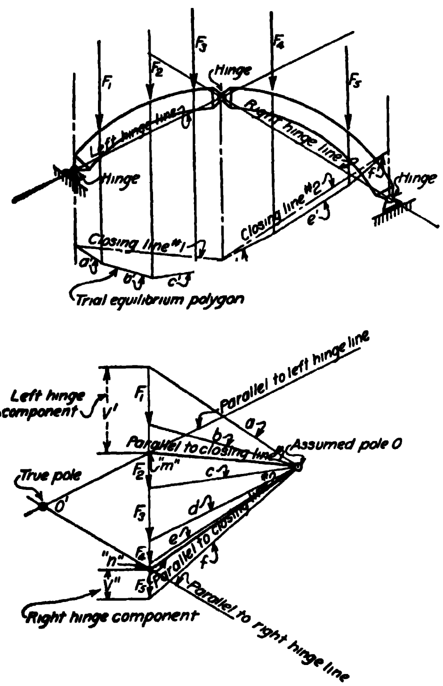


FIG. 5.

3. Linear Arches.—In order to understand the use and significance of the *linear arch* as applied to the selection of the curve for arch rings and ribs, it is first necessary to consider the three-hinged arch rib shown in Fig. 5 under the action of the load system $\Sigma F = F_1 - F_5$ and the induced support reactions.

The loads $F_1 - F_5$, inclusive, are first laid off on a vertical load line (loads on bridge arches are nearly always vertical; if inclined loads are to be considered, the procedure is exactly the same) and an arbitrary point o assumed for a pole, as shown in Fig. 5. With this pole, the trial ray diagram $a - f$ is constructed, and from it, the trial equilibrium polygon $a' - f'$. Through the points where this trial polygon intersects verticals through the respective hinges, closing lines are drawn as shown. Through the assumed pole o, parallels to these closing lines are constructed. These parallels cut off segments V' and V'' on the load line which, from the laws of graphic statics, represent to scale the vertical component of each skewback hinge reaction transmitted to said hinge by virtue of the half rib acting as a simple beam between crown and skewback hinges.

It is apparent that the value of these segments V' and V'' is entirely independent of the position chosen for the trial pole and will be the same for any position of the pole.

If the trial equilibrium polygon had passed through the three hinges, the two closing lines (closing line No. 1 and closing line No. 2) would have been coincident with straight lines joining the two skewback hinges to the crown hinge (the *hinge lines* of Fig. 5).

If, therefore, through the points m ana n on the load line, parallels to these respective hinge lines be drawn, it is at once apparent that their intersection determines another pole o' such that an equilibrium polygon constructed therefrom will pass successively through each of the three hinges.

Proceeding in this manner, the equilibrium polygon a'-f' (of Fig. 6) is now constructed. It should now be clear that segment a' is identical in value and direction with the left reactions R_1; also that segment b' is the resultant of R_1 and F_1; and that segment c' is the resultant of R_1, F_1 and F_2, and so on.

For any section A-A (Fig. 6) considering the left-hand portion of the structure as a *free body in equilibrium*, it must be apparent that the internal thrust on the rib balances the action of the external forces F_1, F_2 and R_1; but the resultant of these forces is segment c' of the equilibrium polygon. Therefore, the internal stress in the arch rib at this point consists of an eccentric thrust T equal and opposite to segment c', coincident in direction and applied at the same point.

By resolving this thrust into tangential and normal components and applying two equal and opposite forces N at the neutral axis (which last addition obviously does not disturb the equilibrium) the stresses in the arch rib at this point may be represented by:

(1) A normal thrust N applied at the neutral axis.

(2) A shearing stress J.

(3) A moment couple $M = N\rho$ (or Td).

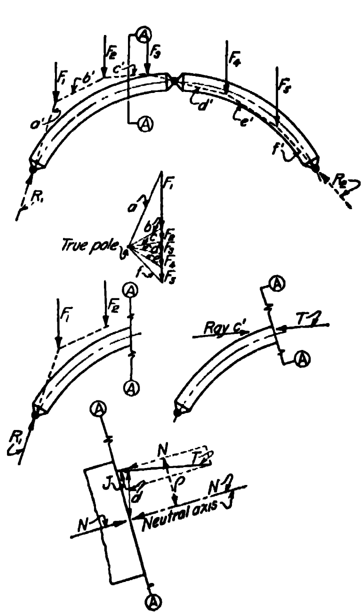


Fig. 6.

It should now be apparent that the true thrust line or equilibrium polygon must always pass through the center of every hinge. If this were not the case and if the thrust at the hinge was eccentric by a distance ρ, then a moment couple $M = N\rho$ would be developed and the arch would immediately begin to rotate about this hinge, coming to rest only when the rib had arrived at a shape such that the thrust line did pass through each hinge point. For all hinged arches in equilibrium, therefore, the thrust line for any load condition must pass through each hinge.

The equilibrium polygon of Fig. 6, therefore, represents the true pressure line under the given load system and the method above described serves for the determination of the thrust line for any condition of loading on a three-hinged arch.

For the fixed arch, the pressure line is not so easily located, both its magnitude, direction and point of application being unknown. To determine this pressure line, therefore, recourse must be had to the so-called "elastic theory" which will be explained and demonstrated in subsequent chapters. For the present suffice it to say that, under any load condition, the location of the thrust line is a function

of the elastic distortion of the rib under load and is interwoven with the shape, the size and the elastic properties of the rib material.

The two-hinged arch has two "control points" for the thrust line while the single hinged arch has one (see Fig. 7).

From the foregoing discussion, it is observed that the bending moment on the arch rib is reduced to zero wherever the thrust line passes through the neutral axis. It is therefore highly desirable so to select the arch ring curve that the thrust line to as great an extent as possible fulfills this requirement.

It is, of course, impossible to secure a condition wherein the thrust line for every load condition passes through the neutral axis inasmuch as the live load is both a varying and a moving load. It is possible, however, to select a curve such that the dead load thrust line will very closely follow the neutral axis throughout. For hinged arches, we have at least one "control" point, and, by the method above outlined, may develop an equilibrium polygon which passes through this control point and any other two selected points. For the fixed arch, we have no control point but may assume for preliminary work that the thrust line passes through the neutral axis at the crown and skewbacks. This is, of course, not true but since the eccentricity is not great at the crown, the method is of value in roughly indicating the shape of the pressure line.

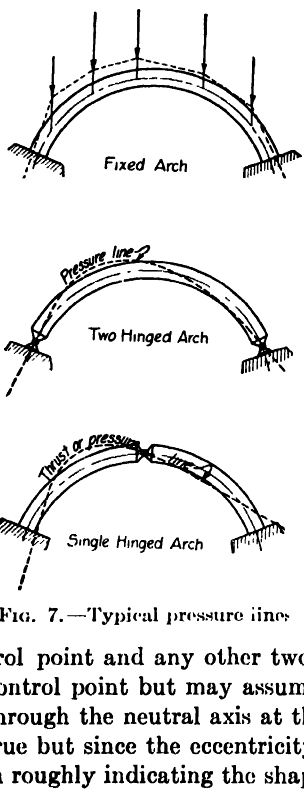


FIG. 7.—Typical pressure lines

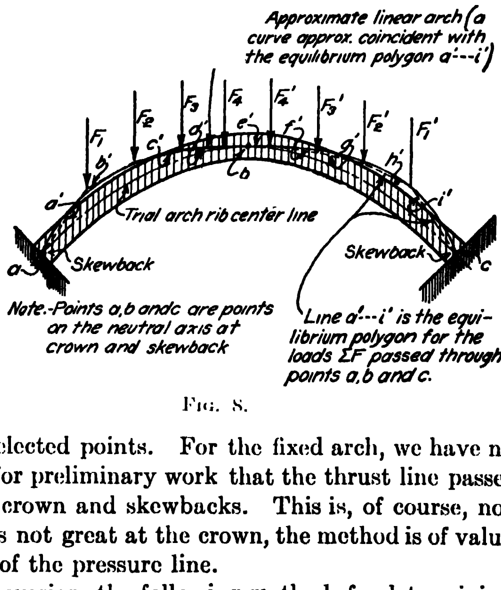


FIG. 8.

Based upon the foregoing discussion, the following methods for determining the most advantageous arch ring curve may be developed:

Trial Equilibrium Polygons.—(1) Assume a trial arch ring, sketch in the rib, (with assumed dimensions), the spandrel posts or walls and the deck, railing, curbs, etc.

(2) Compute the dead load of the arch, filling, deck, etc.

If the arch is an open spandrel arch, the deck loads are concentrated at the spandrel posts; if the arch is a filled spandrel arch, the loads may be assumed as concentrated at equidistant points along the rib.

(3) With these dead loads, pass an equilibrium polygon through the hinges)or if there be no hinges through the neutral axis at crown and skewbacks((see Fig. 8).

(4) Correct the assumed arch rib curve to fit as closely as possible to the above equilibrium polygon.

(5) Re-compute the dead loads if necessary and correct the equilibrium polygon to correspond with these new dead loads.

The above process is repeated if necessary to correct again for the difference between the new dead loads and those first assumed. Generally this is not necessary, one trial being sufficient to arrive at a very close approximation. The arch rib is not designed as yet so that it is impossible to arrive at the exact dead loading, for which reason great refinement at this point is not warranted. After the arch is completely designed and detailed, it is sometimes advisable to re-calculate the dead loads and if the assumed loads were very far in error, to correct the rib design. It is generally possible to assume the dead loads within 10 per cent of the correct value, which is doubtless close enough in view of the inevitable variation in the weight of the earth fill, the weight of the concrete and masonry materials, the variation in formwork and other factors which make extreme refinement in dead load calculation unwarranted.

For the fixed arch, the pressure line will not fall exactly at the neutral axis at the crown as assumed and may also be considerably eccentric at the skewbacks. The above method, however, is the best approximation which can be made and must serve in lieu of better information as a basis for the preliminary selection

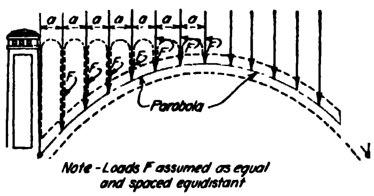


Fig. 9.—Typical loading for parabolic linear arch.

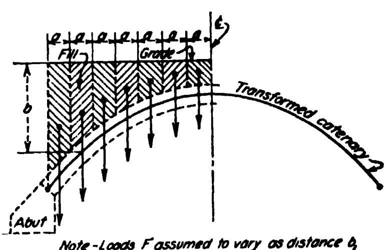


Fig. 10.—Type of loading for catenarian linear arch.

of the arch ring curve. The final analysis by the elastic theory will truly indicate the exact position of the dead load thrust line and further correction can be made if desired after this analysis is made.

It is sometimes considered better to assume the arch axis coincident with the thrust line for full dead plus ½ live load, and sometimes for full dead plus full live load in order to have minimum eccentricity of thrust when the thrust itself is a maximum. This practice is of doubtful propriety in view of the fact that the dead load stresses are of constant duration while the live loadings are only of momentary duration.

For different arrangements of dead loading, the equilibrium polygon or pressure line assumes the shape of certain definite mathematical curves, which fact is of considerable value in first selecting a suitable curve Such curves are termed *linear arches* for the given load condition.

The two linear arches most frequently employed are the parabola and the transformed catenary. When the dead loads are equal and the dead load concentrations equidistantly spaced along the span, as shown in Fig. 9, the linear

arch or pressure line lies very close to a parabola whose vertex is in the crown and whose major axis is vertical. For this type of structure, therefore, the parabolic arch rib results in minimum dead load moments.

When the dead load along the rib varies in direct proportion to the distance b below a horizontal line, which is approximately the condition in a filled spandrel arch (Fig. 10), the linear arch assumes the shape of a transformed catenary.

For the fixed arch, the type of arch curve selected has also an important bearing on the value of the temperature stresses in the rib. The following extracted from a paper by C. S. Nichols[1] and the writer, published in the *Iowa Engineer*, March, 1912, illustrates the effect of intradosal curvature as above noted:

In order to show the effect of the different shapes on the temperature stresses occurring in the arch ring, and also to show the tremendous intensity of this stress in a very flat arch, four arches, each of 50-ft. clear span, and a rise equal to one-tenth of the span (which has represented about the limit in safe design) were analyzed for dead load and temperature, and the results interpreted graphically.

Arch No. 1 has for both intrados and extrados a transformed catenary, whose equation is $y = y_0\left(e^{\frac{x}{a}} - e^{\frac{-x}{a}}\right)$ where y_0 is the distance from the top of the loading (grade) to the curve. This curve may also be written $y = y_0 \cosh \frac{x}{a}$ and the curve may be plotted from a table of hyperbolic cosines. This curve, as is well known, is the equilibrium polygon or linear arch for earth loading below a horizontal line. Arches Nos. 2, 3, and 4 were of circular, elliptical, and parabolic intrados, respectively, and so constructed that the moment of inertia at each point is equal to the moment of inertia at the corresponding point on the catenarian arch. Thus the quantity $G\left(= \frac{ds}{I}\right)$ used in computing the moments, thrusts, etc., was identical for the four arches.

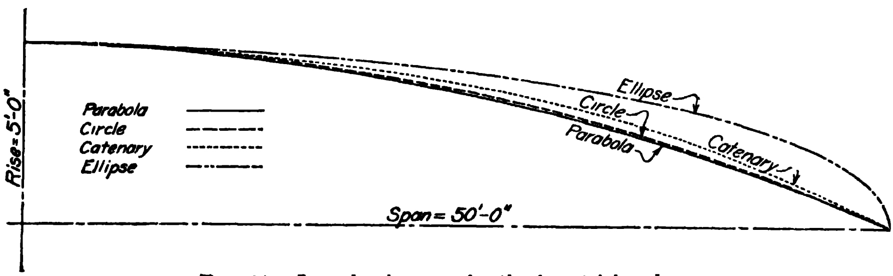


Fig. 11.—Intradosal curves for the four trial arches.

A comparison of the intradosal curves of the four arches (Fig. 11) will show the parabola as falling the lowest, the others in the order, circle, catenary and ellipse, the latter affording the greatest amount of waterway, while a consideration of the stress sheets will disclose the following facts.

As would be expected, the equilibrium polygon for the centenarian arch follows very closely to the neutral or center line: in fact the only divergence is due to the error introduced by replacing the signs of integration by those of summation in the formulas used. The other arches can be placed, as regards bending stress at the spring line (which is largely indicative of the stress in the arch) in the order: Circle, parabola, ellipse, the highest stressed being placed last. Figure 12 shows the dead load moments for the four arches graphically, and Fig. 13, the same for the dead load thrusts. The excessive thrust on the elliptical arch is due to the fact that although the rise of the intrados is the same for the four

[1] Assistant to the Dean and Director, Division of Engineering, Iowa State College, Ames, Iowa.

arches, the rise of the center line is much less in the case of the ellipse than in the other cases. Although this point is possibly too obvious to deserve mention, yet it is one that is apparently overlooked by some bridge companies who send in for approval plans fully as extreme as this.

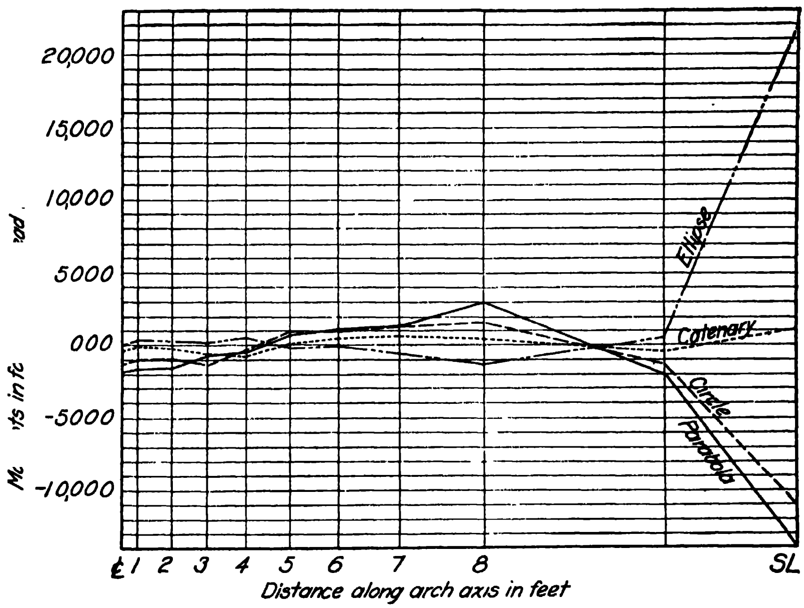


FIG. 12.

From an inspection of Fig. 14, it is seen that for a given ratio of rise to span the thrust due to a rise in temperature will decrease as the arch curve flattens out—that is to say, the nearer the intradosal (or axial curve) approaches a

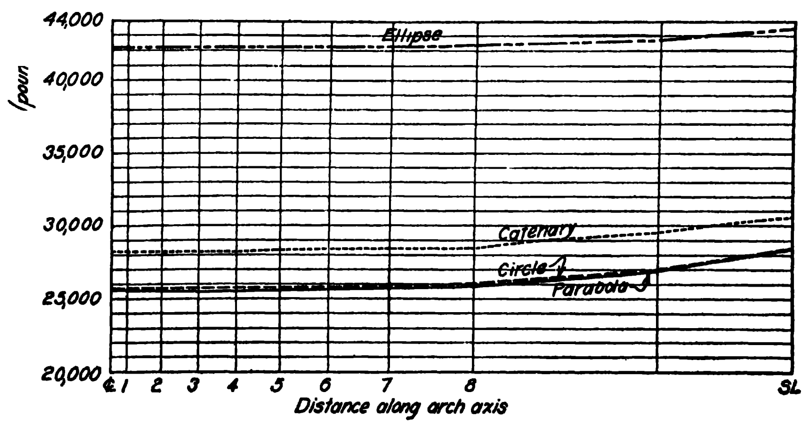
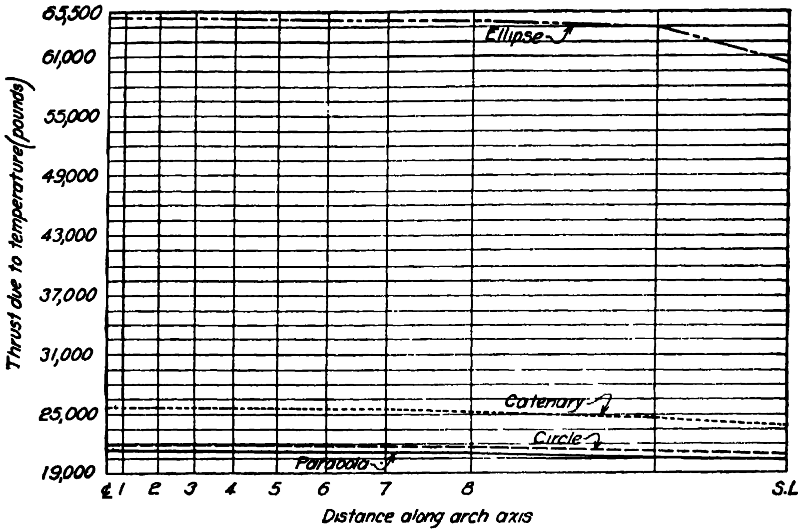


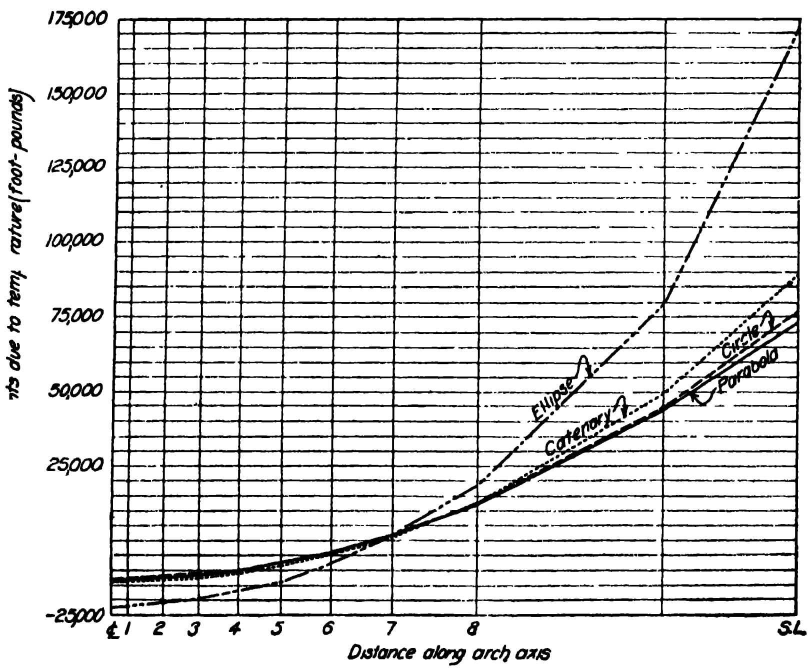
FIG. 13.

straight line joining crown and skewback, the less will be the temperature thrust. Figure 15 indicates a similar variation in temperature moments, particularly those near the skewback.



65,500
61,000
55,000
49,000
43,000
37,000
31,000
25,000
19,000
Thrust due to temperature (pounds)
Ellipse
Catenary
Circle
Parabola
℄ 1 2 3 4 5 6 7 8 S.L.
Distance along arch axis

FIG. 14.



175,000
150,000
125,000
100,000
75,000
50,000
25,000
-25,000
rature (foot-pounds)
Ellipse
Catenary
Circle
Parabola
℄ 1 2 3 4 5 6 7 8 S.L.
Distance along arch axis

FIG. 15.

4. Method of Constructing Various Types of Intradosal Curves.—The three-centered arch curve may be constructed through any three given points P_1, P_2 and P_3 as shown in Fig. 16 by means of the radii formula there given.

The ellipse may be constructed (given the major and minor axes) by the method shown in Fig. 17 as follows:

Construct the major and minor circles as shown and draw any radial line cutting the minor circle in a_1 and the major circle in a_2. A vertical through a_2 and a horizontal through a_1 intersect in point a which is a point on the true ellipse. In this manner, any number of points may be determined and the curve sketched in.

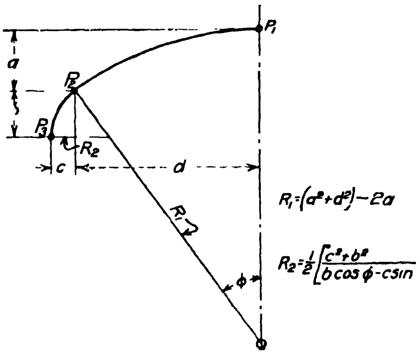


FIG. 16.—Method of drawing three centered arch curve.

The parabola may be constructed as shown in Fig. 17 as follows:

Let it be required to pass a parabola through point a and b with its vertex at a and its major axis vertical (which is the condition desired in open spandrel arch design).

Lay off ac and cb and divide each into an equal number of equal parts.

Draw radial lines from a to the division points on line cb and drop verticals through the corresponding division points on line ac.

These lines intersect to define points on the parabola as shown in the figure (Fig. 17).

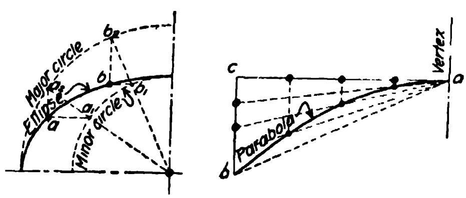


FIG. 17.—Method of constructing the ellipse and parabola.

It is general custom to use a multicentered circular curve approximating the parabola or the ellipse rather than these curves themselves, owing to the ease with which circular curves may be laid out for centering diagrams, etc. Figures 18 and 19 indicate methods of constructing three- and five-centered circular curves approximating the ellipse. It is seen that the assumed points in both cases may be shifted somewhat to accommodate the new curve to the true ellipse.

5. Formulas for Most Advantageous Shape for Arch Rings.—Victor H. Cochrane in a paper before the Engineers' Society of Western Pennsylvania entitled "Design of Symmetrical Hingeless Concrete Arches" (see November, 1916, issue of *Proc.*, vol. 32, No. 8, pp. 647–713) has presented a formula for the

axial curve best suited to arches both of the filled spandrel and open spandrel type. These formulas are given in Fig. 20.

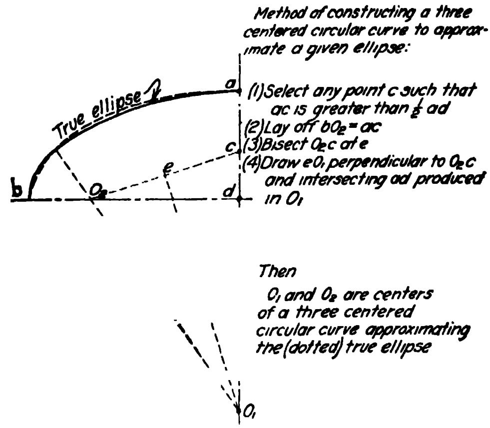


FIG. 18.

6. Depth of Filling Over Arch Cover.—The greater the depth of filling over the crown of a filled spandrel arch, the greater the dead load stresses.

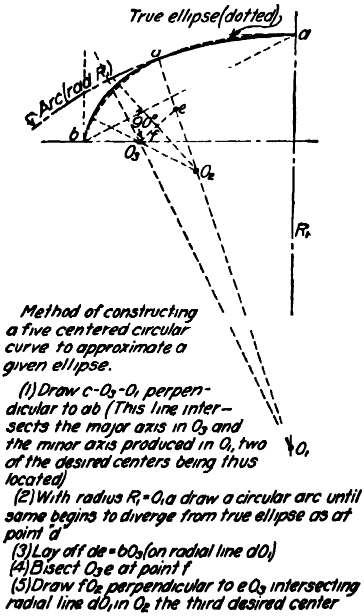


FIG. 19.

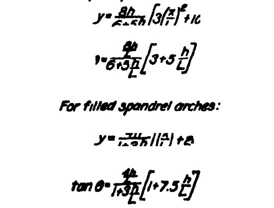


FIG. 20.—Victor H. Cochrane's formulas for most suitable shape of arch ring.

On the other hand, the impact stresses for heavy rolling loads will be greatly diminished and the distribution of load concentrations will be greatly increased.

For highway loadings, a minimum of 12 in. over the crown is needed to avoid excessive impact stresses while for railway loadings at least 2 ft. under the ties is needed for a load cushion.

7. Crown Thickness for Preliminary Assumptions.—Before making an analysis of any masonry arch, the general ring dimensions must be assumed, these being corrected later to conform to stress requirements as indicated by the final analysis.

For this reason, it is well to have certain approximate formulas for crown thickness and for the variation in ring thickness from crown to spring line as a guide to the designer in making his preliminary assumptions.

There have been several ring formulas proposed, the most commonly used of which are discussed in the "Concrete Engineers' Handbook" by Hool & Johnson (p. 656) as follows:

The Weld Formula

$$h = \sqrt{L} + \frac{L}{10} + \frac{w}{200} + \frac{w'}{400}$$

Where h = crown thickness in inches.
L = clear span in feet.
w = live load (uniformly distributed) in pounds per square foot.
w' = dead load above crown in pounds per square foot.

The W. J. Douglass Formula (For Highway Loadings):

Span Length	Values of Crown Thickness
Under 20 ft.	$0.03\ (6 + L)$
20 ft.—50 ft.	$0.015\ (30 + L)$
50 ft.—150 ft.	$0.0001\ (11{,}000 + L^2)$
Over 150 ft.	$0.016\ (75 + L)$

For railway loadings, increase the crown thickness as given above 25 per cent for spans up to and including 50 ft., 20 per cent for spans up to and including 150 ft. and 15 per cent for spans over 150 ft.

The Joseph P. Schwada Formula:

$$h = \frac{L^2}{57.6(r - h)f_cK}\left[\frac{B}{20} + r + 8h + 6F + \frac{w}{20}\right]$$

Where h = crown thickness in feet.
L = clear span in feet.
r = rise of intrados in feet.
f_c = maximum stress which may reasonably be expected if arch ring is designed in accordance with formula. Values of f_c from 550 lb. per so. in. to 600 lb. per sq. in. are recommended by Mr. Schwada.
K = a coefficient of f_c depending upon the ratio $\frac{r}{L}$, the span and the type of loading. Values of K are tabulated for two different load conditions in the table below.
F = depth of fill at crown (feet) exclusive of track and ballast or pavement.
B = weight of track and ballast or pavement in pounds per square foot.
w = uniform live load in pounds per square foot.

VALUES OF COEFFICIENT K

Ratio	Span 20 ft.	30 ft.	40 ft.	50 ft.	60 ft.	120 ft.	150 ft.
0.250	R 0.26 H 0.16	R 0.36 H 0.25	R 0.41 H 0.35	R 0.44 H 0.42	0.46	0.58	0.64
0.200	R 0.32 H 0.20	R 0.42 H 0.30	R 0.48 H 0.40	R 0.52 H 0.49	0.54	0.66	0.72
0.167	R 0.38 H 0.24	R 0.49 H 0.35	R 0.55 H 0.45	R 0.57 H 0.55	0.59	0.71	0.77
0.143	R 0.46 H 0.27	R 0.56 H 0.39	R 0.59 H 0.50	R 0.61 H 0.60	0.63	0.75	0.81
0.125	R 0.55 H 0.31	R 0.62 H 0.44	R 0.64 H 0.55	R 0.65 H 0.64	0.67	0.79	0.85

R = railway loadings. H = highway loadings.

To obtain the approximate trial crown thickness by the above formula, it is necessary first to assume a value of h and solve formula as a check. If the original assumed value of h is greatly in error, correct the assumption and re-solve until the final and assumed values of h check fairly close.

8. Loadings on Arch Bridges.—For railway arches, the rolling loads used in arch design are generally Coopers' standard E series or standard wheel load diagrams of similar character. Most main line structures are now designed for loadings equivalent to Coopers' E55 or E60 and for heavy duty lines E70 or equivalent is now being employed.

Where earth filling is employed over the crown of railway arches, the load concentrations are distributed through the filling material and the arch ring itself may be designed for an equivalent uniform live load. It is generally customary to consider any axle load as being carried by three ties and thence downward through the fill as a truncated pyramid whose side planes have a slope with the vertical whose tangent is equal to 0.50. For railway loadings, equivalent uniform live loads range from 600 to 1,200 lb. per sq. ft.

Ordinary impact formulas for filled spandrel arches may safely be reduced due to the cushioning and distributive effect of the same.

The subject of loadings for highway bridges was quite thoroughly covered in 1914 in a report by the Committee on "Reinforced Concrete Highway Bridges and Culverts," American Concrete Institute, of which the writer was Chairman. This report was presented and adopted by the Institute February, 17, 1914, and the following is extraced therefrom:

Dead Load.—For important construction, or that requiring a nicety of design, field measurements of the actual unit weights of the materials to be used may be warranted.

The ordinary structural materials will not vary materially in unit weight. For filling material under different conditions of drainage, consolidation, etc., the unit weights are not so uniform. Thus the weight of earth filling may vary by more than 40 per cent. By means of small brass cylinders carefully driven into the material in situ, data have been collected concerning the maximum density of earth filling under varying conditions. The range of unit weights which may be expected is illustrated by Table 1 from results of actual measurements as above described:

TABLE 1.—WEIGHTS PER CUBIC FOOT OF SOLID UNDISTURBED SOIL IN PLACE

Kind of Material	Damp	Saturated
Black top soil	94	105
Yellow clay	121	123
Sandy clay	106	115
Clayey sand	123	132
Blue clay	114	118

In view of the fact that a difference in weight may affect the design to a considerable degree (particularly in abutment and spandrel fill arch design) your committee recommends for all important construction accurate measurements to determine the unit weights of filling material.

In the absence of accurate experimental data for the particular case involved, the following unit weights may be taken as representing average values:

Materials	Lb. per Cu. Ft.
Earth filling	120
Plain concrete	145
Reinforced concrete	150
Steel	490
Cast iron	450
Vitrified brick	140
Common brick	125
Granite and limestone masonry	165
Sandstone	140
Macadam-Telford	150
Pine, fir, etc.	42
Oak and yellow pine	48
Creosoted timber	60

Live Loadings.—Conditions governing the selection of a standard live loading over a territory of the size occupied by the membership of this Institute are so widely varied that to design for a loading which in one locality would be no more than safe practice would result in extravagance for less populous localities.

The live load which, in the interest of both safety and economy of first cost, should be selected is that loading to which the bridge structure during its estimated life is likely to be ordinarily subjected. This involves not only a study of the traffic conditions of today, but a prediction of future development, which latter can be based only on a careful study of past growth. To emphasize the magnitude of this question and the tremendous waste of funds which might easily accrue to the taxpayers of the United States through an improper selection of loadings for their highway bridges would be superfluous.

It is hoped that in the near future, through correspondence with manufacturing firms and street railway officials, and through co-operation with the engineering profession in general, more data looking to the selection of proper loading assumptions may be collected by this committee.

After a consideration of the data at hand, the following recommendations are offered pending further investigation:

Class A Bridges.—City bridges and bridges on main thoroughfares leading therefrom.

Concentrated Live Load.—A motor truck of the following dimensions and weights

Total weight	50,000 lb.
Weight on rear axle	33,000 lb
Distance between axles	10 ft.
Width of tread of rear wheels	24 in.
Distance between centers of rear wheels	6 ft.
Roadway space occupied, width	10 ft.
Length	30 to 36 ft.

For city bridges in the areas of heavy traffic (especially if a crossing at either end of the span operates to hold up traffic), it not infrequently happens that trucks of this character follow one another so closely as to constitute a connected string reaching more than across any ordinary single span. Outside the limits of the heavy traffic areas and on the tributary main highways, the possibility of more than two trucks being upon the span at any one time is exceedingly remote.

It is, therefore, recommended that for the former locations as many of the above motor trucks as are necessary for maximum stresses be considered on the structure at one time; for the latter locations the number may in all safety be limited to two.

The following alternate uniform live loadings are recommended:

Span (ft.)	under 80	80-100	100-125	125-150	150-200	over 200
Loads (lb. per sq. ft)	125	110	100	90	85	80

It is further recommended that the above uniform live loading be assumed to occupy all or such portion of the roadway area not occupied by the motor truck loading as is necessary for maximum stresses.

Sidewalk Loadings.—The greatest legitimate loading likely to be imposed upon a bridge sidewalk will rarely exceed 90 lb. per sq. ft. of area.

In this connection it is recommended (especially for city bridges) that special precaution be taken to protect the sidewalk area from any of the street traffic. If the details of the design are such that this cannot be done, it is recommended that the sidewalk area be designed for the maximum street traffic.

Class B Bridges.—The minimum requirements for ordinary highway traffic should probably be within the following limits:

Concentrated Live Load.—A traction engine or motor truck within the following limits:

Total weight	30,000–36,000 lb.
Concentration on rear wheels	66⅔ %
Distance between axles	10–12 ft.
Distance between rear wheels	5–6½ ft.
Width of tread of rear wheels	20–24 in.

Only one such vehicle need be considered upon any single span at one time. The floor area occupied by the above live loading may be taken as 10 ft. in width.

The alternate uniform live loading on Class B spans of varying lengths may safely be assumed as follows:

Span (ft.)	under 80	80-100	100–125	125–150	150-200	over 200
Load (lb. per sq. ft.)	100	90	80	75	65	60

Class E-1.—Bridges carrying ordinary electric railway traffic. It is recommended that these bridges be designed to carry a concentrated live loading of the following dimensions and weights:

Total weight	100,000 lb.
Number of wheels (on two trucks)	8
Spacing of wheels, center to center	7 ft.
Spacing of trucks, center to center	20 ft.

Each axle load distributed over three ties.

Special Bridges.—The electric railway loading above specified will doubtless be exceeded by the heaviest interurban traffic and by heavy electric engines carrying freight, which latter may exceed 100 tons. These, together with loads resulting from traffic from mines, quarries, manufacturies, etc., all constitute a special class and bridges carrying same should, of course, be designed to meet local present and future requirements.

The following is extracted from the specifications used by the writer at the present time for the design of reinforced concrete highway bridge work in Oregon.

For purposes of design, all bridge structures to be constructed in accordance with these specifications shall be grouped under three principal traffic classifications, as follows:

Class A Traffic.

This classification shall include:

(1) All permanent structures on State Highways.

(2) All permanent structures on roads other than State Highways which are or may be, during the estimated life of the structure, paved with a hard surfacing material.

(3) All permanent structures, on roads other than State Highways, falling outside the above classifications, but which may, during the estimated life of the structure, carry a considerable percentage of heavy motor traffic.

(4) All temporary construction where traffic requirements during the estimated life of the structure appear to warrant such traffic loading.

Class B Traffic.

This classification shall include:

(1) Any construction, either permanent or temporary, on lateral and secondary roads which by virtue of its location will never be called upon to carry other than horse-drawn or light motor vehicle traffic.

Class C Traffic.

This classification shall include:

(1) Any special traffic structure such as those on main thoroughfares radiating from large centers of industry, structures carrying interurban, street or suburban railway traffic, etc., etc.

The dead loading shall comprise the actual weight of the structure for which purpose the following unit weights shall be used:

Materials	Unit Weight (Lb. per Cu. Ft.)
Plain concrete	145
Reinforced concrete	150
Earth filling (wet or packed)	120
Gravel filling	135
Steel	490
Cast iron	450
Vitrified paving brick	140
Common brick	125
Stone masonry (ordinary)	165
Stone masonry (granite or like materials)	170
Macadam	150
Untreated timber (fir, Oregon pine, etc.)(green)	48
Untreated timber (oak or like material)	50
Creosoted timber	50
Bituminous surfacing	130
Snow	35
Brass (cast, rolled)	535
Lead	710

Bridges carrying Class A traffic shall be designed for a concentrated loading consisting of a motor truck of the following dimensions and weights:

Total weight	40,000 lb.
Weight on rear axle	28,000 lb.
Weight on front axle	12,000 lb.
Distance center to center wheels	6 ft. 0 in.
Distance center to center axles	10 ft. 0 in.
Width of each rear tire	2 ft. 0 in.
Total roadway space occupied	24 ft. 0 in. × 10 ft. 0 in.

That portion of the road area not occupied by the above concentrated loading shall be considered to carry the following uniformly distributed live load:

Uniform Live Loadings (Class "A" Traffic).

Span Length	Uniform Live Load (Lb. per Sq. Ft.)
0–100 ft	100
100–150 ft	90
150–200 ft	80
Above 200 ft	70

Bridges carrying Class "B" Traffic shall be designed for a live loading consisting of from 50 to 80 per cent of the loading designated for Class "A," depending on the location and probability of increased traffic.

Bridges carrying Class "C" traffic shall be designed to meet the special requirements of each individual case.

All structures having a clear roadway width of 20 ft. or greater shall be designed for two motor trucks passing, headed in the same direction. For roadway widths less than 20 ft., only one truck shall be considered. The space at the side of the truck is not to be considered as loaded with uniform live load.

9. Internal Temperature Range in Masonry Arches.—A change in the internal temperature of the masonry in a fixed arch ring will cause the ring to rise or fall at the crown and (because it has no hinges) thus introduce stress in the material.

During the years 1909–1913, Professor C. S. Nichols (then Assistant Director of the Iowa State College Engineering Experiment Station), H. B. Walker (now State Engineer of Kansas) and the writer conducted a series of investigations looking toward the determination of internal temperature range in masonry structures in Iowa.

Measurements were made by means of both Mercurial Soil Thermometers and Electrical Resistance Coils, the methods used and data secured being given in complete detail in Bulletin No. 30 of the Iowa State College Engineering Experiment Station.

The following is extracted from the conclusions set forth in this investigation:

(1) The yearly range in temperature in a reinforced concrete arch structure, typical of the highway arch construction in this State (Iowa), is, in this latitude, not far from 80 deg. F.

(2) The relation between the depth of concrete covering at any point and the yearly temperature range may be represented by the equation

$$y = 90 - {}^{53}\!/_{100}x$$

where y = the yearly temperature range in degrees Fahrenheit, and
x = the distance from the nearest exposed surface in inches.

However, the effect of the different factors, such as the presence of a water surface, direction of prevailing winds, etc., so modifies the results that the writers prefer to state their conclusions as in (1), giving an average value for points throughout a structure of this type.

(3) The amount of direct sunlight modifies somewhat the actual temperature in the concrete for a considerable distance into the interior of the mass, although, on account of the meager nature of the data gathered, no more definite conclusion can be stated.

(4) The data on the "lag" seem to show that in structures of this type the minimum temperatures are attained in time intervals anywhere from less than one day to four days after the atmospheric minimum. This interval depends upon the position of the portion of the structure considered, and is roughly proportional to the distance from the nearest exposed face.

(5) Because of the high temperature in the concrete when it attains its set, and the effect of atmospheric temperature upon this maximum, other conditions being equal, the pouring of an arch ring at a temperature near the atmospheric mean annual operates materially to lower the stresses in the ring induced by temperature variation.

(6) To render an arch ring structurally safe, provision should be made, in this latitude, for stresses induced by a temperature variation of at least 40 deg. F. each way from an assumed temperature of no stress. Particular circumstances may demand that a greater variation be used for drop in temperature to prevent the appearance of cracks. This will always remain largely a matter of judgment with the designing engineer.

ANALYSIS OF ARCHES

10. Fundamental Theorems Relating to Internal Work in Ribs and Frames.—The analysis of the fixed arch is based upon what is generally known as the mathematical theory of elasticity. The fundamental elastic equations which form the basis of this theory may be derived from a consideration of any one of several basic mathematical concepts, for example:

(1) From a consideration of Castigliano's theorem regarding the partial derivatives of the internal work.

(2) From a consideration of the law of mutual elastic equilibrium or virtual work.

(3) From the laws governing elastic displacements in ribs and frames.

Any one of the above methods yields a series of elastic equations practically identical in form. The last named method of derivation—namely, that based upon a consideration of elastic displacements—has been chosen for this discussion because of its comparative simplicity.

In order that the application of this theory of analysis may be thoroughly understood, it is necessary to preface the treatment of arch analysis proper by a brief consideration of the fundamental laws of internal work in ribs and frames. This consideration forms the subject matter of the subsequent paragraphs.

10*a*. The Laws of Internal Work in Structural Frames.—The structural frame, which as hereinafter treated may be made to include the solid web structure as well (see Art. **10*c***), may be regarded as a machine in motion. The motion is intermittent, occurring only when the equilibrium of the system is disturbed, as when loads are added, altered, or removed from the structure.

Consider any elastic framed structure under zero loading and having zero stress in each of its members. As external loadings are gradually applied, a slight change in form takes place, the points of application of the external loads execute small displacements and the internal stresses, which are now set up in the various members, are moved through small distances equal in each case to the distortion in the member. At the instant of application both external loads and internal stresses have zero values from which they increase gradually and simultaneously, reaching their maximum values at the same instant.

If, at any instant, the value of any external force is F, its average value during the period of application is obviously $(F + 0) \div 2$, or $\frac{1}{2}F$. If the displacement of the point of application of this force parallel to its line of direction be represented by the symbol Δ, it follows from the laws of mechanics that the work done by the force thus far may be represented by the expression $\frac{1}{2}F\Delta$.

The total work performed thus far by all the external forces of the system is obtained by simply summing the terms $\frac{1}{2}F\Delta$ for each external force acting, thus

$$W_E = \tfrac{1}{2}\Sigma F\Delta \tag{1}$$

As the loading has progressed each of the members has been required to change length, by a certain small amount λ, in order to accommodate itself to the new distorted shape of the frame. This linear distortion has in turn induced a stress in each member whose value at this instant is S and whose average value

during the period of loading is obviously $(S + 0) \div 2$, or $\frac{1}{2}S$. The internal work performed by each member in resisting distortion is therefore

$$\tfrac{1}{2}S\lambda$$

and for all the members of the frame, we may write

$$W_I = \tfrac{1}{2}\Sigma S\lambda \tag{2}$$

At the instant of application of loading the sturcture is at rest, the kinetic energy or energy of motion is zero and the condition is one of static equilibrium.

The application of the load disturbs the balance and the machine is set in motion. The term $\frac{1}{2}\Sigma F\Delta$ at any instant represents the externally applied work. The term $\frac{1}{2}\Sigma S\lambda$ represents the work of resistance or the work which must be overcome by the external work. Obviously, the difference between these two values represents the amount of external work available for imparting kinetic energy or energy of motion to the structure, so that we may write for any instant during the motion

$$\tfrac{1}{2}\Sigma F\Delta - \tfrac{1}{2}\Sigma S\lambda = \text{Kinetic energy imparted to structure}$$

As the loading progresses, more and more of the kinetic energy is absorbed by the internal members until a point is reached where the internal and external work just balance. We then have

$$\text{Kinetic energy} = \tfrac{1}{2}\Sigma F\Delta - \tfrac{1}{2}\Sigma S\lambda = 0 \tag{3}$$

and the structure again comes to a point of rest.

This second rest point may be termed a condition of elastic equilibrium, whose fundamental law may be stated thus:

$$W_I = W_E \tag{4}$$

This equality furnishes the basis for the determination of stresses in all structures not susceptible of analysis from the laws of pure statics.

Whenever a force and the displacement of its point of application act in the same direction, the sign of the work is positive. When the direction of the force is opposite to that of the displacement, the sign of the work is negative.

In general, the external forces representing loads are displaced in the same direction in which the force acts, though this is not always true. The internal work, however, is negative as the displacements of the extremities of the members are opposite in direction to the tendency of the stress.

To illustrate, a tensile stress in any member operates to pull the pin points at its extremities together, while the actual displacement is a movement in the opposite direction (a lengthening of the member).

The internal work may also be regarded as negative in the sense that it represents the stored up energy of resistance, the resilient energy of the structure. As the loads are gradually released, it is this energy that operates to bring the structure back to its normal position, and, in changing from a negative value to zero, it performs positive work.

From Hooke's law of stress and strain proportionality, it follows that the term λ in eq. (2) may be replaced by the expression $\frac{Sl}{AE}$, where S represents the stress in the member, l its length under zero stress, A its cross-sectional area and E

the modulus of elasticity of the material of which it is composed. We may therefore write, for bodies in elastic equilibrium

$$\frac{1}{2}\Sigma F\Delta = \frac{1}{2}\sum\frac{S^2l}{AE} \tag{5}$$

which is, in certain cases, a more convenient form for practical use.

For an uniform change in temperature of t degrees, the internal stress in each member of the frame increases gradually from the value S to the value $(S + S_t)$, the average value during the change being $(S + \frac{1}{2}S_t)$. During this time the strain or distortion is represented by the expression $\lambda_t = ctl$, where c is the coefficient of expansion and l represents the length of the member. The total internal work is therefore given by the expression

$$W_{I_t} = \Sigma Sctl + \tfrac{1}{2}\Sigma S_tctl$$

The external work, since the forces F remain at a constant value, is obviously $W_{E_t} = \Sigma F\Delta_t$, where Δ_t represents the displacement of the point of application of each force due to internal temperature strains.

Equating these two, we may write

$$\Sigma F\Delta_t = \Sigma Sctl + \tfrac{1}{2}\Sigma S_tctl \tag{6}$$

for any structural frame at rest and in elastic equilibrium.

In general the term S_t is zero for simple spans and cantilevers. In other words the frame, being free to move, simply adjusts itself to the new lengths of its members and is therefore unstressed under temperature changes.

If any of the supports are elastic, the corresponding reactions are moved through certain displacements Δ and thus perform negative work (negative because the force and its displacement act in opposite directions). We may then write eq. (5)

$$\frac{1}{2}\Sigma F\Delta - \frac{1}{2}\Sigma R\Delta_r = \frac{1}{2}\sum\frac{S^2l}{AE} \tag{7}$$

In other words, a portion of the externally applied work $\frac{1}{2}\Sigma F\Delta$ must be consumed in overcoming the elastic resistance of the supports and consequently the work $\frac{1}{2}\sum\frac{S^2l}{AE}$ absorbed by the internal system will be less by that amount.

The complete work equation, including the effect of temperature changes and reaction displacements, may be written

$$\frac{1}{2}\Sigma F\Delta + \Sigma F\Delta_t - \frac{1}{2}\Sigma R\Delta_r = \frac{1}{2}\sum\frac{S^2l}{AE} + \Sigma Sctl + \frac{1}{2}\Sigma S_tctl \tag{8}$$

The term Δ as herein used does not refer to the actual movement of the point of application of a force, but to the component of this movement parallel to the line of direction of the force, otherwise the above equality does not hold. Unless otherwise specifically defined, the terms Δ and δ are used in this sense through the discussion.

In the application and derivation of the theorems which form the subject matter of this and subsequent articles, it will be understood that:

(1) All deformations are within the elastic limit of the material.

(2) Any change in form, either of a beam or a frame is slight. (This is in accordance with observation of ordinary engineering structures under loads within the elastic limit.)

(3) Tensile stresses and strains of elongation are considered as positive; compressive stresses and strains are considered as negative.

10b. Deflections and Panel Point Displacements in Frames.—Consider the curved cantilever frame, shown in Fig. 21, at rest and under zero loading. If any load F_b acting in any direction, and applied at any panel point b, is imposed upon the frame, the same will cause a distortion of the members and a deflection of the frame.

Let it be required to find the vertical deflection of panel point c, due to this load F_b applied at panel point b.

Let S = the stress in any member of the frame due to the load F_b.

λ = the distortion in such member due to the stress S.

Δ_{bb} = the displacement of panel point b (measured parallel to the line of action of F_b) due to the force F_b.

Δ_{bc} = the desired vertical deflection of panel point c due to this same load.

From the equality of internal and external work stated in eq. (5), we may write (for the load F_b alone)

$$\frac{1}{2}F_b\Delta_{bb} = \frac{1}{2}\sum\frac{S^2l}{AE} \tag{9}$$

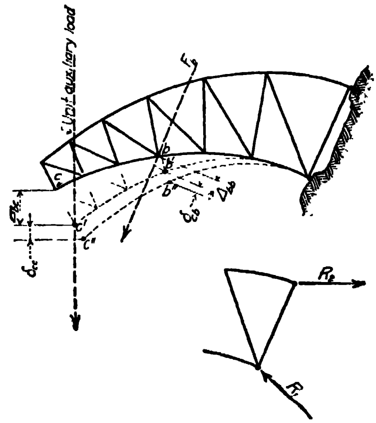


Fig. 21.

Now let us apply an auxiliary load equal to unity at panel point c, acting in the direction in which it is desired to determine the deflection (in this case vertically).

Let s_c = the stress in any member of the frame due to this unit load

δ_{cc} = the vertical displacement at panel point c due to this unit load.

δ_{cb} = the displacement at panel point b due to this unit load measured along the line of action of the force F_b.

The addition of this unit load has increased both the internal and external work as follows:

$$\text{Additional internal work} = \Sigma\left(S + \frac{1}{2}s_c\right)\frac{s_cl}{AE}$$

$$\text{Additional external work} = \frac{1}{2}\,(\text{Unity})\;\delta_{cc} + F_b\delta_{cb}$$

These two according to the preceding article must be equal, therefore

$$\frac{1}{2}\delta_{cc} + F_b\delta_{cb} = \sum\frac{Ss_cl}{AE} + \frac{1}{2}\sum\frac{(s_c)^2l}{AE} \tag{10}$$

If the above two loads were to be applied simultaneously, equating the internal work to the external work would yield the following expression:

$$\frac{1}{2}F_b(\Delta_{bb} + \delta_{cb}) + \frac{1}{2}(\text{unity})(\Delta_{bc} + \delta_{cc}) = \frac{1}{2}\Sigma(S + s_c)^2\frac{l}{AE}$$

or

$$\frac{1}{2}F_b\Delta_{bb} + \frac{1}{2}F_b\delta_{cb} + \frac{1}{2}\Delta_{bc} + \frac{1}{2}\delta_{cc} = \frac{1}{2}\sum\frac{S^2l}{AE} + \sum\frac{Ss_cl}{AE} + \frac{1}{2}\sum\frac{(s_c)^2l}{AE} \tag{11}$$

If all loads were released and the unit load alone applied to the structure equating the internal and external work would yield the following expression:

$$\frac{1}{2}\,(\text{Unity})\delta_{cc} = \frac{1}{2}\,\frac{\Sigma(s_c)^2l}{AE} \tag{12}$$

Substituting from eqs. (9) and (12) in eq. (11) and canceling like terms we have

$$\frac{1}{2}F_b\delta_{cb} + \frac{1}{2}\Delta_{bc} = \frac{\Sigma S s_c l}{AE} \tag{13}$$

Substituting from eq. (12) in eq. (10) and canceling like terms, we have

$$F_b\delta_{cb} = \frac{\Sigma S s_c l}{AE}$$

or

$$\frac{1}{2}F_b\delta_{cb} = \frac{\frac{1}{2}\Sigma S s_c l}{AE} \tag{14}$$

Substituting for $\frac{1}{2} F_b\delta_{cb}$ in eq. (13), canceling and multiplying by 2 on each side, we have

$$\Delta_{bc} = \frac{\Sigma S s_c l}{AE} \tag{15}$$

This is the expression for the desired deflection Δ_{bc} and may be expressed by this important rule:

To find the displacement or deflection of any point in a structural frame in any given direction and under any given set of load conditions, proceed as follows:

(1) Place an auxiliary unit load at the panel point at which the deflection is desired. This auxiliary load is to be assumed as acting in the direction along which the deflection is desired.

(2) Compute the stresses in the given frame due to the given loadings, calling these stresses "S."

(3) Compute stresses in each member of the frame due to the auxiliary unit load, calling these stresses "s."

(4) Compute for each member of the frame the length l and the cross-sectional area A.

(5) The desired deflection Δ is then given by the expression

$$\Delta = \frac{\Sigma S s l}{AE} \tag{16}$$

the summation being for each member of the frame.

The above law is entirely general and can be applied to any frame for any load or set of loads and for the calculation of deflections in any direction. For example, if the unit auxiliary load shown in Fig. 21 had been taken as horizontal the values of s_c would have been entirely different and the result would have expressed the *horizontal* displacement of panel point c.

If the result comes out negative, it simply indicates that the true movement is opposite in direction to that assumed for the unit load in computing the values of "s."

Temperature Displacements.—If instead of the load F_b of the preceding discussion the vertical deflection at panel point c, due to a change in temperature, were desired, we have only to reason as follows:

The deflection Δ_{bc} was caused by the linear distortion in the various members due to the load F_b, which linear distortion was represented by the expression $\frac{Sl}{AE}$. A change in temperature of t degrees will, obviously, distort each member

an amount $\lambda_t = ctl$, where c is the coefficient of thermal expansion, and l is the length of the member. If this distortion is substituted for the distortion $\frac{Sl}{AE}$ due to the load F_b, we may at once write the expression for temperature deflection at panel point c as follows:

$$\Delta_{tc} = \Sigma s_c(ctl) \tag{17}$$

Δ_{tc} represents the displacement of panel point c due solely to a uniform change in temperature of t degrees in the various members of the frame and is measured in the direction assumed for the auxiliary load (in this case vertical).

Effect of Reaction Displacements.—Throughout the foregoing the supports have been considered as inelastic, in which event the work done by the reactions is zero. If, however, these supports do yield under load, the work done by the reactions is not zero and must be considered.

Consider, as an illustration, the case shown in Fig. 21. The effect of the rigid anchorage at the right-hand end of the cantilever beam may be represented by two reactions R_1 and R_2, whose numerical value and direction of action may be easily determined from statics.

If each of these reactions is supplied by supports which are elastic or yielding, it is apparent that the movement of the same will have an effect upon the deflection Δ_{bc}, as determined above.

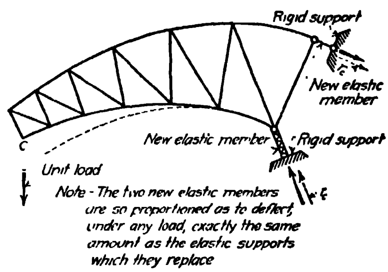


FIG. 22.

If these two elastic supports were to be removed and elastic frame members (resting in turn upon rigid supports) inserted in their stead (as shown in Fig. 22) it is at once apparent that the deflection at any point such as Δ_{bc} remains unaltered—*provided, the movement of these new frame members is the same as that of the yielding supports.*

Let

Δ_r represent the movement of any elastic support under the given loading,

r_c represent the reaction at this same support induced by a unit auxiliary load at point c.

Applying eq. (15) to this new frame we have

$$\Delta_{bc} = \frac{\Sigma S s_c l}{AE} + \frac{\Sigma S' s_c' l'}{A'E} \tag{18}$$

where S', s_c', l', etc., refer to the two new frame members.

Since from the original hypothesis these two new frame members exactly reproduce the movement of the original elastic supports, we may write, for each support

$$\Delta_r = \frac{S'l'}{A'E} \tag{19}$$

Also, from inspection

$$r_c = s_c \tag{20}$$

Therefore

$$\Delta_{bc} \quad \frac{\Sigma S s_c l}{AE} + \Sigma r_c \Delta_r \tag{21}$$

The effect of support displacements, in general, is very slight in well-designed arch construction and ordinarily is not considered in the analysis. For particular cases, however, it becomes necessary to consider the elastic movement of the abutments and piers, in which event the above equation becomes of value.

The general equation including the effect of temperature changes and reaction displacements may be written

$$\Delta + \Delta_t - \Sigma r\Delta_r = \sum \frac{Ssl}{AE} + \Sigma sctl \tag{22}$$

If the desired displacement Δ is an angular rotation instead of a translatory movement as above, the law stated in eq. (22) holds, the only difference being that the auxiliary unit load employed must, in this case, be a unit moment couple, and the stresses s must be taken as the stresses resulting from the application of this unit moment couple. (This may be easily proved from the general principles above developed; the detailed demonstration need not be given here.)

10c. Work Expressions for Solid Web Beams and Cantilevers.—The fundamental principles, the demonstration of which form the subject matter of the foregoing articles, are based upon a consideration of framed structures. Let us now consider the case of the homogeneous beam or solid webbed structure.

Consider the structure shown in Fig. 23, a homogeneous, solid curved beam, hinged at one end and freely supported at the other, under the action of a certain system of external loading ΣF. The structure is at rest and in elastic equilibrium (see Art. **10**a), hence we may write

$$W_I = W_E \text{ (as in eq. (4))}$$

The external work is obviously represented by the expression $\frac{1}{2}\Sigma F\Delta - \frac{1}{2}\Sigma R\Delta_r$ (see eq. (7)). The internal work corresponding to the expression $\frac{1}{2}\sum\frac{S^2l}{AE}$ of eq. (7) will now be evaluated.

Consider any lamina, a-b-c-d, of the beam included between two consecutive cross-sections whose distance apart is represented by the term ds (Fig. 23a).

The stresses induced in this lamina by the gradual application of the external load system ΣF may, obviously, all be resolved into two components, to wit: R_1 representing the resultant of all stresses transmitted to the lamina from that portion of the beam on the right, and R_2 representing the resultant of all those forces transmitted to the lamina from the left. Each of these forces being unknown both in direction, amount and point of application may be represented by a normal force N applied at the neutral axis of the section, a tangential or shearing force V and a system of graded forces f (increasing from zero at the neutral axis to a maximum value at the extreme fibers) representing the stress couple,

$$M = N\rho$$

Figure 23 shows the lamina a-b-c-d as a "free body" under the action of the two resultant forces R_1 and R_2 and also shows these two forces resolved into the six unknown elements M_1, M_2, N_1, N_2, V_1 and V_2.

From the figure it is apparent that

$$N_1 = N_2 \tag{23}$$

$$V_1 = V_2 \tag{24}$$

$$M_1 = M_2 + V_2ds \tag{25}$$

Each of the three forces, N, M and V cause a certain distortion of the lamina and hence the total internal work will comprise

(1) The work of the normal force N.
(2) The work of the bending moment M.
(3) The work of the shearing force V.

The derivation of the expressions for the above work elements is given in complete detail in Art. **12**a. Following are the results of such derivation:.

$$W_{IN} \text{ (the work of the normal force } N) = \frac{\frac{1}{2}N^2 ds}{AE} \tag{26}$$

$$W_{IM} \text{ (the work of the bending moment } M) = \frac{\frac{1}{2}M^2 ds}{EI} \tag{27}$$

$$W_{IV} \text{ (the work of the shearing force } V) = \frac{\frac{1}{2}CV^2 ds}{E_s A} \tag{28}$$

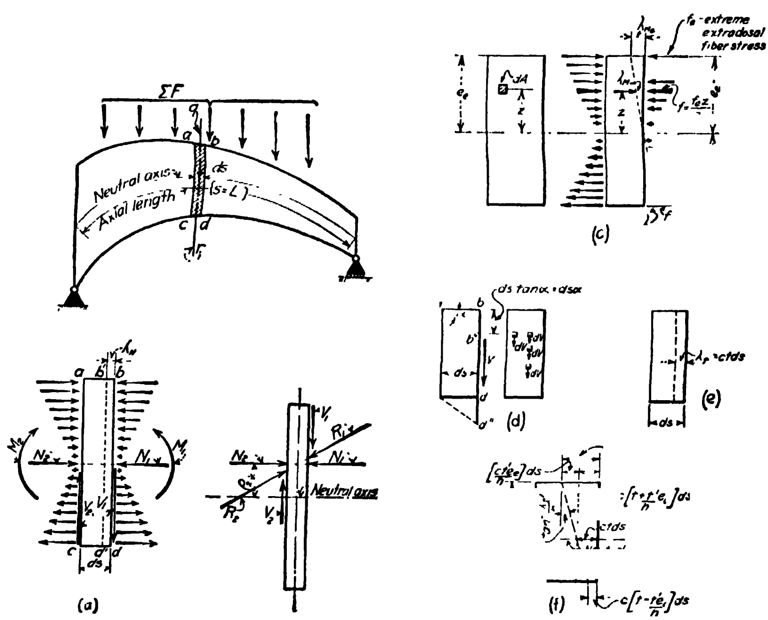


FIG. 23.

In the above expressions:

N, V and M are as above defined.
ds is the length of the lamina of rib included between the two cross-sections *a-c* and *b-d*.
A is the cross-sectional area of the beam.
I is the moment of inertia of the cross-section about the neutral axis (see Fig. 23).
E is the modulus of elasticity of the material in flexure.
E_s is the modulus of elasticity of the material in shearing.
C is a constant depending upon the shape of the beam cross-section.

For rectangular cross-sections........................ $C = \frac{6}{5}$
For circular cross-sections........................... $C = \frac{10}{9}$

For I-beams and riveted plate girders C is generally taken equal to $(A_T \div A_W)$ where A_T is the total cross-section, and A_W the area of the web.

Work Due to a Uniform Change in Temperature.—For a uniform change in temperature, the only movement is a direct shortening of the lamina ds, and therefore the only force which does any work is the axial pressure N, the forces M and V doing no internal work. The total work due to a uniform temperature change is therefore

$$W_{It} = Nctds \text{ (see Art. } \mathbf{12}a) \tag{29}$$

Work Due to a Variable Change in Temperature.—If the upper fibers of the beam are raised or lowered to a different temperature than those of the lower fibers, each lamina will undergo a slight angular distortion and the moment M will undergo a certain displacement. The work of both the axial thrust N and the shearing force V are obviously zero in this case and the entire work done on the lamina is given by the expression

$$W_{It'} = \frac{Mdsct'}{h} \tag{30}$$

where t' is the difference in temperature between the upper and lower fibers of the beam and h is the total depth of the lamina.

The total work done on the lamina a-b-c-d by all the forces active is, therefore, represented by the expression

$$W_I = \frac{N^2ds}{2AE} + \frac{M^2ds}{2EI} + \frac{CV^2ds}{2E_sA} + Nctds + \frac{Mct'ds}{h} \tag{31}$$

The entire internal work for the beam is very clearly the summation of that for each lamina.

Thus for the entire beam:

$$W_I = \frac{1}{2}\sum\frac{N^2ds}{AE} + \frac{1}{2}\sum\frac{M^2ds}{EI} + \frac{1}{2}\sum\frac{CV^2ds}{2E_sA} + \Sigma Nctds + \sum\frac{Mct'ds}{h} \tag{32}$$

This is the fundamental expression for the internal work in any solid homogeneous beam including temperature effects and is entirely general, holding for straight as well as for curved beams under any loading and for any method of support.

The complete work equation may therefore be written:

$$\frac{1}{2}\Sigma F\Delta + \Sigma F\Delta_t + \Sigma F\Delta_t' - \frac{1}{2}\Sigma R\Delta_r = \frac{1}{2}\sum\frac{N^2ds}{AE} + \frac{1}{2}\sum\frac{M^2ds}{EI} + \frac{1}{2}\sum\frac{CV^2ds}{2E_sA} + \Sigma Nctds + \sum\frac{Mct'ds}{h} \tag{33}$$

This corresponds to eq. (8) of Art. **10**a, except that for framed structures the effect of a variable temperature change was not considered separately.

The complete derivation of the above work formulas is given here for reference:

Derivation of Expressions for Internal Work in Ribs and Beams. *Work Due to the Axial Force N.*—The effect of this force is clearly to cause the linear distortion $\lambda_N = \frac{Nds}{AE}$, where A represents the cross-sectional area of the lamina perpendicular to the line of action of the force N. If W_{IN} represents the work due to this axial force, then we may at once write

$$W_{IN} = \frac{1}{2}N\lambda_N = \frac{\frac{1}{2}N^2ds}{AE} \tag{A}$$

Work Done by Moment Couple M.—The effect of this force is clearly to shorten the fibers on one side of the neutral axis and lengthen them on the other, producing the distortion shown in Fig. 23*c*. The graded fiber stresses on the right-hand face of the lamina do not exactly equal those on the left-hand face inasmuch as the two moments differ by the quantity Vds. However, by taking ds sufficiently small the average value $M = \frac{1}{2}(M_1 + M_2) = M_2 + \frac{Vds}{2}$ may be substituted for M_1 or M_2 without material error.

This average value of M represents very closely the moment along the center line q-r of the lamina in question and will be designated by the letter M without subscript.

Consider any element of area dA whose distance from the neutral axis is represented by the term z (Fig. 23*c*).

From ordinary mechanics of flexure the stress on this element is represented by the expression

$$fdA = \frac{f_e z dA}{e_e} \text{ or } \frac{f_i z dA}{e_i}$$

(See Fig. 23*c*.)

The distortion of this fiber is represented by the expression

$$\lambda_M = f\left(\frac{ds}{E}\right) = \left[\left(\frac{f_e z}{e_e}\right)\left(\frac{ds}{E}\right)\right] = \left[\left(\frac{f_i z}{e_i}\right)\left(\frac{ds}{E}\right)\right] \tag{B}$$

and the internal work resisted by this elementary area dA is represented by the expression

$$dW_{IM} = \left(\frac{1}{2}fdA\right)\left(\lambda_M\right) = \frac{\frac{1}{2} f_e^2 z^2 dA ds}{e_e^2 E} \tag{C}$$

The upper surface of a beam or arch rib is hereinafter termed the extrados and the distance to the extreme upper fiber measured from the neutral axis will be designated by the term e_e; the extreme unit fiber stress (due to bending stresses only) will be represented by the term f_e and the corresponding extreme fiber distortion by the term λ_{Me}. The lower surface of a beam or arch rib is hereinafter termed the intrados and the corresponding quantities will be designated by the terms e_i, f_i, λ_{Mi}, etc.

From the fundamental theory of flexure (plane sections maintained during bending) and from Hooke's law of stress and strain proportionality, it follows that, for homogeneous beams and ribs

$$f_e : e_e :: f_i : e_i$$

The internal work resisted by the entire lamina is obviously given by integrating the above expression, whence

$$W_{IM} = \int_{z=-e_i}^{z=+e_e} dW_{IM} = \frac{f_e^2 ds}{2Ee_e} \int_{z=-e_i}^{z=+e_e} z^2 dA = \frac{f_e^2 dsI}{2Ee_e^2} \tag{D}$$

where I represents the moment of inertia of the cross-section about the neutral axis.

From the theory of flexure $\frac{f_e I}{e_e} = M$, substituting which we have

$$W_{IM} = \frac{M^2 ds}{2EI} \tag{E}$$

for the internal work overcome by the average bending moment M active on the lamina a-b-c-d.

Work Done by the Shearing Forces.—The effect of the shearing stress V is shown in Fig. 23*d*. The total shear may be considered as made up of elementary forces dV, each active on an element of area dA. The displacement of the point of application of any one of these elementary forces is, from Fig. 23*d*, equal to bb'' or $ds \tan \alpha$, or, since α is very small, equal to $ds\alpha$. The work resisted by each elementary force dV is therefore represented by the expression

$$dW_{IV} = \tfrac{1}{2}\, dV\, dA\, ds\, \alpha \tag{F}$$

But from the definition of E_s (the shearing modulus of elasticity), $\alpha = \frac{V}{AE_w}$, hence

$$dW_{IV} = \frac{1}{2}\left(\frac{VdVdAds}{AE_s}\right) \tag{G}$$

and the work done by the total shear V active over the entire section is represented by the expression

$$W_{IV} = \int dW_{IV} = \frac{Vds}{2AE_s}\int dVdA \tag{H}$$

If the shear were entirely vertical and uniformly distributed over the cross-section, dV would be constant and equal to $\frac{V}{A}$, whence

$$W_{IV} = \frac{V^2 ds}{2A^2E_s}\int dA = \frac{V^2 ds}{2E_sA} \tag{I}$$

The shear is not uniformly distributed over the cross-section and furthermore is not exactly vertical since lateral distortions developed by the axial stresses induce small shearing stresses at right angles to the plane of the load system. Consequently the above expression must be multiplied by a distribution coefficient C which may be determined for any particular section and we may write

$$W_{IV} = \frac{CV^2ds}{2E_sA} \tag{J}$$

The distribution coefficient is a function of the size and shape of the particular cross-section under consideration and is derived from a theoretical consideration of the distribution and direction of the elementary shearing forces active on any section. The following values of C will suffice for the solution of the problems ordinarily encountered.

For rectangular cross-sections . . . . $C = \frac{6}{5}$
For circular cross-sections . . . . $C = \frac{10}{9}$

For I-beams and riveted plate girders no material error will result if C is taken as $\frac{A_T}{A_W}$ = total area of cross-section ÷ area of the web alone.

Work Due to Uniform Change in Temperature.—If the lamina *a-b-c-d* be subjected to a uniform change in temperature of t deg., the force N will be displaced through a distance equal to $\lambda_t = ctds$, and the resulting work will be represented by the expression

$$W_t = Nctds \text{ (see Fig. 23e)} \tag{K}$$

The forces M and V do not in this case contribute to the internal work.

Work Due to a Variable Change in Temperature.—Assume that the temperature at the extreme lower fiber of the lamina *a-b-c-d* exceeds that of the upper fiber by t' deg. and that the variation from lower to upper fiber is uniform.

In addition to the direct axial distortion (taken care of by the work expression $W_t = Nctds$ (see eq. (K)) the lamina will distort as shown in Fig. 23f. The work at any elementary area dA whose distance from the neutral axis is z is represented by the expression

$$dW_{It'} = \left(\frac{f_e z dA}{e_e}\right)\left(\frac{ct'dsz}{h}\right) = \frac{f_e dsct'z^2dA}{e_e h} \tag{L}$$

and the entire work of the lamina *a-b-c-d* is represented by the expression

$$W_{It'} = \frac{f_e dsct'}{e_e h}\int_{z=-i}^{z=+e} z^2dA = \frac{f_e I dsct'}{e_e h} = \frac{Mdsct'}{h} \tag{M}$$

Total Work on Any Lamina.—The total work done on the lamina *a-b-c-d* by all the various forces active is represented by the expression

$$W_I = W_{IN} + W_{IM} + W_{IV} + W_{It} + W_{It'} = \frac{N^2ds}{2AE} + \frac{M^2ds}{2EI} + \frac{CV^2ds}{2E_sA} + Nctds + \frac{Mct'ds}{h} \tag{N}$$

Entire internal work for the beam is very clearly the summation of the expression given in eq. (N) for every lamina in the beam which is obtained by integration between the limits $s = 0$ and $s = l$. Thus for the entire beam

$$W_I = \int_{s=0}^{s=l}\frac{N^2ds}{2AE} + \int_{s=0}^{s=l}\frac{M^2ds}{2EI} + \int_{s=0}^{s=l}\frac{CV^2ds}{2E_sA} + \int_{s=0}^{s=l} Nctds + \int_{s=}^{s=l}\frac{Mct'ds}{h} \tag{O}$$

10d. Displacements and Deflections in Beams and Ribs.—Proceeding exactly as in the case of the framed structure (Art. **10b**) we may now derive an expression for the displacement or deflection of any point in a solid beam or rib under any condition of loading.

The method of derivation is practically identical with that employed in deriving eq. (15) of Art. **10b** and need not be repeated here. The resulting equation corresponding to eq. (15) of Art. **10b** is as follows:

$$\Delta_{bc} = \sum \frac{Nn_c ds}{AE} + \sum \frac{Mm_c ds}{EI} + \sum \frac{CVv_c ds}{E_s A} \tag{34}$$

Where n_c, m_c and v_c represent, respectively, the axial thrust, bending moment and shear due solely to the action of a unit auxiliary load applied at c, acting in the direction along which the deflection Δ_{bc} is desired.

The expressions $\frac{Nn_c ds}{AE}$, $\frac{Mm_c ds}{EI}$ and $\frac{CVv_c ds}{E_s A}$ for any lamina will be positive when N and n_c, M and m_c, etc. have the same sign, and vice versa.

A positive result for the term Δ_{bc} indicates that the deflection is in the same direction as that chosen for the unit load; a negative value indicates motion in the reverse direction.

In general the displacements due to shearing strains are very small and may be neglected, whence eq. (34) may be written

$$\Delta_{bc} = \sum \frac{N n_c ds}{AE} + \sum \frac{M m_c ds}{EI} \tag{35}$$

Throughout the remainder of this and subsequent discussions the effect of shearing distortions in ribs will be entirely ignored.

Uniform Temperature Effect.—The expression for the displacement due to a uniform change in temperature (corresponding to eq. (17) of Art. **10b**) may be written

$$\Delta_{tc} = \Sigma n_c ctds + \Sigma m_c ctds \tag{36}$$

It is apparent from Fig. 23, that the distortion *ctds* is an axial distortion only, the angular movement being zero. The product (m_c) $(ctds)$ is therefore zero and we may write

$$\Delta_{tc} = \Sigma n_c ctds \tag{37}$$

Variable Changes in Temperature.—It is sometimes necessary to consider a condition wherein the upper and lower fibers of a beam or rib are changed to different temperatures. This condition undoubtedly obtains in masonry arches and to an even greater extent in steel ribs.

For a variable temperature change whose average value at the neutral axis is t deg. and whose maximum and minimum values are $t + \frac{t'e_s}{h}$ and $t - \frac{t'e_i}{h}$, eq. (37) may be written

$$\Delta_{tc} + \Delta_{t'c} = \Sigma n_c ctds + \sum \frac{m_c ct'ds}{h} \tag{38}$$

Where n_c acts in a direction such as to produce a strain in the same direction as *ctds*, the term $n_c ctds$ will be positive and vice versa. Where the moment acts in a direction such as to produce a fiber strain of the same direction as is produced by the variable temperature change t', the term $\frac{m_c ct'ds}{h}$ will be positive and vice versa.

The complete equation for the displacement of any point, including the effect of both variable and uniform temperature changes, may now be written as follows:

$$\Delta + \Delta_t + \Delta_{t'} - \Sigma r\Delta_r = \sum \frac{Nnds}{AE} + \sum \frac{Mmds}{EI} + \Sigma nctds + \sum \frac{mct'ds}{h} \tag{39}$$

11. Development of General Elastic Equations for Rib Arches.—Let Fig. 24 represent any fixed arch rib at rest and in equilibrium under the action of any system of external loads ΣF and the rigid anchorage at each abutment. If the two abutments were to be removed, it is clearly seen that the action of each abutment could be exactly reproduced by the introduction at each support of a moment couple M, an axial thrust N, and a tangential or shearing force V (Fig. 24*b*). It is also apparent that, if the value of these six forces under any given load condition can be determined, the stress in the rib at any point may very easily be found.

There are three fundamental static equations of equilibrium which may be written for any structure, as follows:

$$\Sigma \text{ (Horizontal forces, or components)} = 0$$
$$\Sigma \text{ (Vertical forces, or components)} = 0$$
$$\Sigma \text{ (Moments about any point)} = 0$$

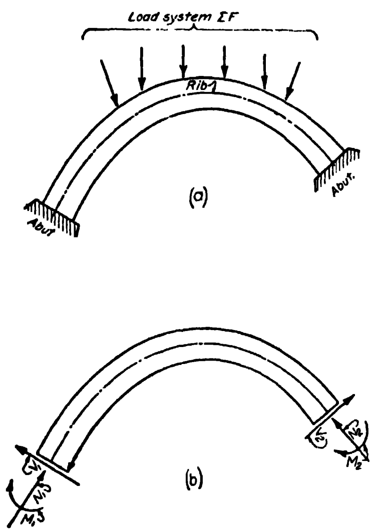


Fig. 24.

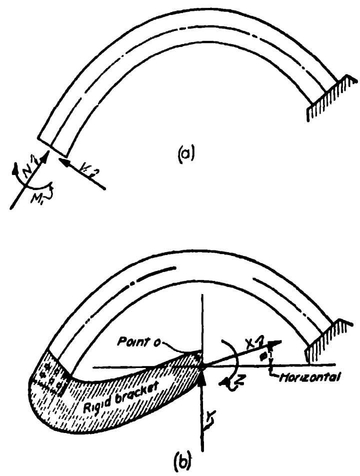


Fig. 25.

These three equations, therefore, suffice for the determination of three of the six above unknown reaction components, leaving three others which must be determined.

If, therefore, either support (say the left support) be removed and the three forces N, V and M be inserted to reproduce the action of this same support as shown in Fig. 25, the problem at once resolves itself into that of evaluating these three unknown forces. Once these are determined, the other three reaction components and the stress in the rib at any point are easily determined from the above static equations.

Carrying the reasoning a step further, let us replace the three forces N, V and M, by a rigid bracket or arm fastened to the rib at the left support and termi-

nating at some point O. At the end of this bracket let us apply three new unknown forces as shown in Fig. 25*b* as follows:

X = the unknown lateral component at point O.
Y = the unknown vertical component at point O.
Z = the unknown moment couple at point O.

It is apparent that these forces may be given a value such that they will exactly reproduce the action of the forces N, V and M at the left abutment and will consequently hold the structure in equilibrium.

It is also apparent that, if these three new unknown forces can be determined, the original reaction components N, V and M can easily be obtained and consequently the other reaction components (and the rib stresses as well) may be readily evaluated from statics.

This last step (the employment of the rigid bracket idea) simply replaces three unknown forces applied at the left abutment with three other unknowns applied at point O. This may seem entirely unnecessary. However, such is not the case, for by properly choosing the location for point O, the terminal point of the rigid bracket, the equations involving the unknowns X, Y and Z (to be derived hereinafter) become very greatly simplified.

If the three forces X, Y and Z were to be entirely removed, the resulting structure would become a cantilever. In any statically indeterminate structure there are always one or more unknown forces or reaction components which may be removed to produce a statically determinate structure. The structure resulting from the removal of these forces is generally termed the "residual frame" and the forces themselves are termed "redundants." In this case X, Y and Z are the "redundants" and the "residual frame" is a cantilever.

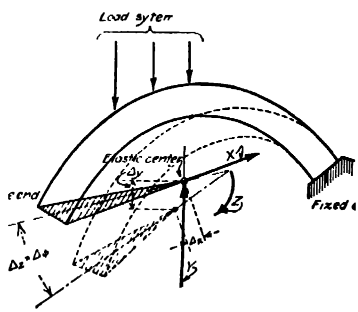


Fig. 26.

Consider the residual frame (a cantilever rib) shown in Fig. 26 under the action of any given system of external loading, together with the resultant redundant forces X, Y and Z. Under the action of these loads the terminal point O of the rigid bracket undergoes certain elastic displacements both angular and linear as shown.

Let

Δ_y = the displacement of point O along the line of action of the redundant Y.

Δ_x = the displacement of point O along the line of action of the redundant X.

Δ_z = the angular displacement of the bracket in the direction chosen for the redundant Z.

Referring back to Art. 10*d*, p. 462, we find these displacements to be given by the following expressions (neglecting the effect of shearing distortions, see eq. 39):

$$\Delta_x + \Delta_{tx} + \Delta_{t'x} - \Sigma r_x\Delta_r \quad \sum\frac{Nn_xds}{AE} + \sum\frac{Mm_xds}{EI} + \Sigma n_xctds + \sum\frac{m_xct'ds}{h} \quad (40)$$

$$\Delta_y + \Delta_{ty} + \Delta_{t'y} - \Sigma r_y \Delta_r = \sum \frac{N n_y ds}{AE} + \sum \frac{M m_y ds}{EI} + \Sigma n_y ctds + \sum \frac{m_y ct' ds}{h} \quad (41)$$

$$\Delta_z + \Delta_{tz} + \Delta_{t'z} - \Sigma r_z \Delta_r = \sum \frac{N n_z ds}{AE} + \sum \frac{M m_z ds}{EI} + \Sigma n_z ctds + \sum \frac{m_z ct' ds}{h} \quad (42)$$

In the above equations the various terms have the following significance:

Δ_x = the displacement as previously defined caused by the load system ΣF applied as shown, in combination with the unknown forces X, Y and Z resulting therefrom. This displacement is measured along the line of action of the redundant X.

Δ_y and Δ_z = similar displacements due to the same loading, but measured along the line of action of the redundants Y and Z (Δ_z is therefore an angular displacement).

Δ_{tx}, Δ_{ty} and Δ_{tz} = displacements measured as above caused by an uniform change in temperature of t deg.

$\Delta_{t'x}$, $\Delta_{t'y}$ and $\Delta_{t'z}$ = similar displacements caused by a variable change in temperature of t' deg. between the upper and lower fibers of the rib.

The terms r_x, r_y, r_z, $\Sigma r_x \Delta_r$, $\Sigma r_y \Delta_r$, etc. = as defined in Art. **10b**, p. 455.

M_o = the moment on the residual cantilever at any point due solely to the load system ΣF.

m_x = the moment at any point on the residual cantilever due solely to a unit load applied at O and acting along the line of action of the redundant X.

m_y = the moment at any point on the residual cantilever due solely to a unit load as above but acting along the line of action of the redundant Y.

m_z = the moment at any point on the residual cantilever due solely to a unit moment couple applied at point O and acting in the direction assumed for the redundant Z.

N_o, n_x, n_y, n_z, etc. are defined as above but refer to axial thrusts in place of moments.

ds, E, I, ctl, A, t' and h as previously defined (Arts. **10c** and **10d**, pp. 458 and 462).

Now it is observed that the unknown redundants X, Y and Z applied at O completely replace and reproduce the action of the left arch abutment. For any point therefore:

The { stress / reaction / displacement / deflection } on the given arch rib due to any given loading is always equal to the corresponding { stress / reaction / displacement / deflection } on the residual cantilever due to this same loading plus the effect of the redundant forces X, Y and Z caused by such loading.

If therefore:

M = the bending moment at any point in the arch rib due to any given loading.

M_o = the bending moment at the same point in the residual cantilever due to this same loading.

Then:

$$M = M_o + M_x + M_y + M_z = M_o + Xm_x + Ym_y + Z\,m_z \qquad (43)$$

In a similar manner

$$N = N_o + Xn_x + Yn_y + Zn_z \qquad (44)$$

We may now write eqs. (40) to (42) as follows:

$$\Delta_x + \Delta_{tx} + \Delta_{t'x} - \Sigma r_x\Delta_r = \sum\frac{N_0 n_x ds}{AE} + \sum\frac{M_0 m_x ds}{EI} + X\left[\sum\frac{(n_x)^2 ds}{AE} + \sum\frac{m_x{}^2 ds}{EI}\right] + Y\left[\sum\frac{n_x n_y ds}{AE} + \sum\frac{m_x m_y ds}{EI}\right] + Z\left[\sum\frac{n_x n_z ds}{AE} + \sum\frac{m_x m_z ds}{EI}\right] + \Sigma n_x ctds + \frac{\Sigma m_x ct' ds}{h} \qquad (45)$$

$$\Delta_y + \Delta_{ty} + \Delta_{t'y} - \Sigma r_y\Delta_r = \sum\frac{N_0 n_y ds}{AE} + \sum\frac{M_0 m_y ds}{EI} + X\left[\sum\frac{n_x n_y ds}{AE} + \sum\frac{m_x m_y ds}{EI}\right] + Y\left[\sum\frac{(n_y)^2 ds}{AE} + \sum\frac{(m_y)^2 ds}{EI}\right] + Z\left[\sum\frac{n_y n_z ds}{AE} + \sum\frac{m_y m_z ds}{EI}\right] + \Sigma n_y ctds + \frac{\Sigma m_y ct' ds}{h} \qquad (46)$$

$$\Delta_z + \Delta_{tz} + \Delta_{t'z} - \Sigma r_z\Delta_r = \sum\frac{N_0 n_z ds}{AE} + \sum\frac{M_0 m_z ds}{EI} + X\left[\sum\frac{n_x n_z ds}{AE} + \sum\frac{m_x m_z ds}{EI}\right] + Y\left[\sum\frac{n_z n_y ds}{AE} + \sum\frac{m_z m_y ds}{EI}\right] + Z\left[\sum\frac{(n_z)^2 ds}{AE} + \sum\frac{(m_z)^2 ds}{EI}\right] + \Sigma n_z ctds + \frac{\Sigma m_z ct' ds}{h} \qquad (47)$$

These are the general elastic equations applicable to the analysis of any fixed rib arch.

The terms $\frac{n_x ds}{AE}$, $\frac{n_x n_y ds}{AE}$, $\frac{(n_y)^2 ds}{AE}$, etc. are relatively very small, representing the distortions due to unit axial stresses. Except for very flat arches, these terms may be disregarded without material error. This will be done hereinafter except in considering the effect of axial stress or "rib shortening" as discussed later on.

Case I—Rigid Supports—Temperature Effects Neglected.—If the supports are rigid and inelastic, the above equations may be very much simplified, inasmuch as the entire left-hand member of each equation becomes zero for such a condition.

Ignoring temperature effects, and disregarding the terms $\frac{(n_x)^2 ds}{AE}$, $\frac{n_x n_y ds}{AE}$, etc. as above set forth, these equations may be written as follows:

$$X\sum\frac{m_x{}^2 ds}{EI} + Y\sum\frac{m_x m_y ds}{EI} + Z\sum\frac{m_x m_z ds}{EI} = -\sum\frac{N_0 n_x ds}{AE} - \sum\frac{M_0 m_x ds}{EI} \qquad (48)$$

$$X\sum\frac{m_x m_y ds}{EI} + Y\sum\frac{(m_y)^2 ds}{EI} + Z\sum\frac{m_y m_z ds}{EI} = -\sum\frac{N_0 n_y ds}{AE} - \sum\frac{M_0 m_y ds}{EI} \qquad (49)$$

$$X\sum\frac{m_x m_z ds}{EI} + Y\sum\frac{m_y m_z ds}{EI} + Z\sum\frac{(m_z)^2 ds}{EI} = -\sum\frac{N_0 n_z ds}{AE} - \sum\frac{M_0 m_z ds}{EI} \qquad (50)$$

Case II—Effect of a Uniform Change in Temperature (Rigid Supports).—The elastic equations for this condition may be readily written from eqs. (45) to (47) inclusive by placing the left-hand member of each equation equal to zero, as in Case I, and also placing the terms M_o, N_o and t' each equal to zero whence we write $\left(\text{ignoring the terms } \frac{(n_x)^2ds}{AE}, \frac{n_xn_yds}{AE}, \text{ etc. as set forth above}\right)$

$$X\sum\frac{(m_x)^2ds}{EI} + Y\sum\frac{m_xm_yds}{EI} + Z\sum\frac{m_xm_zds}{EI} = -\Sigma n_xctds \qquad (51)$$

$$X\sum\frac{m_xm_yds}{EI} + Y\sum\frac{(m_y)^2ds}{EI} + Z\sum\frac{m_ym_zds}{EI} = -\Sigma n_yctds \qquad (52)$$

$$X\sum\frac{m_xm_zds}{EI} + Y\sum\frac{m_ym_zds}{EI} + Z\sum\frac{(m_z)^2ds}{EI} = -\Sigma n_zctds \qquad (53)$$

Case III—Effect of a Variable Change in Temperature.—In a manner exactly analogous to the above these equations become:

$$X\sum\frac{(m_x)^2ds}{EI} + Y\sum\frac{m_xm_yds}{EI} + Z\sum\frac{m_xm_zds}{EI} = -\sum\frac{m_xct'ds}{h} \qquad (54)$$

$$X\sum\frac{m_xm_yds}{EI} + Y\sum\frac{(m_y)^2ds}{EI} + Z\sum\frac{m_zm_yds}{EI} = -\sum\frac{m_yct'ds}{h} \qquad (55)$$

$$X\sum\frac{m_xm_zds}{EI} + Y\sum\frac{m_ym_zds}{EI} + Z\sum\frac{(m_z)^2ds}{EI} = -\sum\frac{m_zct'ds}{h} \qquad (56)$$

The individual terms n_xctds, n_yctds, $\frac{m_xct'ds}{h}$, $\frac{m_yct'ds}{h}$, etc. carry positive signs when the distortions due to the unit thrusts and moments n_x, n_y, m_x, m_y, etc. are in the same direction as those produced by the temperature change, and vice versa.

12. Development of Elastic Influence Lines for Rib Arches.

12a. Development of Formulas.—Consider the fixed arch rib shown in Fig. 27, which, in order to make the problem entirely general, has been taken as unsymmetrical. Through the terminal point O of the assumed rigid bracket, construct two coordinate axes as follows:

Y-Y vertical

X-X acting along the line of action assumed for the redundant X and thus making some angle ϕ (as yet unknown) with the horizontal.

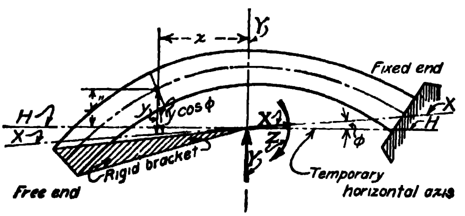


Fig. 27.

Let us measure the abscissae (x) horizontally, calling the signs positive to the right of the Y-Y axis, and let us measure the y ordinates vertically, calling the signs positive above the X-X axis.

Assume the three redundant forces to act as shown in Fig. 27. (These redundants may be arbitrarily assumed to act in either direction. If the true direction is opposite to that assumed, the signs will simply come out negative.)

Moments causing compressive stresses in the upper fiber of the residual cantilever will be assumed as positive, and conversely.

From inspection of the figure

$$m_x = -y \cos \phi \quad (57)$$
$$m_y = x \quad (58)$$
$$m_z = 1.0 \quad (59)$$

Divide the arch ring into small equal segments, as shown in Fig. 28, and compute for each segment the term $\frac{m_z ds}{I} = \frac{ds}{I}$, where ds is the length of the segment and I is the moment of inertia of the rib at the center of said segment. The term $\frac{ds}{I}$ is sometimes called the "elastic weight" of each voussoir or arch block and will hereinafter be designated by the term G.

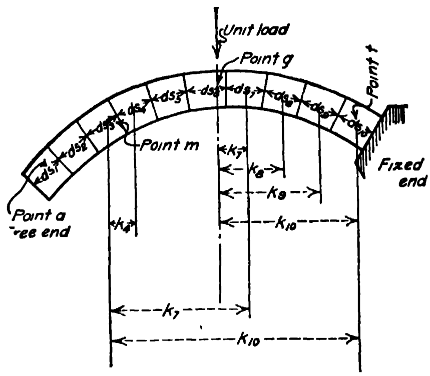


FIG. 28.

We may now write eqs. (48) to (50) inclusive (after multiplying them by the term E) as follows:

$$X \cos \phi^2 \, \Sigma G y^2 - Y \cos \phi \, \Sigma G x y - Z \cos \phi \, \Sigma G y = -\sum \frac{N_0 n_x ds}{A} + \cos \phi \, \Sigma M_0 G y \quad (60)$$

$$-X \cos \phi \, \Sigma G x y + Y \Sigma G x^2 + Z \Sigma G x = -\sum \frac{N_0 n_y ds}{A} - \Sigma M_0 G x \quad (61)$$

$$-X \cos \phi \, \Sigma G y + Y \Sigma G x + Z \Sigma G = -\sum \frac{N_0 n_z ds}{A} - \Sigma M_0 G \quad (62)$$

Now if the terms $G = \frac{ds}{I}$ are considered as weights applied at the center of each voussoir or arch block, and the terminal point O made coincident with the center of gravity of such "elastic" system, we observe at once that

$$\Sigma Gx = O$$
$$\Sigma Gy = O$$

Proceeding a step further it is also observed that the angle ϕ may be so taken as to make the term $\Sigma Gxy\cos\phi$ vanish also. This last procedure is as follows: Through the origin O construct a temporary horizontal axis H-H and let y'' represent the vertical ordinate to the center of each ds segment scaled from such axis (see Fig. 27).

Then

$$y'' - x \tan \phi \quad (63)$$
$$\Sigma Gxy = \Sigma Gxy'' - \Sigma Gx^2 \tan \phi \quad (64)$$

Placing this last expression equal to zero and solving for the angle ϕ we have

$$\phi = \tan^{-1} \frac{\Sigma Gxy''}{\Sigma Gx^2} \tag{65}$$

from which equation the value of ϕ may be readily determined.

Two axes X-X and Y-Y, so located that ΣGxy vanishes, are termed conjugate axes and the operation above described is termed conjugating the redundants.

Applying the redundants X and Y along the conjugate axes above determined and locating the terminal point O at the center of gravity of the "elastic load system, "which point will be hereinafter termed the "elastic center," we may write eqs. (60), (61) and (62) as follows:

$$X = -\frac{\sum \frac{N_o n_x ds}{A} - \cos\phi \, \Sigma M_o G y}{\cos \phi^2 \, \Sigma G y^2} \tag{66}$$

$$Y = -\left[\frac{\sum \frac{N_o n_y ds}{A} + \Sigma M_o G x}{\Sigma G x^2}\right] \tag{67}$$

$$Z = -\left[\frac{\sum \frac{N_o n_z ds}{A} + \Sigma M_o G}{\Sigma G}\right] \tag{68}$$

The first term in the numerator of each of the above expressions represents the effect of the axial thrust and may be disposed of in the same manner as is done with uniform temperature effects. Neglecting this term for the present, therefore, the above expressions may be written:

$$X = \frac{\Sigma M_o G y}{\cos\phi \, \Sigma G y^2} \tag{69}$$

$$Y = -\frac{\Sigma M_o G x}{\Sigma G x^2} \tag{70}$$

$$Z = -\frac{\Sigma M_o G}{\Sigma G} \tag{71}$$

By means of the above equations the values of the redundant forces X, Y and Z may be evaluated for any given condition of loading. It will be observed, however, that this method necessitates the solution of each equation for every possible position of the dead and live loads and is therefore somewhat lengthy. A much easier and simpler method is to assume a unit load at any given point, compute the value of the term M_o for such load and solve for the redundants X, Y and Z induced by such unit load. Then by moving this unit load across the span, values of the redundants may be obtained for each position of the unit load and an influence line constructed for each of the redundant forces.

Placing a unit load at any point g (see Fig. 28) we may at once write

$$\begin{aligned} M_o &= 0 \text{ (for any point between } a \text{ and } g) \\ M_o &= -k \text{ (for any point between } g \text{ and } t) \\ \Sigma M_o G &= \Sigma_a^g G(0) + \Sigma_g^t G(-k) = -\Sigma_g^t G k \end{aligned} \tag{72}$$

Whence eqs. (69) to (71) inclusive above become

$$X_g = \frac{-\Sigma_g^t kGy}{\cos\phi\ \Sigma Gy^2} \tag{73}$$

$$Y_g = \frac{\Sigma_g^t kGx}{\Sigma Gx^2} \tag{74}$$

$$Z_g = \frac{\Sigma_g^t kG}{\Sigma G} \tag{75}$$

(The minus sign is used before the term k because it represents a moment which causes tension in the upper fibers of the residual cantilever.)

Placing a unit load at any other point m and proceeding in an exactly similar manner, we find

$$X_m = \frac{-\Sigma_m^t kGy}{\cos\phi\ \Sigma Gy^2} \tag{76}$$

$$Y_m = \frac{\Sigma_m^t kGx}{\Sigma Gx^2} \tag{77}$$

$$Z_m = \frac{\Sigma_m^t kG}{\Sigma G} \tag{78}$$

By means of the above equations it is possible to plot an influence line for the redundant reaction components X, Y and Z for any arch ring having rigid supports. With these influence lines plotted, the value of the redundants can be easily determined for any given set of load conditions and from these the stress in the rib at any point may be easily computed.

12b. Graphical Solution for Redundant Influence Diagrams.—The foregoing method of calculating the influence line ordinates for the redundants X, Y and Z involves the summation of the terms given in the numerator once for every position of the moving unit load and is therefore somewhat laborious. An easier and more rapid method may be developed graphically as follows:

Consider the given arch rib as a cantilever fixed at the left support and free at the right support (note that this is the reverse of the condition under which the redundants X, Y and Z were first developed). Load each ds segment at its center with the corresponding elastic load $G = \frac{ds}{I}$. With pole distance ΣG construct a ray diagram for these loads and an equilibrium polygon (Polygon A, Fig. 29). The intercept on this polygon by a vertical through any point (as point g) is obviously measured by the term

$$\frac{\Sigma_g^t kG}{\Sigma G} = Z_g \tag{79}$$

Polygon A is therefore the influence line for the redundant Z.

Now load the rib with the load system Gy and construct an equilibrium polygon (Polygon B, Fig. 29) with pole distance $\cos\phi\ \Sigma Gy^2$. The intercept on

this polygon by a vertical through any point (as point g) is obviously measured by the term

$$\frac{-\Sigma_g^t kGy}{\cos\phi\ \Sigma Gy^2} = X_g \tag{80}$$

Polygon B is therefore the influence Diagram for the redundant X.

In a similar manner Polygon C (Fig. 29), using the load line Gx and a pole distance $H = \Sigma Gx$ becomes the influence line for the redundant Y.

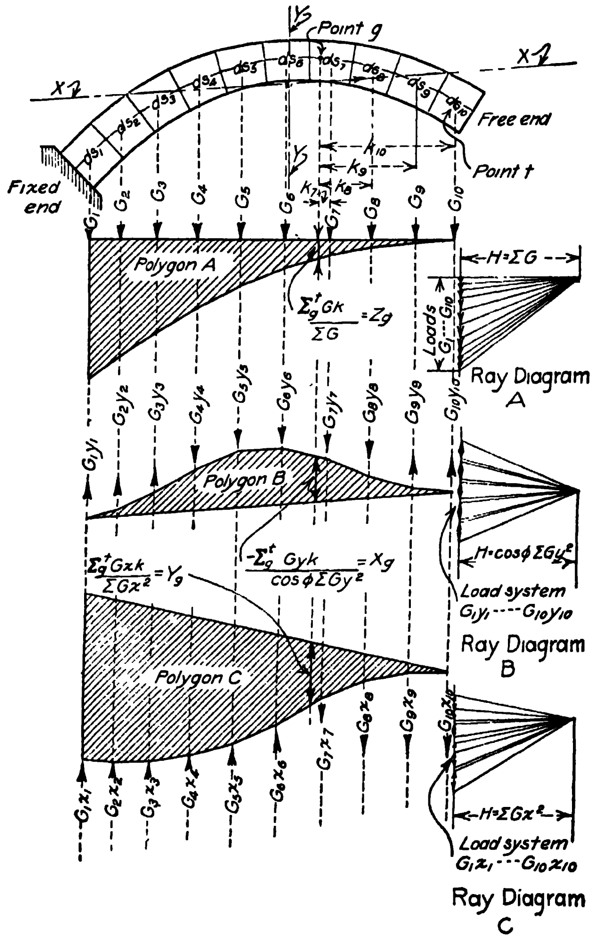


FIG. 29.

Having these influence lines plotted, the determination of stresses at any point in the rib becomes a problem involving ordinary statics. This phase of the work involves only ordinary methods of stress calculation in statically determinate structures and need not be discussed further at this point.

12c. Stresses Due to Uniform Temperature Changes.—With the terminal point of the rigid bracket located at the "elastic center" and with the

redundants X and Y properly "conjugated" the following simplifications for eqs. (51), (52) and (53) (See Case II, Art. **11**, p. 464) may be written:

$$\sum \frac{(m_x)^2 ds}{EI} = \frac{\cos \phi^2}{E} \Sigma G y^2 \tag{81}$$

$$\sum \frac{m_x m_y ds}{EI} = \frac{-\cos \phi}{E} (\Sigma G x y) = 0 \tag{82}$$

$$\sum \frac{m_x m_z ds}{EI} = \frac{-\cos \phi}{E} (\Sigma G y) = 0 \tag{83}$$

$$\sum \frac{(m_y)^2 ds}{EI} = \frac{1}{E} \Sigma G x^2 \tag{84}$$

$$\sum \frac{m_y m_z ds}{EI} = \frac{1}{E} (\Sigma G x) = 0 \tag{85}$$

$$\sum \frac{(m_z)^2 ds}{EI} = \frac{1}{E} \Sigma G \tag{86}$$

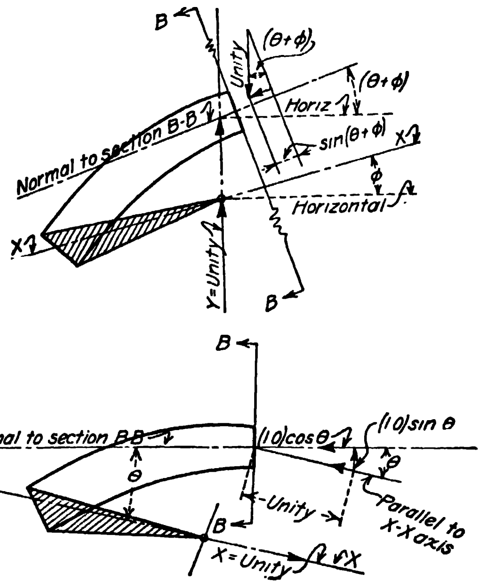


FIG. 30.

Also from Fig. 30

$$n_x = \cos \theta \tag{87}$$

$$n_y = \sin (\theta + \phi) \tag{88}$$

$$n_z = 0 \tag{89}$$

The axial fiber distortion caused by the loading X = unity acting as assumed is a compression or shortening of the rib. Therefore, the individual products $\cos \theta ctds$ will be positive for a temperature crop and negative for a temperature rise. The sign before the individual products $\sin (\theta + \phi)\ ctds$ will obviously be positive for a temperature drop when $(\theta + \phi)$ is positive and negative when $(\theta + \phi)$ is negative.

Applying these formulas

$$X_t = \frac{-E\Sigma \pm \cos \theta\ ctds}{\cos \phi^2\ \Sigma G y^2} \tag{90}$$

Similarly

$$Y_t = \frac{-E\Sigma \pm \sin(\theta + \phi)ctds}{\Sigma Gx^2} \tag{91}$$

$$Z_t = 0 \tag{92}$$

From Fig. 31

$$\Sigma \cos\theta \, ds = L' \tag{93}$$

$$\Sigma \sin(\theta + \phi) \, ds = L'' \tag{94}$$

Whence

$$X_t = \pm \left[\frac{EctL'}{\cos\phi^2 \, \Sigma Gy^2}\right] \tag{95}$$

$$Y_t = \pm \left[\frac{EctL''}{\Sigma Gx^2}\right] \tag{96}$$

Obviously the plus signs are to be used for a temperature rise and the minus signs for a temperature drop.

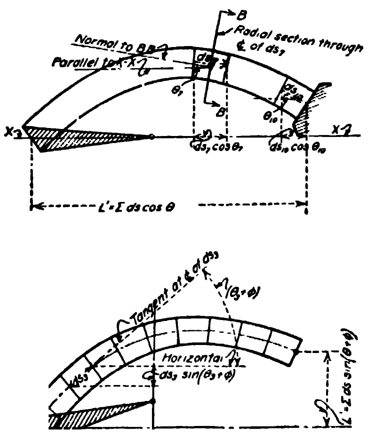


Fig. 31.

12*d*. Stresses Due to a Variable Temperature Change.—Using the simplification identities listed in Art. **12*c***, above eqs. (54), (55), (56) of Case III, p. 468, may be written

$$X_{t'} = \pm \left[\frac{ct'E \, \Sigma yds}{\cos\phi \, \Sigma Gy^2}\right] \tag{97}$$

$$Y_{t'} = \pm \frac{ct'E \, \dfrac{\Sigma xds}{h}}{\Sigma Gx^2} \tag{98}$$

$$Z_{t'} = \pm \left[\frac{ct'E \, \dfrac{\Sigma ds}{h}}{\Sigma G}\right] \tag{99}$$

These equations express the values of the redundants due to a difference in temperature between the upper and lower fibers of the arch rib. When the upper

fibers are at a higher temperature, the distortion of the residual cantilever will obviously be in the same direction as that induced by a vertical loading. Hence the redundant forces will act in the same direction as for vertical loading. When the upper fibers are at a lower temperature than that of the lower fibers, the reverse is true.

12e. Stresses Due to Rib Shortening or Axial Thrust.—In Art. 11, p. 464, the terms $\frac{n_x ds}{AE}$, $\frac{n_x n_y ds}{AE}$, etc. were dropped from the elastic equations (see eqs. (48) to (50) inclusive). While this is perfectly permissible when considering the effect of gravity loadings and of temperature, it involves large errors when considering the effect of axial distortions. We must therefore develop the expressions for the redundants due to axial thrust directly from eqs. (45) to (47) inclusive.

With the terminal point of the rigid bracket system located at the elastic center and with the redundant axes properly "conjugated"

$$\left.\begin{array}{l} \sum \frac{m_x m_y ds}{EI} \\ \sum \frac{m_x m_z ds}{EI} \\ \sum \frac{m_y m_z ds}{EI} \end{array}\right\} = 0$$

For rigid supports, and neglecting all temperature terms and terms involving the moments M_o (since the effect of the thrusts N alone is now desired), we may write eq. (45) as follows:

$$\sum \frac{N_o n_x ds}{AE} = -\left[X \sum \frac{(m_x)^2 ds}{EI} + X \sum \frac{(n_x)^2 ds}{AE} + Y \sum \frac{n_x n_y ds}{AE} + Z \sum \frac{n_x n_z ds}{AE}\right]$$

$$= -\left[X \sum \frac{(m_x)^2 ds}{EI} + \Sigma(Xn_x + Yn_y + Zn_z)\left(\frac{n_x ds}{AE}\right)\right] \tag{100}$$

$$X = \frac{-\Sigma(N_o + Xn_x + Yn_y + Zn_z)\frac{n_x ds}{AE}}{\sum \frac{(m_x)^2 ds}{EI}} \tag{101}$$

But from eq. (44)

$$N_o + Xn_x + Yn_y + Zn_z = N \tag{102}$$

Whence

$$X_N = \frac{-E \sum \frac{N n_x ds}{AE}}{\cos \phi^2 \ \Sigma G y^2} \tag{103}$$

In a similar manner

$$Y_N = \frac{E \sum \frac{N n_y ds}{AE}}{\Sigma G x^2} \tag{104}$$

$$Z_N = \frac{E \sum \frac{N n_z ds}{AE}}{\Sigma G} = 0 \text{ (since } n_z = 0) \tag{105}$$

It will be noted that these equations are in exactly the same form as those representing the effect of a uniform change in temperature if we substitute $\frac{N}{A}$ for Ect. In other words, the stresses due to the effect of the axial thrust are equal to those produced by a temperature drop of t'' degrees where

$$t'' = \frac{N}{AEc} \tag{106}$$

In this manner the axial effects may be regarded in the light of an "equivalent temperature drop" and solved from the temperature stresses by direct proportion. These stresses are generally designated by the term "rib shortening " stresses.

12f. Symmetrical Arch Ribs.—If the arch rib is symmetrical about the center line of the span, it is obvious that the "elastic center" will fall on a

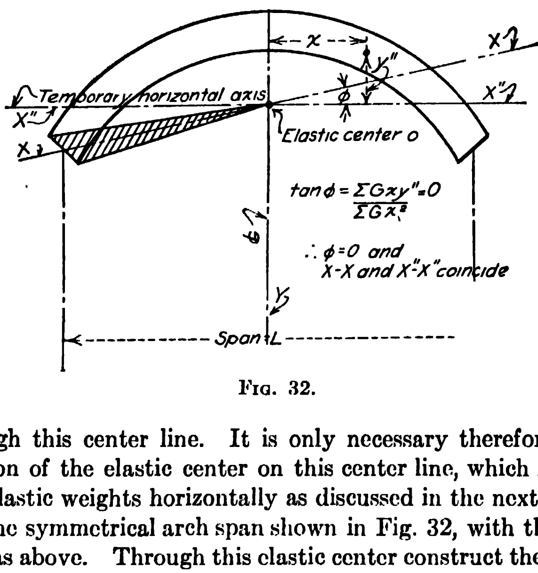


FIG. 32.

vertical through this center line. It is only necessary therefore to locate the vertical position of the elastic center on this center line, which may be done by applying the elastic weights horizontally as discussed in the next article.

Consider the symmetrical arch span shown in Fig. 32, with the elastic center O determined as above. Through this elastic center construct the vertical redundant axis $Y-Y$ and a temporary horizontal axis $X''-X''$.

The true conjugate axis $X-X$ (from eq. (65)) makes some angle ϕ with the axis $X''-X''$ such that

$$\tan \phi = \frac{\Sigma Gxy''}{\Sigma Gx^2} \tag{107}$$

But, since the arch is symmetrical, $\Sigma Gxy''$ must equal zero, whence $\tan \phi = 0$, $\phi = 0$, and the axes $X''-X''$ and $X-X$ coincide. For symmetrical arch ribs, therefore, the conjugate axes are at right angles, which simplifies the equations for determination of the redundants.

We may therefore rewrite the redundant equations in the foregoing articles to apply to symmetrical arches as follows:

Case I—*Unit Load at Any Point g* (*see eqs.* (73) *to* (75) *inclusive*).

$$X_g = -\frac{\Sigma_g^t\, kGy}{\Sigma Gy^2} \tag{108}$$

$$Y_g = \frac{\Sigma_g^t\, kGx}{\Sigma Gx^2} \tag{109}$$

$$Z_g = \frac{\Sigma_g^t\, kG}{\Sigma G} \tag{110}$$

Case II—*Uniform Change in Temperature.*

(Note that for a symmetrical rib, $L' = L$ and $L'' = 0$.)

$$X_t = \pm \left[\frac{EctL}{\Sigma Gy^2}\right] \tag{111}$$

$$Y_t \text{ and } Z_t = 0 \tag{112}$$

Case III—*Effect of Variable Temperature Change.*

$$X_{t'} = \pm \left[\frac{ct'E\ \Sigma y ds/h}{\Sigma Gy^2}\right] \tag{113}$$

$$Y_{t'} = \pm \left[\frac{ct'E\ \Sigma x ds/h}{\Sigma Gx^2}\right] \tag{114}$$

$$Z_t' = \pm \left[\frac{ct'E\ \Sigma ds/h}{\Sigma G}\right] \tag{115}$$

13. Condensed Outline of Method for Analysis of Fixed Rib Arches.—The object of this article is to present in concise form the method of analysis described in the foregoing article; utilizing formulas, the derivation of which form the subject matter of the two preceding articles.

13*a*. Analysis of Symmetrical Reinforced Concrete Arch Spans.—The following method may be used in the design of all fixed symmetrical reinforced concrete arches.

(1) Assume a crown thickness and a tentative center line curve to fit the individual conditions for the span.

(2) Divide the half arch rib, along its center line, into not less than ten equal parts and compute the dead load, including any concentrations from spandrel columns and decking, for each part.

(3) Using the above dead loads, construct an equilibrium polygon for the same, passing this polygon through the center of the rib at crown and at spring line.

(4) Adopt as a final axial curve, a compound circular curve as closely approximating this equilibrium polygon as possible.

(5) Having thus determined the most advantageous curve, sketch in the arch rib. The dimensions chosen for the rib are assumed largely as the result of experience or from comparison with like dimensions for known existing structures. In general the thickness of the rib need not be greatly increased between the crown and the quarter point for ordinary highway loadings. This does not hold true, however, for arches carrying heavy moving live loads, such as railway arches under light fills, arches carrying heavy suburban or interurban traffic, etc.

From the quarter point to the spring line or skew back, the arch rib should be gradually increased so that it will be from 2½ to 3 times the crown thickness at this latter point. Many rules and tables for arch rib dimensions have been devised and published. A discussion of a few of these has already been given (see Art. 7, p. 445). These formulas and the above may serve as a rough guide, tempered of course by a consideration of individual conditions. If the final analysis indicates an improper proportioning of the arch rib dimensions, the design is of course corrected and the analysis re-run using the new corrected dimensions.

(6) Divide the half rib thus determined into not less than ten equal divisions. The length of each division will be termed ds. Compute for each division the quantity $\frac{ds}{I}$. This quantity will be termed the elastic weight of such division and designated hereinafter by the term G. (For spans over 150 ft. the number of linear segments ds should be taken greater than 10.)

In Fig. 33 suppose the half arch axis scales 480 in. If 10 equal divisions are used, then

$$ds_1 = ds_2 = \qquad .ds_{10} = \frac{480}{10} = 48 \text{ in.}$$

Suppose that the reinforcement consists of one 1-in. square bar at the intrados and one 1-in. square bar at the extrados, and that the arch rib is 12 in. wide (see Fig. 33).

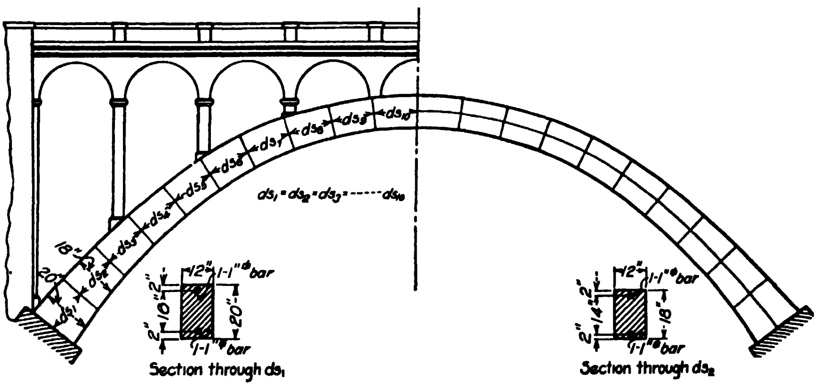


FIG. 33.

The moment of inertia for any cross-section is equal to the moment of inertia of the plain concrete section plus the moment of inertia of the steel times the term $\frac{E_s}{E_c}$. This latter term is generally taken equal to 15.

$$\therefore I_1 = I_1 \text{ (concrete)} + 15 I_1 \text{ (steel)}$$

$$I_1 \text{ (concrete)} = \frac{bh^3}{12} = \frac{(12)(20)^3}{12} = 8{,}000$$

$$15 I_1 \text{ (steel)} = 15\,(2)(8)^2 = 1{,}920$$

$$\text{Total } I_1 = 8{,}000 + 1{,}920 = 9{,}920$$

$$\therefore G_1 = \frac{ds}{I_1} = \frac{48}{9{,}920} = 0.0048$$

In a similar manner:

$$I_2 = \frac{(12)(18)^3}{12} + 15(2)(7)^2 \quad 7{,}302$$

$$G_2 \quad \frac{48}{7{,}302} = 0.0066$$

In this manner the elastic weight G for each "Voussoir" or arch block is calculated.

(7) Having the elastic weights computed, proceed now to find the center of gravity of these weights, which point is termed the "elastic center" of the arch. The method of doing this is as follows:

Since the arch is symmetrical, the center of gravity will lie on a vertical through the crown. Its vertical position may be determined in two ways, as follows:

Algebraic Method.—Select any horizontal axis M-M as shown in Fig. 34. Carefully scale the vertical distances, m, m_2, m_3, m_4, etc., from this axis to the center of each voussoir. The center of gravity is located on a horizontal line whose distance below the axis M-M is equal to

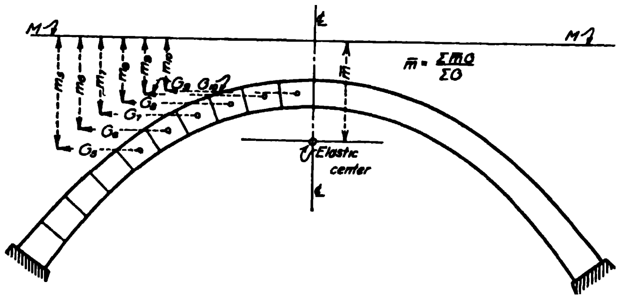


FIG. 34.

$$\frac{Gm_1 + Gm_2 + \ldots Gm_{10}}{G_1 + G_2 + \ldots G_{10}}$$

Call this distance

$$m = \frac{\Sigma Gm}{\Sigma G}$$

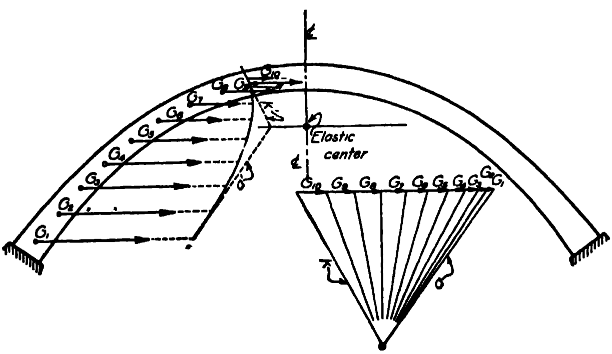


FIG. 35.

Graphic Method.—The elastic center may be determined graphically as follows (see Fig. 35).

(1) Lay off the elastic loads G on a horizontal load line.

(2) Assume each load to act horizontally and construct an equilibrium polygon for the same.

(3) Intersect the first and last rays of this polygon. This point of intersection is on the horizontal containing the required center of gravity, and the intersection of this horizontal with the vertical through the crown is the elastic center of the arch.

(8) Through the elastic center thus determined, construct two coordinate axes, X-X (horizontal) and Y-Y (vertical). Carefully scale the distance x and y for each voissoir and compute and tabulate the quantities Gx, Gy and Gx^2 and Gy^2 for the entire arch ring. Also determine the summations ΣGx, ΣGy, ΣGx^2 and ΣGy^2 (see Fig. 36).

(9) Next consider the arch as a cantilever fixed at the *left end*, as shown in Fig. 37, and construct the following three equilibrium polygons:

Polygon A with loads G and pole distance ΣG.
Polygon B with loads Gx and pole distance ΣGx^2.
Polygon C with loads Gy and pole distance ΣGy^2.

(10) Now consider the arch as a cantilever fixed at the *right end* and conceive a rigid bracket fastened to the free end and terminating at the elastic center O. Also conceive three forces, one (X) horizontal, one (Y) vertical and a moment couple (Z) acting at O as shown in Fig. 38.

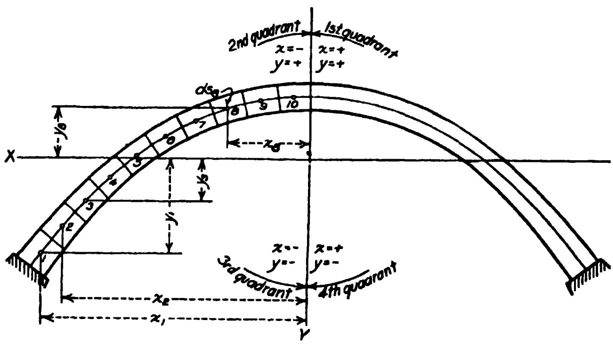


FIG. 36.

Assume the direction of these forces to be as shown. It has been proved that polygon A is the influence line for moment couple Z. Also that polygon B is the influence line for the force Y and that polygon C is the influence line for force X. From these three influence lines, the influence lines for the thrust and bending moment at any point on the arch rib may be easily drawn.

For example, suppose it is desired to draw the influence line for the bending moment at section A-A, Fig. 38. Take a section at A-A and treating the left hand portion of the structure as a free body in equilibrium (see Fig. 39), we find, by taking moments about the neutral axis A-A that

For a unit load P to the left of A-A

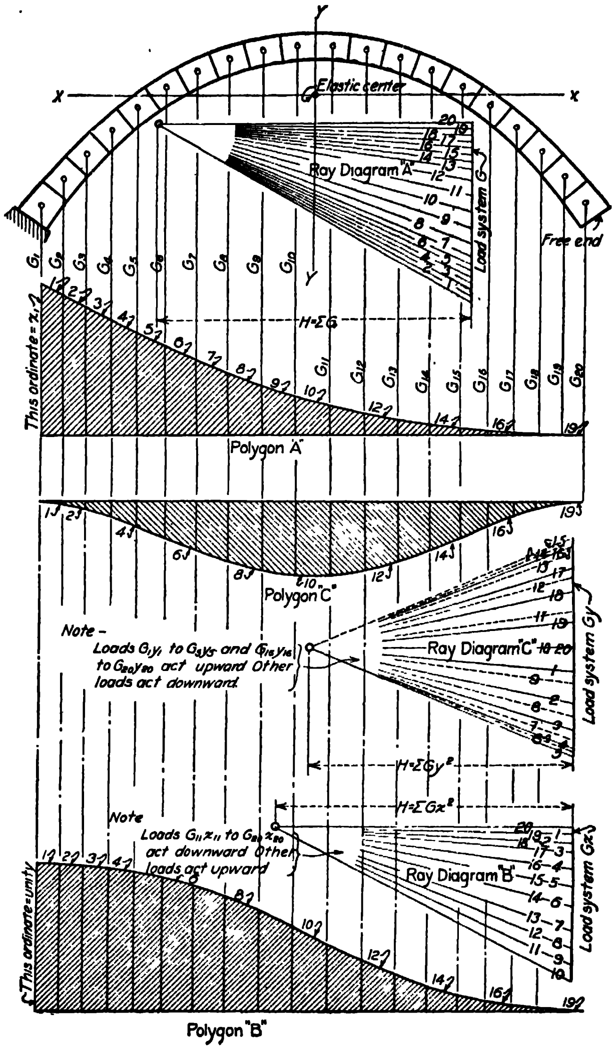
$$M_A = Z - Ya - Xb - Pr$$

For a unit load to the right of A-A

$$M_A = Z - Ya - Xb$$

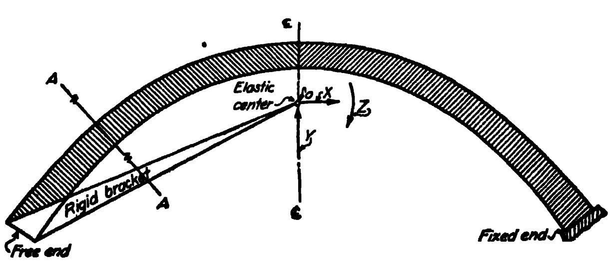
Note that the action of M_A is always assumed in such a direction as to produce compression in the top fibers of the rib. A negative value of M_A simply indicates that the action is reversed and the upper fibers are in tension.

The normal thrust at section A-A may be determined in a similar manner as follows:



Elastic center
Ray Diagram "A"
Load system G
Free end
This ordinate = x_1
$H = \Sigma G$
Polygon "A"
Polygon "C"
Note - Loads G_1y_1 to G_5y_5 and $G_{16}y_{16}$ to $G_{20}y_{20}$ act upward. Other loads act downward.
Ray Diagram "C"
Load system Gy
$H = \Sigma Gy^2$
$H = \Sigma Gx^2$
Note Loads $G_{11}x_{11}$ to $G_{20}x_{20}$ act downward. Others loads act upward
Ray Diagram "B"
Load system Gx
This ordinate = unity
Polygon "B"

FIG. 37.



Elastic center
Rigid bracket
Free end
Fixed end

FIG. 38.

From Fig. 40 it is seen that:

For a unit load P to the left of section A-A

$$T_A = X \cos\theta + Y \sin\theta - P \sin\theta$$

For a unit load to the right of section A-A

$$T_A = X \cos\theta + Y \sin\theta$$

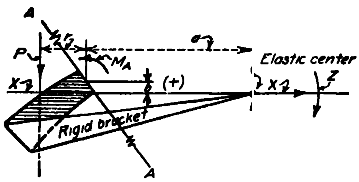


FIG. 39.

In this manner the moment and thrust at any point, due to unit loading, may be computed and the influence lines for the same plotted.

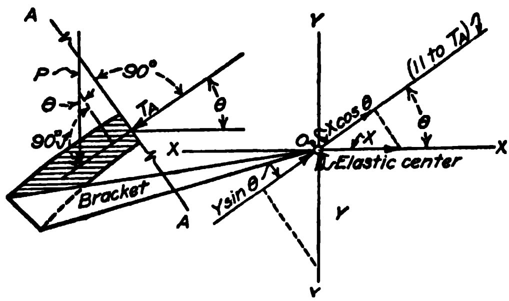


FIG. 40.

The data should be collected in tabular form as follows:

INFLUENCE LINE DATA—SECTION A-A

Unit load at point	Moment					Thrust			
	Z	Ya	Xb	Pr	M_A	$P \sin\theta$	$X \cos\theta$	$Y \sin\theta$	T_A
2									
3									
4									
5									
etc.									

Note, that for any section, x, y and θ are constant, while Z, Y and X are variable quantities and are obtained from polygons A, B and C. Note also that P = unity.

Temperature Stresses.—A rise of t deg. in temperature induces a reaction X, but for symmetrical spans induces no reaction Y or moment couple Z.

For a rise in temperature of t deg.

$$X_t = \frac{EctL}{\Sigma Gy^2} = \frac{12tL}{\Sigma Gy^2} \text{ (For concrete arches)}$$

where L = total span in inches.

The moment at any section as A-A is $X_t b$, and the thrust is $X_t \cos\theta$ (see Fig. 41). For a drop in temperature, the direction of the reaction X_t is reversed.

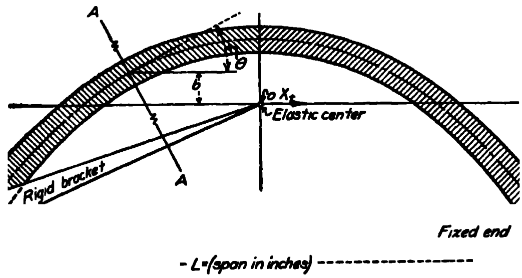


FIG. 41.

Stresses Due to Axial Compression or "Rib Shortening."—It is general practice to compute and consider the stresses resulting from the axial compression due to dead load only the axial compression under live load being small and well taken care of by the factor of safety. Using the dead load compressive stresses the method of procedure is as follows:

(1) Compute the dead load thrust N at various points in the arch ring. These are the thrusts N (or T) on the arch rib, not the values N_0 for the residual cantilever (see Art. **12e**, p. 475).

(2) Using the average value of N found as above, solve for the equivalent temperature drop t'', where

$$t'' = \frac{N}{AEc}$$

(3) Using this value the rib shortening stresses may be quickly obtained from the temperature stress by the direct proportion

$$\frac{t''}{t}$$

Note that these stresses are always of the same sign as those produced by a *drop* in temperature.

Stresses Due to a Variable Change in Temperature.—The distribution of temperature throughout the interior of the arch rib is so problematical that these stresses are rarely calculated.

Calculation of Stresses in the Material.—The influence lines for the moment and thrust at the crown, spring line and any other points which may be deemed necessary having been plotted, the dead and live load moments and thrusts are

computed and combined with the temperature effects. Care should be taken to see that for each moment, the thrust that occurs simultaneously be used and not some other thrust caused by a different system of loading.

Care must also be exercised to try every combination of loadings in order that a true maximum condition will be obtained.

After the combinations are made and tabulated, the stresses in the material are calculated in the usual manner.

13b. Analysis of Unsymmetrical Reinforced Concrete Arch Spans.—Where the arch axis is not symmetrical the above method is slightly modified—the following procedure being followed:

(1) Assume tentative dimensions for the arch rib and a tentative axial curve to fit the particular conditions of the problem.

(2) Divide the entire rib along its center line into not less than 20 equal parts, compute the dead loads, construct a dead load equilibrium polygon and adopt an axial curve to fit as closely as possible to this dead load equilibrium polygon. (The procedure thus far is practically identical with that outlined under items 1, 2, 3 and 4 for symmetrical arches.)

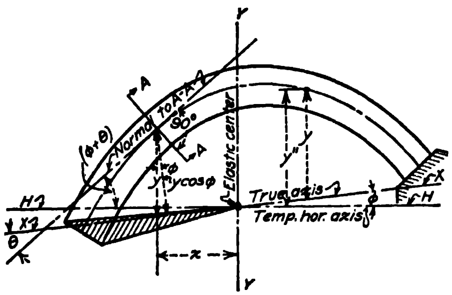


Fig. 42.

(3) Sketch the arch rib in as before.

(4) Divide the entire rib into not less than 20 equal divisions, calculate the elastic loads G and determine the position of the true "elastic center" exactly as in the case of the symmetrical arch. In this case it will be necessary to determine the vertical as well as the horizontal containing the elastic center. This may be done by applying the elastic weights vertically as well as horizontally, either algebraically or graphically, as described under the preceding article.

(5) Through the elastic center construct the true Y-Y axis (vertical) and a temporary horizontal axis H-H. The true redundant axis X-X "conjugate" to Y-Y will make an angle ϕ with H-H wherein

$$\tan \phi = \frac{\Sigma Gxy''}{\Sigma Gx^2} \qquad \text{(see Fig. 42)}$$

(6) Calculate the terms Gxy'', Gx^2 and ΣGx^2 and determine the value of ϕ.

(7) Plot the true axis X-X and scale or calculate the values of y.

(8) Compute for each voussoir the terms Gx, Gy, Gx^2, Gy^2 and tabulate the summations ΣG, ΣGx, ΣGy, ΣGx^2 and Gy^2.

(9) Next consider the arch as a cantilever fixed at the left support and construct the following three equilibrium polygons:

Polygon A with loads G and pole distance ΣG

Polygon B with loads Gx and pole distance ΣGx^2

Polygon C with loads Gy and pole distance $\Sigma Gy^2 \cos \phi$

These are the three redundant influence lines and the analysis may proceed from this point exactly as in the case of the symmetrical span.

Temperature Stresses (Uniform Change).—Equations (95) and (96), p. 474, may be employed to determine the value of the redundants X_t and Y_t. Having these values, the moment at any section A-A (Fig. 42) is clearly

$$M_{t_A} = \pm(X_t y \cos \phi + Y_t x)$$

The thrust at section A-A is clearly

$$T_{t_A} = \pm(X_t \cos \theta + Y_t \sin (\theta + \phi).$$

Stresses Due to Variable Temperature Effects.—These may be determined from eqs. (97) to (99) inclusive, p. 474, thus determining the redundants $X_{t'}$, $Y_{t'}$ and $Z_{t'}$ which may be applied in a manner analogous to the above. It is not customary to consider the effect of a variable temperature change for ordinary arch spans —this effect being somewhat problematical due to the lack of accurate data concerning the actual distribution of internal temperature throughout the interior of the rib.

MULTIPLE SPAN ARCHES ON ELASTIC PIERS[1]

14. General Considerations.—The general elastic equations for arch spans under any loading and for any support condition whatsoever are written for rib arches on p. 467 (eqs. (45) to (47) inclusive).

A consideration of the elastic yielding of the arch supports is necessitated in practice only in the case of rib arch structures, as a general rule, the employment of the fixed framed arch type being generally limited to long single spans sprung from massive and (to all practical intents and purposes) unyielding abutments.

In view of the above, therefore, the present discussion will be restricted to rib arch structures only. Should it ever be necessary to consider the case of yielding supports in connection with framed arches, a treatment entirely analogous to that hereinafter developed for the rib arch may be employed for deriving the formulas for the redundant reaction components.

It has been shown on p. 464, Art. **11**, and p. 475, Art. **12***e*, that the terms $\sum\frac{(n_x)^2ds}{AE}$, $\sum\frac{n_xn_yds}{AE}$, $\sum\frac{(n_y)^2ds}{AE}$, etc., represent the effect of the axial distortion or shortening of the rib and that such effect commonly known as "rib shortening" may be evaluated by treating the same as an "equivalent temperature drop." Omitting these terms therefore, and substituting for the terms $\sum\frac{M_om_xds}{EI}$, $\sum\frac{(m_x)^2ds}{EI}$, etc., in terms of the elastic weights G (as was done in Art. **12***a*, p. 468), we may write the general elastic eqs. (45) to (47) inclusive as follows:

Gravity Loadings Only (*Temperature Effects Neglected*)

$$E(\Delta_x - \Sigma r_x\Delta_r) = -\cos\phi\Sigma M_oGy + \cos^2\phi X\Sigma Gy^2 - \cos\phi Y\Sigma Gxy - \cos\phi Z\Sigma Gy \tag{116}$$

$$E(\Delta_y - \Sigma r_y\Delta_r) = \Sigma M_oGx - \cos\phi X\Sigma Gxy + Y\Sigma Gx^2 + Z\Sigma Gx \tag{117}$$

$$E(\Delta_z - \Sigma r_z\Delta_r) = \Sigma M_oG - \cos\phi X\Sigma Gy + Y\Sigma Gx + Z\Sigma G \tag{118}$$

Uniform Change in Temperature.

$$E(\Delta_{tx} - \Sigma r_x\Delta_{tr}) = E\Sigma n_xctds + \cos^2\phi X\Sigma Gy^2 - \cos\phi Y\Sigma Gxy - \cos\phi Z\Sigma Gy \tag{119}$$

$$E(\Delta_{ty} - \Sigma r_y\Delta_{tr}) = E\Sigma n_yctds - \cos\phi X\Sigma Gxy + Y\Sigma Gx^2 + Z\Sigma Gx \tag{120}$$

$$E(\Delta_{tz} - \Sigma r_z\Delta_{tr}) = E\Sigma n_zctds - \cos\phi X\Sigma Gy + Y\Sigma Gx + Z\Sigma G \tag{121}$$

From these general expressions a method of analysis for multiple span rib arches on elastic piers may now be developed.

15. Two Span Arch with Elastic Center Pier.—Consider the two-span arch shown in Fig. 43, the same being supported on rigid abutments at either end, but on a central pier which is elastic or yielding under load. Any load over the arch rib A_2 will be transmitted to the abutment at the right and to the elastic pier P_1 at the left. This pier distorting under load will in turn throw certain stresses into rib A_1 so that both arch ribs and also the pier are stressed a certain amount for any load from point s to point t over the entire length of both spans.

[1] The consideration of elastic yielding of supports in arch design will not be necessary for ordinary structures. For special construction involving exceptionally high and slender piers and for research and experimental work the theory demonstrated in this chapter will prove of value.

By means of the section *B-B* the above structure may be divided into two elastic systems as shown in Fig. 44 as follows:

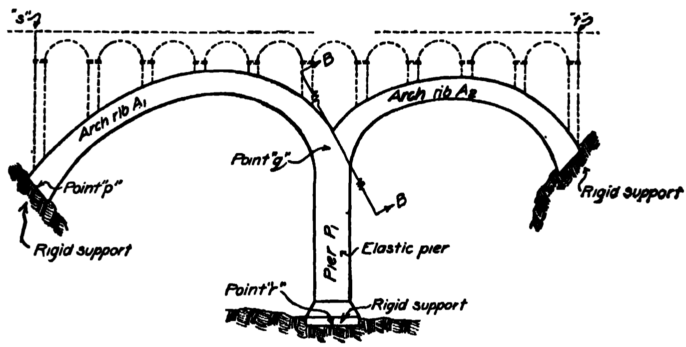
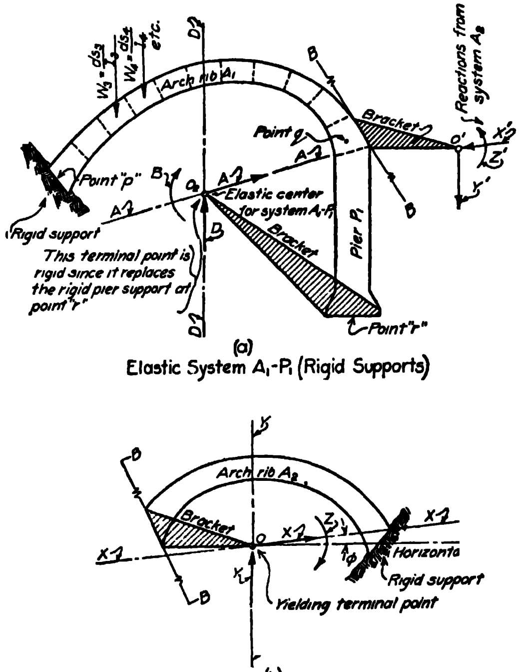


Fig. 43.

(1) The elastic system A_2 resolved into a residual cantilever as shown (Fig. 44*b*). This system is rigidly supported at the right abutment, but the terminal



(a)
Elastic System A_1-P_1 (Rigid Supports)

(b)
Elastic System A_2 (Yielding Left Support)

Fig. 44.

point *o*, of the bracket which replaces the left support, is not fixed as has been the case heretofore, but yields an amount Δ_x, Δ_y and Δ_z under the action of the redundants *X*, *Y* and *Z* in combination with the external loading.

(2) The elastic system A_1-P_1. This may be treated as an arch rigidly supported at the left abutment and also at the bracket terminal point o_2. This second elastic system is exactly like any other arch rib sprung from rigid supports except that the pier P_1 is considered as part of the arch ring. It must be observed, however, that for any load over any portion of either span there will be in addition to the three principal redundants A, B and D (acting at point o_2) three other forces, viz., X', Y' and Z' as yet unknown. These forces may be considered as applied at the end of a second rigid bracket attached to the rib A_1-P_1 along the section B-B and terminating at point o'. These last three forces obviously represent the action of the arch rib A_2 on the elastic system A_1-P_1. Point o' must therefore coincide with point o, the terminal point of this same bracket when considered as applied to the arch A_2. The forces X', Y' and Z' in Fig. 44*a* (for any given loading) must be equal and opposite to the corresponding forces X, Y and Z of Fig. 44*b* (action and reaction are equal and opposite).

It should now be clear that, if for any given loading the redundant forces X, Y and Z can be determined for the arch A_2, the elastic system A_1-P_1 may then be readily analyzed by the methods discussed in the sections giving the development of general elastic equations and influence lines for rib arches, since this system is simply an arch supported on rigid foundations except for the three additional unknown forces, $X' = (-X)$, $Y' = (-Y)$ and $Z' = (-Z)$.

Let us first, therefore, consider the arch system A_2 developing the residual cantilever and inserting the three redundant forces as shown in Fig. 44*b*.

For a unit load at any point, therefore, we may write: $\Sigma r\Delta = O$, since the left support (which is the only support for this residual cantilever) is fixed and rigid.

Now considering the elastic system A_1-P_1

Let δ_{xx} = the displacement of point o measured in the direction assumed for the redundant X due solely to the loading X = unity.

δ_{yx} and δ_{zx} = the displacement of point o measured in the same direction but due solely to the loadings Y = unity and Z = unity respectively.

δ_{xy}, δ_{yy} and δ_{zy} = similar displacements, but measured in the direction assumed for the redundant Y.

δ_{xz}, δ_{yz} and δ_{zz} = similar displacements, but measured in the direction assumed for the redundant Z.

Also let:

δ_{ox}, δ_{oy} and δ_{oz} = the displacements of the terminal point o (in the respective directions X, Y and Z) caused by a unit load at any point on the elastic system A_1-P_1.

It is now apparent that

$$\Delta_x = X\delta_{xx} + Y\delta_{yx} + Z\delta_{zx} + \Sigma F\delta_{ox}$$
$$\Delta_y = X\delta_{xy} + Y\delta_{yy} + Z\delta_{zy} + \Sigma F\delta_{oy}$$
$$\Delta_z = X\delta_{xz} + Y\delta_{yz} + Z\delta_{zz} + \Sigma F\delta_{oz}$$

where ΣF represents the summation of loads on the elastic system A_1-P_1.

Equations (116) to (118) inclusive may now be written:

$$-\cos\phi\Sigma M_oGy - E\Sigma F\delta_{ox} + X[\cos^2\phi\Sigma Gy^2 - E\delta_{xx}] - Y[\cos\phi\Sigma Gxy + E\delta_{yx}] - Z[\cos\phi\Sigma Gy + E\delta_{zx}] = 0 \quad (122)$$

$$\Sigma M_oGx - E\Sigma F\delta_{oy} - X[\cos\phi\Sigma Gxy + E\delta_{xy}] + Y[\Sigma Gx^2 - E\delta_{yy}] + Z[\Sigma Gx - E\delta_{zy}] = 0 \quad (123)$$

$$\Sigma M_oG - E\Sigma F\delta_{oz} - X[\cos\phi\Sigma Gy + E\delta_{xz}] + Y[\Sigma Gx - E\delta_{yz}] + Z[\Sigma G - E\delta_{zz}] = 0 \quad (124)$$

To evaluate the terms δ_{ox}, δ_{xx}, δ_{zx}, etc., we may now consider the elastic system A_1-P_1 under the successive action of the various auxiliary unit loads which cause such displacements as follows:

Consider first the evaluation of the term δ_{zz}. This is, obviously, the angular movement of the terminal point o' (Fig. 44a) caused by the application of a unit moment couple, $Z = 1.0$ or $Z' = -1.0$, at this point o' on the elastic system A_1-P_1. The forces active are the unit couple $Z' = -1.0$ and the three redundant forces A, B and D induced by the same at point o_2, this point o_2 being taken as the elastic center of the system A_1-P_1.

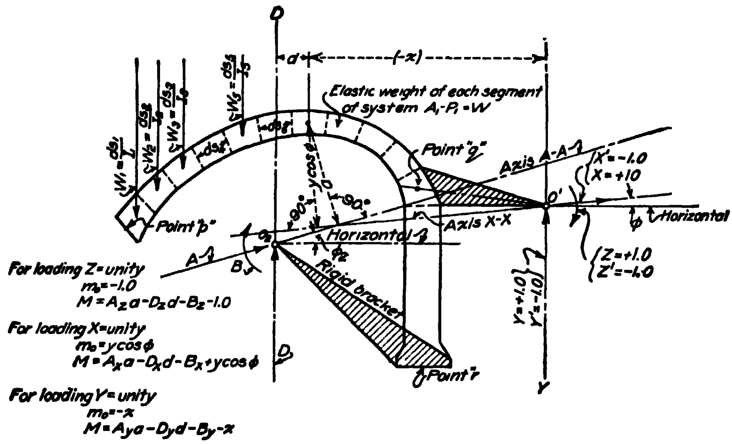


FIG. 45.

From the considerations which form the subject matter of the first chapter in this section, we may, at once, write

$$E\delta_{zz} = \sum_{\text{point } p}^{\text{point } q} Mm_oW \quad (125)$$

where

M = moment on the *arch rib* A_1-P_1 at any point due to the loading $Z = 1.0$ (or $Z' = -1.0$).

m_o = corresponding moment on the residual cantilever.

W represents the elastic weight of any segment of the system A_1-P_1.

From a consideration of Fig. 45.

$$m_o = -1.0 \quad (126)$$

$$M = A_za - D_zd - B_z - 1.0 \quad (127)$$

If the angle ϕ_2 is so chosen that the two axes A-A and D-D are conjugate axes,

$$A_z = \frac{\Sigma_p^q Wa}{\Sigma_p^r Wa^2} \tag{128}$$

$$D_z = \frac{-\Sigma_p^q Wd}{\Sigma_p^r Wd^2} \tag{129}$$

$$B_z = \frac{-\Sigma_p^q W}{\Sigma_p^r W} \tag{130}$$

Therefore

$$E\delta_{zz} = -\left[\frac{\Sigma_p^q Wa}{\Sigma_p^r Wa^2}\right]\Sigma_p^q Wa - \left[\frac{\Sigma_p^q Wd}{\Sigma_p^r Wd^2}\right]\Sigma_p^q Wd - \left[\frac{\Sigma_p^q W}{\Sigma_p^r W}\right]\Sigma_p^q W + \Sigma_p^q W \tag{131}$$

Let

$$\bar{d} = \frac{\Sigma_p^q Wd}{\Sigma_p^r W} \text{ and } \bar{a} = \frac{\Sigma_p^q Wa}{\Sigma_p^q W} \tag{132}$$

We may then write:

$$E\delta_{zz} = -\Sigma_p^q W\left[\left(\frac{\Sigma_p^q Wa}{\Sigma_p^r Wa^2}\right)\bar{a} + \left(\frac{\Sigma_p^q Wd}{\Sigma_p^r Wd^2}\right)\bar{d} + \left(\frac{\Sigma_p^q W}{\Sigma_p^r W}\right) - 1.0\right] \tag{133}$$

The term inside the brackets may be evaluated from a consideration of the elastic system A_1-P_1 alone (that is to say, this term is independent of the arch system A_2). We may evaluate this term therefore and write

$$E\delta_{zz} = -C_o\Sigma_p^q W \tag{134}$$

where C_o represents the expression inside the brackets in eq. (133).

In an exactly similar manner

$$E\delta_{zx} = +C_o\Sigma_p^q W(y \cos \phi) \tag{135}$$

$$E\delta_{zy} = -C_o\Sigma_p^q Wx \tag{136}$$

$$E\delta_{xx} = -C_o\Sigma_p^q W(y \cos \phi)^2 \tag{137}$$

$$E\delta_{yy} = -C_o\Sigma_p^q W(x)^2 \tag{138}$$

$$E\delta_{xy} = +C_o\Sigma_p^q Wxy \cos \phi \tag{139}$$

The terms δ_{ox}, δ_{oz}, etc.; are displacements at point o' caused by a unit gravity loading applied at any point on the elastic system A_1-P_1.

If we let m represent the moment at any point in the residual cantilevers of the system A_1-P_1 caused by such unit load, then clearly

$$E\delta_{oz} = -C_o\Sigma_p^q Wm \tag{140}$$

$$E\delta_{ox} = +C_o\Sigma_p^q W(y \cos \phi)m \tag{141}$$

$$E\delta_{oy} = -C_o\Sigma_p^q Wxm \tag{142}$$

It is also clear that

$$E\Sigma F\delta_{oz} = -C_o\Sigma_p^q W(F_1m_1 + F_2m_2 + \ldots \text{ etc}) = -C_o\Sigma_p^q WM_o,$$

where $M_o = F_1m_1 + F_2m_2 +$ etc. for each load acting on the system A_1-P_1. With these substitutions, we may write eqs. (122) to (124) inclusive as follows:

$$-\cos\phi\,[\Sigma M_oGy + C_o\Sigma_p^q M_oWy] + X\cos^2\phi[\Sigma Gy^2 + C_o\Sigma_p^q Wy^2] \\ -Y\cos\phi\,[\Sigma Gxy + C_o\Sigma_p^q Wxy] - Z\cos\phi[\Sigma Gy + C_o\Sigma_p^q Wy] = 0 \quad (143)$$

$$[\Sigma M_oGx + C_o\Sigma_p^q M_oWx] - X\cos\phi\,[\Sigma Gxy + C_o\Sigma_p^q Wxy] \\ + Y[\Sigma Gx^2 + C_o\Sigma_p^q Wx^2] + Z[\Sigma Gx + C_o\Sigma_p^q Wx] = 0 \quad (144)$$

$$[\Sigma M_oG + C_o\Sigma_p^q M_oW] - X\cos\phi\,[\Sigma G_y + C_o\Sigma_p^q Wy] + Y[\Sigma Gx + C_o\Sigma_p^q Wx] \\ + Z[\Sigma G + C_o\Sigma_p^q W] = 0 \quad (145)$$

It must now be clear that the effect of the elastic supporting system A_1-P_1 may be represented by the substitution of an elastic system $\Sigma_p^q C_oW$ as shown in Fig. 46. This system is composed of the elastic weights of the arch system A_1-P_1 included between points p and q, each elastic weight acting as its

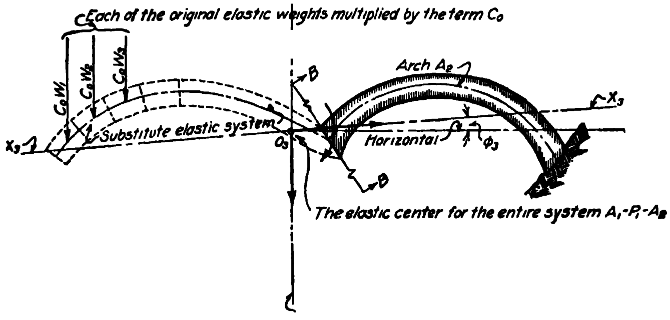


FIG. 46.

original gravity center, but having a value equal to its original value multiplied by the constant C_o.

If this substitute system is constructed and a new point O_3 located at the elastic center of the system $(\Sigma G + \Sigma_p^q C_oW)$, and if the new axes X_3-X_3 and Y_3-Y_3 are conjugated for this new elastic system, we may write:

$$X_3 = \left[\frac{\Sigma M_oGy + C_o\Sigma_p^q M_oWy}{\cos\phi_3\,[\Sigma Gy^2 + C_o\,\Sigma_p^q Wy^2]}\right] \quad (146)$$

$$Y_3 = -\left[\frac{\Sigma M_oGx + C_o\Sigma_p^q M_oWx}{\Sigma Gx^2 + C_o\Sigma_p^q Wx^2}\right] \quad (147)$$

$$Z_3 = -\left[\frac{\Sigma M_oG + C_o\Sigma_p^q M_oW}{\Sigma G + C_o\Sigma_p^q W}\right] \quad (148)$$

The coordinates x and y in the above equations refer, of course, to the new axes X_3-X_3 and Y_3-Y_3.

From these equations influence lines may be plotted for the new redundant forces X_3, Y_3 and Z_3 applied at point O_3.

For a unit load at any point on the arch span A_2 (as, for example, point g, Fig. 47)

$$\Sigma M_o Gy = \sum_{\text{point } g}^{\text{point } t}(Gy)k \tag{149}$$

$$C_o \sum_p^q M_o Wy = 0 \tag{150}$$

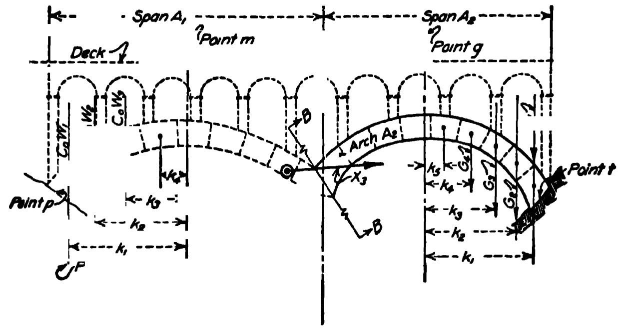


Fig. 47.

For a unit load at any point on the arch span A_1(as for example point m)

$$\Sigma M_o Gy = 0 \tag{151}$$

$$C_o \sum_p^q M_o Wy = \sum_{\text{point } m}^{\text{point } p}(C_o Wy)k \tag{152}$$

The influence line for the redundant X_3 may therefore be determined by considering the arch system as a double cantilever loaded with the elastic load system

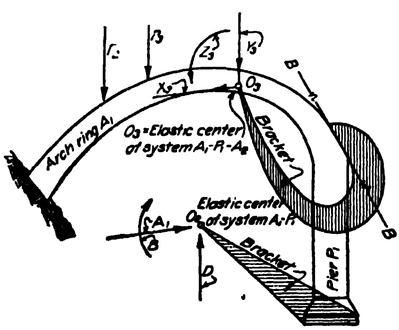


Fig. 48.

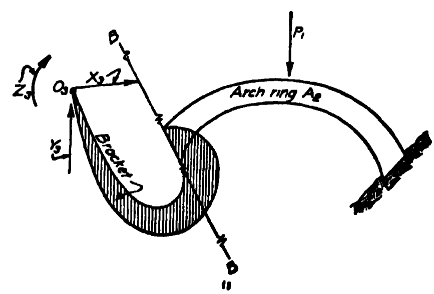


Fig. 49

Gy on the right, and $C_o Wy$ on the left. An equilibrium polygon, for this load system with pole distance equal to the term, $\cos \phi_3 \, [\Sigma Gy^2 + C_o \, \Sigma_p^q Wy^2]$ is obviously the influence diagram for this redundant.

In a like manner the other influence lines are readily constructed. Having the influence lines for the redundants X_3, Y_3 and Z_3, the stresses in either arch ring, and also in pier P_1 may be obtained by analyzing each system separately. For example, suppose there are two loads P_2 and P_3 on the span A_1 and one load P_1 on the span A_2. From the above influence lines, values of X_3, Y_3 and Z_3 for this loading

may be at once determined. Having these values, the system A_1-P_1 becomes a cantilever under the action of five known forces, P_2, P_3, X_3, Y_3 and Z_3 and three unknown redundant forces A, B and D applied at O_2, the elastic center of the system A_1-P_1 (see Fig. 48). This system can therefore be analyzed by the methods ordinarily employed for fixed single-span arches on rigid supports.

The system A_2 is clearly a cantilever under the action of four known forces X_3, Y_3, Z_3 and P_1 and can be analyzed by statics (see Fig. 49).

In the above manner stresses at any point in the system may be determined for any load condition.

16. Series of Three Spans on Elastic Piers.—If the structure shown in Fig. 43 were to be supported at the right, on an elastic pier and a third arch

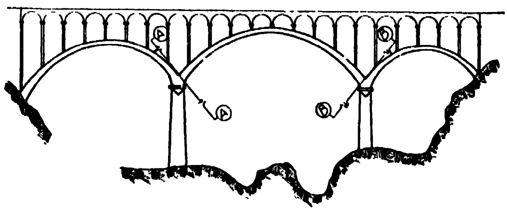


FIG. 50.

span, as shown in Fig. 50, the terms $\Sigma r_x\Delta_r$, $\Sigma r_y\Delta_r$ and $\Sigma r_z\Delta_r$ of eqs. (116) (117) and (118) would no longer equal zero, and would have to be evaluated.

This may be done as follows: Divide the structure into three elastic systems:

$$\left.\begin{array}{l}\text{System } A_1\text{-}P_1\\ \text{System } A_2\\ \text{System } A_3\text{-}P_2\end{array}\right\}\text{(see Fig. 51)}$$

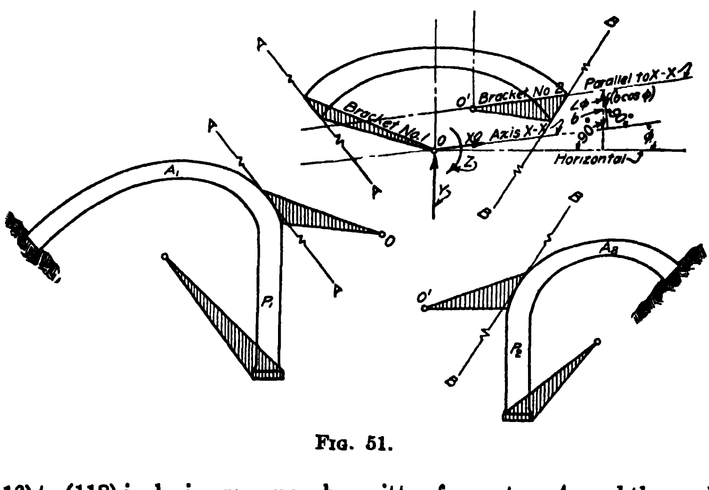


FIG. 51.

Eqs. (116) to (118) inclusive may now be written for system A_2 and the evaluation of the terms $\Sigma r\Delta$ developed in the following manner:

Assume the three redundants X, Y and Z acting on the arch A_2, the support from the system A_1-P_1 being replaced by bracket No. 1 and the support from

system A_3-P_2 replaced by bracket No. 2. Let o and o' represent the assumed terminal points on these brackets. Also let

r_{xx} = the component of the reaction at point o' caused by the auxiliary loading X = unity and measured in the direction X-X.

r_{xy} = the component of this reaction measured in the direction Y-Y.

r_{xz} = the reaction moment at this point due to this same loading.

Δ_{rx}, Δ_{ry} and Δ_{rz} = components of the displacement of the support o' measured along the directions assumed for the redundant forces X, Y and Z respectively.

It is apparent, therefore, that

$$\Sigma r_x \Delta_r = r_{xx}\Delta_{rx} + r_{xy}\Delta_{ry} + r_{xz}\Delta_{rz} \tag{153}$$

Now from the figure (Fig. 51)

$$r_{xx} = \text{unity} \tag{154}$$

$$r_{xy} = \text{zero (for conjugated axes)} \tag{155}$$

$$r_{xz} = b \cos \phi \tag{156}$$

Also

$$\Delta_{rx} = X\delta'_{xx} + Y\delta'_{yx} + Z\delta'_{zx} + \Sigma F\delta'_{ox} \tag{157}$$

$$\Delta_{rz} = X\delta'_{xz} + Y\delta'_{yz} + Z\delta'_{zz} + \Sigma F\delta'_{oz} \tag{158}$$

The accent marks above are used to distinguish the terms, δ'_{xx}, δ'_{xy}, etc., which are displacements of the system A_3-P_2 under certain auxiliary unit loadings from the corresponding terms δ_{xx}, δ_{xy}, etc., which represent displacements of the system A_1-P_1 as described in the preceding article.

If the point o is so chosen that $b = o$ we may write

$$\Sigma r_x \Delta_r = X\delta'_{xx} + Y\delta'_{yx} + Z\delta'_{zx} + \Sigma F\delta'_{ox} \tag{159}$$

Substituting back in eq. (116) and also substituting for Δ_x as in eq. (122), we may write

$$-\cos\phi\,\Sigma M_oGy - E\,\Sigma F\delta_{ox} - E\,\Sigma F\delta'_{ox} + X[(\cos\phi)^2\Sigma Gy^2 - E(\delta_{xx} + \delta'_{xx})]$$
$$- Y\cos\phi[\Sigma Gxy + E(\delta_{yx} + \delta'_{yx})] - Z[\cos\phi\,\Sigma Gy + E(\delta_{zx} + \delta'_{zx})] = 0 \tag{160}$$

In an exactly similar manner the other two elastic equations may be developed as follows:

$$\Sigma M_oGx - E\,\Sigma F\delta_{oy} - E\,\Sigma F\delta'_{oy} - X[\cos\phi\,\Sigma Gxy + E(\delta_{xy} + \delta'_{xy})] + Y[\Sigma Gx^2$$
$$- E(\delta_{yy} + \delta'_{yy})] + Z[\Sigma Gx - E(\delta_{zy} + \delta'_{zy})] = 0 \tag{161}$$

$$\Sigma M_oG - E\,\Sigma F\delta_{oz} - E\,\Sigma F\delta'_{oz} - X[\cos\phi\,\Sigma Gy + E(\delta_{xz} + \delta'_{xz})] + Y[\Sigma Gx -$$
$$E(\delta_{yz} + \delta'_{yz})] + Z[\Sigma G - E(\delta_{zz} + \delta'_{zz})] = 0 \tag{162}$$

It is at once seen that this arch system may be readily analyzed by substituting an equivalent elastic system, ΣC_oW, to replace A_1-P_1 on the right and another system, $\Sigma C_o'W_r'$ to replace A_3-P_2 on the left.

The entire procedure is very much the same as that already outlined for the two-span arch system. Briefly outlined, the steps are as follows:

(1) Let it be required to analyze the three-span arch system shown in Fig. 50, the end abutments being rigid and the center piers elastic.

(2) By means of the sections A-A and B-B divide this arch system into the central arch and the flanking elastic supporting systems A_1-P_1 and A_3-P_2 (see Fig. 52).

(3) Treating each of the flanking arch systems as a single fixed arch span with rigid supports, determine the constants C_o and C_o' (as outlined for C_o in the preceding article (see Fig. 52).

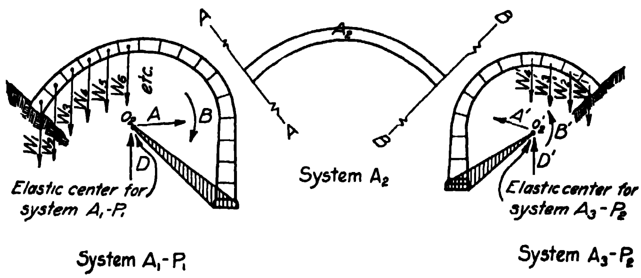


Fig. 52.

(4) Having the values of C_o and C_o', determine the substitute elastic systems ΣC_oW and $\Sigma C_o'W'$ (as shown in Fig. 53) and determine the true elastic center o for the entire system A_1-P_1-A_2-P_2-A_3.

(5) Determine the value of ϕ such that the axes XX and YY are conjugate axes for the entire elastic system.

(6) The redundant forces X, Y and Z for a load at any point on the span may now be determined from formulas analogous to eqs. (146) to (148) inclusive.

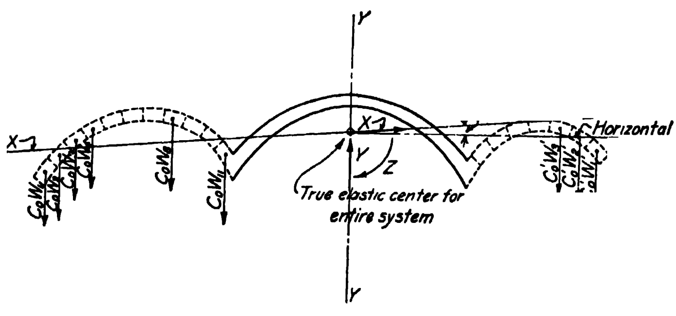


Fig. 53.

(7) With these redundants evaluated, the stresses in any portion of either pier or any arch rib of the system are easily obtained. For example, under a single load P on the span A_1 the values of X, Y and Z are first determined as shown in Fig. 54.

The system A_1-P_1 is clearly a fixed arch under the action of the four loads P, X_p, Y_p and Z_p and may be readily analyzed as a fixed arch on rigid supports. The system A_2 is a cantilever under the action of the three forces X_p, Y_p and Z_p applied at o, and is even more easily susceptible of analysis. The system A_3-P_2 is an arch system similar to A_1-P_1 except that there are in this case only three loads, X_p, Y_p, Z_p.

For a load on either of the other spans the procedure is exactly the same. The load P is obviously considered only for the span in which it acts, its effect on the other spans being measured by the values of X_p, Y_p and Z_p.

In the above manner the influence line for the thrust moment or shear may be developed for any point on either arch rib or either pier.

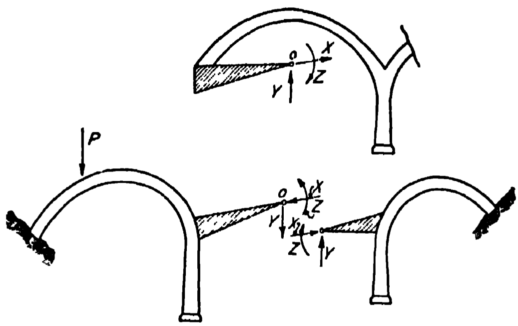


FIG. 54.

17. Temperature Effects.—If it be desired to consider the effect of the elastic yielding of the supports on temperature stresses, an entirely analogous procedure (based upon the simplification of eqs. (119) to (121) inclusive instead of (116) to (118) inclusive as before) may be readily worked out.

If all of the spans are of equal length and rise, the results will obviously be identical with those obtained by considering the piers as rigid, since the temperature thrusts in this case will exactly balance and there will be no movement of the piers.

Unless there be a marked change in span length, the error resulting from considering all pier supports as rigid is exceedingly small, and in general the consideration of elastic pier yielding for temperature effects may be regarded as a needless refinement, especially in view of the uncertainty which exists as to the actual distribution of internal temperature throughout the interior of the rib.

18. More Than Three Spans over Elastic Supports.—If there be more than two intermediate elastic piers for any series of arch spans, it is apparent that the above method must be somewhat modified. The effect of any load, however, is very small except for the span in which it is applied and the two adjacent spans for which reason it is hardly ever necessary to consider a series of more than three arches at one time.

THE ELLIPSE OF ELASTICITY IN ARCH ANALYSES

19. General.—The subject matter of this chapter has to do with certain general properties and laws pertaining to the ellipse of inertia for any system of elastic loads ΣG and is the basis of a method of arch analysis somewhat similar to that presented in the foregoing chapters. This method, however, is a distinct refinement over the preceding method and is moreover especially advantageous as adapting itself to ready graphical interpretation.

20. The Ellipse of Inertia.—Through the center of gravity of any homogenous solid, such as shown in Fig. 55, let two coordinate axes X-X and Y-Y be drawn at right angles to each other. Let x and y denote the coordinates of any elementary area dA, x being measured perpendicular to the axis X-X and y being measured perpendicular to the axis Y-Y.

The term $\int x^2dA$ represents the moment of inertia of the figure about the axis X-X, the term $\int y^2dA$, the moment of inertia about Y-Y, and the term $\int xydA$, a quantity known as the product of inertia about the axes X-X and Y-Y.

In treatises on mechanics, it is proved that there will always exist two axes at right angles to each other in reference to which the product of inertia is zero. Such axes are termed the principal inertial axes.

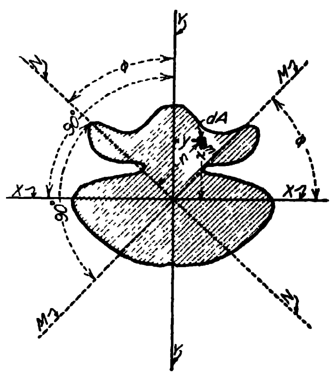


Fig.

Let X-X and Y-Y be the principal inertial axes and let M-M and N-N be any other axes at right angles to each other and making with the principal axes the angle ϕ. From Fig. 55

$$m = x\cos\phi - y\sin\phi \tag{163}$$

$$n = y\cos\phi + x\sin\phi \tag{164}$$

$$\int m^2dA = (\cos^2\phi \int x^2dA) - [2(\cos\phi)(\sin\phi)\int xydA] + (\sin^2\phi \int y^2dA) \tag{165}$$

$$= \cos^2\phi \int x^2dA + \sin^2\phi \int y^2dA \tag{166}$$

(since by hypothesis $\int xydA = 0$).

But $\int x^2dA = I_x$ the moment of inertia about the axis X-X.
$\int y^2dA = I_y$ the moment of inertia about the axis Y-Y.
$\int m^2dA = I_m$ the moment of inertia about the axis M-M.
$\int n^2dA = I_n$ the moment of inertia about the axis N-N.

Whence

$$I_m = \cos^2\phi\, I_x + \sin^2\phi\, I_y \tag{167}$$

In a similar manner is derived

$$I_n = \cos^2\phi\, I_y + \sin^2\phi\, I_x \tag{168}$$

Dividing through by A and denoting the radii of gyration about the several axes by the terms r_x, r_y, r_m and r_n, we have

$$r_m^2 = \cos^2\phi\, r_x^2 + \sin^2\phi\, r_y^2 \tag{169}$$

$$r_n^2 = \cos^2\phi\, r_y^2 + \sin^2\phi\, r_x^2 \tag{170}$$

The locus of the last two equations is an ellipse whose semi-axes lie in the lines X-X and Y-Y and are represented in length by the terms r_x and r_y. This ellipse is known as the ellipse of inertia for the figure under consideration.

If eqs. (169) and (170) be added, the following expression is derived:

$$r_m^2 + r_n^2 = r_x^2 + r_y \tag{171}$$

Since one of the intrinsic properties of the ellipse is that the sum of the squares of the semi-axes is equal to the sum of the squares of any two conjugate semi-diameters, eq. (171) indicates that the radii of gyration about any set of rectangular axes, as MM and NN, are equivalent in value to the length of the conjugate semi-diameters of the inertial ellipse corresponding to said axes MM and NN. Thus in Fig. 56, OB and OC are conjugate semi-diameters corresponding to the axes MM and NN. Whence $r_n = OB$ and $r_m = OC$. Since the eccentric angles for any two conjugate semi-diameters always differ by 90 deg., the points C

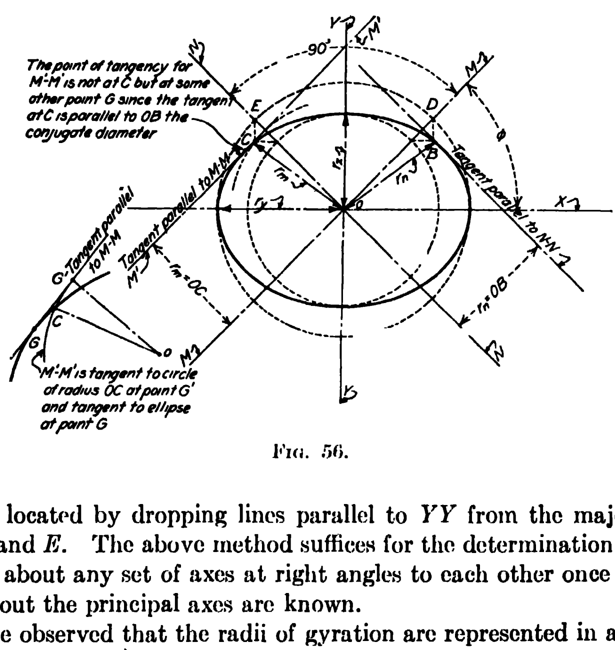


FIG. 56.

and B are located by dropping lines parallel to YY from the major auxiliary circle at D and E. The above method suffices for the determination of the radii of gyration about any set of axes at right angles to each other once the radii of gyration about the principal axes are known.

It will be observed that the radii of gyration are represented in amount, but not in direction by the linear segments OB and OC. Thus the radius r_m is equal quantitively to OC, but its direction of action is perpendicular to the axis MM (see Fig. 56).

It may further be shown from the general properties of the ellipse that a perpendicular dropped from the center O on to a tangent parallel to any axis, as MM, is equal to the radius of gyration r_m about this axis (see Fig. 56).

For any area such as shown in Fig. 55, the principal axes may be located in the following manner.

Assume any two axes M-M and N-N at right angles to each other and making with the principal axes the unknown angle ϕ. Let X-X and Y-Y be the principal axes whose location in reference to the assumed axes is to be determined.

From Fig. 55

$$x = m \cos \phi + n \sin \phi \tag{172}$$

$$y = n \cos \phi - m \sin \phi \tag{173}$$

$$\int xydA = 0 = [(\cos^2 \phi - \sin^2 \phi) \int mndA] + [(\sin \phi \cos \phi) \int n^2dA - \int m^2dA] \tag{174}$$

Whence

$$\tan 2\phi = \frac{2\int mndA}{\int m^2dA - \int n^2dA} \tag{175}$$

With the angle ϕ known the principal axes are at once located and the ellipse of inertia may be constructed thereon.

21. Inertial Ellipse for System of Load Concentrations in a Plane.— The theory of the inertial ellipse may be more readily understood by means of its application to the structural frame although the same laws hold for homo-

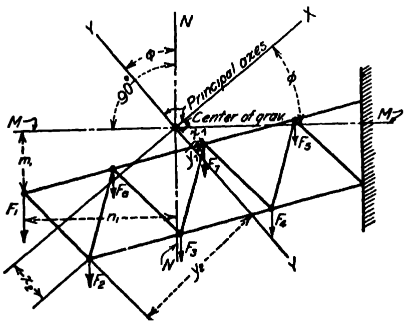


Fig. 57.

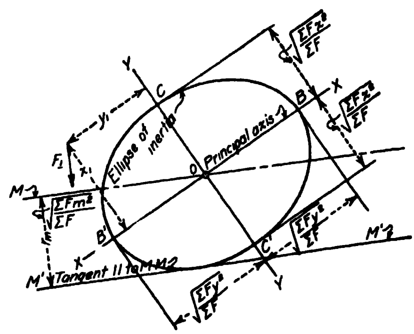


Fig. 58.

geneous ribs as will be seen later. For this reason, the theory has been illustrated by the construction of a structural frame, as shown in Fig. 57. Consider the structural frame shown in Fig. 57 loaded with the panel point concentrations $\Sigma F = F_1, F_7$ and let O be the center of gravity of the load system. If through this center O a set of rectangular coordinate axes M-M and N-N be arbitrarily chosen, the angle ϕ which these axes make with the principal axes is at once determined from the formula

$$\tan 2\phi = \frac{2\Sigma Fmn}{\Sigma Fn^2 - \Sigma Fm^2} \tag{176}$$

The term ΣFx^2 represents the inertial moment of this load system about the principal axis X-X. The term ΣFy^2 that about the axis Y-Y and the term ΣFxy the product of inertia of the load system about the two axes. The inertial ellipse for such load system may be constructed in the same manner as for any homogeneous solid figure the principal semi-axes being given by the expressions

$$OC \quad \sqrt{\frac{\Sigma Fx^2}{\Sigma F}} \text{ and } OB \quad \sqrt{\frac{\Sigma Fy^2}{\Sigma F}} \tag{177}$$

(see Fig. 58). It will also be noted that the perpendicular distance between any axis M-M through the center of the ellipse and a parallel tangent M'-M' (Fig. 58) is given by the expression

$$r_m = \sqrt{\frac{\Sigma Fm^2}{\Sigma F}} \tag{178}$$

22. The Inertial Ellipse for a System of Elastic Loads (The Ellipse of Elasticity).—If the load system ΣF be replaced by a system of joint loads representing the elastic weights $G = l \div AE\rho^2 \left(\frac{ds}{I}\right.$ in case of rib structures$\Big)$, an inertial ellipse for the same may be constructed in the manner as hereinbefore outlined. Such an ellipse will have its center coincident with the center of gravity of the elastic load system ΣG or the elastic center of the frame. This ellipse has been termed the ellipse of elasticity and possesses certain intrinsic properties which render it exceedingly useful in the analysis of elastic arch structures—both framed arches and arch ribs.

23. General Properties of the Ellipse.—Before entering upon a discussion of the properties of this ellipse of elasticity in its application to arch analysis, certain fundamental theorems relative to the general properties of the ellipse

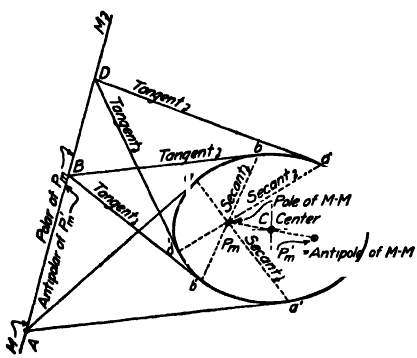


FIG. 59.

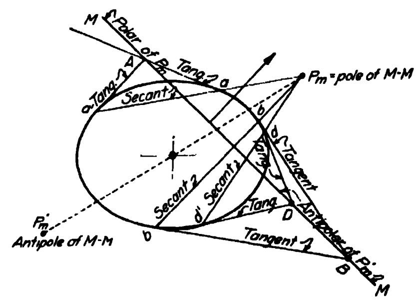


FIG. 60.

will be stated. The complete demonstration of such theorems form a portion of the subject matter of Analytical and Projective Geometry.

Consider the ellipse shown in Fig. 59. If through any point P_m secants to the ellipse be drawn (as aa', bb' and dd', Fig. 59), tangents to the ellipse at the extremities of these secants intersect in the points A, B, D, etc., all of which lie on a line (as M-M, Fig. 59). The line M-M is called the polar of the point P_m with respect to the ellipse in question. Conversely the point P_m is known as the pole of the line M-M with respect to this ellipse. The point P'_m symmetrical with P_m about the center C of the ellipse is termed the antipole of the line M-M and conversely the line M-M is the antipolar of the point P'_m. From Fig. 60 it will be observed that the pole and antipole of an ellipse may fall either within or without the same.

The following properties of poles, polars, antipoles and antipolars are of value in the succeeding discussion.

(*A*) If the pole P_m (Fig. 61) of any line M-M lies on another line N-N, then the pole P_n of the line N-N lies on M-M. In other words, any line as N-N drawn through P_m has its pole on the line M-M. Also any line as S-S drawn through P_n has its pole on the line N-N.

(*B*) If the antipole P'_m (Fig. 62) of any line M-M lies on the line N-N, then the antipole P'_n of the line N-N will lie on M-M, and conversely. In

other words, the same relationship exists between antipoles and polars or between poles and antipolars as set forth above.

(*C*) In Fig. 60, it will be noted that as the line *M-M* moves parallel to itself in the direction indicated by the arrow, the point P_m, its pole, moves toward the conic until *M-M* becomes tangent to the curve at which instant P_m lies on *M-M*. Therefore the polar of any point on the ellipse is simply the tangent to the curve at that point.

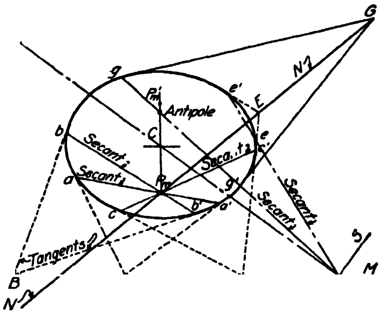


Fig. 61.

(*D*) Any line passing through the center of the conic is called a diameter. From Fig. 63, it will be observed that the pole and antipole of any diameter as *CD* lie at infinity on a line parallel to the tangents *A′-B′* and *A″-B″*, at its extremities. Two diameters each parallel to the tangents at the extremities of the other, are called conjugate diameters. We may therefore state the above relationship as follows: *The pole and antipole of a diameter fall upon its conjugate diameter at infinity.*

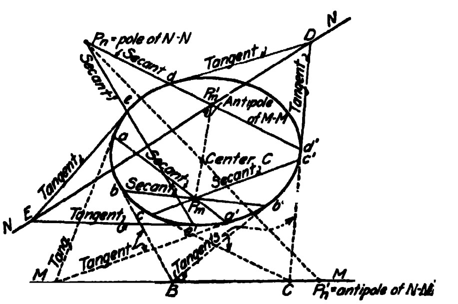


Fig. 62.

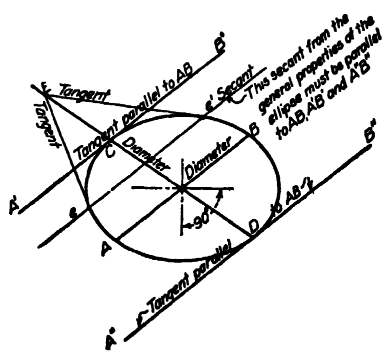


Fig. 63.

(*E*) If in Fig. 61 the line P_n-*C* be drawn through the center of the ellipse, the polars of all points on this line pass through a single point, namely the pole of P_n-*C*. But since P_n-*C* is a diameter, its pole lies on the conjugate diameter at infinity. Therefore the line P_n-*C* and a parallel to *N-N* through the center of the ellipse are conjugate diameters. Furthermore the line P_n-*C* is the locus of poles and antipoles of all lines parallel to *N-N*. ***The pole and antipole of any line therefore lie on a diameter whose conjugate is parallel to the given line.***

Any line joining a pole or antipole to the center of an ellipse is conjugated with a parallel to the corresponding polar.

(*F*) The polar of an ellipse is the locus of points which divide harmonically secants through the pole. Thus in Fig. 64, if A be any point on the polar of P with respect to the ellipse there shown, the points A, B, P and D form an harmonic range, that is

$$AB:BP::-AD:DP \tag{179}$$

In Fig. 65

$$\frac{PB}{BA} = \frac{-PD}{DA} \tag{180}$$

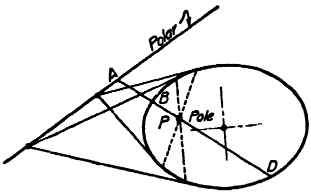


Fig. 64.

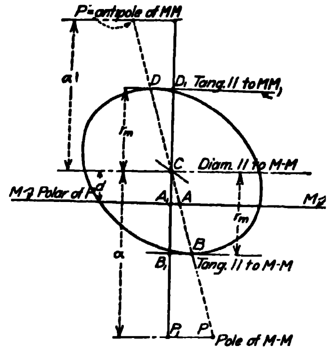


Fig. 65.

Likewise from direct proportion

$$\frac{P_1B_1}{B_1A_1} = \frac{-P_1D_1}{D_1A_1} = \frac{P_1D_1}{A_1D_1} \tag{181}$$

Now

$$\begin{aligned} P_1B_1 &= a - r_m \\ B_1A_1 &= r_m - d \\ P_1D_1 &= (a + r_m) \\ A_1D_1 &= (r_m + d) \end{aligned} \tag{182}$$

Whence

$$(r_m)^2 = ad \tag{183}$$

24. Properties of Instantaneous Centers for Elastic Loadings.—Bearing in mind the general properties of the ellipse as above set forth, we may now take up a general study of certain elastic functions hereinafter to be described.

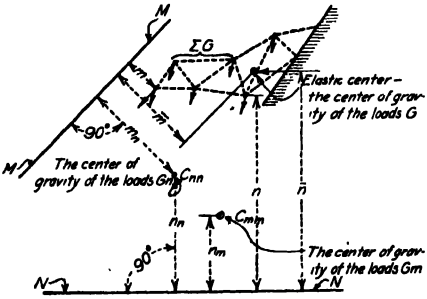


Fig. 66.

The elastic loads G, regarded as weights, each have a certain static moment about any assumed axis M-M. For example in Fig. 66, the static moment of these elastic loads about the axis M-M are given in value by the term Gm. These elastic moments regarded as loads obviously have their own center of gravity which *is not* in general coincident with the elastic center O. This point may be termed the center relative to the axis M-M. To illustrate more fully, C_{mm} the center relative to the axis M-M is merely the center of gravity of the elastic moments Gm

considered as loads; the center C_{nn} relative to the axis N-N is the center of gravity of the elastic moments Gn and so on.

Consider the elastic frame shown in Fig. 67 under the action of the load concentration F applied at panel point a. In consequence of the action of this force, the point a will rotate about some instantaneous center O (whose location is to be determined) and through some angle ϕ whose value, from the considerations stated in eq. (15), p. 456, is given by the expression

$$\phi = \sum \frac{Ssl}{AE} \tag{184}$$

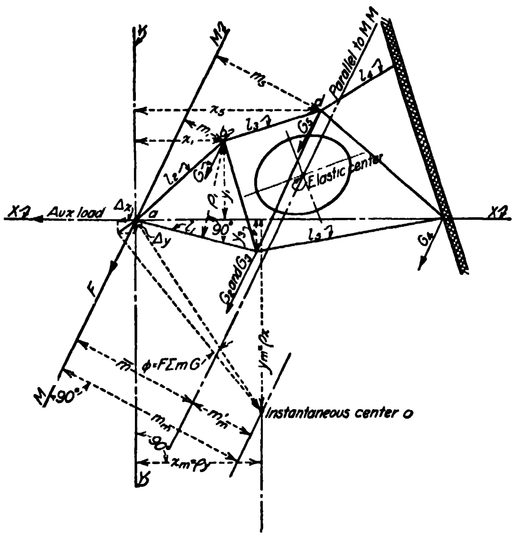


FIG. 67.

where S = the stress in any member of the frame due to the load concentration.
s = the stress in any member of the frame due to a unit auxiliary moment couple applied at a, that is to say

$$S = \frac{Fm}{\rho} \quad \text{and} \quad s = \frac{1}{\rho} \tag{185}$$

$$\therefore \phi = F\Sigma m\left(\frac{l}{AE\rho^2}\right) = F\Sigma Gm \tag{186}$$

Here $G = \dfrac{l}{AE\rho^2}$

If now two rectangular coordinate axes be arbitrarily drawn through point a as origin, it is at once observed that Δ_x (the displacement of a along the axis X-X) is given by the expression

$$\Delta_x = \rho_x\phi = \rho_x F\Sigma Gm \tag{187}$$

Similarly

$$\Delta_y = \rho_y F\Sigma Gm \tag{188}$$

(see Fig. 67).

Now if an auxiliary load of unity acting along the line X-X be applied at point a, we have

$$\Delta_x = \sum \frac{Ssl}{AE} \tag{189}$$

where $S = \frac{Fm}{\rho}$ as before, and $s = \frac{y}{\rho}$ = the stress in any member of the frame due to a unit auxiliary load at a acting along the line X-X.

$$\therefore \Delta_x = F\Sigma my \frac{l}{AE\rho^2} = F\Sigma Gmy \tag{190}$$

Similarly

$$\Delta_y = F\Sigma Gmx \tag{191}$$

Equating the two values of Δ_x and Δ_y from eqs. (187) to (191) inclusive, and solving for ρ_x and ρ_y, we have

$$\rho_x = \frac{\Sigma Gmy}{\Sigma Gm} \tag{192}$$

$$\rho_y = \frac{\Sigma Gmx}{\Sigma Gm} \tag{193}$$

If the elastic moments Gm about the line of action (M-M) of the force F be regarded as weights, the coordinates of their center of gravity parallel to the axes X-X and Y-Y are given respectively by the terms

$$x_m = \frac{\Sigma Gmx}{\Sigma Gm} \tag{194}$$

$$y_m = \frac{\Sigma Gmy}{\Sigma Gm} \tag{195}$$

whence

$$x_m = \rho_y \text{ and } y_m = \rho_x \tag{196}$$

Expressed in words

The instantaneous center for any terminal point a of an elastic system under the action of a force F acting along any line M-M is coincident with the "center relative to the line M-M" of the elastic load system ΣG.

If instead of the frame structure as above considered, the rib or solid webbed structure had been chosen and the same had been divided into segments ds having elastic weights G equal to $\frac{ds}{I}$, the same law could easily have been demonstrated.

25. Displacements by Means of the Ellipse of Elasticity.—Considering Fig. 68, the displacement Δ_m of point a measured along the line M-M is given by the expression

$$\Delta_m = \sum \frac{Ssl}{AE} \tag{197}$$

where $S = \frac{Fm}{\rho}$ as before, and s = the stress in any member of the frame due to an auxiliary unit load applied at point a, and acting along the line M-M (that is to say, $s = \frac{m}{\rho}$).

$$F\Sigma \frac{m^2 l}{AE\rho^2} = F\Sigma Gm^2 \tag{198}$$

If through the elastic center O, a parallel to M-M be drawn, and if m' denotes the perpendicular distance from any panel point to such parallel, we may substitute $m = \bar{m} + m'$ in the above expression and write

$$\Delta_m = F(\Sigma G\bar{m}^2 + 2\Sigma G\bar{m}m' + \Sigma Gm'^2 \tag{196}$$

Since O is the center of gravity of the elastic load system, the term $2\Sigma G\bar{m}m,$ vanishes and we may write

$$\Delta_m = F\Sigma G\bar{m}^2 + F\Sigma Gm'^2$$

or since $\bar{m}$ is constant

$$\Delta_m = F\bar{m}^2\Sigma G + F\Sigma Gm'^2 \tag{200}$$

From the relationship expressed in eqs. (194) and (195), we may also write

$$\Sigma Gm^2 = m_m\Sigma Gm = m_m\bar{m}\Sigma G \tag{201}$$

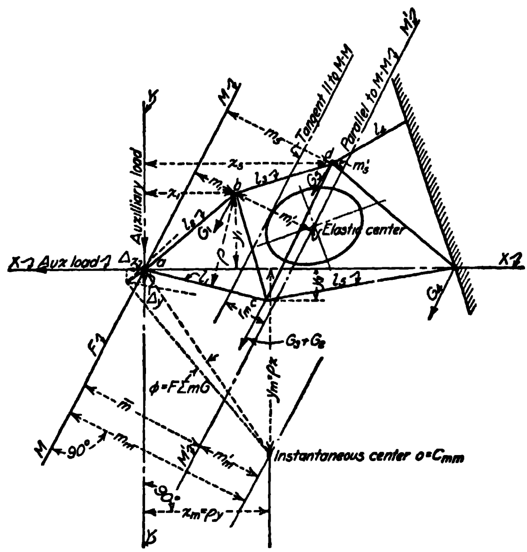


FIG. 68.

Substituting $m_m = m_m' + \bar{m}$ (see Fig. 68)

$$\Delta_m = Fm'_m\bar{m}\Sigma G + F\bar{m}^2\Sigma G \tag{202}$$

Equating the two expressions for Δ_m (eqs. (200) and (202))

$$F\Sigma Gm'^2 = Fm'_m\bar{m}\Sigma G \tag{203}$$

Dividing through by $F\Sigma G$

$$m'_m\bar{m} = \frac{\Sigma Gm'^2}{\Sigma G} \tag{204}$$

If a tangent parallel to $M'M'$ be drawn to the inertial ellipse, and if r_m represents the perpendicular distance of such tangent from the center of the ellipse, then

$$m'_m\bar{m} = \frac{\Sigma Gm'^2}{\Sigma G} = r^2_m \text{ or } m'_m = \frac{r_m^2}{\bar{m}} \tag{205}$$

From eq. (183), the distance from the center of the ellipse to the pole and antipole of the line M-M is given by the expression

$$a = a' = \frac{r_m^2}{\bar{m}} \tag{206}$$

$$\therefore a \text{ or } a' = m_m' \tag{207}$$

That is to say, the instantaneous center for the force F is either the pole or antipole of the line of action of this force with respect to the ellipse of elasticity. It cannot be the pole for, if such were the case, when the force F were in a direction such as to bring its line of action tangent to the ellipse, the instantaneous center would be on the line of action of the force itself. (A polar tangent to an ellipse passes through its own pole.) As this is manifestly impossible, the instantaneous center must be on the *antipole.*

The above relationship constitutes the fundamental theory underlying the entire method of the ellipse of elasticity. Expressed in words:

In any elastic structural frame if the elastic loads ΣG be employed to construct an inertial ellipse, the instantaneous center for any force F acting along a line M-M will be the antipole of such line M-M with respect to this ellipse.

The same law obviously holds for the elastic rib structure as well as the elastic frame.

Based on this conception, the following fundamental properties and uses of the ellipse of elasticity may be stated:

(1) Any force whose line of action passes through the elastic center can cause motion of translation only, as its instantaneous center lies at infinity (the antipole of a diameter lies at infinity).

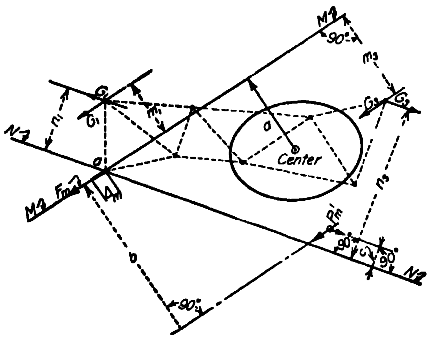


FIG. 69.

(2) The direction of the resulting displacement is perpendicular to the diameter conjugate to the line of action of this force (the antipole of a diameter lies upon a conjugate diameter at infinity).

(3) The displacement Δ_{mm} of any force F_m along its line of action is equal to the product of the force by the moment of inertia of the elastic load system about the line of action of the force. That is to say,

$$= F\Sigma Gm^2 \tag{208}$$

This term is equal to the product of the force by the expression $a.b.\Sigma G$ (see Fig. 69) where a represents the perpendicular distance from the line of action of F_m to the center of the ellipse and b the perpendicular distance to P'_m, the antipole of the line of action of F_m with respect to this ellipse.

(4) The displacement Δ_{mn} of any force F_m in the direction N-N is equal to the product of the force by the product of inertia of the elastic load system about axes M-M and N-N. That is to say

$$\Delta_{mn} = F\Sigma Gmn \tag{209}$$

This term is equal to the product of the force by the expression $a.c.\Sigma G$. (see Fig. 69).

(5) If the line N-N passes through P'_m, then c becomes zero and the displacement Δ_{mn} is zero; that is to say, the total displacement Δ_m is perpendicular to N-N.

From the reciprocal relationship between polars and antipoles, the center P'_n must in this last case fall on the line M-M. Consequently the displacement Δ_n

due to a force F_n acting along the line N-N would be perpendicular to M-M. In other words, if any axis N-N passes through the point a and the instantaneous center P'_m for the line M-M, then P'_n (the instantaneous center for any force acting along the line N-N) will fall on M-M.

(6) Since the displacement of any force passing through the elastic center is perpendicular to the conjugate diameter, the product of inertia of any two conjugate diameters will (from (4) above) always be zero. This offers a ready method of locating conjugate diameters.

(7) The angular movement of any terminal point such as a is given by the expression

$$\phi = F\Sigma Gm \tag{210}$$

representing the product of the force by the static moment of the elastic load system about its line of action.

(8) There are but two directions along which a force acting through the elastic center will produce a displacement parallel to its line of action—namely, the two principal axes of the ellipse of elasticity. That this is true follows at once from the fact that the major and minor axes of the ellipse are the only conjugate axes at right angles.

26. Method of Constructing Ellipse of Elasticity.—There are two methods which may be used to locate the antipole of the ellipse of elasticity. The

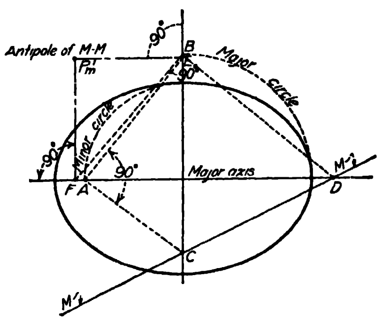


Fig. 70.

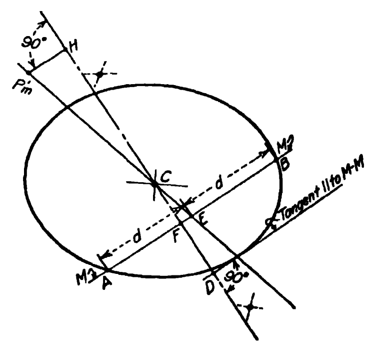


Fig. 71.

first, a purely graphical method, presents the advantage that the ellipse itself need not be drawn, only *the principal* axes being necessary to the construction.

The method is as follows: On the two axes construct the major and minor circles, as shown in Fig. 70, thus determining on the principal axes the points A and B. Join A and C, and B and D. Draw AE perpendicular to AC and BF perpendicular to DB. Parallels to the principal axes through E and F intersect to locate the antipole P'_m of the line M-M (see Fig. 70).

The second method, a semi-graphical one, requires the ellipse itself to be accurately drawn (with an ellipsograph, this may be readily accomplished).

The construction is as follows (see Fig. 71): Bisect the chord A-B of the polar M-M intercepted by the conic. The line joining this mid point and the center C of the conic is evidently conjugated with a diameter parallel to M-M and is hence the locus of the antipole P'_m. From C drop a perpendicular upon M-M

and draw the tangent parallel to M-M intersecting this perpendicular in the point D. The point H is now located from the relation

$$CH = \frac{(DC)^2}{FC} \tag{211}$$

and a perpendicular through H cuts the locus EC produced in P'_m, the antipole of the line M-M as required.

The ellipse of elasticity may be very conveniently employed in the analysis of both arch ribs and framed arches. For the solid rib, moreover, such method of analysis offers a distinct refinement over ordinary methods as will be hereinafter pointed out. To illustrate the application of this theory, analysis will be made of a fixed highway arch span of reinforced concrete. It will be noted that the fundamental operations are no different from those employed in the analysis of any fixed arch as described in preceding articles.

27. Analysis of a Fixed Arch Rib by Means of the Ellipse of Elasticity.

27*a*. General.—The span under consideration is shown in Fig. 72, being the unsymmetrical end span of a series of three arches. We shall assume that the piers are of sufficient dimension to warrant their consideration as *fixed* supports and will proceed first with a consideration of the end span. The strength of the rib being within small limits directly proportional to its width, we may first investigate a strip 12 in. in width, afterward determining by direct proportion the necessary width for each rib to carry its requisite pro rata of the load. Each 12-in. strip is reinforced with 1 sq. in. of intradosal metal and 1 sq. in. of extradosal metal.

27*b*. Detailed Analysis of End Span.—The analysis may now proceed as follows:

Operation No. 1.—Divide the arch rib into any arbitrary number of "voussoirs" or arch blocks of equal length and compute for each voussoir the term $G = \frac{ds}{EI}$ representing the "elastic weight" of the segment exactly as set forth in preceding articles. Note that the elastic weight represents the angular distortion $d\phi$, due to a unit auxiliary moment couple. That is to say,

$$G = d\phi = \frac{m^2 ds}{EI} = \frac{(\text{unity})ds}{EI} = \frac{ds}{EI}$$

For framed arches

$$G = d\phi = \frac{s^2 l}{AE} = \frac{\left(\frac{m}{\rho}\right)^2 l}{AE} = \frac{l}{AE\rho^2}$$

Since the term E except for temperature effects always occurs in both the numerator and denominator of all expressions for the redundant members, this term may be given any value for convenience. For example, in Fig. 72, E was taken equal to unity $\left(\text{whence } G = \frac{ds}{I}\right)$. In any case, the true value of E must of course be employed when solving for temperature stresses.

In this particular case, the arch rib has been divided into but twelve voussoirs in order to avoid the use of a large number of lines on the drawing. In actual practice for the desired precision, it would doubtless be desirable to use at least twenty divisions.

Where the divisions are comparatively short and where the cross-section of the rib does not change materially throughout the length of the voussoir, the point of application of the elastic load G may be considered as coincident with the midpoint on the segment of axis included within the limits of the block. For longer divisions, or where the cross-section of the block is rapidly changing, as in the case of voussoirs No. 1, No. 2, No. 11 and No. 12, it is better to subdivide the voussoir into smaller component blocks and, considering the elastic load of each of these subdivisions applied at its center, calculate or determine graphically the more exact elastic center or center of gravity of the component elastic loads. For example in *a*, *b* and *c* of Fig. 72, block No. 1 has been divided into four equal linear subdivisions 1*a*, 1*b*, 1*c* and 1*d* and the elastic weight G of each subdivision (considered as applied at the center of that subdivision) used graphically to construct a ray diagram and equilibrium polygon, thus determining in the usual manner the "elastic center" of the entire voussoir (see Figs. *a*, *b*, and *c*, Polygons A and B).

The elastic weights for each voussoir are given in Table 1 below.

TABLE 1.—CALCULATION OF ELASTIC WEIGHTS

Voussoir No.		h (ft.)	$I_t = (I_c + nI_s)$ (foot units)	ds (ft.)	$G = \frac{ds}{I}$	
1	1*a*	7.90	43.12	2.53	0.058	
	1*b*	7.30	34.87	2.53	0.072	
	1*c*	6.70	27.07	2.53	0.093	
	1*d*	6.00	19.63	2.53	0.129	
	Total.....	....	.....	.....		0.35
2		4.45	8.19	10.12		1.24
3		3.00	2.59	10.12		3.92
4		2.35	1.28	10.12		7.92
5		2.18	1.02	10.12		9.92
6		2.15	0.98	10.12		10.32
7		2.12	0.94	10.12		10.76
8		2.15	0.98	10.12		10.32
9		2.18	1.02	10.12		9.92
10		2.40	1.36	10.12		7.45
11		3.25	3.28	10.12		3.08
12	12*a*	4.00	6.00	2.53	0.422	
	12*b*	4.42	8.04	2.53	0.316	
	12*c*	4.72	9.75	2.53	0.260	
	12*d*	5.05	11.86	2.53	0.214	
	Total.....	....	.....	.....		1.21

Operation No. 2—Location of Elastic Center of Entire Arch.—With the elastic load system determined as above, the elastic center of the entire span is located as follows: Lay off load line ΣG vertically and with any pole distance H_e (Fig. 72) construct ray diagram C and equilibrium polygon C. Segments a_1' and m_1' of this polygon intersect in o_1' on a vertical containing the elastic center o. Next draw horizontals through the elastic center of each voussoir and upon these

construct equilibrium polygon *D*, each segment of which is perpendicular to the corresponding ray of ray diagram *C*. The first and last segments a_1'' and m_1'' of this polygon intersect in o'' on a horizontal containing the elastic center *o*.

Thus is located the elastic center of the entire span which, according to Art. **22**, p. 500, is the center of the "ellipse of elasticity" for the arch span as a whole.

Operation No. 3—Determination of Conjugate Redundant Axes.—In the discussion of arch analysis which forms the subject matter of the preceding chapters, it has been shown that the redundant forces applied at the elastic center of any arch system should always be in a direction such that the term ΣGxy would vanish. In other words, these axes must be so located that the product of inertia of the elastic load system about the same must equal zero. This clearly means, from a consideration of the preceding chapter, that these axes may be any pair of conjugate diameters of the ellipse of elasticity. Therefore, we may arbitrarily select any diameter of the ellipse. (The vertical diameter is preferable for reasons which will be apparent as the analysis is developed.) Selecting this vertical diameter, the conjugate diameter remains to be located as follows:

Select any arbitrary vertical axis as *M-M*, Fig. 72, and produce the segments of equilibrium polygon *C* to cut this line in segments as shown. These segments are clearly equal to the terms *Gm* (times some constant) where *Gm* represents the static moment of any elastic load about the axis *M-M*. Considering these intercepts, *Gm* (K_2) (or $Gm \div H_c$), construct a ray diagram for the same (ray diagram *E*, Fig. 72) and the corresponding equilibrium polygons (polygons *E* and *F*, Fig. 72). Each segment of the equilibrium polygon *F* is drawn perpendicular to the corresponding ray of ray diagram *E*. Thus in exactly the same manner as was previously adopted for the load system ΣG, the center of gravity of this new load system (*K*) ΣGm is determined.

From the preceding chapter, this last determined point is the "center relative to the axis *M-M*," for the elastic load system ΣG and hence from Art. **25**, p. 504, is also the antipole of the line *M-M* with respect to the ellipse of elasticity for the system $\Sigma G = G_1 \ . \ . \ . \ . \ G_{12}$.

The line joining the elastic center *o* with this newly determined antipole is (from Art. **23**, p. 500) conjugated with the assumed vertical diameter (see paragraph *E*, Art. **23**—"Any line joining a pole or an antipole to the center of the ellipse is conjugated with a diameter parallel to the corresponding polar").

Operation No. 4—Evaluation of Redundants.—Exactly as in former problems, the left support is now removed and replaced by a rigid bracket terminating at the elastic center *o*.

Using for the redundant axes the conjugate diameters above determined, the formulas stated in eqs. (73) to (75) inclusive may be employed. Thus for a unit load at any point *g*

$$X_g = \frac{\Sigma_g^l kGy}{\cos \phi \Sigma Gy^2} \tag{73}$$

If for convenience in this case, we measure the ordinate *y* perpendicular to the axis *X-X* rather than vertically as heretofore, we may write the above equation as follows:

$$X_g = \frac{\Sigma_g^l kGy}{\Sigma Gy^2} \tag{73A}$$

In a similar manner:

$$\frac{\Sigma_g^t kGx}{\Sigma Gx^2} \tag{74}$$

$$Z_g = \frac{\Sigma_g^t kG}{\Sigma G} \tag{75}$$

Operation No. 5—Redundant Influence Lines.—The influence line for the redundant Z is constructed exactly as described on p. 510, to wit: The pole distance H_e used in drawing ray diagram C is taken equal to the quantity ΣG. The ordinate to the equilibrium polygon C intercepted by the vertical through any point (for example point g) measures to the scale of the arch diagram the term

$$Z_g = \frac{\Sigma_g^t Gk}{\Sigma G}$$

The area included between the polygon C and its final segment m_1' is, therefore, the influence line for the redundant Z.

For the determination of the influence lines for the X and Y redundants, the method herein presented involves a distinct refinement over the method outlined in previous articles. It is noted that the expressions for these forces (eqs. (74) and (73A)) involve the use of the elastic loads Gx and Gy.

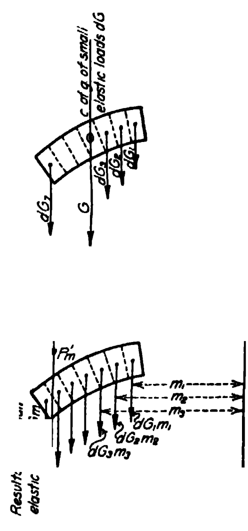


Fig. 73.

Heretofore these loads have been assumed as acting at the center of gravity of the voussoir or arch block in question, which assumption is not strictly true as shown from a consideration of Fig. 73, which figure represents any single voussoir or rib. If this segment, whose elastic weight is G, be divided into smaller segments, each of whose elastic weights be represented by the term dG, the center of gravity of the small component elastic weights dG is very closely coincident with the center of gravity of the entire segment. The weight G may therefore be considered as applied through the center of gravity of the entire segment without material error except when the segment is long or of rapidly changing cross-section, in which case, it is split up into smaller sections and the true elastic center determined as was done with segments No.1 and No.12 for the arch under consideration. For the redundant Z therefore (where only the loads ΣG are involved) the method above outlined introduces no material error.

If each one of the above component weights be multiplied by its distance m from any axis M-M, the center of gravity of the component moments dGm do not by any means fall at the gravity center of the section but at some point P_m' whose location is yet to be determined.

The load systems Gx and Gy should not, therefore, be applied at the gravity centers of the various voussoirs but at some other definite point which must be found for the load Gx and also for the load Gy for each segment or voussoir of the span.

From the considerations set forth on p. 502, the center of gravity of the elastic moments Gx is coincident with the center relative to the axis Y-Y for all the infinitesimal elastic weights dG included in one voussoir. These centers are coincident with the antipole of the axis Y-Y relative to the ellipse of elasticity of each voussoir block. These individual ellipses are drawn for each voussoir in Fig. 72 and are shown cross-hatched.

Since a force applied along the arch axis will cause a linear distortion in the same direction it follows from the considerations which form the subject matter of the preceding chapter that one of the principal axes of the ellipse of elasticity for each voussoir lies in the axis of the arch. The value of this distortion (Art. **25**, p. 504) is given by the expression

$$\Delta = F\Sigma(dG)m^2 = FGb^2$$

where b is the semi-axis of the ellipse of elasticity perpendicular to the arch axis. But

$$\Delta = F\left(\frac{ds}{A}\right) = F\left(\frac{ds}{I}\right)\left(\frac{I}{A}\right) = FG\left(\frac{I}{A}\right)$$

therefore b, the semi-axis perpendicular to the arch axis, is given by the expression

$$b = \sqrt{\frac{I}{A}} = h\sqrt{\frac{1}{12}}$$

In a similar manner a, the semi-axis lying in the arch axis, is given by the term

$$a = ds\sqrt{\frac{1}{12}}$$

The major axis in this case will be constant for each voussoir since ds is constant; the minor axis shown will vary with the depth of arch rib.

The antipoles P_y' are located by means of the graphical construction shown in Fig. 70, p. 507. In this manner the antipoles P_{y1}' . . . . . P_{y12}' are located and the elastic moments Gx for each voussoir considered as acting through these points. This, as above stated, is a distinct refinement over the ordinary method wherein the term Gx is considered as a load acting at the center of each voussoir.

Using the above refinement therefore the procedure is as follows: Produce the rays of polygon C to intersection with the axis Y-Y, the intercepts on this axis measure the terms $\frac{Gx}{H_c}$ or K_2Gx $\left(\text{where } K_2 = \frac{1}{H_c} = \frac{1}{\Sigma G}\right)$. Applying these elastic moments as weights through the corresponding antipoles P_y', another ray diagram (ray diagram N, Fig. 72) is drawn using an arbitrarily selected pole distance H_N. With this pole distance, equilibrium polygon N is constructed any ordinate to which through any point (say point g) represents (to the scale of the arch rib drawing) the term

$$\frac{K_2}{H_N}\sum_g^t kGx \text{ or } \left(\frac{K_2}{K_1}\right)\left(\frac{\Sigma_g^t kGx}{\Sigma Gx^2}\right)$$

where $K_1 = \frac{H_N}{\Sigma Gx^2}$, a constant as yet unknown. In other words, the cross-hatched area of equilibrium polygon N is the influence line for the redundant Y with the unknown factor $\frac{K_2}{K_1}$.

This factor may be determined by computing the term ΣGx^2, algebraically and from this the term K_1, but such a procedure is not necessary since the intercept of polygon N on the axis Y-Y, included between the first and last segments (a_6' and m_6,' Fig. 72) represents the term

$$\frac{K_2}{K_1}\left(\frac{\Sigma_a^t Gxk}{\Sigma Gx^2}\right) = \left(\frac{K_2}{K_1}\right)(\text{unity})$$

This ordinate therefore determines the value of $\frac{K_2}{K_1}$, thus making completely determinate the influence line for the redundant Y.

The X influence line remains to be determined, which may be done as follows: The elastic loads ΣG are laid off on a load line parallel to the axis X-X. With pole distance H_g, ray diagram G is constructed and from it equilibrium polygon G. These loads are assumed to act on lines parallel to the axis X-X through the elastic center of each voussoir.

Segments of equilibrium polygon G produced intercept on the axis X-X, segments representing to scale the terms $\frac{Gy}{H_g}$ (= $0.05\,Gy$ in this case). Using these segments as weights applied at the antipole of the line X-X with respect to the ellipse of elasticity for each voussoir, ray diagram K and equilibrium polygon K are next drawn. The first and last sides of this equilibrium polygon intercept on the axis X-X a segment repesenting to scale the term $\frac{\Sigma Gy^2}{H_K H_G} = \frac{\Sigma Gy^2}{400}$ (see Fig. 72).

The vertical load line for the elastic moments Gy is next drawn and with pole distance $H_M = K_3 \Sigma Gy^2$ (in this case $K_3 = 0.20$ has been used) ray diagram M and equilibrium polygon M are constructed. Note that the elastic moments Gy are applied vertically through the antipoles Px' of the axis X-X. An ordinate to equilibrium polygon M through any point g represents the term

$$\frac{\Sigma_g^t kGy}{K_3 \Sigma Gy^2} = \frac{X_g}{K_3} = (\text{in this case } 5X_g)$$

This equilibrium polygon is, therefore, the influence area for the redundant X with the factor 5.

As a general check on the accuracy of the graphical work, the first and last rays of equilibrium polygon G should intersect on the axis X-X. Also segments a_5' and m_5' of equilibrium polygon M coincide.

With the redundant influence lines drawn the values of X, Y and Z may be readily determined for any position of loading and from these values the shear, thrust or bending moment at any point as explained in previous articles. From these redundant influence lines, stress influence lines, moment, shear, or thrust influence lines for any point in the structure may be readily constructed. As this phase of the work has already been fully discussed, the problem herein given need be carried no further.

It will be noted that the segments determined by equilibrium polygons G and K are quite small, which greatly increases the liability of error. For this reason, the graphical work should, by all means, be checked by scaling the dis-

tances x and y and algebraically calculating the terms Gx, Gy, Gx^2, Gy^2, ΣGx^2 and ΣGy^2.

Operation No. 6—Uniform Temperature Effects.—The value of the redundant forces X_t, Y_t and Z_t are given directly from eqs. (92), (95) and (96).

As a check on the above calculations, it will be noted that since a uniform temperature change causes the rib to change dimensions equally in all directions, the distortion due to the resultant R_t, of X_t and Y_t, must be along the line joining the extremities of the arch axis (the line q-p, Fig. 72). From the theory of the ellipse of elasticity therefore the resultant R_t must act along a diameter of this ellipse whose conjugate is perpendicular to this line q-p.

The ellipse of elasticity for the entire arch rib may be readily constructed from the considerations of this and the preceding chapter.

Operation No. 7—Effect of Direct Axial Stress.—These stresses may be evaluated in exactly the same manner and by means of the same formulas as set forth in Art. **12c**, p. 475.

Operation No. 8—Stresses Due to Variable Temperature Effects.—These stresses may be evaluated by means of the formulas given in Art. **12d**, p. 474.

It should be noted that in the problem under discussion, the ordinate y has (for convenience) been measured perpendicular to the axis X-X while in the chapters giving the development of general elastic equations and influence lines for rib arches, this ordinate was measured vertically. In using formulas (57) to (115), inclusive, therefore the present term y replaces the term $y \cos \phi$ of the chapters above mentioned. For y ordinates measured perpendicular to the X-X axis therefore the formulas become:

For uniform temperature:

$$X_t = \pm\left[\frac{EctL'}{\Sigma Gy^2}\right]$$

$$Y_t = \pm\left[\frac{EctL''}{\Sigma Gx^2}\right]$$

$$Z_t = 0$$

For variable temperature:

$$X_{t'} = \pm\frac{ct'E\Sigma\frac{yds}{h}}{\Sigma Gy^2}$$

$$Y_{t'} = \pm\frac{ct'E\Sigma\frac{xds}{h}}{\Sigma Gx^2}$$

$$Z_{t'} = \pm\frac{ct'E\Sigma\frac{ds}{h}}{\Sigma G}$$

27c. Analysis of Symmetrical Center Span.—Assuming a rigid support from each of the intermediate piers, the central span may be analyzed in a manner analogous to that hereinabove described for the unsymmetrical end span.

In this case, however, the symmetry of the span clearly indicates that the product of inertia about two axes, one vertical and one horizontal is zero.

Since there are but two conjugate axes of the ellipse of elasticity which can be at right angles (to wit: the semi-axes), the ellipse of elasticity for the entire

span is at once determined. The major axis of this ellipse is horizontal and is given in length by the expression

Semi-major axis = $\sqrt{\dfrac{\Sigma G\bar{x}^2}{\Sigma G}}$

Similarly: Semi-minor axis = $\sqrt{\dfrac{\Sigma G\bar{y}^2}{\Sigma G}}$

27*d*. Effect of Elastic Yielding of Piers.—A solution taking into account the elasticity of the intermediate piers may be effected in much the same manner as described in the chapter entitled "Multiple Span Arches on Elastic Piers." The refinements as regards the point of application of the elastic moments Gx and Gy render this method more precise than that outlined in the chapter just mentioned.

For a further and more complete consideration of this subject, reference is made to Chapter VIII of Hool's "Reinforced Concrete Construction," Vol. III.

ABUTMENTS AND PIERS

28. Arch Abutments.—An arch abutment is held in equilibrium by the following forces:

The thrust of the arch ring.

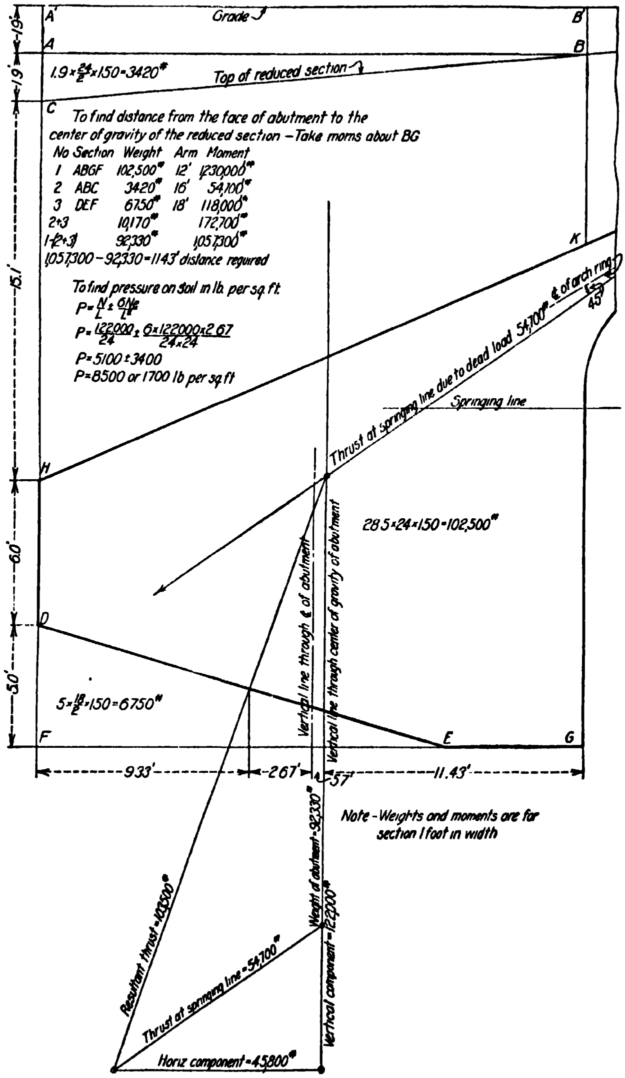


Fig. 74.—Analysis of abutment for barrel arch span.

The weight of the abutment and filling material superimposed thereon.
The pressure from the foundation.
The friction of the foundation.

Consider, as an illustration, the abutment shown in Fig. 74. We will analyze this abutment for dead load thrust only as it is apparent that stresses produced by any given live load thrust may be obtained in an exactly similar manner.

The section *HKGED* is composed of concrete and may be assumed to weigh 150 lb. per cu. ft.

The section *A'B'KH* is the fill over the abutment proper which may be assumed to weigh 120 lb. per cu. ft.

Lay off $KB = \frac{120}{150}$ of KB', and also $HC = \frac{120}{150}$ of HA'.

The section *HCBK* is therefore a section, which, if it were of masonry, would weigh exactly the same as the fill section *A'B'KH*. We may, therefore, analyze the structure considering the abutment to be of concrete throughout and having the section *BCDEGB*.

The center of gravity of this section is found to be 0.57 ft. to the right of the center line of the base as shown in Fig. 74 and the total weight found to be 92,330 pounds.

The arch ring thrust, 54,700 lb., is combined with this force (applied at the gravity center) and the resultant determined.

This resultant passes through the base at a point 2.67 ft. to the left of the center of the same, and has a vertical component of 122,000 lb. and a horizontal component of 45,800 lb.

The maximum pressure will obviously occur on the outer or left-hand corner of the base and is approximately given in amount by the term

$$P\frac{N}{A} + \frac{Me}{I}$$

where N = the total vertical load.
A = the area of the base (in vertical projection).
M = the moment of the vertical load about the center of the base.
e = the distance from the center line of the base to the extreme toe fiber.
I = the moment of inertia of the base about its lateral center line.

In this case

N = 122,000 lb.
A = 24 × 1.0 (since a strip 1.0 wide is being considered)
M = N(2.67)
e = 12 ft.
$I = \frac{bh^3}{12} = \frac{(24)^3}{12}$ (approximately).

$$P = \frac{122{,}000}{24} + \frac{122{,}000(2.67)12}{\frac{(24)^3}{12}} = \frac{122{,}000}{12} + \frac{(6)(122{,}000)(2.67)}{(24)^2} = 8{,}500 \text{ lb.}$$

The pressure on the extreme inner face of the abutment is obviously

$$\frac{N}{A} - \frac{Me}{I} = 5{,}100 - 3{,}400 = 1{,}700 \text{ lb.}$$

No account has been taken of the fact that the plane *DE* is not horizontal. This would operate to change the value of I very slightly. The principal reason for cutting the corner *DEF* is to increase the resistance to lateral movement and to save material.

In the above analysis the structure chosen was a solid barrel arch whose width was practically the same as the width of the abutment for which reason a strip 1 ft. in width was chosen for analysis. In this case, therefore, the thrust, 54,700 lb., represents the dead load thrust per foot of arch ring.

For structures of this type, we may at once write:

$A = L \times 1.0 = L$ where L is the length of the abutment base.

$M = Nc$ where c represents the eccentricity of the thrust measured from the center of base.

$$I = \frac{L^3}{12}$$

$$L$$

Whe

$$P = \frac{N}{L}\left(1 \pm \frac{6c}{L}\right)$$

If the arch is of the rib type sprung from a solid abutment, it is customary to assume a certain width of abutment as carrying the thrust from each rib; thus in Fig. 75, we may assume that the width W_1 carries the thrust of rib No. 1, W_2 that of rib No. 2, etc. In this case, the value of the thrust used is that for one rib and the value of N is derived from the resultant of such total rib thrust and the weight of the abutment for the width W.

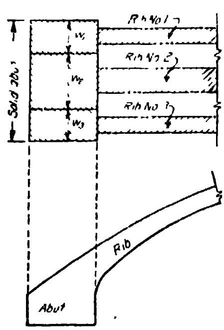


FIG. 75.

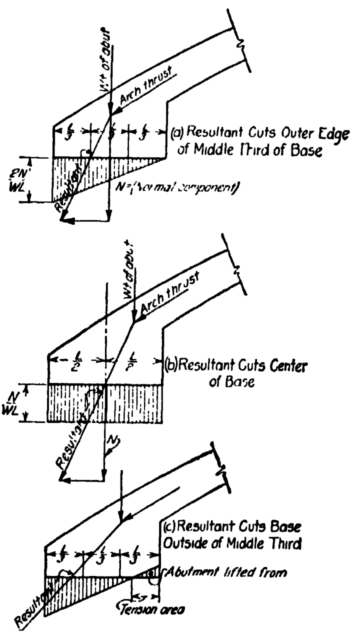


FIG. 76.

The formula then becomes

$$P = \frac{N}{LW} \pm \frac{6Nc}{WL^2} = \frac{N}{LW}\left(1 \pm \frac{6c}{L}\right)$$

In any case, therefore, the toe pressures are functions of the terms $\left(1 \pm \frac{6c}{L}\right)$.

If c is less than $\frac{L}{6}$, it is apparent that both of these terms will be positive, or,

expressed in words: If the resultant abutment pressure passes through the middle third of the base, the entire area will be in compression.

If the resultant cuts the edge of the middle third, the pressure on one toe is zero and on the other $\frac{2N}{LW}$ as shown in Fig. 76*a*. If it cuts the center line, the pressure on the base is uniformly distributed and equal in value to $\frac{N}{LW}$ or one-half the above value (see Fig. 76*b*). If the resultant falls outside the middle third, one toe is lifted from the foundation as the connection between abutment and foundation cannot take tension (see Fig. 76*c*). Such an arrangement will cause the entire load to be concentrated on an area less than that of the full abutment base, and, therefore, result in heavy toe pressures. In this case, it is seen that a portion of the base is idle as far as helping to sustain the load is concerned and represents a waste of material. If the pressure line falls entirely without the base, the abutment will obviously overturn.

The design of abutments, therefore, resolves itself into the selection of a mass of size and weight such that the resultant pressure cuts the base as near as possible to the center line of the same, and in no case outside the middle third.

Toe pressures should be determined for:

(1) The dead load thrust as above described.

(2) Dead load thrust in combination with the extreme upper position of the live load thrust.

(3) Dead load thrust in combination with the extreme lower position of the live load thrust.

(4) Dead load thrust in combination with temperature thrusts and tensions.

All of these cases differ only in the value of arch thrust used for combining with the weight of the abutment and superimposed fill.

In addition to the above determinations, each abutment should be investigated to determine its resistance to sliding laterally.

In the case illustrated above, the horizontal component of the resultant base pressure is 45,800 lb. Assuming a coefficient of friction of 50 per cent for masonry against the foundation material, the resistance to sliding amounts to 50 per cent of 122,000 lb. = 61,000 lb.

The abutment, therefore, will not slide under dead load for a friction coefficient of 50 per cent.

The coefficient of friction for masonry on dry clay is about 55 per cent where the surfaces are fairly smooth. If the surfaces are roughened or serrated the frictional resistance is greatly increased.

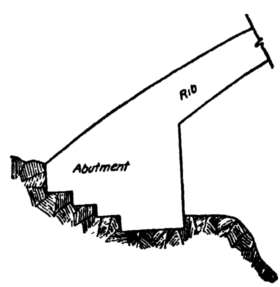


FIG. 77.—Serrated abutment footings.

In the above analysis, no account has been taken of the passive resistance of the earth back of the abutment, nor of the frictional resistance of the filling material over the masonry, both of which tend to increase greatly the resistance to lateral movement so that the above design may be held quite safe against lateral movement for ordinary foundation materials.

Where rock foundations are encountered, which is usually the case in masonry arch construction, serrating or stepping the rock as shown in Fig. 77 greatly increases the resistance to lateral displacement.

Where the water surface comes above the bottom of the abutment footings, the buoyancy of the water decreases the weight of the submerged portion by 62.5 lb. per cu. ft. This buoyancy factor operates to decrease the value of N, but also to increase the value of c where the resultant falls outside the center line of the abutment. It is, therefore, necessary in such cases to compute the toe pressures for both low-water and high-water conditions.

Where pile foundations are employed, it is generally considered better resultant practice to drive the pile on a batter approximately parallel to the dead load thrust line in order to insure against lateral movement due to bending in the pile. It is not always possible to secure a batter as great as this, and if the difference in angle between piling and thrust line is very great, a re-design of the abutment to pull down the thrust line is warranted.

29. Arch Piers.—For equal arch spans, the horizontal components of the dead load thrusts balance so that the resultant is vertical and passes through the center of the base. Where the spans are unequal, unequally loaded, or of different shape, the resultant is inclined and eccentric (see Fig. 78). The analysis of foundation pressures on piers is thus a problem exactly similar to that for abutments.

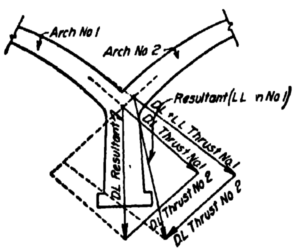


Fig. 78.

Whenever a bridge structure is to comprise a large number of arch spans in series, it is customary to design every fifth or sixth pier in such manner as to withstand the total thrust from either side considered alone. Such a pier is termed an abutment pier and is employed to take up the effect of any slight movement of an intermediate pier which would otherwise transmit some stress from end to end of the bridge.

Ordinarily piers are so designed as to be practically fixed and unyielding under the thrusts from adjacent arches for all conditions of loading. In such a case, the arch ribs are analyzed on the assumption of absolute rigidity at spring lines. For exceptionally high piers, however, it may be necessary at least to investigate the effect of elastic displacement of the piers under load. The analysis of arches on elastic piers has already been discussed.

DETAILS OF REINFORCED CONCRETE ARCH BRIDGES

30. Reinforcement.—Reinforcing for arch ribs and rings generally consists of a series of longitudinal bars parallel to and from 2 to 3 in. from the intrados and a similar series of extradosal bars in combination with transverse distribution or temperature bars, stirrups, etc. In a few instances, single line reinforcement consisting of intradosal bars at crown, bent up at or near the quarter points, and thence running as extradosal bars to the abutment have been employed. Such a system, however, is highly unsatisfactory and has been practically discarded by the profession at this time.

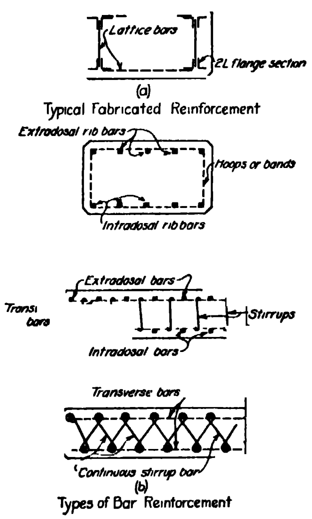


FIG. 79.

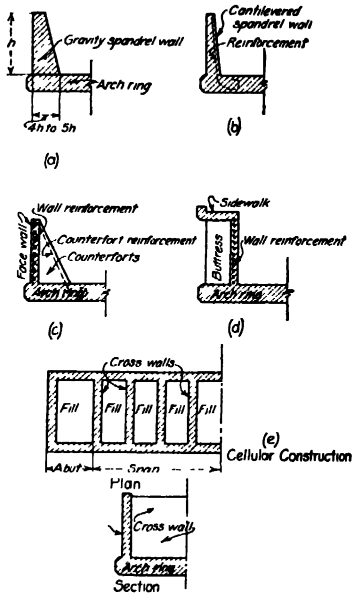


FIG. 80.

Figure 79 shows a few typical schemes used for placing reinforcement both in rib and barrel arches; these being only a few of a great variety of arrangements.

Figure 79*a* is typical of a class of reinforcements known as fabricated or structural reinforcement. The main rib bars are of angles, angles and plates or channels and are generally connected by batten plates and lattice bars to form a complete rib unit. The lattice bars distribute the stresses and act as shear reinforcement and also as a bonding agent.

When the intradosal bars are thrown into tension, there is a tendency for the same to straighten and thus operate to crack loose the concrete underneath. Lattice bars tying the intradosal and extradosal system together operate to counteract the above tendency.

The use of fabricated reinforcement is not always limited to rib arches, but may be employed for barrel arches as well. Sometimes it is convenient and

desirable to design the rib reinforcement sufficiently stiff to carry the dead load of the green masonry and attach the rib form work thereto, thus dispensing with the use of centering.

The bar type of reinforcing in every case consists of longitudinal rib bars near both intrados and extrados. For arch ribs, these bars are generally held in position by means of spacing hoops which also act as shear reinforcement. For barrel arches, the main bars are used in conjunction with transverse bars top and bottom (as shown in Fig. 79*b*) whose function is to distribute load concentrations and to act as temperature and shrinkage reinforcement. Stirrup bars either individual or continuous, are also employed (Fig. 79*b*).

31. Spandrel Walls.—For filled arches, the longitudinal walls (termed spandrel walls) may be designed either as gravity walls (see Fig. 80*a*), as cantilevered walls (Fig. 80*b*), as counterforted walls (Fig. 80*c*), or as buttressed walls (Fig. 80*d*). Cellular construction, as shown in Fig. 80*e*, may also sometimes be profitably employed.

32. Drainage of Back Filling.—For filled spandrels, provision should always be made for adequate drainage of the back filling. This is generally accomplished by means of tile or pipe lines laid parallel to the extradosal face of the arch and running down the same, and through pier or abutment walls to a convenient outlet. No drain should be used whose diameter is less than 3 in. and preferably 4 in. should constitute the minimum diameter of the drain.

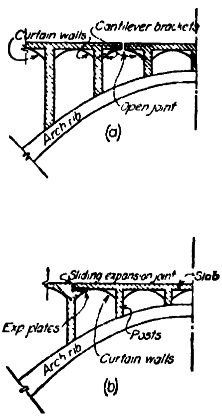


FIG. 81.—Types of expansion joints for arch spandrels.

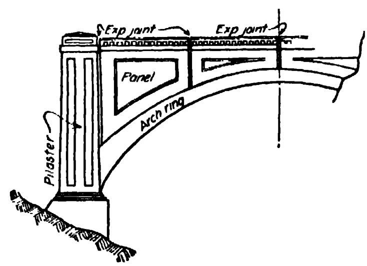


FIG. 82.—Expansion joints in filled spandrel walls.

33. Expansion Joints.—Under the action of temperature, vertical movement takes place in all masonry arch construction; thus operating to throw high bending stress into deck slabs, spandrel walls, etc. It is necessary, therefore, that the superstructure which rests upon the arch proper be cut by expansion joints to permit the necessary freedom of movement. Such joints must be so fashioned as to permit both horizontal and vertical movement. The open cantilevered joint of Fig. 81*a* is much to be preferred over the sliding joint of Fig. 81*b* for this reason.

Spandrel walls are generally cut vertically to provide a clear open joint, as shown in Fig. 82.

For arch spans up to 70 ft. in length, an expansion joint at crown and at springing lines is generally employed; for longer spans, five or more expansion joints are frequently used.

Where counterforted spandrel walls are used, the expansion joint is best provided by means of a double counterfort, as shown in Fig. 83.

Joints may be formed by means of steel or timber plates, or bulkheads inserted during the pouring of the adjacent sections and subsequently removed; by means of prepared asphaltic felt inserted between adjacent sections and left in place; or by means of so-called sand joints. The sand joint is formed by filling the space between two thin plates (held to position by temporary battens) with sand. Upon removal of the lower form, the sand runs out and the thin plates are readily collapsed and removed.

One of the unfortunate effects of expansion joints in a concrete wall surface is the tendency to stain and discolor the concrete surface due to the leeching of water laden with suspended matter (earth, rust or street refuse). This can be eliminated to a large extent by the use of asphaltic felt as a joint filler. A better method, perhaps, is the employment of a copper strip or flashing bent to permit

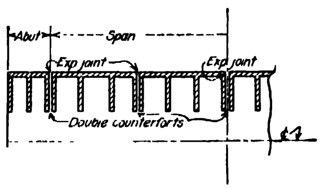


FIG. 83.—Expansion joints in counterforted spandrel walls. (Plan view.)

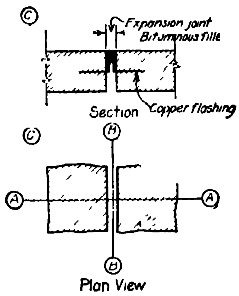


FIG. 84.—Typical method of waterproofing expansion joints.

movement, as shown in Fig. 84. Such a joint permits free movement in the direction A-A, a certain amount of relative movement in the direction c-c; but very little in the direction B-B without shearing the copper flashing strip.

34. Waterproofing.—The upper surface (or back) of barrel arches carrying a superimposed earth fill, together with the inner faces of the spandrel walls on filled arch structures, should be treated with waterproofing.

Two general systems are employed designated as the "membrane" system and the "plaster" treatment.

The membrane system generally consists in the placement of a waterproof blanket by means of alternate layers of bituminous material and asphalt, saturated cotton fabric or other similar material.

The waterproofing medium, generally an asphalt, is first applied hot to the surface of the masonry upon which coat is placed a layer of the saturated fabric. This layer is swabbed or coated with hot bitumen and another layer applied—thus building up a multi-ply membrane over the masonry. The number of plies or layers varies from two to five or six depending upon conditions.

The plaster treatment as the name implies consists of the application of a waterproofing plaster coat. Bitumiuous coatings, prepared waterproofing paints or mortar treated with integral waterproofing compounds have been used for this purpose.

CONSTRUCTION OF MASONRY ARCHES

It is the intention in this chapter to mention only the principal construction features peculiar to this type of structure.

35. Method of Pouring Concrete.—Small arch structures (spans up to 75 or 80 ft.) can generally be poured in one operation with the ordinary construction plant likely to be used on a job of this size. On work of this character, the pouring must be so regulated as to load centering evenly and symmetrically, the masonry being brought up from both spring lines simultaneously, closing at the crown.

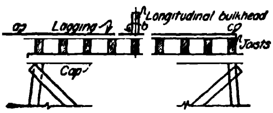


FIG. 85.

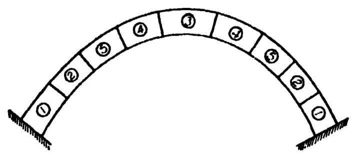


FIG. 86. Sequence of pouring arch voussoirs.

For spans of greater dimension, the pouring is not done in one operation on account of the introduction of shrinkage stresses and because of the length of time required for pouring. Such spans may be poured either in longitudinal sections, or by the "voussoir" or block method. The longitudinal section method is obviously applicable only to the barrel arches and is accomplished by dividing the arch barrel by means of longitudinal bulkheads. For long spans this method of pouring is apt to introduce high stresses in the ring due to the shrinkage of the concrete, and is therefore not to be recommended. The method also has the disadvantage of loading the centering unequally in a lateral direction,

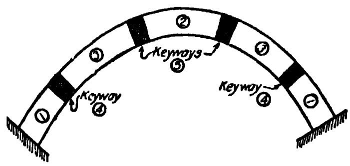


FIG. 87.—Method of pouring arch rib using keyways.

thus causing progressive settlement of the centering. For example, in Fig. 85 if the right half of the arch span is first poured, the centering under this span deflects a certain amount and the lagging takes the position shown by the dotted line a-b'-c'. If the left half of the arch is now poured, the lagging surface takes the position a''-b''-c' and the centering is pulled away from the first arch section causing the same to take weight. If the concrete is still plastic, no harm is done as the distortions are very small. If, on the other hand, the right-hand section has attained its initial set, which is usually the case, partial dead load stresses are thrown into the green concrete. To eliminate the above development, each

longitudinal section should be carried by independent lagging independently supported. This, however, is apt to result in a very ragged and uneven joint due to uneven settlement of the adjacent sections.

The "voussoir" or arch block method contemplates the division into lateral sections by means of transverse bulkheads. The transverse sections or "voussoirs" are poured alternately, generally in about the order shown in Fig. 86, thus eliminating shrinkage stresses and also unequal loading on the centering. It is also noted that the arch rib cannot take stress as an arch at any time before the key section is placed so that any slight settlement of the centering will throw no load on to the green concrete.

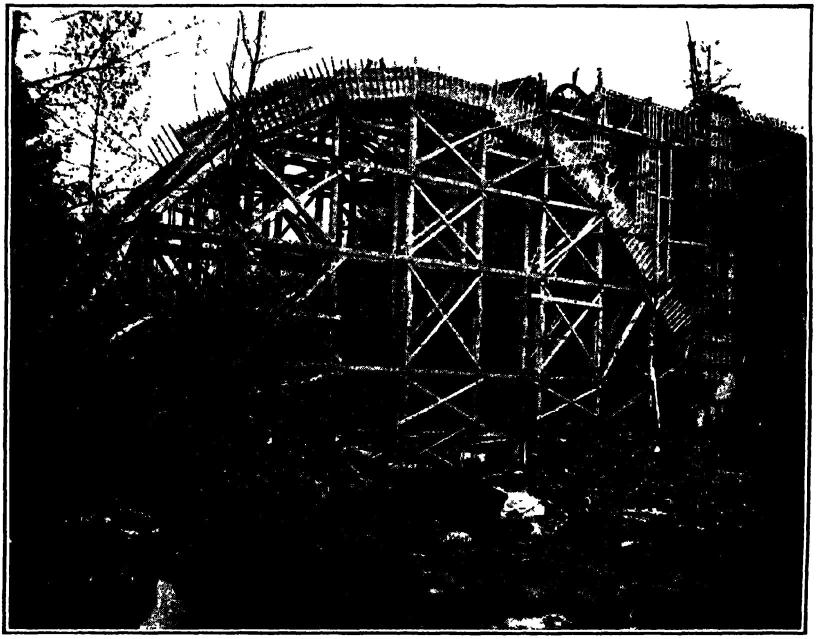


Fig. 88.—Centering for Oswego Arch—Clackamas County, Oregon,—trestle type of rib centering.

Arches poured in the above manner may be poured either with or without key ways, as shown in Figs. 86 and 87.

36. Centering.—The falsework used to support the concrete of the arch rib or barrel is known as the *centering*. There are several methods of constructing arch centering, as follows:

36a. Trestle Centering.—In this method of construction, the arch forms are supported on transverse caps which in turn are supported by posts or piles sway braced and line girted as is an ordinary trestle. Figure 88 illustrates this type of construction, the same being a view of a 130-ft. open spandrel arch structure built by the writer in 1919. Figure 89 is another example of construc-

tion of this type, being a construction view of a 75-ft. arch span built during the same year over "Dry Canyon" in Wasco County, Oregon.

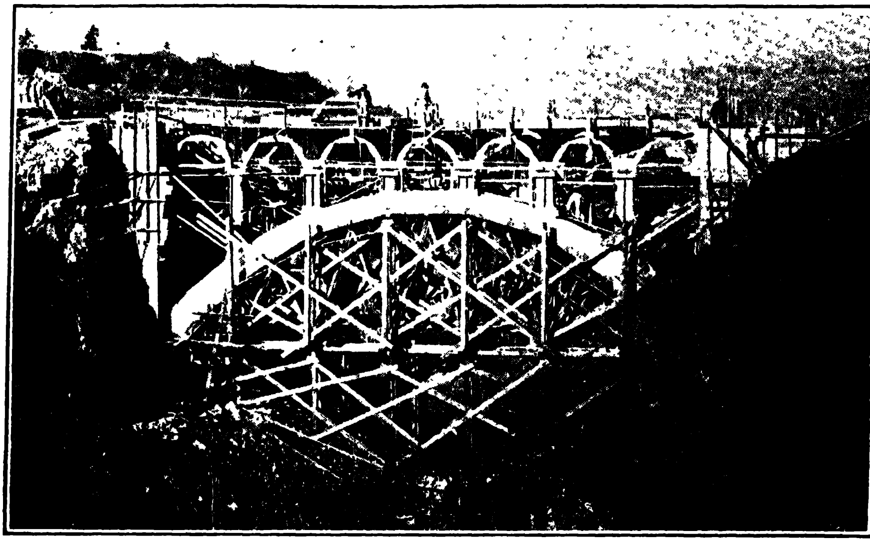


Fig. 89.—Trestle centering for 75-ft. arch span—Dry Canyon Arch—Wasco County, Oregon.

36*b*. Trussed Centering.—Figure 90 illustrates the type of centering to which this name is applied. This being a construction view of an arch bridge

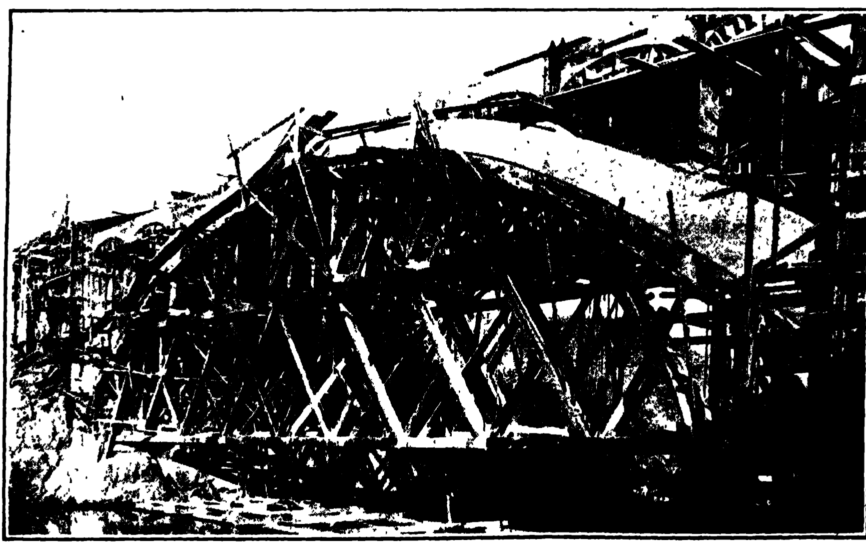


Fig. 90.—Trussed centering—Rogue River Bridge near Gold Hill, Oregon.

constructed across the Rogue River near Gold Hill, Oregon, and described by the writer in *Engineering News*, April 29, 1920. Here the rib forms are supported

on short bents resting on the panel points of a timber Howe truss, the details being clearly explained by the photographs.

36c. Suspended Centering.—It is sometimes necessary to dispense with the supporting structure underneath the arch forms and support the same from a suspension cable, as shown in Fig. 91. The main cables are anchored on shore by means of concrete anchorages or "dead men" and run over towers and across the opening carrying hangers or supporting bents upon which is built

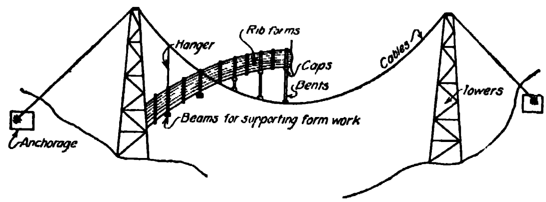


FIG. 91.

forms for the arch rib proper. Figure 92 illustrates centering of this type applied to the construction of the large steel arch described in the volume on "Movable and Long-span Steel Bridges."

36d. Integral Centering.—It is sometimes advisable to arrange the reinforcement for a concrete arch rib or barrel in such manner as to enable the formwork to be suspended therefrom, thus dispensing with falsework in the stream. Centering of this type must, of course, consist of stiff fabricated reinforcement, bar reinforcement being obviously not suitable.

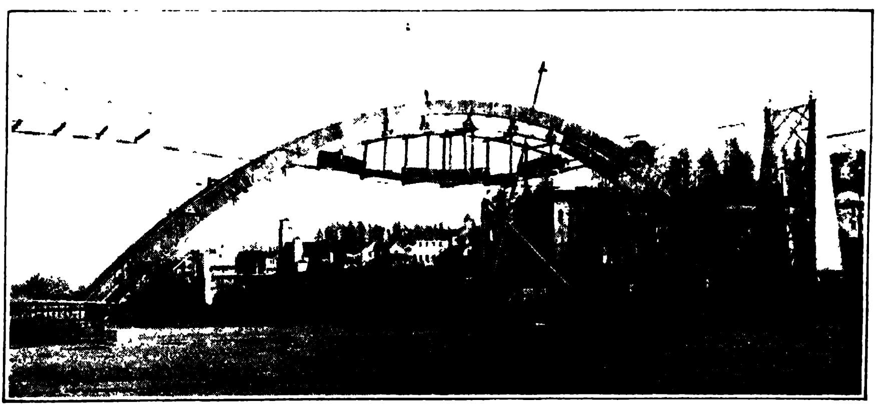
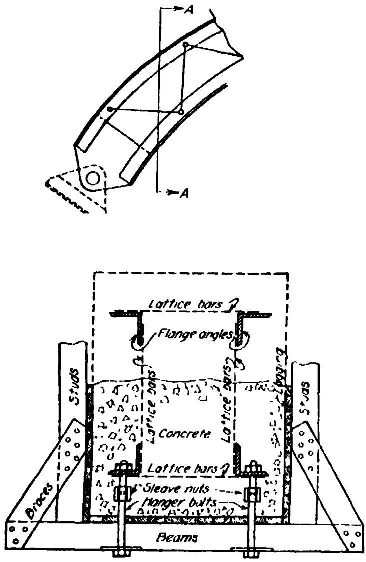


FIG. 92.—Suspended centering for steel arch rib—Oregon City Bridge over Willamette River.

The steel ribs are first erected and swung, after which the formwork is suspended therefrom by means of suitable hanger bolts, as shown in Fig. 93. These hanger bolts are usually fitted with sleeve nuts so that the exposed end may be removed after the formwork is stripped, and the hole filled with concrete or mortar. Integral centering of this character is usually swung as a three-hinged arch for convenience in erection. The concrete arch bridge over the Connecticut River at Springfield, Mass., described in the *Engineering News Record,* March 30, 1922, is an example of construction of this type.

The disadvantage of this type of centering lies in the relatively high cost and the fact that initial stresses of varying magnitude are introduced in the steel reinforcing system as the span is progressively loaded; by proper design, however, this latter objection may be removed and the metal and masonry stressed in an economic ratio. The principal advantage of this method is the elimination of falsework in the stream with the attendant danger from drift and ice. The economy for this type of construction increases with the number of equal spans to be constructed.



Section A-A

FIG. 93.

36e. Steel Centering Frames.—These are steel trusses or arch frames used to support the lagging and not a part of the reinforcing system. Such centering has the advantage of freedom from drift and ice dangers, and is generally stiffer and more certain in action than timber centering. The disadvantage lies in the relatively high cost, but if there are several spans of equal dimension so that the steel centers may be used several times, the cost is greatly decreased.

36f. General Data.—In erecting arch centers, allowance should be made for camber, and for shrinkage and settlement of the centering. The camber is generally computed and added to grade elevation on the plans so that plan elevations include provision for the necessary camber. If this is not the case, grade elevations should be raised to provide enough camber so that under full live loading, the structure will not deflect below a straight line. This is the absolute minimum. In general, highway structures are cambered much more than this for appearance sake. In addition to the above, the elevations shown should be increased by a margin sufficient to provide for shrinkage and settlement of the centering under load. The amount to provide for this will, of course, depend upon the loads, and the type of centering used, its method of support, etc.

Centering is usually carried on wedges or sand boxes so that the same may be gradually released when the masonry has set up and is ready to be swung. Sand boxes are not used to any great extent at present, but, when used, great care must be exercised to keep the sand dry. This type of release will prove more especially adapted to large arch construction.

For barrel arches, the lagging is generally supported on longitudinal curved beams or joists supported at the falsework panel points. These curved beams

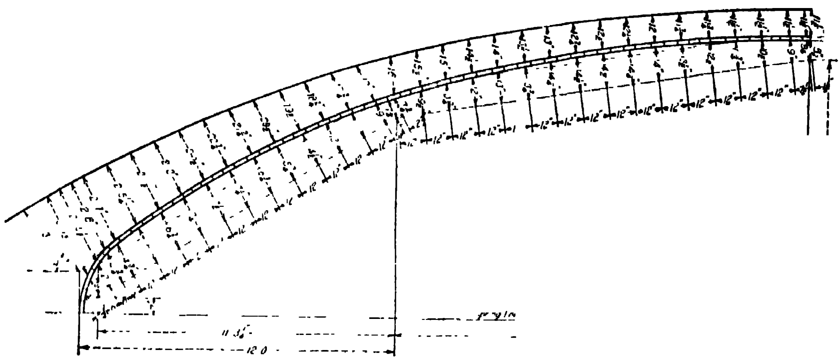


Fig. 94. - Curved lagging beams for barrel arch centering.

are cut to the true curve of the intrados and may either be laid off from the radii in a loft or yard, or cut from a line scribed from ordinates, as shown in Fig. 94.

Great care should be exercised to insure that the centers are gradually released. The crown should be released first and then the two flanks simultaneously. In a series of arches, all centers between abutments or abutment piers should be lowered simultaneously. Centering for concrete masonry arches should not in general be released until the concrete has set for at least 28 days, and in cold weather a longer period is highly desirable and sometimes absolutely essential

SECTION 9

HYDRAULIC STRUCTURES

MASONRY DAMS

BY WILLIAM P. CREAGER

1. Definition and Types.—A dam is an impervious barrier built across the bed of a stream for the purpose of raising the level of the surface of the water. Its principal uses are to create an artificial fall for the generation of water power; to divert the water into a conduit for water supply, irrigation, power or other uses; and to store flood waters for use during the dry seasons.

Masonry dams are divided into the following three general types: (1) Solid gravity dams, (2) hollow gravity dams, and (3) arch dams.

Solid gravity and large arch dams may be composed of ashlar masonry, mortar rubble masonry or concrete. Large stones are usually imbedded in the concrete for these types, and the concrete so constructed is termed "cyclopean concrete." The mix is usually in the proportion of 1:3:6 throughout, but thinner and richer mixtures have been used.

Thin arch dams are composed of 1:2¼:4½ to 1:2½:5 plain or reinforced concrete.

The buttresses of hollow dams are 1:2½:5 to 1:3:6 reinforced concrete with decks, aprons and struts composed of 1:2:4 reinforced concrete.

2. Nomenclature.—Unless specifically mentioned, all forces are in pounds and all distances in feet.

Let

W = vertical force, positive when directed downward.

P = horizontal force, positive when directed towards the left.

P_i = ice pressure per linear foot of dam.

R = resultant of forces.

$\Sigma(W)$ = algebraic summation of the vertical components of all forces acting above a given level including uplift at that level but not the upward reaction; positive when directed downwards.

$\Sigma(P)$ = algebraic summation of the horizontal components of all forces acting above a given level excluding the reaction at that level, positive when directed towards the left.

$\Sigma(Wx)$ = moment about a given point, of the summation $\Sigma(W)$; positive when counter-clockwise.

$\Sigma(Px)$ = moment, as above, for the summation $\Sigma(P)$.

A = area in square feet.

a = distance from the top of the dam to water surface.
c = ratio of the area subjected to uplift to the whole area.
e = distance from the center of gravity of the base to the point of application of the loading.
E = subscript used to represent empty reservoir.
F = subscript used to represent full reservoir.
f = coefficient of static friction as indicated by well-dressed specimens of the materials.
h = vertical distance.
H = total height of a dam above a given level.
I = moment of inertia of a figure in feet units.
k = proportion of voids in earth.
L = top width of a non-overflow dam.
l_o = known length of a horizontal joint.
l = unknown length of a horizontal joint.
m = distance to the right or left of the center of gravity of a figure.
p = unit pressure or compressive stress in pounds per square foot.
p_r' and p_r'' = unit *vertical reaction*, exclusive of uplift, at the toe and heel of a joint.
p_v and p_v' = unit *vertical compressive stress* at the toe and heel of a joint.
p_i' and p_i'' = unit *maximum inclined compressive stress* at the toe and heel of a joint.
p_u' and p_u'' = unit *effective uplift* at the toe and heel of a joint.
p_n' and p_n'' = unit normal pressure of water and earth at the toe and heel of a joint.
q = discharge in cubic feet per second per foot length of crest.
q' = unit load in pounds per square foot.
r = up-stream radius of arch dams.
s = factor of safety.
t = thickness of arch dams.
u = horizontal distance from the down-stream extremity of a joint to the point of intersection of the resultant R.
w = unit weight.
w_1 = unit weight of masonry.
w_2 = unit weight of water.
w_3 = unit weight of earth.
x = lever arm of moments.
y = horizontal distance from the origin of moments to the upstream extremity of the joint.
z = horizontal distance from the origin of moments to the point of intersection of the resultant R with the joint.
α = angle of repose of earth.
θ = angle of inclination of the resultant R with the vertical. (If the base of the dam is inclined, θ is measured from a normal to the base.)
ϕ' and ϕ'' = angle of inclination, with the vertical, of the face of the dam at the toe and heel.

3. Forces Acting on Dams.—A consideration of the following forces is necessary in the complete design of masonry dams:

(*a*) Water pressure including uplift.
(*b*) Earth or silt pressure.
(*c*) Ice pressure.
(*d*) Weight of the dam.
(*e*) Reaction of the foundation.

Some of these forces do not admit of exact determination and certain assumptions must be made for designing purposes, which must be based on the exercise of the engineer's best judgment and experience, and confirmed by what precedent has shown to be safe.

3*a*. Water Pressure.—In Fig. 1, let 1–4 represent a submerged vertical rectangular plane, of area A, and width b. The total pressure, P, of quiet water on each side of this plane is

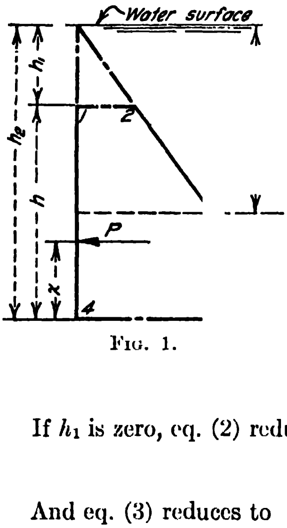


FIG. 1.

$$P = w_2 A h_3 \tag{1}$$

where w_2 is the weight of 1 cu. ft. of water (usually assumed as 62.5 lb. per cu. ft.) and h_3 is the vertical distance from the center of the plane to water surface (all dimensions in feet). This may be reduced to

$$P = \frac{bw_2}{2}\left(h_2^2 - h_1^2\right) \tag{2}$$

The force P will be horizontal and will be located a vertical distance above the bottom of the plane equal to

$$x = \frac{3h_1h + h^2}{6h_1 + 3h} \tag{3}$$

If h_1 is zero, eq. (2) reduces to

$$P = \frac{bw_2h_2^2}{2} \tag{2a}$$

And eq. (3) reduces to

$$\frac{h}{3} \tag{3a}$$

The impact of the approaching water against the upstream face of the dam is approximately

$$P' = \frac{w_2v^2h}{g}$$

where v is the average velocity in the middle of the channel of approach in feet per second, g is the acceleration of gravity = 32.2 and w_2 is the weight of 1 cu. ft. of water. Impact need be considered only for low dams having large discharges. P' may be assumed to act at a distance of $\frac{h}{2}$ above the base.

In the design of dams, it is found convenient to deal with horizontal and vertical forces only. The horizontal component of water pressure on an inclined plane is equal to the water pressure on the vertical projection of the plane.

The vertical component of water pressure on an inclined plane is equal to the weight of water directly above the plane.

Thus, in Fig. 2, the horizontal water pressure P on plane 1-2-3-4 is equal to the pressure on the projected plane 5-4, as indicated by eq. (2), and its distance x above the foundation is found from eq. (3). The vertical component of water pressure w on plane 3-4 is equal to the weight of water within the limits 7-3-4-6 and this force passes through the center of gravity of the area 7-3-4-6.

The vertical component of the water pressure on plane 1-2 as well as the water pressure on the crest and downstream face of the dam due to the spilling water is neglected as the jet approaches very nearly spouting velocity.

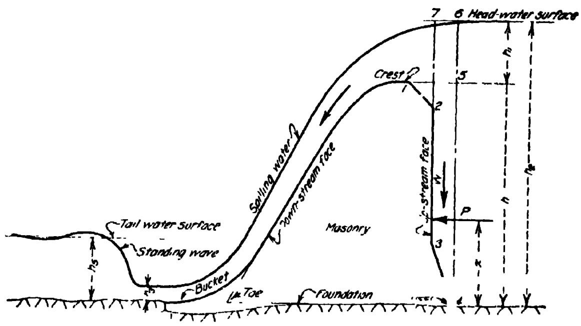


FIG. 2. Solid spillway dam with dimension exaggerated to clarify discussion.

The horizontal and vertical components of tail-water pressure are found in the same manner; but, in spillway dams, the energy of the spilling water may be sufficient to reduce the depth of tail-water by the creation of a "standing wave" as indicated in Fig. 2, thereby eliminating tail-water pressure entirely. This may occur if h_5 is equal to or less than about

$$h_5 \qquad \frac{\quad}{16.1h_3} + \frac{h_3^2}{4} - \frac{h_3}{2} \tag{4}$$

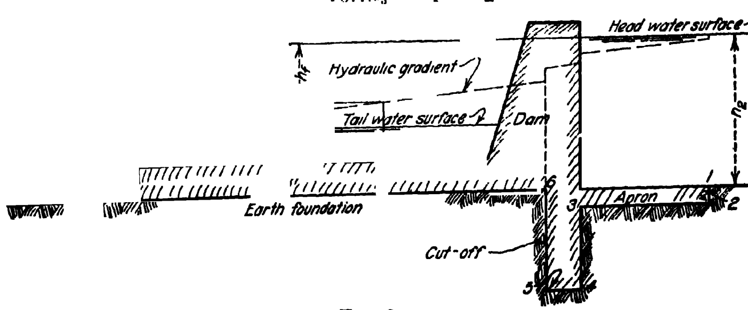


FIG. 3.

where q is the discharge in cubic feet per second per foot length of crest, and other notation as indicated in Fig. 2. The dimension h_3 may be obtained as indicated in Art. 7, Fig. 13.

The presence of tail-water may assist stability or, under certain conditions, may have the opposite effect. Equation (4) is only approximately correct owing to certain factors not possible of consideration in its derivation. Therefore, if the depth of tail-water is within 20 per cent of the value given in eq. (4), its resultant pressure on the dam should be considered only if it tends to reduce stability.

In pervious foundations, an upward water pressure, or "uplift" will occur on the base of the dam. In order to understand better the characteristics of uplift, consider the general case shown in Fig. 3 which indicates a dam on an earth foundation having a cut-off and both an up-stream and a down-stream apron. It has been proven by experiments that the flow of water under dams has pressure characteristics exactly similar to the flow of water in pipes. The pressure at point 1 is equal to head-water pressure h_2, and at point 8 equal to tail-water pressure h_5. The difference, h_2-h_5, is the head lost in friction. The friction loss varies directly as the length of the "path of percolation" between the masonry and the foundation as indicated by the "hydraulic gradient" in Fig. 3. The friction loss between point 1 and any point on the base is therefore proportional to the distance the water has traveled. Thus at point 7 the friction loss is

$$h_f = (h_2 - h_5)\frac{l_7}{l_8}$$

where l_7 is the length of the path of percolation 1–2–3–4–5–6–7 and l_8 the total path of percolation or corresponding distance from point 1 to point 8.

The uplift pressure at any point on the base is therefore equal to the vertical distance from that point to the elevation of head-water surface less the friction head h_f.

In the case of rock foundations, it is assumed that only a percentage, c, of the base is subject to uplift pressure. If the foundation has no cut-off, as in Fig. 2, the resultant uplift on the base may be expressed by the equation

$$W = cw_2bl\frac{h_2 + h_5}{2} \tag{5}$$

The resultant pressure will be located a distance from the heel equal to

$$x = \frac{l(h_2 + 2h_5)}{3(h_2 + h_5)} \tag{6}$$

Absolutely impervious foundations are impossible of attainment. The amount of uplift depends upon the *relative* resistance to flow at the up-stream end of the base to that over the rest of the base. For this reason a special effort is often made to obtain greater resistance at the up-stream end of the base by one of the following methods:

(1) To increase the resistance at the heel.

(2) To decrease the resistance below the heel.

By the first method, trenches are often excavated at the heel and refilled with masonry, or holes may be drilled in the rock and grouted to provide an efficient cutoff. For earth foundations sheet piling cutoffs are common and sometimes upstream aprons are used as indicated in Fig. 3.

By the second method, drainage systems have been installed between the dam and the foundation, down-stream from the heel, to allow free exit of the water which passes the heel.

Percentages of c for rock foundations as high as 0.66 have been used for important structures; but ordinarily a much lower value is commonly adopted. For earth foundations c is always unity.

Uplift should also be considered to exist on all horizontal joints in the masonry above the base. For dams on rock foundations this is usually assumed to be the same as the unit value adopted for the base.

Uplift due to head water is assumed not to exist in hollow dams as the cavities in the structure effectively relieve the buttresses of all such pressure.

For a complete discussion of the subject of uplift see *Transactions*, Am. Soc. C. E., Vol. LXXV, pp. 142–225.

If it is expected that fine silt will be deposited by the stream against the up-stream face and silt pressure included in the forces acting on the dam, it is reasonable to assume that uplift from head-water will not exist. The stability of the dam should be tested with and without the silt pressure.

3b. Earth Pressure.—In practically all streams, considerable quantities of sand, gravel or silt, washed down by floods, are deposited against the up-stream face of dams built across them, unless special sluicing provisions are made to prevent their accumulation.

The horizontal component of earth pressure may be taken from Rankine's well-known equation,

$$P = \frac{w_3h^2}{2}\left(\frac{1 - \sin\alpha}{1 + \sin\alpha}\right) \tag{7}$$

where

P = total pressure in pounds, in addition to the water pressure.
w_3 = weight, in pounds per cubic foot, of the submerged earth.
α = its angle of repose.
h = depth of the earth, in feet.

P is located a distance from the surface of the earth equal to $\frac{2h}{3}$. The unit weight of submerged earth is

$$w_3 = w'_3 - w_2(1 - k) \tag{8}$$

where

w'_3 = its weight in air.
w_2 = unit weight of water.
k = proportion of voids in the earth

The nature of the materials deposited against dams varies considerably and should be investigated for each case. Usual values for w_3 are from 60 to 70 lb. per cu. ft.; but α varies from about 30 deg. for sand and gravel to zero for liquid mud.

The vertical component of earth pressure on an inclined face is equal to the weight of earth directly above that face as indicated for water pressure.

3c. Ice Pressure.—Expansion of ice on the surface of the reservoir due to a change in temperature may exert an overturning force on the dam. The thrust of ice is impossible of exact determination. An extended discussion of ice pressure is given in *Transactions*, Am. Soc. C. E., Vol. LXXV, p. 142.

Important municipal dams in this country have been designed to resist ice pressures varying from 24,000 to 47,000 lb. per lin. ft. of dam; but much lower values may be used, particularly if the dam is in a narrow gorge.

Considerable ice pressure seldom exists at times of flood. It is therefore seldom necessary to design for ice pressure when water surface is above the level of the spillway crest.

3d. The Weight of the Dam.—The weight of masonry varies considerably, depending upon the ingredients of which it is composed. A common value for use in the design is 145 lb. per cu. ft. In important structures careful determination should be made of the weight of the masonry to be used, as the

amount of materials involved for solid dams varies almost directly as the unit weight.

3e. The Reaction of the Foundation.—In Fig. 4 let $\Sigma(W)$ represent the resultant of all vertical forces acting on the dam, including uplift; and let $\Sigma(P)$ represent the resultant of all horizontal forces. The resultant R of $\Sigma(W)$ and $\Sigma(P)$ must be balanced by an equal and opposite reaction of the foundation, consisting of a vertical reaction $\Sigma(W)$ and a horizontal shearing or frictional resistance $\Sigma(P)$. In the following equations all forces are in pounds and dimensions in feet.

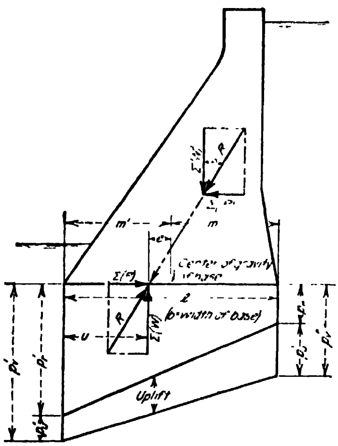


FIG. 4.

The variation of unit vertical reaction is assumed to be linear as indicated in the figure. The unit vertical reaction at the heel and toe is

$$p_r' = \frac{2\Sigma(W)}{bl}\left(2 - \frac{3u}{l}\right) \tag{9}$$

and

$$p_r'' = \frac{2\Sigma(W)}{bl}\left(\frac{3u}{l} - 1\right) \tag{10}$$

If the base is divided into three equal parts, the middle part is termed the "middle third." When the resultant R, intersects the base at the upstream extremity of the middle third,

$$\frac{2l}{3}, \quad p_r' = 0 \quad \text{and} \quad p_r'' = \frac{2\Sigma(W)}{bl} \tag{11}$$

When the resultant intersects the base at the downstream extremity of the middle third,

$$\frac{l}{3}, \quad p_r'' = 0 \quad \text{and} \quad p_r' = \frac{2\Sigma(W)}{bl} \tag{12}$$

When the resultant intersects the base outside of the middle third, tension will exist at the opposite end of the base. Because of the fact that tension in masonry is unreliable and objectionable, it is the usual practice in the design of dams, to provide for the resultant intersecting the base within the middle third. The exception to this rule is in the case of spillway dams where it is impossible to provide sufficient masonry at the top to force the resultant within the middle third. In such cases, tension is allowed if it is not great; although in some cases steel reinforcement has been provided. Figure 7 indicates the location of the resultant near the top of a typical spillway dam. In this case reinforcement would not be necessary.

The foregoing equations indicate the unit vertical *reaction;* but the unit vertical *compressive stress* exceeds the values given by the amount of the uplift pressure.

For the resultant within the middle third

$$p_v' = \frac{2\Sigma(W)}{bl}\left(2 - \frac{3u}{l}\right) + p_u' \tag{9a}$$

$$p_v'' = \frac{2\Sigma(W)}{bl}\left(\frac{3u}{l} - 1\right) + p_u'' \tag{10a}$$

For the resultant at the upstream extremity of the middle third

$$p_v' = 0 + p_u' \tag{11a}$$

$$p_v'' = \frac{2\Sigma(W)}{bl} + p_u'' \tag{11b}$$

For the resultant at the downstream extremity of the middle third

$$p_v' = \frac{2\Sigma(W)}{bl} + p_u' \tag{12a}$$

$$p_v'' = 0 + p_u'' \tag{12b}$$

The foregoing equations are also applicable to the stresses in horizontal joints above the foundations, but to rectangular bases and joints only.

For bases and joints which are not rectangular, the following equations apply:

$$p_v' = + \frac{\Sigma(W)em'}{I} + \frac{\Sigma(W)}{A} + p_u' \tag{13}$$

$$p_v'' = - \frac{\Sigma(W)em''}{I} + \frac{\Sigma(W)}{A} + p_u'' \tag{14}$$

where m' and m'' = distances in feet from center of gravity to the extremities of the base.

I = moment of inertia of the base or joint, in feet units, about an axis through its center of gravity perpendicular to the dam section.

e = eccentricity of the loading in feet.

A = area of the base or joint in square feet.

For no tension to exist, e must have the following value:

$$e = \frac{I}{Am''} \tag{15}$$

The unit *vertical* compressive stresses p_v' and p_v'' are not the maximum compressive stresses, but only the vertical component of such stresses.

Considerable disagreement exists among engineers regarding the maximum inclined stresses. They do not permit of exact determination; but approximate values may be obtained from the following equations:

At the toe of the dam,

$$p_i' = (p_v' \sec^2 \phi' - p_n' \tan^2 \phi') \text{ or } p_n' \text{ or } p_v' \sec^2 \theta \tag{15}$$

At the heel of the dam,

$$p_i'' = (p_v'' \sec^2 \phi'' - p_n'' \tan^2 \phi'') \text{ or } p_n'' \text{ or } p_v'' \sec^2 \theta \tag{16}$$

In the foundation, at the toe,

$$p_i' = p_v' \sec^2 \theta \tag{17}$$

In the foundation, at the heel,

$$p_i'' = p_v'' \sec^2 \theta \tag{18}$$

The distribution of the force $\Sigma(P)$ over the base is not possible of determination. The resistance of the dam to sliding is therefore made to depend solely upon friction which, as will be shown hereinafter, is represented by the inclination θ of the resultant with the vertical.

4. Designing Rules.

4a. Causes of Failure.—There are only two direct ways in which a gravity masonry dam will fail as a whole—that is, by sliding or by overturning.

It may fail by sliding or overturning on a plane above the base, at the base, or on a plane below the base. The last case is apt to occur when the erosive force of spilling water scours out the rock down-stream from the dam.

The cause of sliding is the existence of horizontal forces greater than the combined frictional and shearing resistance at the plane of failure.

Overturning occurs when the resultant R passes outside the limits of the dam and the masonry is not capable of resisting tensile stresses. With the resultant well inside the dam, a failure of the masonry by crushing may result in a narrowing of the limits of the dam an amount sufficient to cause overturning.

4b. **Rule 1: Location of the Resultant.**—The first requirement in a gravity dam is that the resultant shall fall within the limits of the section. It is further required that the resultant shall be so located as to prevent tension in any joint of the dam under all conditions of loading. Equations (11) and (12) indicate that tension in rectangular joints is impossible if the resultant intersects within the middle third. For irregular joints the maximum distance of the resultant from the center of gravity of the joint is given in eq. (15).

4c. **Rule 2: Inclination of the Resultant.**—In order to resist sliding, it is customary to provide that the frictional resistance alone shall be sufficient to resist the resultant of all horizontal forces with ample margin of safety. The frictional resistance is equal to $f\Sigma(W)$ and this should exceed the total horizontal forces $\Sigma(P)$. Expressed algebraically

$$f\Sigma(W) = S\Sigma(P)$$

where S is the factor of safety desired.

Therefore

$$\frac{\Sigma(P)}{\Sigma(W)} = \tan\,\theta = \frac{f}{s} \qquad (19)$$

where $\tan\,\theta$ is the inclination of the resultant R with the vertical.

For rock foundations and horizontal joints in the masonry, s equal to unity may be used provided that f is taken from the results of experiments on well-dressed specimens of like materials. Roughening of the surface of the foundation and building joints during construction and the considerable, though indeterminate, shearing resistance is considered to provide a requisite factor of safety.

For masonry on masonry and for masonry on rock, values of f have been assumed variously between 0.6 and 0.75. For best rock and good workmanship, 0.75 is not excessive; but due allowance must be made for bad conditions and particularly for possible clay seams in the foundations if the rock downstream is apt to scour away and eliminate the toe hold of the dam.

For gravel, sand and clay, approximate values of f are 0.5, 0.4, and 0.3 respectively. A factor of safety, s, of 3 or more should be adopted for earth foundations unless the dam is anchored by deep cut-off walls or piles.

4d. **Rule 3: Compressive Stresses.**—The compressive stresses in the dam and in the foundation should not exceed allowed limits. Because of a disagreement among engineers regarding the amount of inclined compressive stresses, it has been the practice in the past to prescribe certain allowable vertical compressive stresses and design for these stresses only. Such vertical stresses, however, have been chosen low enough to compensate for the fact that they do not represent the maximum stresses which exist.

Equations (9a) to (18) inclusive indicate the vertical and inclined stresses in the dam and the foundation. The angle ϕ will not exceed 45 deg. for well-designed high dams, and θ will be considerably less. Consequently it is seen, from eqs. (15) to (18) that the inclined stresses will not be greater than about twice the vertical stresses. Greater vertical stresses have been allowed at the heel than at the toe of solid dams because, as ϕ'' is much less than ϕ' and as θ is zero for pond empty, the condition of loading giving the greatest stress at the heel, the inclined stresses at the heel will be much less than at the toe.

The following values of working vertical compressive stresses for masonry dams on good rock foundations will ordinarily result in maximum inclined stresses within safe limits:

At the toe of solid dams $\frac{1}{21}$ of the ultimate strength.

At the heel of solid dams $\frac{1}{15}$ of the ultimate strength.

At the toe and heel of hollow dams $\frac{1}{15}$ of the ultimate strength.

For earth foundations, the requirements for Rule 2 necessitate a very small value of θ. Therefore the vertical and inclined stresses in the foundation are approximately equal as indicated by eqs. (17) and (18), and the stresses in earth foundations govern the design. Common values for allowed stresses in earth foundations are:

Clay	8,000 lb. per sq. ft.
Coarse sand	4,000 to 8,000 lb. per sq. ft.
Fine silt	2,000 to 4,000 lb. per sq. ft.

4e. Rule 4: Tension in Vertical Planes.—The inclination ϕ' of the downstream force with the vertical must be limited to prevent failure by tensile stresses in vertical planes. It is obvious that, if ϕ' is very large, the toe of the dam will be long and tapering and as such would not be capable of transferring the proper proportion of the total load to the foundation without tending to crack off. The writer's empirical equations governing the maximum allowed values of ϕ' are

For earth or pile foundations,

$$\tan \phi' = < \sqrt{\frac{10}{H}} \tag{20}$$

where H is the height of the dam.

For rock foundations,

$$\tan \phi' = < \frac{4}{3} f \tag{20a}$$

or

$$\tan \phi' = < \sqrt{\frac{10}{H}} \tag{20b}$$

whichever allows the greater value.

5. General Equations for Design of Solid Gravity Dams.—Gravity dams are designed joint by joint, beginning at the top, making each joint conform to the foregoing designing rules. Assuming the dam to have been designed from the top to the horizontal joint 1-2, Fig. 5, equations may be written to determine the length and position of the joint 3-4 next below. The designing joints are generally adopted a distance apart equal to about 15 per cent of the distance from the joint to the top of the dam; but a considerably greater distance apart

will result in little increased error. The design is thus made progressively from the top to the foundation.

In Fig. 6, which represents a typical non-overflow dam, each zone corresponds to a portion of the structure the design of which is governed by a particular designing rule or combination of rules.

The top width of non-overflow dams is usually adopted 10 to 14 per cent of the maximum height of the dam, although a greater width may be required for low dams to withstand shock from floating bodies or to provide a roadway or platform. A super-elevation above high water surface is sometimes adopted for various purposes. A super-elevation of 5 per cent of the maximum height of the dam may not prove uneconomical.

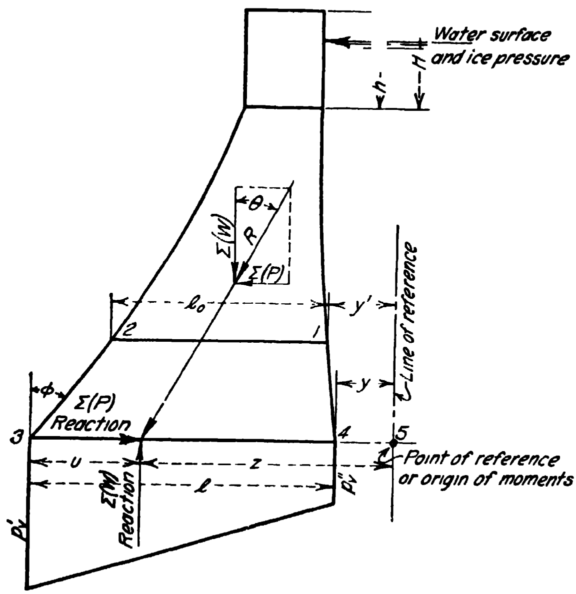


FIG. 5.

Where ice pressure is considered, the weight of masonry in Zone I is fixed by Rule 2 and must be sufficient to prevent that portion from sliding. Either the width of top or the super-elevation must be adjusted, if necessary, to this end.

As the top width is always greater than necessary to conform to Rule 1, Zone II represents that portion of the structure in which the resultants, reservoir full and empty, lie within the middle third of the joints, both faces remaining vertical. At the bottom of Zone II, the resultant, reservoir full, is at the down-stream extremity of the middle third; while, for reservoir empty, the resultant is at the middle of the joint.

In Zone III the down-stream face must begin to batter as indicated in order to accord with Rule 1, the up-stream face remaining vertical until the resultant, reservoir empty, intersects at the up-stream extremity of the middle third.

In Zone IV both up-stream and down-stream faces are battered in order that the resultants, reservoir full and empty, will lie at the exact extremity of the middle third.

As the design is continued, the pressures at the toe and heel will increase. The allowed pressures are usually reached first at the toe. In Zone V, therefore, the design is governed by Rule 3 for full reservoir, Rule 1 still influencing the design for empty reservoir. The resultant, reservoir full, will therefore lie well within the middle third in the lower part of this Zone in order to have sufficient width of joint to distribute the pressures as required by Rule 3.

In Zone VI the allowed working pressures, for both full and empty reservoir, govern, and the design is influenced entirely by Rule 3. In this Zone the resultants

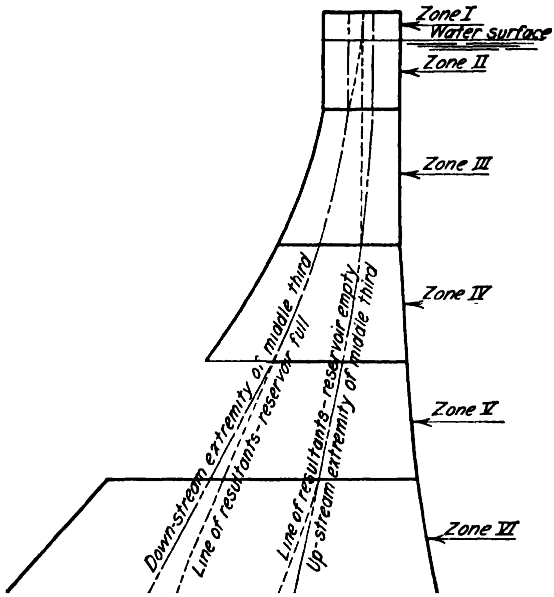


Fig. 6.

for both full and empty reservoir lie well within the middle third. Zone VI will continue until the foundation is reached.

If, at any stage of the design, it is found that tan θ exceeds the value allowed by Rule 2, it will be necessary to re-design the section using an increased batter on the upstream face in order to include a larger vertical component of water pressure to assist in the resistance to sliding. Ordinarily for allowed values of tan θ not less than 0.7 to 0.75, Rule 2 will not be a governing condition for solid dams except in Zone I as previously indicated.

There is considerable opportunity, in dams of several hundred feet in height, for the batter of the down-stream face in the lower part of Zone VI to be flatter than that allowed by Rule 4. If this should occur, the remedy also lies in an increase in the batter of the up-stream face.

In Fig. 7, which represents a typical solid spillway dam, the shape of the crest and downstream face is fixed to conform to the shape of a free spilling weir jet as explained later.

In Zone I the requirements of Rules 1 and 2 must of necessity be violated because it is impossible, near the top of a spillway dam, to provide sufficient masonry to withstand sliding by friction alone or to throw the resultant, reservoir full, within the middle third. In Zone I*a* the section still lacks the necessary weight to accord with Rule 2, but the resultant, reservoir full and empty fall within the middle third in conformity with Rule 1. Unless considerable ice pressure is considered, the height of Zones I and I*a* is small and the tension and horizontal shear can be provided against by monolithic construction above the bottom of Zone I*a*.

When ice pressure occurs, Zones I and I*a* may extend for a considerable distance below the crest. In such cases steel reinforcement may be required to

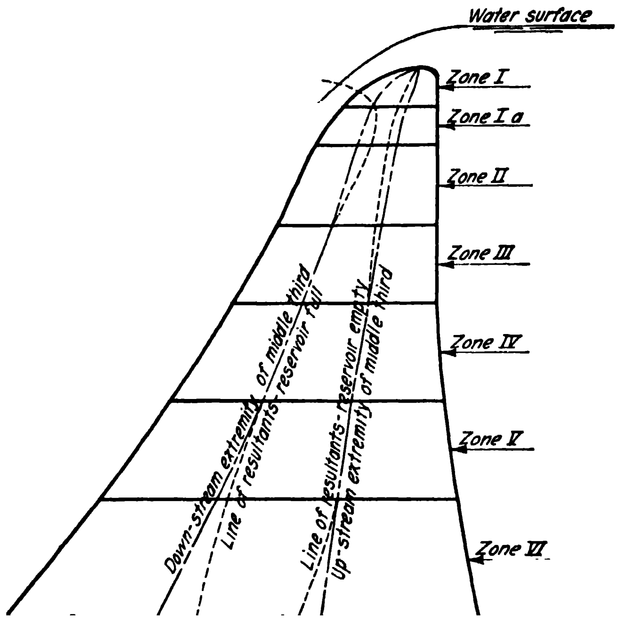


FIG. 7.

resist the tensile stresses and particular care taken with horizontal building joints to provide numerous large projecting stones or special keyways to increase the shearing resistance.

The conditions affecting Zones II to VI inclusive are as previously described for non-overflow dams. In Zone II, however, the downstream face of the spillway sections batters only because of the necessary special shape of the crest and downstream face. The shape of the downstream face, as so fixed, will extend to the bottom of Zone II and its batter will be increased in Zone III.

The arrangement of zones as hereinbefore described may vary somewhat in practice for different types of dams; but the general theory will still apply. In all cases the part of the dam between two horizontal designing joints should be proportioned in accordance with the designing rules which are thought to govern that particular part of the dam and then tested for stability in accordance with all of the other designing rules.

The basic equations for design, termed "equations of investigation" are employed principally to test any predesigned section for conformity with the designing rules. Further equations, derived from the basic equations and termed "equations of determination," are necessary for use in determining the length and location of the horizontal designing joints as the design progresses.

5a. Equations for Rule 1—*Equations of Investigation.*—Referring to Fig. 5, the moment of the resultant R of all forces above the joint 3-4, about the point 5, is $\Sigma(Wx) + \Sigma(Px)$. This is equal to the moment of the reaction at the base, or $\Sigma(W)z$, the moment of the force $\Sigma(P)$ being zero. Therefore

$$\Sigma(Wx) + \Sigma(Px) = \Sigma(W)z$$

or

$$z = \frac{\Sigma(Wx) + \Sigma(Px)}{\Sigma(W)} \tag{21}$$

This is the basic equation for Rule I. It gives the location of the resultant relative to the point 5. According to Rule I, the distance z-y must be less than $\frac{2l}{3}$ for full reservoir and greater than $\frac{l}{3}$ for empty reservoir.

Equations of Determination—Case 1.—The most general equation is applicable to Zone IV where the location of the resultant for both full and empty reservoir governs the design, and the resultants for both cases are required to intersect the joint at opposite extremities of the middle third. The two general equations for this case are

For full reservoir

$$y + \frac{2l}{3} = \frac{\Sigma(Wx)_F + \Sigma(Px)_F}{\Sigma(W)_F} \tag{22}$$

For empty reservoir

$$y + \frac{l}{3} = \frac{\Sigma(Wx)_E + \Sigma(Px)_E}{\Sigma(W)_E} \tag{23}$$

Referring to Fig. 5, it is assumed that the dam has been designed down to joint 1-2. Equations (22) and (23) will assist in the determination of the length l and location y of the next joint 4-3.

In these two equations we have two unknowns, y and l. The values $\Sigma(Wx)$ and $\Sigma(W)$, for both full and empty reservoir, are, however, depiendent upon y and l. The solution of the equations is therefore made by tr(al. Tentative values of y and l are assumed, from which tentative values of $\Sigma Wx)$ and $\Sigma(W)$ may be calculated and substituted in eqs. (22) and (23) to derive more accurate values of y and l. One or two trials will ordinarily be sufficient to determine y and l close enough for all practical purposes.

Case 2.—For Zone III, in which the upstream face is vertical, y is a constant and l can be determined from eq. (22) alone by the method previously described. The length l of the joint having been thus determined, the location of the resultant for reservoir empty can be determined from eq. (21).

Case 3.—The location of the lower extremity of Zone II, for solid non-overflow dams, can be obtained from the following equation, reference being made to Fig. 5.

$$w_2h^3 + (cw_2L^2 + 6P_i - w_1L^2)h = w_1L^2a \tag{24}$$

The location of the bottom of Zones III and IV can be obtained only by trial as the design progresses.

5b. Equation for Rule 2—*Equation of Investigation.*—Equation. (19) is the basic equation of investigation for Rule 2.

Equation of Determination.—If, at any stage of the design it is found that Rule 2 is a governing condition as in Zone I, the required value of $\Sigma(W)$ can be found from eq. (19) using the adopted maximum allowed value of tan θ.

5c. Equations for Rule 3.—*Equation of Investigation.*—Equations (9*a*) and (10*a*) are the basic equations of investigation for Rule 3.

Equations of Determination.—For Zone V, where Rule 3 at the toe and Rule 1 at the heel govern the design, the following equations of determination apply:

For full reservoir:

$$\frac{p_u' - p_v'}{6} l^2 - \frac{\Sigma(W)_F}{3} l = \Sigma(W)_F\, y - \{\Sigma(Wx)_F + \Sigma(Px)_F\} \tag{25}$$

For empty reservoir

$$y + \frac{l}{3} = \frac{\Sigma(Wx)_E + \Sigma(Px)_E}{\Sigma(W)_E} \tag{26}$$

Referring to Fig. 5, it is assumed that the dam has been designed down to joint 1-2. Equations (25) and (26) will assist in the determination of the length l, and location y of the next joint 4-3. In these two equations we have two unknowns l and y, but the values $\Sigma(Wx)$ and $\Sigma(W)$ for both full and empty reservoir, are dependent upon l and y. The solution of the equations is therefore made by trial, adopting tentative values of l and y and solving for closer values as previously described for eqs. (22) and (23).

For Zone VI, where Rule 3 at both toe and heel governs the design, the following equations apply:

For full reservoir

$$\frac{p_u' - p_v'}{6} l^2 - \frac{\Sigma(W)_F}{3} l = \Sigma(W)_F\, y - \{\Sigma(Wx)_F + \Sigma(Px)_F\} \tag{27}$$

For empty reservoir

$$\frac{p_v'' - p_u''}{6} l^2 - \frac{2\Sigma(W)_E}{3} l = \Sigma(W)_E\, y - \{\Sigma(Wx)_E + \Sigma(Px)_E\} \tag{28}$$

These equations can also be solved for l and y by the foregoing tentative method.

5d. Equations for Rule 4.—Equations (20), (20*a*) and (20*b*) may be used for equations for investigation for Rule 4. No equations of determination can be written. If, at any stage of the design, it is found that the inclination of the face is too small, adjustments must be made in the section, as previously described, in order to steepen the face to the desired angle.

6. Design of Solid Non-overflow Gravity Dams.—The design is always started at the top. The super-elevation of the top of the dam above high-water surface is generally made sufficient to get beyond the reach of waves and to provide sufficient weight to resist ice thrust. The width of top is usually from 10 to 14 per cent of the maximum height unless a greater width is required for a roadway, walkway or other purpose.

The top of the dam having been fixed, the design proceeds according to the foregoing methods.

Illustrative Problem.—In this example we will assume:

w_1 = weight of masonry = 145 lb. per cu. ft.
w_2 = weight of water = 62.5 lb. per cu. ft.
p_v' = maximum allowed vertical compressive stress at the toe = 18,000 lb. per sq. ft.
p_v'' = maximum allowed vertical compressive stress at the heel = 25,000 lb. per sq. ft.
H = maximum height of the dam above the foundations = 200 ft.
c = area of joints and base subjected to uplift, assumed to vary uniformly from head-water pressure at the heel to zero at the toe (no tail-water being assumed) = 50 per cent.
f = working value of coefficient of friction of joints and base = 0.75.
L = top width = 12 per cent of the maximum height = 24 ft.
a = distance of top of dam to level of highest head-water surface = 10 ft.
a' = distance from top of dam to level of the spillway crest = 20 ft.
P_i = ice pressure = 40,000 lb. per lin. ft. of dam acting at level of the spillway crest.

The assumption of ice thrust at or below the spillway crest is reasonable in consideration of the fact that, during freshets, the ice sheet is considerably broken up in the vicinity of the dam.

We can at once write another assumption regarding the maximum allowable value of ϕ' to fulfill the condition of Rule 4. Assuming the foundation to be good rock, we have from eq. (20a)

$$\tan \phi' = \frac{4}{3} f = \frac{4 \times 0.75}{3} = 1.0$$

Therefore the maximum allowed value of ϕ' is 45 deg.

Calculations to determine the shape of the section will now be given. The final design is indicated in Fig. 8, taken from the writer's "Engineering for Masonry Dams."[1]

There are, for this example, two conditions of loading:

Loading 1.—Low water and ice pressure.

Loading 2.—High water and no ice pressure.

The dam must be stable above any level for whichever one of these loadings imposes the severest condition. It will be found by trial that loading No. 1 will govern the design above level 126 and No. 2 below that level. This example will be worked out only for that loading which governs the design. Sufficient calculations for each loading should be made as the design progresses to determine which loading imposes the severest conditions.

For reasons not pertinent to the case in hand, this example has been worked out with considerable accuracy; but, in practice, all calculations may be made on a 10-in. slide rule with accuracy well within the precision of the designing assumptions. A 1-ft. length of dam will be considered in the design.

Having fixed the width of top and super-elevation, the first step is to test the section at the level of the bottom of Zone I for conformity with Rule 2 as described in Art. 5b. For substitution in eq. (19) we have, above level 20,

$$\Sigma(P) = \text{ice pressure} = 40{,}000 \text{ lb.}$$
$$\Sigma(W) = \text{weight of masonry} = (24)(20)(145) = 69{,}600 \text{ lb.}$$

Therefore

$$\frac{40{,}000}{69{,}600} = 0.574 = \tan \theta = f$$

since s for rock foundations may be taken as unity. This is well within the allowed value of 0.75.

The next step is to fix the bottom of Zone II, which is determined by the requirements of Rule I described in Case 3 of Art. 5a.

From eq. (24) we have

$$[(0.5)(62.5)(24)(24) + (6)(40{,}000) - (145)(24)(24)]h + 62.5h^3 = (145)(24)(24)(20)$$

from which we find, by successive trials, that $h = 9.27$. Therefore the bottom of Zone II will be 9.27 ft. below the water surface or at level 20 + 9.27 = 29.27.

As explained before, each step of the calculations to determine the shape of the section must be based on the designing rule which is thought to govern, after which the dam above that level must be tested for conformity with all the other designing rules.

[1] John Wiley and Sons, 1917.

Equation (24), which was used to fix the level of the bottom of Zone II, embodies the condition that the resultant reservoir full, shall intersect the joint at the extremity of the middle third. With reservoir empty it is evident that the resultant intersects at the center of the middle third. Hence all conditions of Rule 1 are fulfilled.

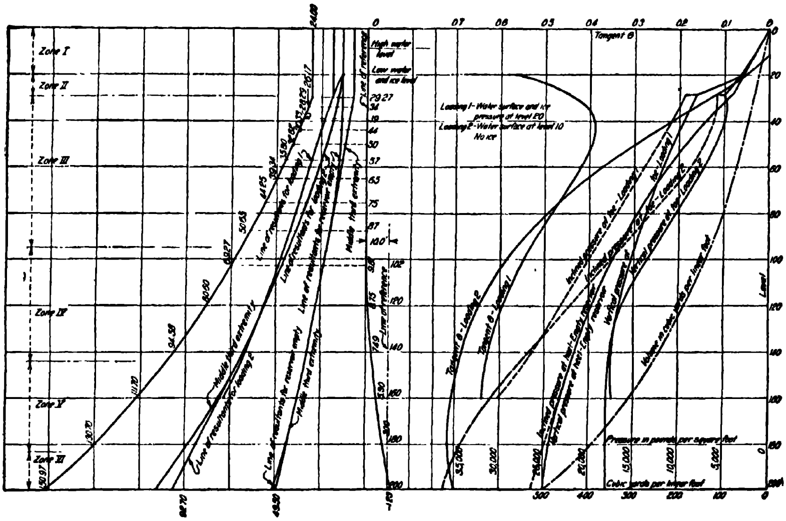


Fig. 8.—Typical 200-ft. solid non-overflow dam.

For Rule 2 we have for substitution in eq. (19).

$$\text{Ice pressure} = \qquad 40{,}000$$

$$\text{Water pressure} = \frac{(9.27)(9.27)(62.5)}{2} = \quad 2{,}685$$

$$\Sigma(P) = \qquad 42{,}685$$

$$\text{Weight of masonry} = (24)(29.27)(145) = 101{,}800$$

$$\text{Uplift} = \frac{(9.27)(62.5)(24)(0.5)}{2} = -3{,}475$$

$$\Sigma(W) = \qquad 98{,}325$$

$$\tan\theta = \frac{42{,}685}{98{,}325} = 0.434$$

which is well within the allowed value of 0.75.

Compressive stresses are never severe for solid dams on rock foundations less than 100 ft. high. Therefore, at level 29.27, Rule 3 cannot be a governing condition. As the down-stream face is vertical, Rule 4 does not govern.

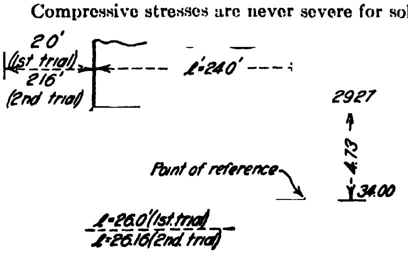


Fig. 9.

The section above level 29.27 is therefore stable.

The height of the section will now be divided into a number of convenient parts by imaginary joints as indicated in Fig. 8 at levels 34, 39, etc. and we will now proceed with the determination of their length and location.

The joint at level 34 is in Zone III and Case 2 of Art. 8a applies. Choosing the point of reference at the upstream face of the dam, y in eq. (22) is zero and we have

$$\frac{2l}{3} = \frac{\Sigma(Wx)_F + \Sigma(Px)_F}{\Sigma(W)_F} \tag{22a}$$

TABLE 1.—COMPUTATIONS FOR JOINT AT LEVEL 34.00—FIRST TRIAL

Item		Factors		Force	Lever		Moment
Vertical forces.	Masonry above level 29.27	From prevous calculation:		101,800	12.00		1,221,600
	Masonry below level 29.27	24 × 4.73 × 145		16,460	12.00		197,600
		2 × 4.73 × 145 ÷ 2		686	24.67		16,920
			$\Sigma(W)_E$	118,946		$\Sigma(Wx)_E$	1,436,120
	Uplift pressure	14 × 62.5 × 26 × 0.5 ÷ 2		−5,690	8.67		−49,230
			$\Sigma(W)_F$	113,256		$\Sigma(Wx)_F$	1,386,800
Horizontal forces	Ice pressure	By assumption:		40,000	14.00		560,000
	Head-water pressure	14 × 14 × 62.5 ÷ 2		6,120	4.67		28,500
			$\Sigma(P)_F$	46,120		$\Sigma(Px)_F$	588,600
					$\Sigma(Wx)_F + \Sigma(Px)_F$		1,975,400

TABLE 2.—COMPUTATIONS FOR JOINT AT LEVEL 34.00—SECOND TRIAL

Item		Factors		Force	Lever		Moment
Vertical forces	Masonry above level 29.27	From previous calculation:		101,800	12.00		1,221,600
	Masonry below level 29.27	24 × 4.73 × 145		16,460	12.00		197,600
		2.16 × 4.73 × 145 ÷ 2		744	24.72		18,410
			$\Sigma(W)_E$	119,004		$\Sigma(Wx)_E$	1,437,510
	Uplift pressure	14 × 62.5 × 26.16 × 5 ÷ 2		−5,725	8.72		−49,900
			$\Sigma(W)_F$	113,279		$\Sigma(Wx)_F$	1,387,610
Horizontal forces	Ice pressure	By assumption:		40,000	14.00		560,000
	Head-water pressure	14 × 14 × 62.5 ÷ 2		6,120	4.67		28,600
			$\Sigma(P)_E$	46,120		$\Sigma(Px)_F$	588,600
					$\Sigma(Wx)_F + \Sigma(Px)_F$		1,976,210

To solve this equation we must assume tentatively a length of joint with which to derive factors for substitution in the second term. Assume, for the first trial $l = 26$ ft. as indicated in Fig. 9. The necessary calculations are indicated in Table 1, substitutions from which in eq. (22a) give

$$\frac{2l}{3} = \frac{1{,}975{,}400}{113{,}256} = 17.44$$

$$l = 26.16$$

Repeating the calculations with a new value of $l = 26.16$ for the second trial, as indicated in Table 2, and substituting in eq. (22a) we have

$$\frac{2l}{3} = \frac{1{,}976{,}210}{113{,}279} = 17.45$$

$$l = 26.17$$

This value of l is near enough to the assumed value of 26.16 and will be called final.

This joint has now been designed for the resultant, reservoir full, to intersect joint at the down-stream extremity of the middle third on the assumption that the resultant, reservoir empty, is within the middle third. To test for this assumption, we have eq. (21). Placing the subscript $_E$ to represent the condition of empty reservoir, we have

$$z = \frac{\Sigma(Wx)_E + \Sigma(Px)_E}{\Sigma(W)_E} \tag{21a}$$

Substituting from Table 2, we have

$$z = \frac{1{,}437{,}510 + 0}{119{,}004} = 12.07$$

This is the distance from the point of reference to the location of the resultant. As the distance from the point of reference to the upstream extremity of the middle third is 26.17 $\div$ 3 = 8.72, it is seen that the resultant, for this case, is well within the middle third and both conditions of Rule I have been fulfilled.

To test for Rule 2 we have, from eq. (19),

$$\tan\theta = \frac{\Sigma(P)_F}{\Sigma(W)_F} = \frac{46{,}120}{113{,}279} = 0.407$$

which is within the allowed value of 0.75.

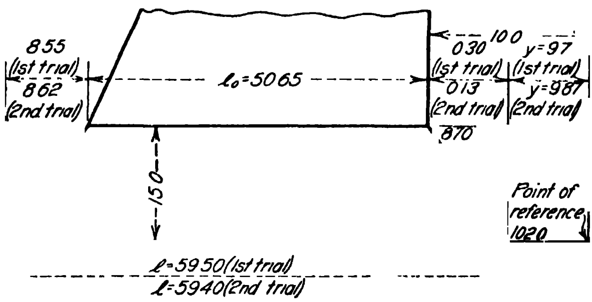


FIG. 10.

As explained before, Rule 3 is not a governing condition for this level.

For Rule 4, we have

$$\tan\phi' = \frac{2.17}{4.73} = 0.459$$

which is within the allowed value of 0.75.

Thus the dam is designed and tested above level 34.0.

The design is continued in this manner, the results of all calculations being tabulated as indicated in Table 7 and plotted in Fig. 8. The calculations and plotting should be made to an extent sufficient to predict when it becomes necessary to change from one zone to another and from one condition of loading to another. For the reader's information, the calculations and plottings have been made, for this example, to a greater extent than would ordinarily be necessary in practice. It is, in many cases, perfectly obvious which designing rule should be used to fix the dimensions of the sections at the various levels. The section would therefore have to be tested for other designing rules only at levels relatively far apart.

It is seen from Fig. 8 that, when level 87 is reached, the resultant, reservoir empty, is about to pass outside the extremity of the middle third. Therefore the next joint at level 102 is in Zone IV and Case 1 of Art. 5*a* will apply. It will now be found convenient to locate the point of reference well outside the base, say 10 ft., as in Fig. 8.

As explained heretofore, tentative values of y and l must be adopted for calculating approximately the second terms of eqs. (22) and (23). From the general trend of the downstream face and the knowledge that the upstream face must begin to batter slightly, tentative values of $y = 9.7$ and $l = 59.5$ will be adopted as indicated in Fig. 10.

With these values, Table 3 is computed, substitutions from which in eqs. (22) and (23) give

$$y + \frac{2l}{3} = \frac{22{,}460{,}100}{453{,}982} = 49.47$$

$$y + \frac{l}{3} = \frac{15{,}675{,}730 + 0}{528{,}826} = 29.67$$

Solving these two equations there results

$$l = 59.40$$
$$y = 9.87$$

Using these for new tentative values for a second set of calculations as in Table 4, there results

$$l = 59.46$$
$$y = 9.86$$

These values are close enough to the tentative values of $l = 59.40$ and $y = 9.87$ and will be called final.

Since eqs. (22) and (23) embody the condition that the resultants, reservoir full and empty, are at the extremity of the middle third, the joint fulfills the conditions of Rule 1.

For Rule 2 we have, for loading 1,

$$\tan \theta = \frac{\Sigma(P)_F}{\Sigma(W)_F} = \frac{250{,}100}{453{,}553} = 0.552$$

However, tan θ, determined in the same way for loading 2, will be found to be 0.595 as indicated in Fig. 8 and is the governing condition for Rule 2, although both are well within the allowed limit of 0.75.

The compressive stresses, affecting Rule 3, are greatest at the toe for full reservoir and at the heel for empty reservoir.

For full reservoir we have for substitution in eq. (9*a*)

$$\Sigma(W) = \Sigma(W)_F = 453{,}550 \text{ from Table 4}$$
$$l = 59.46$$
$$b = 1.0$$
$$u = \frac{l}{3} = 19.82$$
$$p_u' = 0, \text{ as there is no tail-water}$$

Therefore

$$p_v' = \frac{(2)(453{,}550)}{(1.0)(59.46)}\left[2 - \frac{(3)(19.82)}{59.46}\right] + 0 = 15{,}250$$

For empty reservoir we have for substitution in eq. (10*a*)

$$\Sigma(W) = \Sigma(W)_E = 528{,}720$$
$$b = 1.0$$
$$u = \frac{2l}{3} = 39.64$$
$$p_u'' = 0, \text{ as, for empty reservoir there is no head-water pressure.}$$

Therefore

$$p_v'' = \frac{(2)(528{,}720)}{(1.0)(59.46)}\left[\frac{(3)(39.64)}{59.46} - 1\right] + 0 = 17{,}770$$

These stresses are well within the allowed values of $p_v' = 18{,}000$ and $p_v'' = 25{,}000$ and Rule 3 is followed.

As a matter of interest, the maximum inclined stresses will now be computed from eqs. (15) and (16). Equations (17) and (18) are not necessary unless the strength of the foundation is less than that of the masonry.

TABLE 3.—COMPUTATIONS FOR JOINT AT LEVEL 102.0—FIRST TRIAL

	Item	Factors		Force	Lever		Moment
Vertical forces	Masonry above level 87.0	From previous calculation:		409,000	27.35		11,190,000
	Masonry below level 87.0	0.3 × 15 × 145 ÷ 2		326	9.90		3,230
		50.56 × 15 × 145		110,200	35.32		3,892,000
		8.55 × 15 × 145 ÷ 2		9,300	63.50		590,500
			$\Sigma(W)_E$	528,826		$\Sigma(Wx)_E$	15,675,730
	Vertical component of head-water pressure	0.3 × 67 × 62.5		1,256	9.85		12,370
		0.3 × 15 × 62.5 ÷ 2		100	9.80		1,000
				530,182			15,689,100
	Uplift pressure	82 × 59.5 × 62.5 × 0.5 ÷ 2		−76,200	29.53		−2,250,000
			$\Sigma(W)_F$	453,982		$\Sigma(Wx)_F$	13,439,100
Horizontal forces	Ice pressure	By assumption:		40,000	82.00		3,280,000
	Horizontal component of head-water pressure	82 × 82 × 62.5 ÷ 2		210,100	27.33		5,471,000
			$\Sigma(P)_F$	250,100		$\Sigma(Px)_F$	9,021,000
					$\Sigma(Wx)_F + \Sigma(Px)_F$		22,460,100

TABLE 4.—COMPUTATIONS FOR JOINT AT LEVEL 102.0—SECOND TRIAL

	Item	Factors		Force	Lever		Moment
Vertical forces	Masonry above level 87.0	From previous calculation:		409,000	27.35		11,190,000
	Masonry below level 87.0	0.13 × 15 × 145 ÷ 2		141	9.96		1,410
		50.65 × 15 × 145		110,200	35.32		3,892,000
		8.62 × 15 × 145 ÷ 2		9,380	63.52		596,000
			$\Sigma(W)_E$	528,721		$\Sigma(Wx)_E$	15,679,410
	Vertical component of head-water pressure	0.13 × 67 × 62.5		871	9.94		8,660
		0.13 × 15 × 62.5 ÷ 2		61	9.92		600
				529,653			15,688,760
	Uplift pressure	82 × 59.4 × 62.5 × 0.5 ÷ 2		−76,100	29.67		−2,257,000
			$\Sigma(W)_F$	453,553		$\Sigma(Wx)_F$	13,431,670
Horizontal forces	Ice pressure	By assumption:		40,000	82.00		3,280,000
	Horizontal component of head-water pressure	82 × 82 × 62.5 ÷ 2		210,000	27.33		5,741,000
			$\Sigma(P)_F$	250,100		$\Sigma(Px)_F$	9,021,000
					$\Sigma(Wx)_F + \Sigma(Px)_F$		22,452,670

Factors for use in eqs. (15) and (16) are:

At the toe of the dam, $\tan \phi' = \frac{8.67}{15} = 0.577$; $\sec^2\phi' = 1.330$

At the heel of the dam, $\tan \phi'' = \frac{0.12}{15} = 0.008$; $\sec^2\phi'' = 1.00$

For full reservoir, $\tan \theta = 0.552$; $\sec^2\theta = 1.305$

For empty reservoir, $\tan \theta = 0.0$; $\sec^2\theta = 1.00$

p_n' is zero for full reservoir.

p_n'' is zero for empty reservoir.

Therefore, from eq. (15), there results

$$p_v' = [(15,250)(1.33) - 0] \text{ or } 0, \text{ or } (15,250)(1.305)$$
$$p_v' = 20,300 \text{ the greatest of these values.}$$

And from eq. (16)

$$p_v'' = [(17,770)(1) - 0] \text{ or } 0, \text{ or } (17,770)(1)$$
$$p_v'' = 17,770, \text{ the greatest of these values.}$$

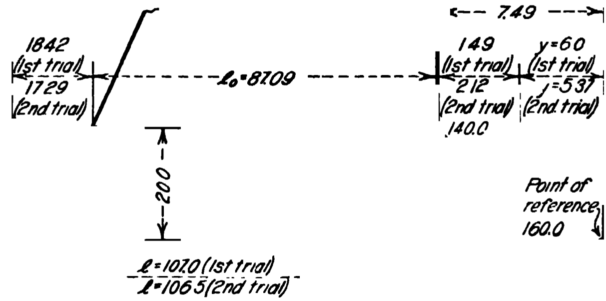


FIG. 11.

For Rule 4, we find that the values of $\tan \phi' = 0.577$, derived above, does not exceed the allowed value of 1.0. The section is therefore designed and tested above level 102. The succeeding joints are fixed in the same manner.

Referring to Fig. 8, it is seen that, at level 120, the lines of resultants for loadings 1 and 2 will cross before the next joint to be designed is reached. Therefore the joints below level 120 should be designed for loading 2.

After the joint at level 140 is designed, it will be seen that the vertical pressures at the toe for loading 2 is about to exceed the allowed value of 18,000 lb. per sq. ft. It is evident, therefore, that the next joint lies in Zone V and eqs. (25) and (26) of Art. 5c apply.

Assuming tentative values of $l = 107.0$ and $y = 6.0$, the factors for use in these equations are shown in Table 5. Making the proper substitutions we have, for eq. (25)

$$\frac{18,000}{6}l^2 - \frac{991,900}{3}l = 991,900y - 74,513,440$$

$$l^2 + 110.21l = -330.63y + 24,838 \qquad (25a)$$

And, for eq. (26),

$$y + \frac{l}{3} = \frac{49,580,000 + 0}{1,212,780} = 40.87 \qquad (26a)$$

Solving for l and y from eqs. (25a) and (26a), there results

$$l = 106.5$$
$$y = 5.37$$

Using these for closer values of l and y in Table 6 and proceeding as before, we find

$$l = 106.4$$
$$y = 5.30$$

which are near enough to be adopted.

The joint should now be tested for the other designing rules as previously indicated. The necessary calculations will not be repeated.

TABLE 5.—COMPUTATIONS FOR JOINT AT LEVEL 160.0—FIRST TRIAL

	Item	Factors		Force	Lever		Moment
Vertical forces.	Masonry above level 140	From previous claculation:		931,200			33,985,000
	Masonry below level 140	1 49 × 20 × 145 ÷ 2		2,160	6 99		15,100
		87 09 × 20 × 145		252,700	51.03		12,890,000
		18.42 × 20 × 145 ÷ 2		26,720	100.72		2,690,000
			$\Sigma(W)_E$	1,212,780		$\Sigma(Wx)_E$	49,580,000
	Vertical component of head-water pressure	Above level 140, from previous calculation:		16,870			145,740
		1 49 × 130 × 62 5		12,110	6 74		81,650
		1 49 × 20 × 62.5 ÷ 2		930	6.50		6,050
				1,242,700			49,813,400
	Uplift pressure	150 × 107 × 62.5 × 0.5 ÷ 2		−250,800	41.67		−10,450,000
			$\Sigma(W)_F$	991,900		$\Sigma(Wx)_F$	39,363,440
Horizontal forces.	Horizontal component of head-water pressure	150 × 150 × 62.5 ÷ 2	$\Sigma(P)_F$	703,500	50.00	$\Sigma(Px)_F$	35,150,000
					$\Sigma(Wx)_F + \Sigma(Px)_F$		74,513,440

TABLE 6.—COMPUTATIONS FOR JOINT AT LEVEL 160.0—SECOND TRIAL

	Item	Factors		Force	Lever		Moment
Vertical forces.	Masonry above level 140	From previous calculation:		931,200			33,985,000
	Masonry below level 140	2.12 × 20 × 145 ÷ 2		3,070	6.78		20,830
		87.09 × 20 × 145		252,600	51.03		12,890,000
		17.29 × 20 × 145 ÷ 2		25,080	100.34		2,515,000
			$\Sigma(W)_E$	1,211,960		$\Sigma(Wx)_E$	49,410,830
	Vertical component of head-water pressure	Above level 140, from previous calculation:		16,780			145,740
		2.12 × 130 × 62.5		17,220	6.43		110,800
		2.12 × 20 × 62.5 ÷ 2		1,320	6.08		8,060
				1,247,270			49,675,430
	Uplift pressure	150 × 62.5 × 106.5 × 0.5 ÷ 2		−249,800	40.87		−10,200,000
			$\Sigma(W)_F$	997,470		$\Sigma(Wx)_F$	39,475,430
Horizontal forces.	Horizontal component of head-water pressure	150 × 150 × 62.5 ÷ 2	$\Sigma(P)_F$	703,000	50.00	$\Sigma(Px)_F$	35,150,000
					$\Sigma(Wx)_F + \Sigma(Px)_F$		74,625,430

TABLE 7.—RESULTS OF CALCULATIONS FOR EXAMPLE NO. 1 OF ART. 38

Vertical forces

Level of joint	Weight of masonry $\Sigma(W)_E$	Vertical component of head-water pressure		Uplift pressure		Summation of vertical forces, reservoir full, $\Sigma(W)_F$	
	Reservoir empty	Loading 1	Loading 2	Loading 1	Loading 2	Loading 1	Loading 2
29.27	101,800			3,475	7,230	98,320	94,570
34	119,000			5,725	....	113,300	
39	138,700			8,400	12,820	130,300	125,900
44	160,000			11,380	....	148,600	
50	187,500			15,400	20,500	172,100	167,000
57	222,300			20,700	....	201,600	
65	265,900			27,700	33,830	238,200	232,100
75	326,500			38,050	....	288,500	
87	409,000			53,050	60,950	356,000	348,100
102	528,700	932		76,100	... ...	454,000	
120	700,700	,096	7,700	112,800	123,600	596,000	584,800
140	931,300	,770	16,780	163,200	176,800	784,800	771,300
160	1,212,000	,450	35,320	232,500	249,800	,011,000	997,500
180	1,551,000		56,320		338,300		1,296,000
200	1,957,000		103,300		453,000		1,607,000

Horizontal forces

Level of joint	Horizontal ice pressure	Horizontal component of head-water pressure		Summation of horizontal forces, reservoir full, $\Sigma(P)_F$	
	Loading 1	Loading 1	Loading 2	Loading 1	Loading 2
29.27	40,000	2,865	11,600	42,685	11,600
34	40,000	6,120	... . ..	46,120	
39	40,000	11,280	26,280	51,280	26,280
44	40,000	18,000	....	58,000	
50	40,000	28,120	50,000	68,120	50,000
57	40,000	42,750	... .	82,750	
65	40,000	63,300	94,550	103,300	94,550
75	40,000	94,600	....	134,600	
87	40,000	140,200	185,300	180,200	185,300
102	40,000	210,100	....	250,100	
120	40,000	312,500	378,000	352,500	378,000
140	40,000	450,000	528,100	490,000	528,100
160	40,000	612,000	703,000	652,000	703,000
180	40,000	....	903,000	. ....	903,000
200	40,000	....	1,128,000	....	1,128,000

TABLE 7.—RESULTS OF CALCULATIONS FOR EXAMPLE NO. 1 OF ART. 38.—(*Continued*)

Level of joint	Moments of vertical forces: Moment of weight of masonry $\Sigma(Wx)_E$	Moment of vertical component of head-water pressure		Moment of uplift pressure	
	Reservoir empty	Loading 1	Loading 2	Loading 1	Loading 2
29.27	1,222,000			27,800	57,850
34	1,438,000			49,900	
39	1,706,000			79,200	120,900
44	2,018,000			115,000	
50	2,451,000			168,400	221,300
57	3,050,000			247,000	
65	3,869,000			363,000	443,500
75	5,137,000			561,000	
87	7,098,000			894,000	1,029,000
102	15,680,000	9,260		2,257,000	
120	22,930,000	75,190	72,010	3,688,000	4,040,000
140	33,980,000	115,700	145,700	5,965,000	6,465,000
160	49,410,000	258,700	264,600	9,180,000	10,200,000
180	70,860,000		352,400	. . .	15,420,000
200	99,750,000		393,400	.	22,490,000

Level of joint	Moments of horizontal forces: Arm and moment of ice pressure		Arm and moment of horizontal component of head-water pressure				Total moments: Summation of moments of all forces, full reservoir $\Sigma(Wx)_F + \Sigma(Px)_F$	
	Loading 1		Loading 1		Loading 2		Loading 1	Loading 2
29.27	9.27	370,800	3.09	8,300	6.42	74,450	1,573,000	1,238,000
34	14.00	560,000	4.67	28,600	....	....	1,976,000	
39	19.00	760,000	6.33	71,400	9.67	254,200	2,458,000	1,839,000
44	24.00	960,000	8.00	144,000	.. ..	....	3,007,000	
50	30.00	1,200,000	10.00	281,200	13.33	666,500	3,764,000	2,894,000
57	37.00	1,480,000	12.33	527,500	.....	....	4,810,000	
65	45.00	1,800,000	15.00	950,000	18.33	1,731,000	6,256,000	5,159,000
75	55.00	2,200,000	18.33	1,734,000	.....	....	8,510,000	
87	67.00	2,680,000	22.33	3,133,000	25.67	4,755,000	12,020,000	10,820,000
102	82.00	3,280,000	27.33	5,741,000	.....	....	22,450,000	
120	100.00	4,000,000	33.33	10,420,000	36.67	13,860,000	33,730,000	32,820,000
140	120.00	4,800,000	40.00	18,000,000	43.33	22,890,000	50,970,000	50,560,000
160	140.00	5,600,000	46.67	28,600,000	50.00	35,150,000	74,390,000	74,630,000
180	......	....	.. ..	....	56.67	51,220,000	....	106,800,000
200	....	.. ....	.....	....	63.33	71,450,000	....	148,900,000

TABLE 7.—RESULTS OF CALCULATIONS FOR EXAMPLE NO. 1 OF ART 38.—(*Continued*)

Level of joint	Location of resultants: Distance from point of reference, Reservoir empty	Distance from point of reference, Loading 1	Distance from point of reference, Loading 2	Length of base	Distance of heel from point of reference	Tangent φ: heel	Tangent φ: φ'	Tangent θ: Loading 1	Tangent θ: Loading 2
29.27	12.00	16.01	13.04	24	0	0	0	0.434	0.123
34	12.07	17.45	.....	26.	0	0	0.459	0.407	
39	12.31	18.86	14.61	28	0	0	0.424	0.394	0.209
44	12.60	20.22	.....	30	0	0	0.408	0.390	
50	13.07	21.88	17.33	32.	0	0	0.415	0.396	0.299
57	13.72	23.87		35	0	0	0.426	0.410	
65	14.55	26.33		39	0	0	0.442	0.434	0.407
75	15.73	29.50		44	0	0	0.491	0.467	
87	17.35	33.77		50	0	0	0.533	0.506	0.533
102	29.68	49.50		59.	9.87	0.009	0.575	0.552	
120	37.70	56.65	56.15	71	8.75	0.059	0.630	0.592	0.646
140	36.52	64.90	65.55	87.	7.49	0.063	0.699	0.624	0.685
160	40.80	73.30	74.80	106	5.30	0.019	0.856	0.643	0.705
180	45.60		84.20	127	3.00	0.115	0.950		0.712
200	50.85		92.70	152	−1.23	0.212	1.013		0.702

Maximum pressure, in thousands of pounds per square foot

Level of joint	At heel: Reservoir empty, Vertical	At heel: Reservoir empty, Inclined	At toe: Loading 1, Vertical	At toe: Loading 1, Inclined	At toe: Loading 2, Vertical	At toe: Loading 2, Inclined
29.27			8.2	8.2	5.0	5.0
34						
39			9.2	10.9	4.9	5.8
44						
50	9.2		10.5	12.3	5.9	7.0
57						
65	12.0		12.1	14.5	8.2	9.8
75						
87	15.7		14.1	18.1	11.6	14.8
102	17.8	17.8	15.3	20.3		
120	19.5	19.5	16.6	23.1	15.9	22.2
140	21.4	21.4	17.6	26.3	17.7	26.4
160	22.8	23.1	17.5	30.3	18.0	31.2
180	24.3	24.6			18.1	34.4
200	25.0	26.2			18.0	36.4

* It will be noticed that the last value of tan φ' was slightly in excess of the allowed value of 1.0. The difference, however, was not considered important enough to warrant correction.

After the joint at level 180 is reached, it is seen that the vertical pressure at the heel of the dam is about to exceed the allowed limit of 25,000. The joint at level 200 is therefore in Zone VI and can be calculated from eqs. (27) and (28) of Art. **5c** and then tested in the manner previously explained for the joint at level 160.

All angles on the faces may now be smoothed up with fillets to obtain an unbroken surface.

In this example the design has not been influenced by the maximum allowed values of tan θ, or tan ϕ' as affecting Rules 2 and 4. If, at any stage of the design, these allowed values are exceeded, the design can be carried through only by a cut and try method as explained in Art. **5**.

7. Details of Design of Solid Spillway Gravity Dams.—The first step in the design of spillway dams is the determination of the elevation of head-water above the spillway crest corresponding to the maximum flood to be expected.

Francis' equation for discharge over dams is:

$$Q = ql_n = C_1\ Cl_n\{(h_c + h_v)^{3/2} - h_v^{3/2}\} \tag{29}$$

where Q = total discharge in cubic feet per second.

q = discharge per linear foot of effective crest.

l_n = net or effective length of crest *i.e.*, the total length of crest corrected for end contractions due to piers and sharp-cornered abutments.

h_c = actual or measured head on the crest, taken at a point sufficiently remote from the dam to avoid the surface curve.

h_v = head corresponding to the velocity of approach.

C = coefficient which depends on the shape of the crest and the head on the crest.

C_1 = correction for submergence.

The head h_v corresponding to the velocity of approach is equal to $\frac{v^2}{2g}$, where v is the mean velocity of approach.

The coefficient C for use in determining the head, corresponding to the maximum flood to be expected, may be taken as 3.9 provided the crest is given the shape recommended hereinafter.

The net length of crest is the total clear length between piers and abutments corrected for contractions which will occur at each face of the piers and abutments. Experiments have indicated that contractions are proportional to the head on the crest and, therefore,

$$l_n = l_t - h_c(C_an_a + C_bn_b\ .\ .\ .\ C_nn_n) \tag{30}$$

where l_t = total clear length of crest.

C_a, C_b, etc. = contraction coefficients applicable to the several different contractions which may be expected.

n_a, n_b, etc. = number of contractions having contraction coefficients, C_a, C_b, etc., respectively. There are two contractions for each pier and one for each abutment.

The contraction coefficient C varies from 0.1 for sharp-cornered abutments and thick, blunt piers, to practically zero for well-rounded corners of abutments and for piers of very small thickness and very pointed upstream. It is not practical to provide very sharp-pointed piers on account of structural weakness and danger of damage from floating ice and debris. For the usual type of piers, having a 90-

deg. upstream point, as indicated in Fig. 12, a value of $C = 0.04$ is usually adopted.

If tail-water rises above the elevation of crest of the dam, the dam becomes entirely submerged and the discharge is reduced. The value of C_1 depends upon the extent of submergence as indicated in Table 8. In this table, h_s represents the depth of tail-water above the crest of the dam.

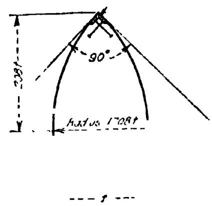


FIG. 12 Typical sharp nose pier.

TABLE 8.—VALUES OF THE COEFFICIENT OF SUBMERGENCE[1]

$\frac{h_c}{h_s}$	C_1	$\frac{h_c}{h_s}$	C_1
0.0	1.000	0.5	0.937
0.1	0.991	0.6	0.907
0.2	0.983	0.7	0.856
0.3	0.972	0.8	0.778
0.4	0.956	0.9	0.621
		1.0	0.000

[1] U. S. Water Supply and Irrigation Paper No. 200 by R. E. HORTON.

Having fixed the head on the crest, the shape of the upper part of the dam must be determined. If the sheet of water, spilling over the crest of a dam, jumps clear of the downstream face, a tendency to form a vacuum exists under the sheet, unless it has ample access to the atmosphere. A partial vacuum will add to the overturning forces acting on the dam. For this reason, and because it is advisable to avoid impact and scour at the toe, the top of the dam and the downstream face, under usual conditions, should be shaped to correspond to the curve of the underside of the sheet of water, due to the maximum flood to be expected. That is to say, the downstream face should have a slope at all elevations no steeper than that curve. For low dams on good rock foundations, this condition may not be required if the depth of head-water over the crest is small and provision is made to prevent a vacuum.

Figure 13 and Table 9 indicate the recommended shape of the crest and the maximum slope of the downstream face for spillway dams. The dotted lines in Fig. 13 show the upper and lower nappe of the freely flowing water. The full line, indicating the recommended masonry line is not as steep as the theoretical water lines on account of the necessity of providing a margin of safety, the water lines being projected from experiments relatively close to the crest.

The requirements of Rules 1 and 2 can never be met within Zone I and stability must be provided by the special construction described in Art. 5.

The lower limit of Zone II can only be obtained by calculating the location of the resultant, reservoir full, at successive levels until a level is reached where the resultant intersects the joint at the exact extremity of the middle third. The downstream face of the dam will have the shape indicated in Table 9 until this level is reached, below which the dam, in consideration of other requirements,

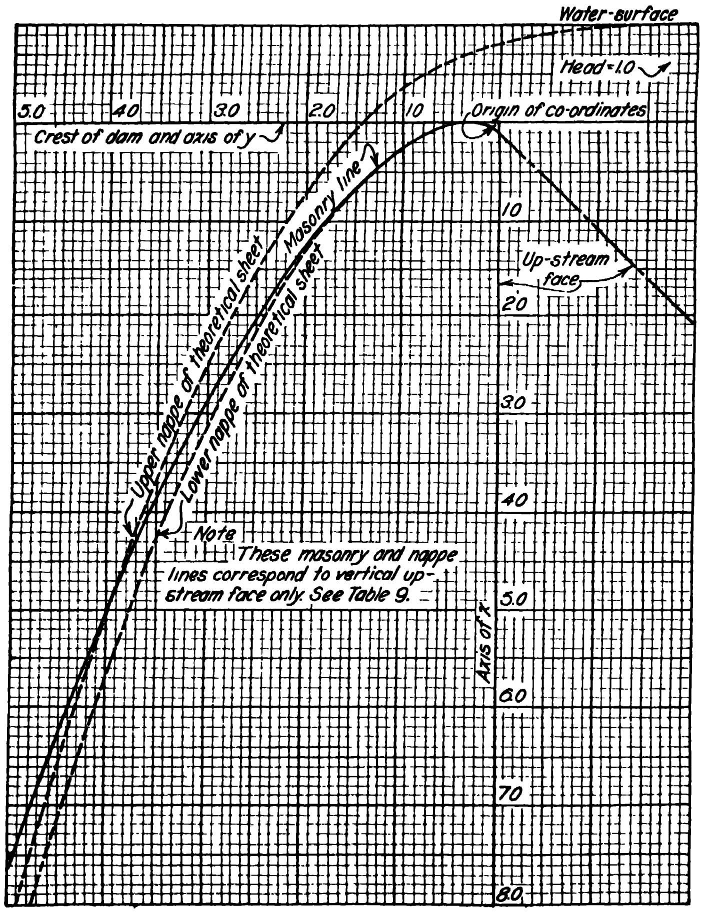


FIG. 13.—Recommended shape of crest and downstream face for spillway dams.

will have a greater thickness and a flatter downstream face than would be required to fit the sheet of spilling water.

Below the lower limit of Zone II the design is carried out exactly as previously described for solid non-overflow dams until the level of the foundation is reached. After the shape of the section is determined, a bucket (Fig. 2, p. 533) should be added at the toe to deflect the spilling water away from the dam. The bucket is sometimes omitted for small dams and for small maximum discharges if the foundation is hard rock.

TABLE 9.—VALUES OF COORDINATES IN FIG. 13 FOR UNITY HEAD ON CREST
(Multiply all quantities in the table by actual head on crest)

	x for vertical upstream face			x for upstream face inclined 45°		
y	Masonry line	Theoretical water sheet		Masonry line	Theoretical water sheet	
		Upper nappe	Lower nappe		Upper nappe	Lower nappe
0.0	0.126	−0.831	0.126	0.043	−0.781	0.043
0.1	0.036	−0.803	0.036	0.010	−0.756	0.010
0.2	0.007	−0.772	0.007	0.000	−0.724	0.000
0.3	0.000	−0.740	0.000	0.005	−0.689	0.005
0.4	0.007	−0.702	0.007	0.023	−0.648	0.023
0.6	0.060	−0.620	0.063	0.090	−0.552	0.090
0.8	0.142	−0.511	0.153	0.189	−0.435	0.193
1.0	0.257	−0.380	0.267	0.321	−0.293	0.333
1.2	0.397	−0.219	0.410	0.480	−0.121	0.500
1.4	0.565	−0.030	0.590	0.665	0.075	0.700
1.7	0.870	0.305	0.920	0.992	0.438	1.05
2.0	1.22	0.693	1.31	1.377	0.860	1.47
2.5	1.96	1.50	2.10	2.14	1.71	2.34
3.0	2.82	2.50	3.11	3.06	2.76	3.39
3.5	3.82	3.66	4.26	4.08	4.00	4.61
4.0	4.93	5.00	5.61	5.24	5.42	6.04
4.5	6.22	6.54	7.15	6.58	7.07	7.61

8. The Design of Hollow Dams.—While hollow dams of many types have been constructed, the usual type consists of a series of parallel equidistant concrete buttresses covered by a water-tight deck on the upstream side and, for spillways, a downstream apron and bucket to support the sheet of spilling water.

The general theory of design given heretofore for solid dams will apply also to hollow dams. The latter, however, admit of no direct economical methods of design and the shape of the buttresses, type of decking, and other details are worked up in accordance with the judgment of the designer and the structure tested for conformity with the designing rules heretofore given.

Hollow dams have been constructed on good rock foundations to heights considerably in excess of 100 ft. Only conservatively designed low hollow dams should be built on earth foundations as unequal settlement is sure to cause stresses not possible to provide for in the design.

Hollow dams are considered more economical than solid gravity dams for remote locations where the unit cost of concrete is quite high and formwork relatively inexpensive.

The choice of top width and super-elevation above water surface for hollow non-overflow dams is fixed according to judgment as outlined for solid dams. The shape of the crest and the upper part of the downstream face of spillway dams is

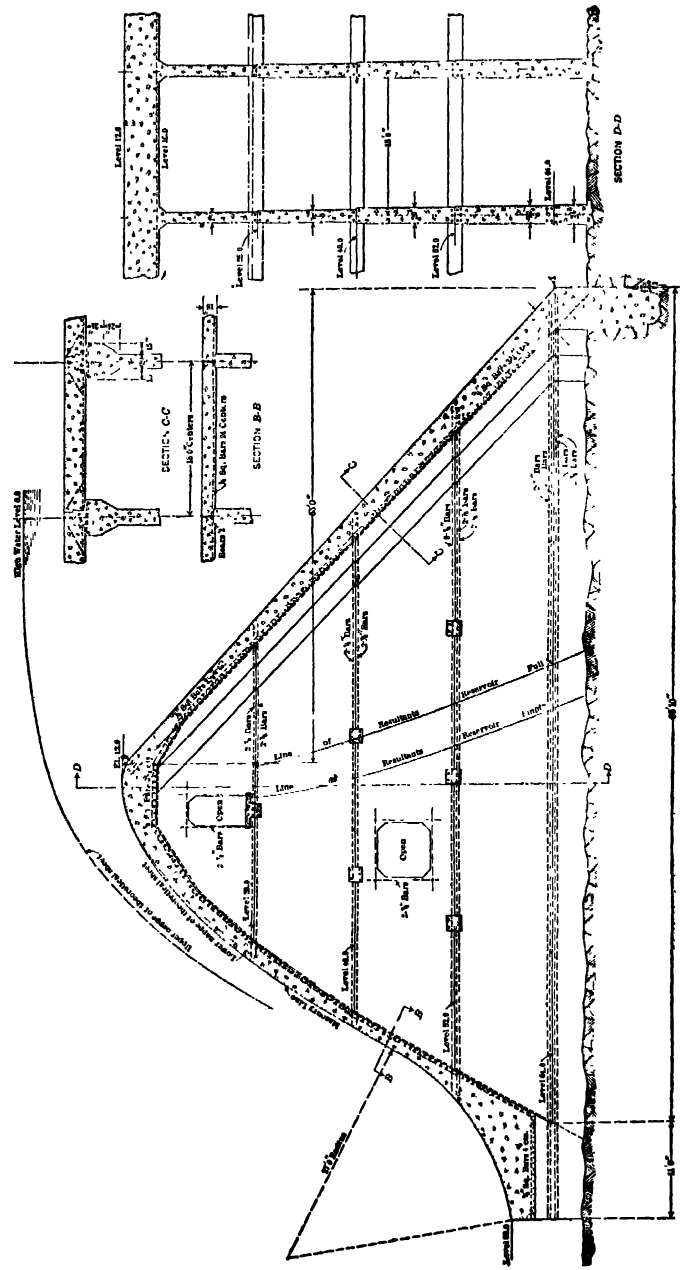


FIG. 14.—Typical hollow flat slab spillway dam.

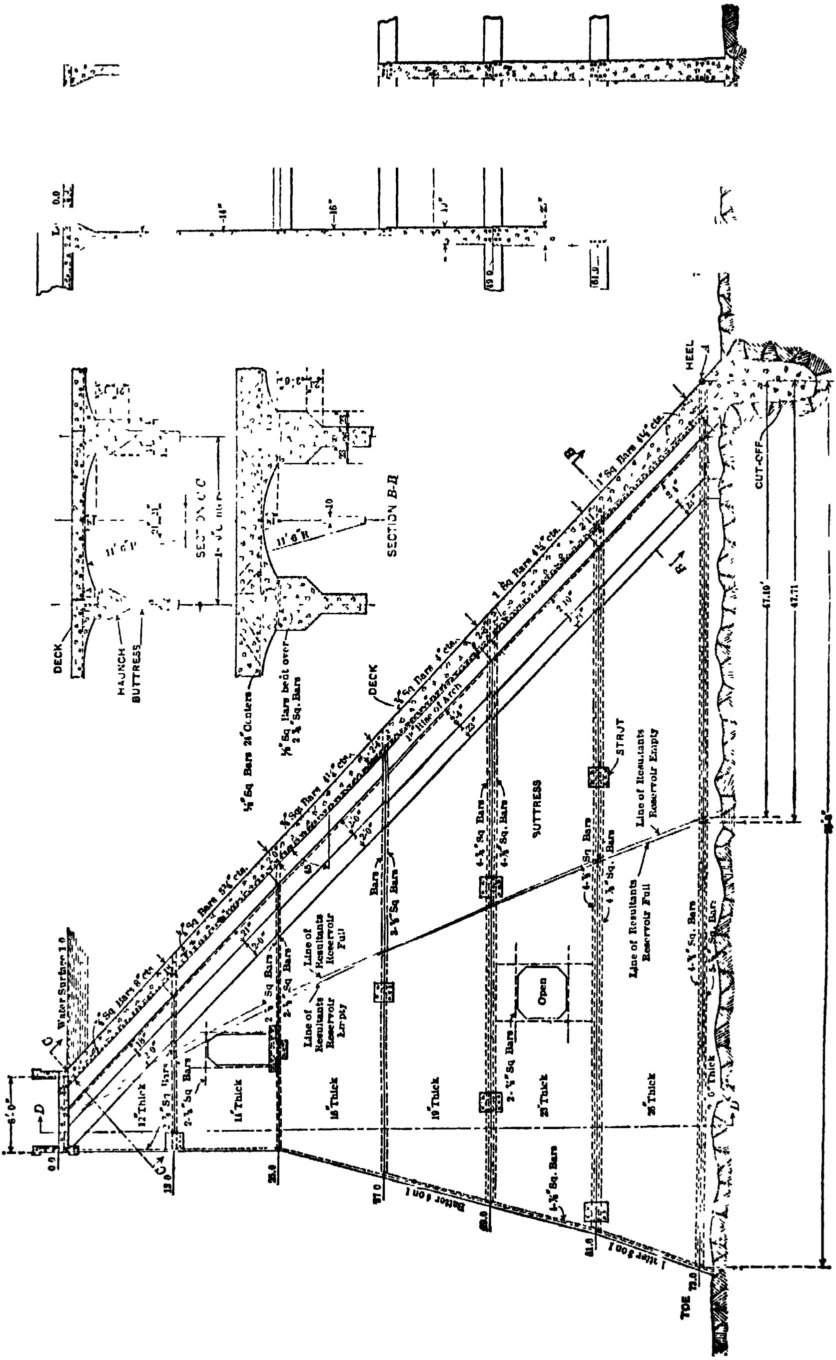


FIG. 15.—Typical hollow flat slab non-overflow dam.

governed by the requirements, previously explained, to conform to the shape of the spilling water corresponding to the maximum flood to be expected.

The upstream ends of the buttresses are built on a slope of 45 to 60 deg. with the horizontal, thus allowing the utilization of a large vertical component of the water pressure to assist stability. This feature results in a considerable saving of concrete as compared with that required for a solid dam, although the unit cost of concrete for the former type is the more expensive.

The downstream ends of those buttresses having no downstream apron are made vertical or slightly battered. This batter usually starts from about mid-height to three-quarters-height of the dam. A downstream batter may be necessary to conform to the requirements of Rule 1 and, in some cases, it may be found advantageous to limit the controlling stresses in the concrete by extending rather than thickening the buttresses.

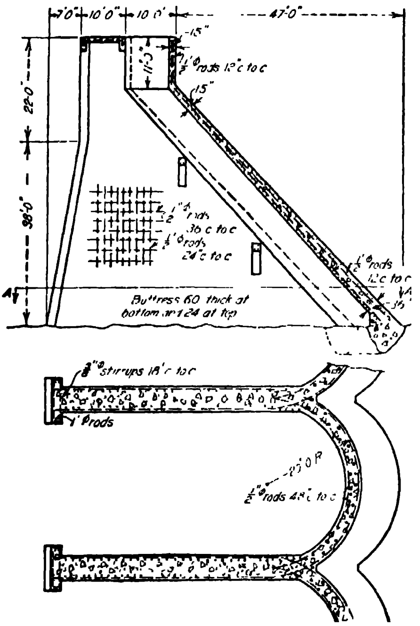


FIG. 16.—Typical hollow multiple arch dam.

The buttresses are in reality thin walls, quite heavily loaded. They are usually well reinforced for stiffness with steel bars extending into the deck and provided with struts at intervals extending throughout the length of the dam. The struts are usually spaced center to center about twelve times the thickness of the buttresses.

The struts are reinforced for compression, tension and also for bending due to their own weight and the weight of any interior walkways which they may be required to support. The horizontal building joints in the buttresses are located on the center line of struts.

On soft foundations the buttresses may be provided with plain or reinforced concrete spread footings. These footings are sometimes of sufficient width to provide a continuous concrete mattress under the dam. Such mattresses should be provided with large weepholes to prevent uplift from head-water.

Open holes in the buttresses are convenient for the passage of men and materials during construction and an inspection gallery is sometimes provided unless, in low dams, access to the interior may be had from the downstream side at ground level.

There are two general types of hollow dams, the "flat deck" type and the "multiple arch" type. Each type is characterized by the decking. The deck for the former type consists of flat, reinforced concrete slabs as indicated in Figs. 14 and 15 and that for multiple arch dams a series of arches as shown in Fig. 16.

The loading for the flat slab deck is uniform for each elevation. Arch decks, however, are designed in normal slices and, for each slice, the crown is at a higher elevation than the haunch. Consequently the loading will be less per square

foot at the crown than at the haunch. This is an important consideration, particularly near the top of the dam where the maximum percentage difference occurs. Mainly for this reason the top part of multiple arch dams are usually provided with vertical decks and some recent dams are designed and built with the arches circular in horizontal section.

The component of the weight of the deck, acting normal to the upstream edge of the buttresses, should be included in the deck loading.

The arches of multiple arch dams should always be reinforced for stiffness and well tied into the buttresses, even if such reinforcement is not needed to resist the unequal loading of the arch. A central angle of 120 deg. is considered most economical for the arches as explained hereinafter for arch dams.

With flat decks the buttresses are spaced from 15 to 25 ft. centers; but for arched decks a greater spacing is more economical, 30 to 40 ft. being usually adopted. Much depends upon the average height of the dam and other considerations. The most economical spacing can only be obtained by comparisons of alternative designs.

For hollow spillway dams of considerable height and those on foundations susceptible of erosion, the dam is provided with a downstream apron, shaped to fit the sheet of spilling water corresponding to the maximum flood to be expected. The proper shape is indicated in Table 9 and Fig. 13. The apron should terminate in a bucket at the toe of the dam to deflect the water properly into a horizontal direction. The apron and particularly the bucket should be of ample thickness and well reinforced to withstand the impact of ice, trees and other objects which may pass over the crest.

If an apron is provided, provision should be made for well ventilating the interior of the dam by means of openings in the buttresses and an open shaft at each end. The interior of the dam should be well drained to prevent the accumulation of leaks from head-water.

In the design of hollow dams the choice of type is the first consideration. This varies with local conditions and the judgment of the designer.

The spacing, thickness, inclination and other details of the buttresses and deck are then adopted tentatively and the dam is tested for conformity with the designing rules hereinbefore given.

Rule 1 may necessitate a change in the length of the buttresses, either by providing flatter slope to the downstream end to increase the length of the base or to the upstream end to increase the vertical component of the water pressure, whichever seems the more expedient.

Rule 2 may require additional weight which can best be obtained by providing a flatter batter to the upstream face.

Rule 3 may require an increase in area of buttresses by thickening or lengthening or both. When calculating the unit compressive stresses in horizontal joints and the base, the horizontal area of the deck not being in monolith with the buttresses, should be included if, by doing so, the calculated stresses are found to be larger. Referring to Art. **3e**, it is seen that the addition of the area of the deck to the area of the buttresses will decrease the direct stress but the eccentricity c may be lengthened sufficiently to increase the flectural stress to a greater extent, resulting in an increased total stress at the extremity of the base.

Rule 4 is not a governing condition in the usual type of hollow dams.

Uplift due to head-water is never considered for hollow dams on account of the thorough draining of the interior and the inclined reinforcement in the decks which is carried down into the masonry cut-off.

The designing conditions and characteristics of the hollow dams indicated in Figs. 14 and 15 are given in Table 10. The weight of concrete was taken at 150 lb. per cu. ft.

TABLE 10.—CHARACTERISTICS OF HOLLOW DAMS INDICATED IN FIGS. 14 AND 15

Levels	Spillway dam				Non-overflow dam					
	61	52	40	28	73	61	49	37	25	13
Deck reaction, lb sq. ft	39,000	35,000	30,000	26,000	37,500	35,000	31,000	30,500	23,500	13,000
Max. vertical stress . . .	29,000	27,000	22,000	21,000	31,000	31,000	31,000	29,000	24,000	11,000
Max. inclined stresses	33,000	29,000	31,000	35,000	42,000	42,000	42,000	38,000	30,000	17,000
Tan θ	0.57	0.55	0.53	0.49	0.60	0.60	0.58	0.55	0.48	0.33
Cu. yd. per lin ft of dam[1]	21.5	15.0	9.5	5.0	26.5	19.5	13.0	8.5	5.0	2.5
Lb. steel per lin. ft of dam[1]	1,850	1,350	850	500	1,700	1,300	950	650	450	200

[1] Not including bucket nor cut-off Includes downstream apron to bottom of bucket concrete. Not including steel in bucket

9. The Design of Arch Dams.—Arch dams are designed to resist the force of the water and silt by horizontal arch action and are adaptable only to those sites where the length is small in comparison with the height and the sides of the valley are composed of good rock to resist the arch thrust at the haunches.

The dam will not act as a true arch due to the restraint at the base where it is in contact with the relatively rigid foundation. Because of this restraint the dam acts partly as a vertical cantilever which reduces the load to be carried by the lower part of the arch; but transfers an additional load to the upper part.

Many attempts have been made to devise methods of design to take into consideration the affecting conditions of the elasticity of the vertical cantilever beam action, the weight of the concrete, varying span, varying radius, and expansion and contraction of the masonry due to changes in temperature, moisture contents and the setting of concrete.

So far, however, such methods of design have not met with general approval as it is considered that the effect of most of these conditions is indeterminate.

Most arch dams have therefore been designed in accordance with the following equation which applies to thin, submerged, empty cylinders

$$p = \frac{q'r}{t} \tag{31}$$

where, for a given level,

p = stress in the arch in pounds per square foot.
q' = load on the arch in pounds per square foot.
r = up-stream radius in feet.
t = thickness of the arch in feet.

It must be realized that this equation is inaccurate for arch dams and gives results which are not necessarily on the safe side. Therefore the allowed arch

stress p for use in the design must be extremely conservative. Arch stresses in 34 existing dams, computed by eq. (31), range from 23,000 to 70,000 lb. per sq. ft. and in two extreme cases, 120,000 lb. per sq. ft. is indicated.

There is no record of a failure of an arch dam. For this reason it is not known how close the actual stresses in these dams approach the ultimate strength of the masonry. It is recommended that the value p, for use in eq. (31), should not exceed $\frac{1}{8}$ to $\frac{1}{12}$ of the ultimate strength of the masonry depending upon the importance of the structure.

The designer should make a detailed study of existing arch dams and conform to the usual relative dimensions of such structures or reduce his allowed stresses accordingly as any deviation from existing types should be considered in the nature of an experiment.[1]

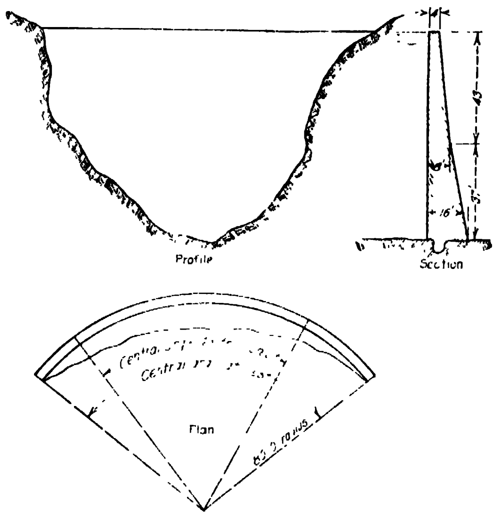


FIG. 17. Typical constant radius arch dam.

As an arch dam is in reality a long column, receiving lateral support only through its connection to the foundation, a maximum ratio of curved length to thickness should not be exceeded. Using as precedent a study of a number of existing dams, it is recommended that this ratio should not exceed 25 at mid-height nor 65 at the top of the dam and preferably should be 20 and 50 respectively. The ratio at the top may be somewhat increased if the ratio at mid-height is considerably decreased, particularly if vertical reinforcement is provided.

The rock at the abutments should be well stepped to safely withstand the arch thrust without sliding at the arch haunch.

Horizontal and vertical reinforcement has been used in small, thin arch dams but seldom to the extent necessary to affect materially the strength of the structure.

Ice thrust, if probable, may be considered as acting over a portion of the arch at least equal to twice the arch thickness plus the assumed thickness of ice and considerably more if vertical steel reinforcement is used.

Early arch dams were constructed mostly with a constant radius at all levels, necessitating a central angle (Fig. 17) which gradually decreased towards the base of the dam. Jorgensen[2] has shown that the greatest theoretical economy obtains when the central angle is 133 deg. 34 sec. at all elevations. If the cost of formwork and similar items were included, this angle would probably not exceed 120 deg. or even less.

[1] For detailed dimensions of 34 arch dams see Table XXIV, "Engineering for Masonry Dams," by W. P. CREAGER, John Wiley and Sons, 1917.

[2] *Transactions*, Am. Soc. C. E., vol. 78, p. 685.

It is not always possible to use a constant central angle where the configuration of the rock surface is such that a constant angle would result in less thickness at the bottom than at higher elevations. Therefore the dam should be designed to as near a constant angle dam as practical considerations will allow.

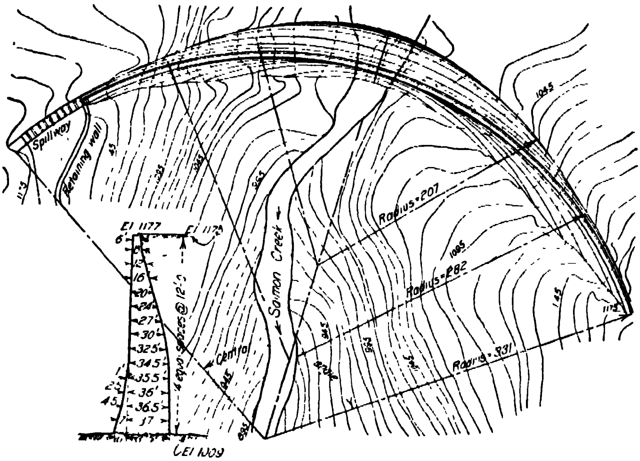


FIG. 18.—Typical constant angle arch dam. 168-ft. dam for Salmon Creek, Alaska.

For the dam shown in Fig. 18 the central angle reduces materially towards the base to keep the structure from overhanging in places. However, the dam more nearly conforms to the characteristics of a constant angle dam than one of

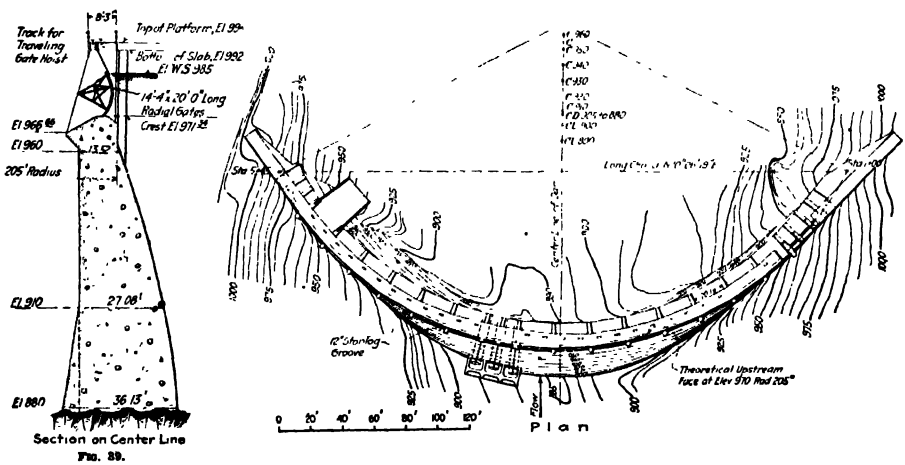


FIG. 19.—The Kerckhoff arch dam. B. F. Jakobsen in Proc. Am. Soc. C. E., December, 1920, p. 21.

a constant radius. The maximum stress in this dam, claculated from eq. (31) is 47,500 lb. per sq. ft.

The weight of masonry in arch dams is not included in the forces resisting overturning and is therefore ample to resist all possible uplift pressure. In

large arch dams, the base is thickened by providing a downstream batter, as indicated in Fig. 18, in order to safeguard against excessive vertical compressive stresses due to vertical beam action.

The downstream face of constant-angle arch dams is usually vertical, or nearly so. A discharge over the crest will therefore leave the downstream face and cannot be directed horizontally away from the dam by a bucket as in the case of gravity dams. Therefore arched spillway dams of this type are not practical for large discharges unless the foundation is not easily eroded, or proper precautions are taken to protect it. Figure 19 indicates an arch dam designed for a large flood discharge.

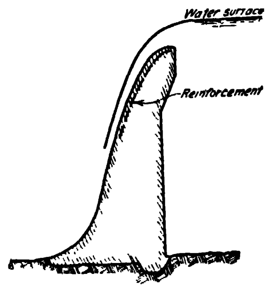


FIG. 20.

The safety of an arch dam is dependent mainly upon the strength of the abutments and particular care should be taken to protect that part from erosion. In some instances, the downstream face has been shaped to fit the underside of the spilling sheet, as indicated in Fig. 13, but this results in a section much too thick for economy as an arch, although some saving can be made by over hanging the lip of the crest as indicated in Fig. 20.

RESERVOIRS

BY S. C. HOLLISTER

Although all forms of structures for retaining liquids may be classed as *reservoirs*, the term is usually taken to apply to those large structures which in themselves are used as liquid containers, the smaller ones being classed as *tanks*.

10. General Types.—There are in general two types—open and closed reservoirs. Covers are usually of concrete, with a topping of earth. They are of advantage in preventing undesirable evaporation, warming or freezing of water, pollution, and the spread of certain objectionable growths requiring sunlight.

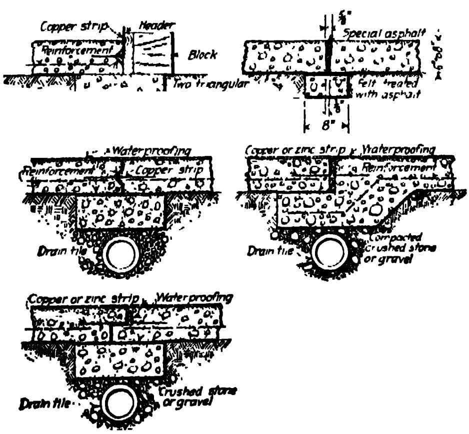


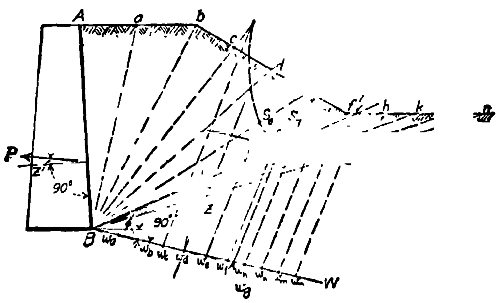
FIG. 21.

11. Open Basins with Embankment Walls.—The simplest form of reservoir is an earth basin lined with a reinforced concrete pavement. The supporting strength in such a structure must be supplied by the embankment and subsoil. Compacting and thorough settling of the embankment is imperative. The inner slopes should preferably be 3:1 and never steeper than 2:1. Since settlement is likely even under the most favorable circumstances, it is preferable to reinforce the pavement, and it is especially important to provide expansion and contraction joints in both directions. Reinforcement is usually a mesh. The joints should be provided with a crimped flexible gusset to insure

water-tightness. Details of such joints are shown in Fig. 21. Spacing of joints should not exceed 60 ft., and if the dry reservoir is exposed to the sun or winter weather, the spacing should not exceed 30 ft.

All pipes and fixtures passing through the floor should be flanged within the floor slab and the joint surrounding it well waterproofed.

12. Open Basins with Concrete Walls.—Another common type of reservoir has gravity type retaining walls, with or without an earth embankment against the outside of the wall. The floor is a reinforced pavement of the form described above. If earth fill is against the outside of the wall, the wall must be designed to withstand the earth pressure from without when it is empty, and the water pressure from within when it is full. If the earth does not shrink from the face of the wall, it will help to resist the water pressure. The wall pushing outward against the earth fill would, however, tend to force out a prism of earth. The largest thrust which the wall can exert without actually forcing out this prism is called the *passive thrust.*



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In Fig. 22 let AB be a given wall with an embankment whose earth surface is $Abfn$ tending to resist a pressure acting against the wall from the left. The friction between wall and earth will act downward, since the earth will tend to move upward. The reaction of the earth P will thus be directed upward making an angle above the normal to the wall of Z (angle of friction of the material on the wall), but never greater than ϕ (angle of internal friction). Lay off on the surface line points a, b, c, etc. arbitrarily. Draw lines from these points to B, thus bounding a series of prisms of length 1 ft. perpendicular to the drawing. Compute the weights of these prisms and to any convenient scale lay them off in order, on the line BW which makes an angle ϕ with the horizontal as w_a, w_b, w_c, etc. Draw through one of these points a line, as tw_d, which makes an angle Z to the right of a normal to BW. Then through each point on BW draw a line parallel to this direction tw_d, to an intersection with corresponding ray through B, as w_e to B_e at S_e; w_f to B_f at S_f; etc. Through the points S draw a smooth curve. (This curve will have breaks at rays connecting B with points of change of slope.) Draw a tangent v to the curve parallel to BW, and through the point of tangency draw a ray from B, as BS_gg. This line will represent the plane of rupture of the embankment if the passive thrust is exceeded. The distance w_gS_g between BW and the tangent v, measured parallel to the direction tw_d and to the same scale as that used in laying off the weights on BW, is the magnitude of the passive thrust.

The point of application of P, the reaction of the earth against the wall, will be one-third of AB above B.

Proof of the construction may be at once seen by revolving the force triangle BS_gw_g to such a position that BW_g lies on AB. Bw_g is then seen to be the weight of the block of earth $AbfgB$ lying above the plane of rupture Bg; BS_g is the

resultant pressure on the plane Bg and making the internal friction angle ϕ thereto; and as previously noted, w_0S_0 denotes the passive thrust, at an angle (90 deg. $\pm$ Z) to the wall.

The moment of the passive thrust is added to the moment of resistance, taken about the third point toward the earth, when the reservoir is full; and the moment of the *active* thrust is taken as the attacking moment about the third point toward the water, when the reservoir is empty.

13. Roofs and Floors of Covered Reservoirs.—The covers or roofs of reservoirs are supported on columns. The roof design may be of beam and girder construction, flat slab, or of groined arch construction. The floors may be formed of concrete pavement with independent footings for the columns, or they may be of flat slab or inverted groined arch construction.

In the event that the columns are supported on separate footings, not a part of the floor slab, the footings should be so designed as to exert upon the undersoil a pressure of intensity as near as possible to that exerted by the floor slab when the reservoir is full. Such a precaution will minimize settlement. Great care must be taken in properly waterproofing the joint of the floor pavement at its junction with the column base.

Inverted groins are favored by many engineers because of the continuous inverts formed in the two directions by the arch construction, thus giving suitable drainage. There is a further advantage in such construction in that a maximum spread to the column load is afforded through this type. The plane of contact between the floor and the undersoil is, of course, flat.

A modification of inverted groined arches consists of a thick slab over the floor with spread footings cast monolithic with it. The space between the column footings is then troweled to form a slight depression thus giving the advantage of draining, yet minimizing the difficulty of formwork experienced in the groined arch construction. Groined arches, on the other hand, may not need to be reinforced.

Flat slab roof construction gives the simplest formwork and the greatest head room. Its construction is similar to that for a floor. Beam-and-girder construction for the roof would be similar to that for a floor.

Groined arch construction is seldom used to form the roof. In such construction it is necessary to build the roof in sections, each acting as an "umbrella" spreading out from the column at its center. Such arches are usually not reinforced but are given a steep pitch. Some trouble has been experienced by cracks forming along the crown of the barrels in both directions, thus following the construction joints. These cracks are usually caused by uneven settlement of the footings. They permit seepage to enter the reservoir and thus thwart one of the main purposes of a cover.

14. Partition and Outside Walls.—Reinforced concrete walls may be used both for outside walls and for partitions. In the case of square or rectangular reservoirs, these walls act as cantilevers. They must be designed to withstand pressure from either side, where such pressure exists.

Vertical joints in reinforced concrete walls, or partitions, are necessary at frequent intervals. The spacing must be determined by the condition of use. If the reservoir is covered, the wall should have a vertical expansion joint spaced at intervals of about three times the height. If, however, the wall is subject to open

exposure with the reservoir empty, the spacing between joints should in no case exceed 30 ft. for reinforced concrete retaining walls, and about twice the height for gravity walls. The tongue-and-groove type of keyway with a folded metal strip, as in slabs, makes a satisfactory joint when thoroughly waterproofed.

The walls of circular reservoirs should be investigated for hoop action. The decision as to whether the wall acts as a cantilever, or as a vertical cylinder, lies in the extent of the radial deflection of the wall when acting under hoop tension alone. If this deflection from hoop tension would exceed the deflection likely from retaining walls, then the wall should be designed as a retaining wall. It will

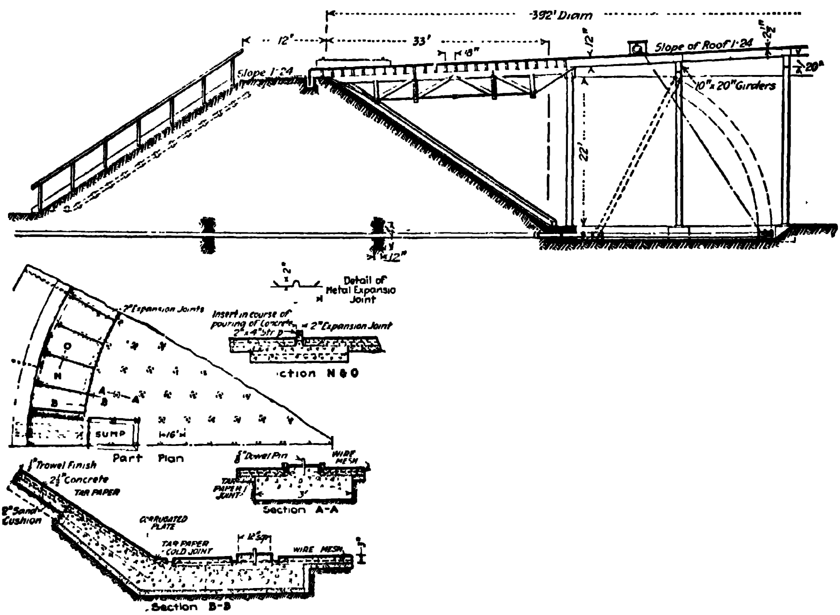


FIG. 23.—Oklahoma oil reservoir formed by lining circular embankment with concrete.

be apparent from this that the lower portion of the wall, in any event, will not act through hoop tension because of the restraint exerted upon the wall by the base. A further study of such restraining action will be found in Art. **18.**

15. Quality of Concrete for Reservoir Masonry.—The concrete to be used in reservoir construction should be designed for maximum density as well as for the desired strength. The selection and gradation of the aggregates, the amount of water, and the method employed in placing are all important factors in obtaining a dense, tight wall. If the space between forms is small, the placing should be done in such a way as to deposit the concrete as near as possible to the point it is to occupy.

16. Examples of Construction.—Figure 23 shows a large reservoir for fuel oil, built at Gainesville, Texas. The embankment was built in 3-in. layers from earth taken from the pit by fresnos and wheel scrapers. Both pit and embankment were thoroughly wetted to aid in compacting the soil before placing the

lining. The drainage from the roof, the seepage stops along the supply line, and the detail of expansion joints should be carefully noted.

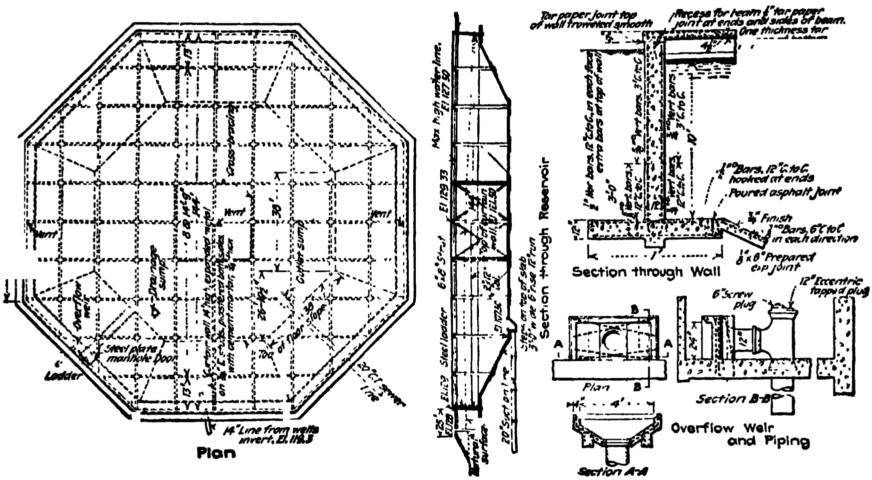


Fig. 24. Octagonal concrete reservoir at Enid, Oklahoma.

Figure 24 shows a 2,000,000-gal. reservoir for water at Enid, Oklahoma. The slope embankment, with a low retaining wall at the top, is a very economical structure, since the slope replaces what otherwise might be a very heavy

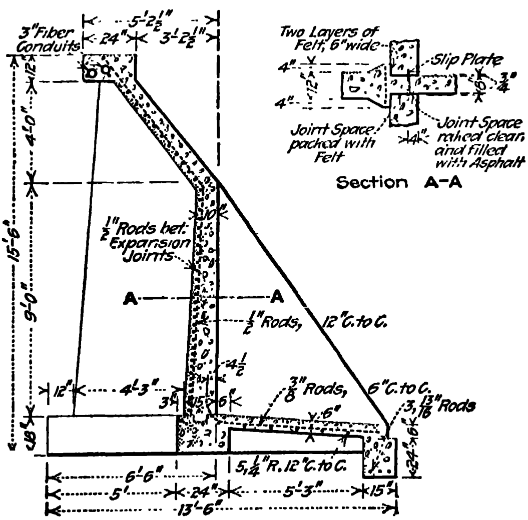


Fig. 25.—Slip plates used in expansion joints at counterforts.

pressure against a high, thick wall. Note the expansion joint between wall and roof.

Figure 25 shows slip plates used in expansion joints at counterforts in a large reservoir with reinforced concrete walls built at Highland Park, Michigan.

STANDPIPES AND SMALL TANKS

By S. C. Hollister

A standpipe is a cylindrical tank usually having a height greater than its diameter and resting directly upon its foundation. The common use of standpipes is for the storage of water.

Standpipes of concrete require particular care in construction. The walls are relatively thin and the head of water is usually quite large. It is therefore essential to use concrete of such proportions as to produce great density, and that all possible care be exercised in placing the concrete to prevent porous areas or improperly constructed work planes.

17. Analysis of Stresses in Standpipes.- Since the pressure of the fluid varies in intensity with the depth, the unit pressure at a depth y (see Fig. 26) is

$$p_y = wy$$

Suppose a circumferential strip 1 ft. wide be cut from the shell at the depth y; then the total tension on any diameter of this ring is

$$T = p_y r = wry$$

where p_y is in pounds per square foot; r is the inner radius of the tank in feet; and T the pull on one side of the tank in pounds per foot of height. Thus it is seen that the hoop tension in the shell increases in intensity with increase in depth.

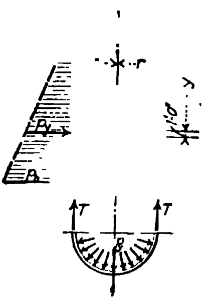


Fig. 26.

Circumferential reinforcement is necessaryto accommodate this bursting stress. The tensile resistance offered by the concrete is low. Concrete of good quality will crack at an elongation of 0.00010 to 0.00020 in. per in. The value 0.00010 corresponds to a unit stress of 200 lb. per sq. in. in the concrete and (when steel bars are present) 3, 000 lb. per sq. in. in steel. Elongations in excess of this amount should be considered as transferring all of the tensile stress to the steel, in which case sufficient steel should be provided. Where high heads are used, low tensile stress should be employed, but in tanks having a head of 30 ft. or less, steel stresses as high as 10,000 lb. per sq. in. have proven successful. This is undoubtedly due to the greater amount of the hoop tension being taken by the concrete. Computations should be made assuming the concrete to take tension, to determine the possible elongation and thus to determine whether vertical cracks would be likely.

An ingenious type of construction has recently been employed in a tank of considerable proportions. A shell of concrete was first erected, after which steel hoops provided with turn-buckles were placed around the shell at spacing dependent upon the pressure. The hoops were then drawn up by means of the turn-buckle, until they sustained a tension of 15,000 lb. per sq. in. Concrete was then built up on the outside of the hoops to embed them and to entirely cover the shell first constructed. By this method it will be seen that the initial compression in the original concrete shell produced by the tension in the hoops, must first

be overcome before the inner shell of concrete would crack. The use of higher hoop stresses are thus possible.[1]

18. Restraint at Base.—The deformation of a standpipe is not wholly due to the elongation of the hoops under stress. At the base the rigidity of the bottom prevents hoop expansion and introduces a restraining moment on a vertical element. If the side of the tank were of uniform thickness and a homogeneous material were employed, the deformation of the rings would vary as the depth of water. But in the design of a concrete standpipe with steel reinforcement, the hoop elongation will be limited to that elongation which corresponds to the adopted working unit stress in the steel. Thus in Fig. 27, the portion *BC* of the exaggerated deformation is a constant for any depth between *B* and *C*. The deformation to *AB* may be a straight line or not; however, it is the gradual increase of deformation up to that corresponding to the working hoop stress.

The restraint of the bottom is such that at *D* no hoop stress exists, but rather a fixity, similar to a vertical beam fixed at the lower end. This restraint decreases

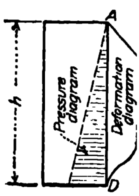


Fig. 27.

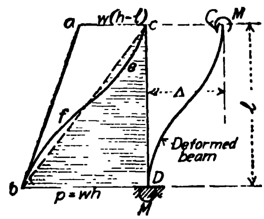


Fig. 28.

so that at some point *C* only hoop stress, and likewise hoop deformation, prevails. It is evident that at this point, *C*, the tangent to the deformation curve is again vertical, thus parallel to the tangent to the curve at the bottom.

Suppose the lower portion of the shell of the tank or standpipe to be made up of vertical beams restrained at their lower end, and having a length sufficient to include the deformation *CD*, Fig. 27. Then (Fig. 28) the water pressure varies from its upper end with $w(h\text{-}l)$ to its lower end with wh. Let the beam be acted upon at its upper end by moment M', sufficient to maintain the slope at that end parallel to that at the base. Then the slope at *C* is zero with respect to that at *D*. The deformation Δ at *C* must be equal to that caused by ring-working stress only.

Now imagine the elastic hoops existing between the levels *C* and *D* to restrain the deformation of the beam somewhat. Let the deformation of the beam thus modified be that shown by *DC*, Fig. 28. The hoops stresses between the levels *C* and *D* will thus vary as abscissas to the deformation curve *CD*. Before we may again consider the vertical beams separately, the effect produced by these hoop stresses should be used to modify the water pressure on this beam. The deformation area *DcC* may be converted to a corresponding stress area, and subtracted from the pressure trapezoid *abDc* by placing *cD* upon *ab*. The loading left to the restrained cantilever beam to be resisted by beam action is therefore represented by the area *bfecD*.

[1] See *Proceedings* American Concrete Institute, 1923.

Since the actual form of the curve *bfec* is still unknown, the resultant loading is likewise unknown. So far as concerns the deflection Δ of the end C a very close approximation is apparent by using the triangular loading *bcD*. It should be noted that this triangle exceeds the actual loading at *e* (curve *bfce*) but is less than that at *f*; but since the upper portion of the beam contributes more than half of the deflection, the approximation becomes, if anything, slightly on the safe side.

Now, from the theory of deflections of reinforced-concrete beams

$$\Delta = \frac{1}{2{,}212E_s}\cdot\frac{hl^4}{d^3}\cdot\frac{n}{\alpha}$$

in which h = depth of water at bottom of tank, in inches.
l = length of beam element CD, in inches.
d = effective depth of beam element CD, in inches.
E_s = modulus of elasticity of steel, pounds per square inch.
$\frac{n}{\alpha}$ = numerical coefficient dependent upon p and n, in which n is recommended by Turneaure and Maurer to be 8 or 10 (see Fig. 29).
Δ = deflection of end C, in inches.

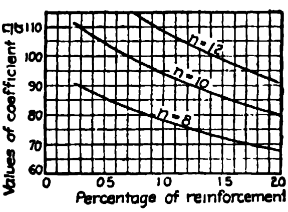


Fig. 29.

Since at the point C, this deflection of the vertical element must equal the radial deflection of hoop tension, which may be written

$$\Delta = \frac{f_s}{E_s}r$$

By equating these two deflections, we obtain the following design formulas:

$$M = 2.55\sqrt{\frac{f_s r d^3 h}{\frac{n}{\alpha}}} \qquad \text{(in.-lb.)}$$

$$M' = \tfrac{1}{3}M$$

$$l = 6.86\sqrt[4]{\frac{f_s r d^3}{h\frac{n}{\alpha}}} \qquad \text{(in.)}$$

The point of inflection is at $0.37l$ above the bottom. At this point the reinforcement may swing from the inner to the outer face. Although only a third of the steel is needed at the outer face, the remaining steel should be carried up sufficiently to provide ample bond.

The moment above the point C may be assumed to vary as a straight line from the value M' at C to zero at the top. This is not strictly true but is sufficiently close to provide a means of cutting some of the steel near the outer face if desired.

19. Shear at Base.—The shear at the base may be seen from Fig. 28 to be equal to the triangular loading on the beam. Thus, the shear per foot of circumference becomes

$$V = 0.217hl$$

where h and l are in inches.

20. Small Tanks.—Tanks and reservoirs of small size are very readily constructed and have been put to a wide variety of uses. They have commonly been used for the storage of water and of fuel oil, both above and below ground. Their use in various manufacturing processes has proven them adaptable to many uses where the contained solution is not strongly destructive of the cement. Concrete tanks have been used successfully for containing mineral oils, and many vegetable and fatty oils; for tanning solutions; for substances in fermentation, as sauerkraut; and when coated with acid-resisting substances, for pickling vats and other metal baths. Carefully graded silicious aggregates should be employed in tanks containing active agents.

Small circular tanks are designed for hoop stress in the same manner as are the sides of standpipes. Restraint of the base must also be considered, since in most small tanks the diameter and height are not great enough to escape this restraining effect on the full height of the tank. When computations, as for a standpipe, indicate a length l greater than the depth h of the tank, the design of vertical elements should follow the loadings of Fig. 30, in addition to the design for hoop stress. The effect of hoop stress will be small near the bottom of the wall, when a sliding joint is not provided. If a construction joint or keyway is provided at the foot of the wall, without provision for moment at that section, loading (b) of Fig. 30 should apply.

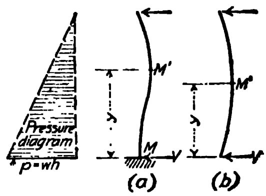


Fig. 30.

(a) Walls Continuous with Base.—The moment at the base (Fig. 30a) is

$$M = 0.0667\,wh^3$$
$$M' = 0.0298\,wh^3$$
$$V = 0.4\,wh^2$$
$$y = 0.553\,h$$

(b) When the Base is Separated by a Joint or by Cracking (Fig. 30b).—

$$M' = 0.0642\,wh^3$$
$$V = 0.333\,wh^2$$
$$y = 0.423\,h$$

In the above formulas, w is the weight per cubic foot of the liquid, and h is the height of the tank in feet. All moments are in foot-pounds.

21. Construction Details of Standpipes and Tanks.—Piping for standpipes and tanks must be so placed that they will not interfere with the movement of the structure as it takes its load. They should be protected from freezing, where necessary.

Covers lessen the chance of ice formation. In extremely cold climates provision must be made for ice formation and expansion, or it must be prevented.

Thin walls are more difficult to cast than thick walls. On the other hand, seepage is less likely to occur with thick walls.

Hoop splicing should be staggered, and where possible should be effected mechanically rather than by lapping.

Figure 31 shows a large tank with a sliding base. This feature was patented by Wm. Meuser. It is claimed that there is small lateral restraint of the sides

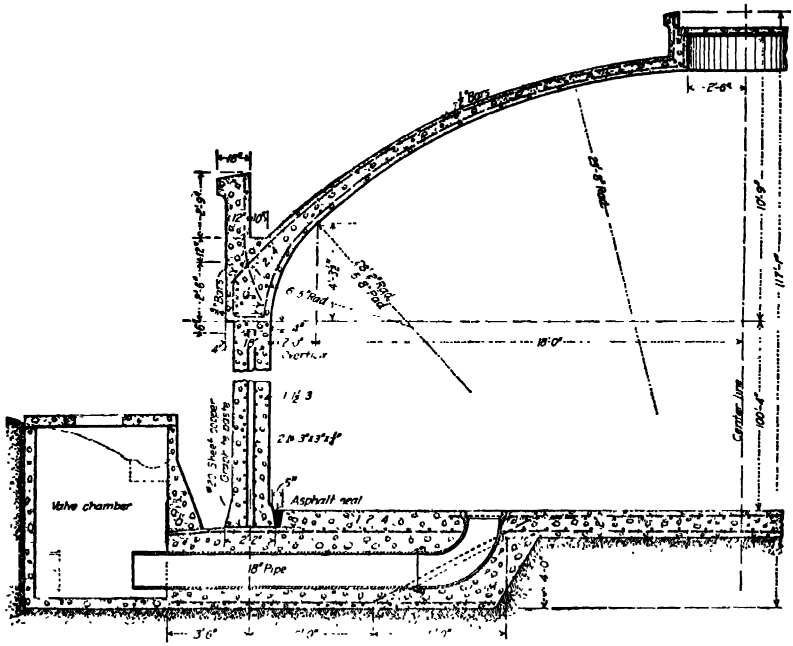


FIG. 31.—Standpoint at Fulton, N. Y., having disconnected base and flexible seal

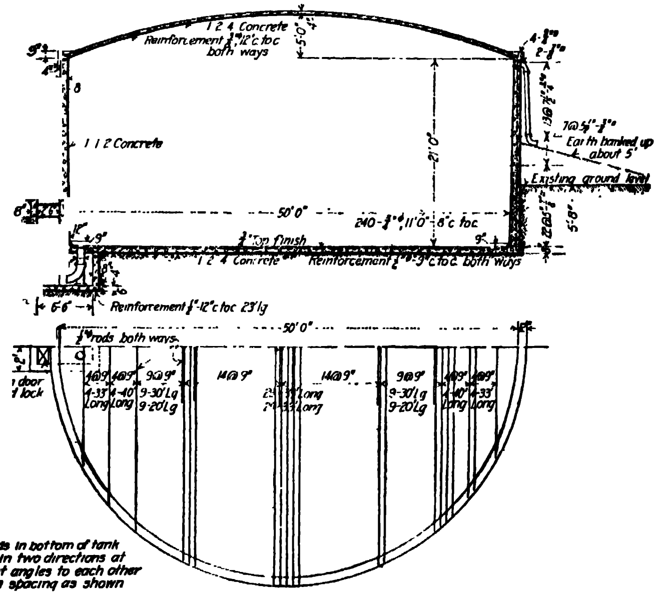


FIG. 32.—Low standpipe at Penetanguishene, Ont.

at the base, thus eliminating the secondary stresses otherwise caused by such restraint. The joint must be so constructed as to offer small frictional resistance.

Figure 32 shows the details of a low standpipe at Penetanguishene, Ont. Its capacity is 300,000 gal. The wall is thicker than required for strength, to prevent the formation of a heavy ice crust. The connection between wall and bottom was analyzed for restraint by an application of Grashof's theory.

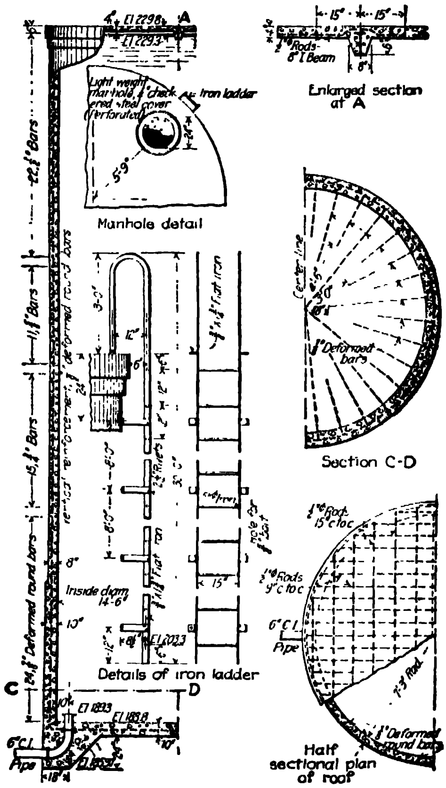


Fig. 33.—Details of a small standpipe at Merrimack, N. H.

Figure 33 shows the details of a small standpipe of 14½ ft. inside diameter and 40 ft. high, built at Merrimack, N. H. The standpipe has a capacity of 50,000 gal. The concrete was poured in one continuous operation which lasted 39½ hr.

CULVERTS

By S. C. Hollister

A culvert is a conduit constructed through embankments, for the purpose of conducting small streams or surface water. Culverts may range in size from sectional pipe up to structures which are in themselves practically small bridges. Culverts having spans in excess of 25 to 30 ft. are usually considered as bridges.

The form of culvert depends upon the available clearance, the load to be carried, the nature of the foundation, and general appearance desired.

22. Waterway Required.—The waterway required in the culvert depends entirely upon the conditions existing at its location. The culvert must necessarily dispose of the accumulated run-off on the upstream side of the embankment, and, except under very unusual flood conditions, should not depend upon accumulated head in order successfully to pass the water. The run-off for the particular location should be ascertained in order to determine the desired waterway and cross-section.

23. Direction Through Embankment.—If a culvert is intended to carry flood water only, its direction through the embankment is not especially essential. It should, however, be placed at a position of maximum drainage of the area on the upstream side of the embankment and should have a fall along the floor of the barrel approximating the general lay of the ground at that point.

If the culvert is to conduct a stream, or is located on a stream bed, it should take the general direction through the embankment that was originally taken by the stream. Both upper and lower ends should have an elevation similar to the original bed in order to prevent collection of waste.

24. Design of Upstream End.– The upstream end of the culvert barrel must receive careful treatment as to design. If the barrel is small, a wall extending both ways at right angles to the barrel, or parallel to the axis of the roadway, will serve as a protection to the embankment surrounding the barrel. It is essential, especially when building on an old stream bed, that a toe-wall be projected downward from the floor of the barrel sufficient to prevent undercutting or scouring. In the design of large culverts, oblique wingwalls are substituted for the simple type described above. The top of these wingwalls conforms to the slope of the embankment. A toe-wall, similar to that described above, should project downward below the barrel floor. If desired, the space between the wingwalls may be floored over, in which event the toe-wall should be located at the upstream edge of such floor. The wingwalls are usually designed as retaining walls carrying the thrust of the embankment which they sustain. Their footings should be so proportioned as to produce a uniform soil pressure over the whole length of the wing, thus preventing a common and serious breaking away of the wingwall from the main structure and giving opportunity for the beginning of progressive seepage along the outside of the barrel.

25. Design of Downstream End.—The downstream end of the barrel, likewise, should be provided with wingwalls to protect the embankment, and the space between these walls should be floored over to carry the water well over the embankment line before being discharged from the floor. Care should be taken to avoid back-eddies when designing the apron, or floor, of the downstream end.

26. Pipe Culverts.—Reinforced concrete culvert pipe in pre-cast units is used in sizes from 15-in. diameter up to about 6-ft. diameter. The length of these units varies from 4 to 8 ft., depending upon the diameter. The sections are usually of the bell-and-spigot type.

Two systems of reinforcement are employed. One consists of two concentric rings of reinforcement, one near the inner face and the other near the outer face of the pipe. The other consists of a single ring of reinforcement so placed that it is near the inside face at the ends of a vertical diameter, and near the outside face at the ends of a horizontal diameter. In the latter system it is necessary, of course, to be certain that the pipe will be installed in the proper position because of

the existence of the single line of reinforcement. The amount of reinforcement depends upon the load or embankment to be carried by the pipe.

26a. Strength of Pipe.—A theoretical analysis of the stresses of thin elastic rings will suffice as the basis of design of the cross-section of reinforced concrete pipe. For convenience, the vertical pressure of the earth is assumed to act over a horizontal projection and no account is taken to the advance in depth of earth from the ground to the half height.

Analysis made for the following three cases of loading was carried on by the general method employed for arches and the values given are the bending moments at the ends of horizontal diameters and require reinforcement placed near the outer face of the pipe. The diameter of the pipe to the center of the shell is called d.

Case I

For single concentrated load on vertical axis top and bottom:

$$M = 0.091Pd$$

Case II

Uniformly distributed load over entire horizontal projection, top and bottom:

$$M = 0.0625Wd$$

Case III

Uniformly distributed load over the top fourth of the circumference, the pipe being supported on the bottom earth:

$$M = 0.077Wd$$

The bending moments at the ends of the vertical diameters, under the above three conditions of loading, are:

$$\begin{aligned} \text{Case I} &= 0.159Pa \\ \text{Case II} &= 0.0625Wd \\ \text{Case III} &= 0.0845Wd \end{aligned}$$

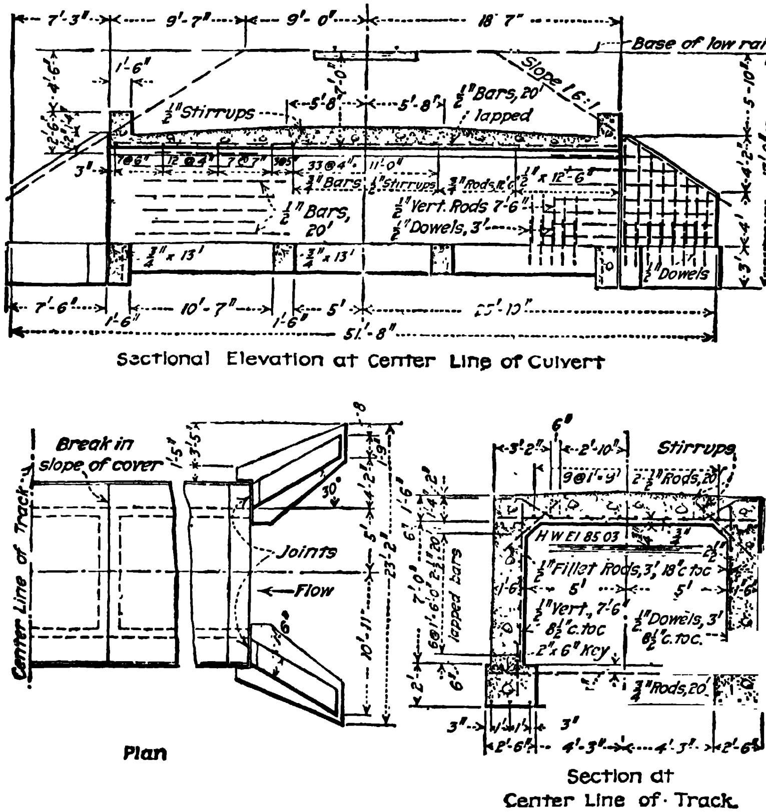
Reinforcement for these moments should be located near the interior face of the pipe at both top and bottom.

The above bending moments will be reduced by the action of lateral pressure. It would be seen from the above formulas that if the lateral pressure exactly equalled the vertical pressure for uniformly distributed loads, the bending moment at all points in the ring would equal zero. Lateral pressure, however, is very difficult to maintain and in order to be effective, it would be necessary that such lateral resistance be offered that the concrete ring could not expand along its horizontal diameter the very small amount that would take place under the vertical loading. Since it is not practical to expect such lateral rigidity of the embankment, it is safest to neglect lateral strength.[1]

26b. Circular Culverts Cast in Place.—Culverts of circular sections cast in place usually are cast with a flat outside bottom, a circular outside face at the top, and with a circular opening. It is preferable that they be reinforced in order to prevent their being cracked through settlement of the embankment. Collapsible core forms may be used, and by their use this form of culvert may be constructed very economically.

[1] For tests which agree closely with mathematical analysis, see *Bulletin* 22, Engineering Experiment Station, University of Illinois, by Prof. A. N. Talbot.

27. Box Culverts.—Culverts having a box-shaped or rectangular cross-section are common, because of simple formwork, and hence less cost of construction. A full distribution of load over the foundation is possible with this form of cross-section, making it suitable for construction on earth foundations. Where clearance is restricted, this form gives maximum waterway for a given width of opening. In cases where the height of opening desired is small in comparison with the width, partitions may be used to subdivide the roof and floor into two or more spans.



34. Single box culvert, C. M. & St. P. Ry.

27*a*. Forms of Box Culverts.—Two forms of box culverts are common, the closed-box and the open-box. The first contains a concrete floor built as part of the structure, and the second does not. Since the wall footings of the open type are required to carry the culvert and its load, it is essential that rigid foundations be available. Proper protection against scour is also necessary. The floor of the enclosed-box type, on the other hand, is available for footings—hence, the desirability of this type when building on earth or other material not capable of supporting heavy load.

27*b*. Loading.—The loading on a culvert consists of the embankment and pavement or track, which together may be called the dead load; and

the traffic or live load which passes over the culvert. Impact may be considered as an augment to the live load.

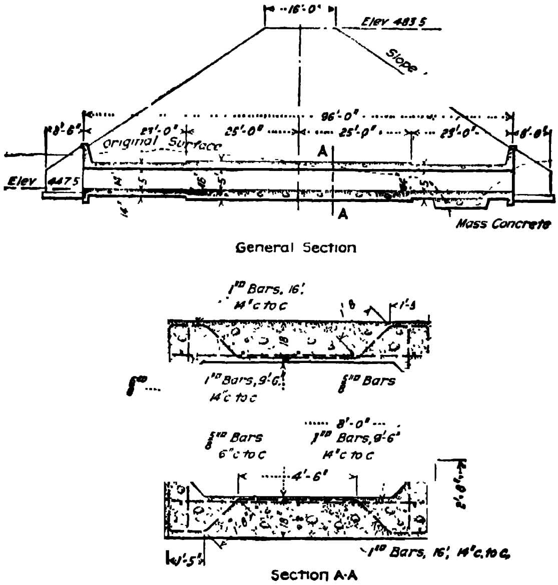


FIG. 35.—Details of box culvert on lines of Eastern-Texas Traction Co.

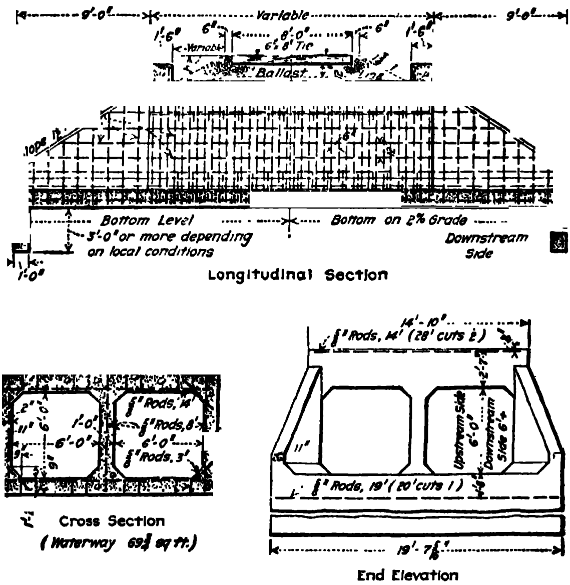


FIG. 36.—Double 6 × 6-ft. culvert, A. T. & S. Fe. Ry. system.

The extent to which the embankment directly above the culvert is actually being carried by the roof slab is dependent upon many factors. It is quite

common to construct the culvert as a part of new construction; and in such instances the newly placed embankment acts over the roof slab for the full depth of the fill. If the culvert is placed in a trench through a previously existing embankment, it is plain that a considerable part of the back-fill, if deep, will be carried through arching, by the sides of the trench. However, the great danger in assuming arch action lies in the fact that very few embankments are not subject to vibratory live loads, such as trains or trucks; and where such vibrations exist, there is a tendency to destroy cohesion and likewise arch action. Current practice, therefore, employs as a dead load the full weight of embankment including track or pavement directly over the roof slab, for fills of a depth over the roof equal to the roof span. Some railroads assume a percentage of this full weight when the depth of embankment exceeds some arbitrarily fixed depth. A safe rule would be, in the case of earth possessing cohesive properties, to include that volume of earth lying over the roof slab between two upward sloping lines inclined toward each other and having a pitch of ¼ horizontal to1 vertical. Thus, fills of such material having a depth over the roof slab greater than twice the roof width would be considered to exert no more pressure than one just equal to twice the roof width, since in either case the two sloping lines just intersect. This reduction should not be practiced in the case of non-cohesive fills, such as sand.

The live load, or pressure of trucks or trains transmitted to the fill through the roadway, is spread over an area increasing with the depth below the roadway. The spread transversely to the direction of the roadway is usually taken as a uniform distribution over a width contained between lines sloping downward and outward at a pitch of ½ horizontal to 1 vertical. In the direction of the roadway the live load may be assumed over the area immediately over the volume of earth assumed as dead load. Thus, if the fill is a cohesive material, and its depth is equal to, or greater than, twice the width of culvert roof, no live load need be considered.

Impact is considered as a percentage increase of the live load. In railroad structures, it is common to figure impact at 50 per cent on all fills up to 40 ft. deep. Some designers use 100 per cent on fills of 2 ft. or less; 75 per cent on fills 2 to 4 ft. deep; and 50 per cent on fills over 4 ft.

Lateral pressure on the sides of culverts is commonly taken as a fluid pressure of 30 lb. per sq. ft., or about one-fourth the weight of earth. To this must be added 25 per cent of the live-load pressure used on the roof design.

27c. Design of Cross-section.—Closed-box culverts may be designed as being made up of simple-span slabs resting or bearing against one another, or they may be designed as monolithic and acting as a closed frame. Open-box types are subject to the same two treatments, due account being taken of the lateral thrust on the wall footings, by using cross-struts at suitable intervals. Whether so designed or not, box culverts will tend to act as continuous frames, forming cracks at corners—which may be on critical shear planes—unless the continuity is provided for by proper reinforcement. Detailed analysis is not justified unless the section is large and material saving may be gained thereby.

The following analysis covers the usual cases of loading of the culvert. The equations for moment may be computed for the separate cases of loading and then combined to give the total effect of the loading conditions assumed.

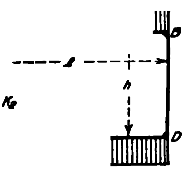
It will be noted that the equations for moment at the corners of frames result in a percentage of the moment that would occur at the same end of the loaded member if the loaded member were a fixed beam. Likewise it should be noted that since the corners do not offer perfect fixity at the ends of the loaded member, the positive moment is increased by the amount of decrease at the ends.

The following notation will be used:

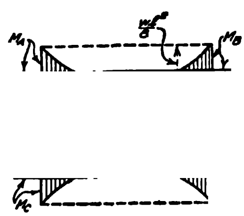
l = span of frame.
h = height of frame.
I_1 = moment of inertia of top or bottom slab (assumed alike) for 1-ft. length of barrel (in.[4]).
I_2 = moment of inertia of either side wall (assumed alike) for 1-ft. length of barrel (in.[4]).
$K_1 = \frac{I_1}{l}(12 \times \text{in.}^3)$.
$K_2 = \frac{I_2}{h}(12 \times \text{in.}^3)$.
w = uniform load per unit area of top or bottom slab (lb. per sq. ft.).
p = uniform lateral pressure per unit area of side wall (lb. per sq. ft.).
P = concentrated load (lb.).
M = moment, subscript denoting location (ft.-lb.).

TYPE A

Case 1



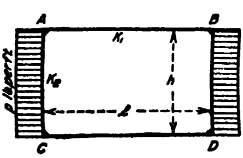
Load Diagram



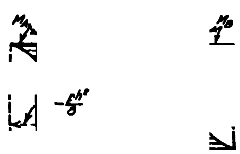
Moment Diagram

$$M_A = M_B = M_C = M_D$$
$$M_A = \frac{K_2}{K_1 + K_2} \cdot \frac{wl_2}{12}$$

Case 2



Load Diagram



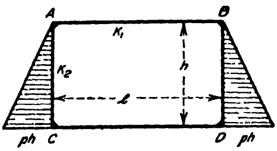
Moment Diagram

$$M_A = M_B = M_C = M_D$$
$$M_A = \frac{K_1}{K_1 + K_2} \cdot \frac{ph^2}{12}$$

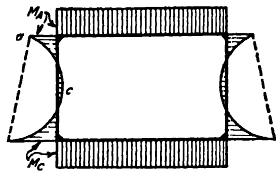
Case 3

$$M_A = M_B = -\frac{(2K_1K_2 + 4K_2^2)}{4K_2^2 - (2K_1 + 4K_2)^2}\cdot\frac{ph^3}{30}$$

$$M_C = M_D = -\frac{(6K_1^2 + 8K_1K_2)}{4K_2^2 - (2K_1 + 4K_2)^2}\cdot\frac{ph^3}{30}$$



Load Diagram

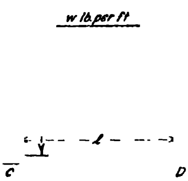


Moment Diagram

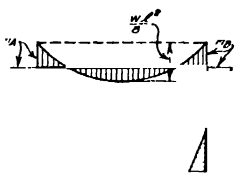
NOTE:—Area *abca* on moment diagram is moment diagram for simple beam of length h and loaded identically as AC, laid off from line ab.

TYPE B

Case 1



Load Diagram

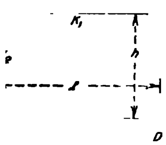


Moment Diagram

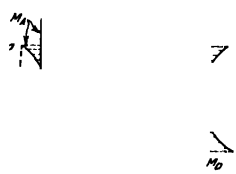
$$M_A = M_B = -\frac{2K_2}{K_1 + 2K_2}\cdot\frac{wl^2}{12}$$

$$M_C = M_D = \frac{K_2}{K_1 + 2K_2}\cdot\frac{wl^2}{12}$$

Case 2



Load Diagram



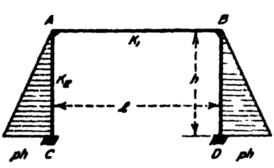
Moment Diagram

$$M_A = M_B = -\frac{K_1}{K_1 + 2K_2}\cdot\frac{ph^2}{12}$$

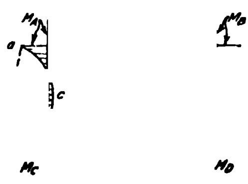
$$M_C = M_D = \frac{K_1 + K_2}{K_1 + 2K_2}\cdot\frac{ph^2}{12}$$

NOTE:—See note under type A, Case 3.

Case 3



Load Diagram



Moment Diagram

$$M_A = M_B = -\frac{K_1}{K_1 + 2K_2}\cdot\frac{ph^3}{30}$$

$$M_C = M_D = \frac{K_1 + K_2}{K_1 + 2K_2}\cdot\frac{ph^3}{20}$$

NOTE:—See note under Type A, Case 3.

27d. Construction Details.—Construction of a box culvert begins with casting the floor slab or footings. A keyway in the concrete should be provided into which the side walls are cast, to give proper bearing for the walls

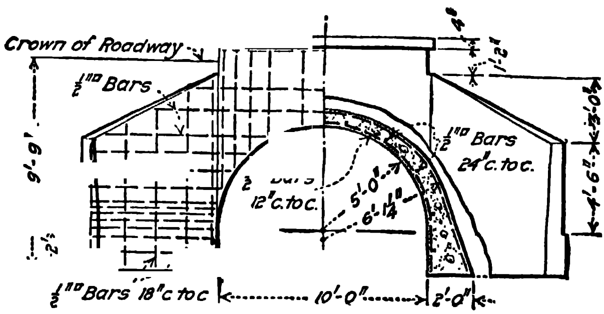


Fig. 37.—Standard design for 10-ft. arch culvert, State of Missouri Highway Department (Actual dimensions and shape of foundations governed by conditions of soil at location of site.)

to prevent the lateral earth pressure from forcing the walls inward. Dowel rods adequate for resisting the corner moments should be placed before casting the base. A keyway similar to that in the bottom should be provided in the

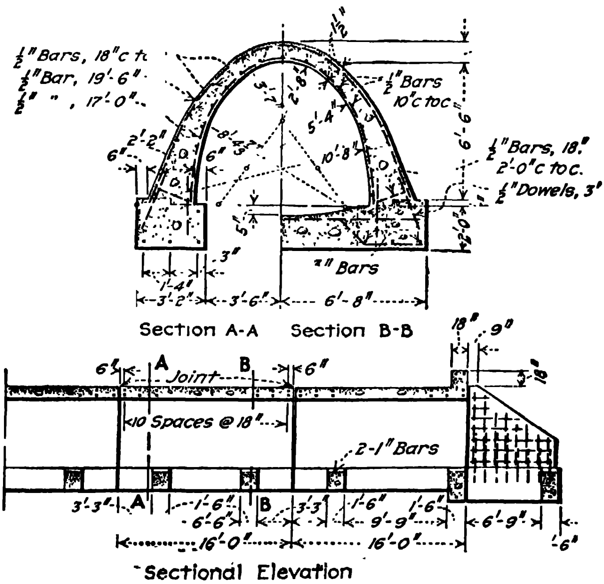


Fig. 38.—Standard arch culvert for fills up to 40 ft. high, C. M. & St. P. Ry.

roof slab to resist movement of the side walls at the top. Reinforcement for the upper corners should be placed before the walls are completed. Brackets extending at least 6 in. along each member should be used at all corners.

28. Arch Culverts.—Where the foundations permit and where the embankment is exceedingly heavy, or where artistic treatment is desired, the arch form of culvert is superior to all other types. The general considerations as to waterway and design of both ends of the barrel are the same as for other forms of culverts. The design of the cross-section is the same as that for an arch of the same span.

If the foundation is doubtful, a footing slab should be used which also forms the invert or floor of the barrel. Suitable keyways should be formed into which the springing of the arch is then cast. The footing slab should be reinforced, if necessary, to distribute the pressure properly. The arch may be

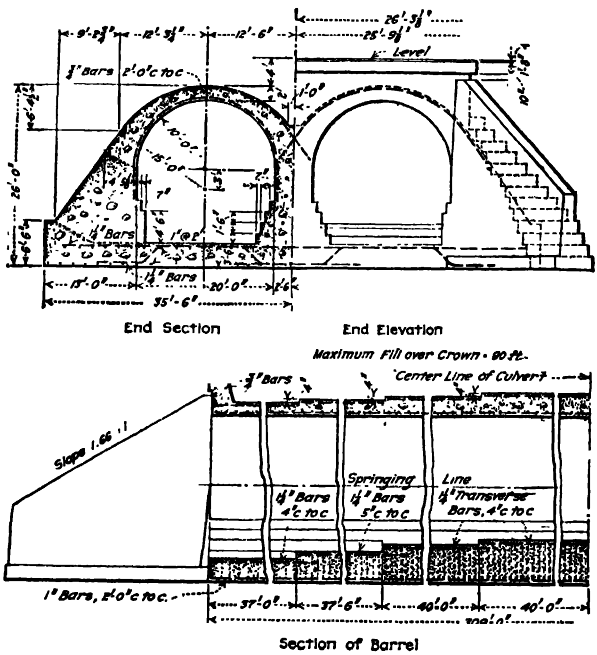


FIG. 39.—Double-barrel culvert, D. L. & W. R. R.

reinforced, depending on the results of the analysis. The construction of the arch would follow the methods enployed for arch bridges.

If good foundations are available, no special treatment of the barrel need be made. If the foundations are poor, the central portion of the barrel will be deflected downward. To resist this bending it is necessary to design the barrel as a box girder, with longitudinal steel in the bottom slab. The loading is not accurately determinable, and must be approximated. The bending tendency is due to the increase in loading from the ends toward the center; and bending takes place until the resistance offered by the foundation offsets further tendency to deflection of the barrel.

A minimum of 0.4 per cent of reinforcement should be placed longitudinally in sides, roof and floor, to offset shrinkage.

CONDUITS AND SEWERS

By S. C. Hollister

Conduits usually include pipe lines carrying internal pressure, with or without external pressure. Sewers, like culverts, should not carry hydraulic pressure and hence are designed to support external load only.

29. Stresses Due to Internal Pressure.—If the internal pressure on a pipe be P lb. per sq. in., then the total tension at any longitudinal section through one side of the pipe for a 1-in. length of pipe is PR. This is resisted by the stress in the steel; whence

$$A_s f_s = ptf_s = PR$$

$$p = \frac{PR}{tf_s}$$

Steel stresses have been used from 8,000 lb. to 16,000 lb. per sq. in. with good results in all cases. If the pipe is carefully made from rich mixtures, pressures up to 100 lb. per sq. in. can be accommodated easily. Steel fabric or deformed bars are more suitable than plain bars for reinforcing, since they distribute the stresses more evenly.[1]

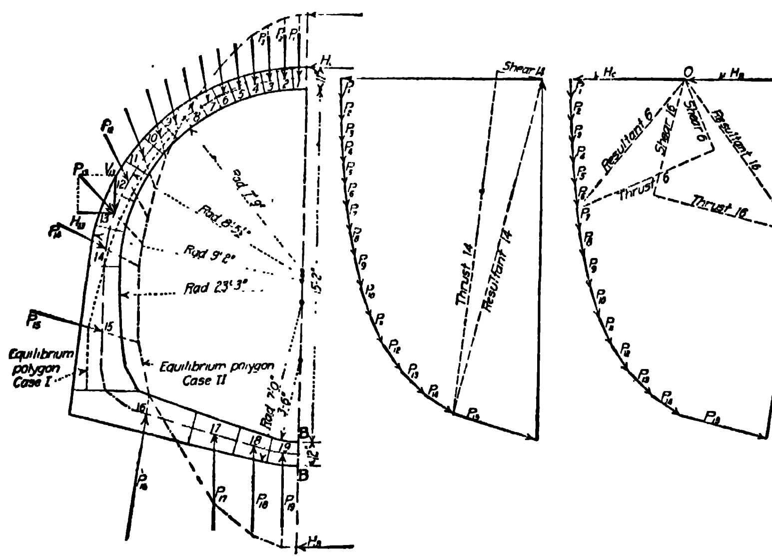


Fig. 40.

30. External Earth Pressures on Circular Pipe.—External earth pressure on pipes has been treated in Art. **26a**.

31. Comparative Estimates of Cost of Various Kinds of Pipe.—Comparative estimates of cost of wood-stave, concrete and cast-iron pipe is to be found in *Engineering News-Record*, Vol. 85, p. 309 (August 12, 1920).

32. Large Conduits and Sewers Not Circular.—For hydraulic reasons the cross-sections of sewers are seldom circular. The cross-section is divided into two

[1] See *Proceedings* American Concrete Institute, 1923 for further discussion of the design of concrete pressure pipe.

parts—namely, the invert, or floor of the barrel, and the arch, or walls and roof. A discussion of various forms of invert and arch may be found in Metcalf and Eddy's "American Sewerage Products," Vol. I, Chap. XI.

When the sewer rests directly upon rock, the invert simply becomes a pavement and the arch may be designed as an arch bridge resting upon rock, the ends being considered to be fixed.

When the sewer rests upon yielding material, the arch and invert must be monolithic. The analysis is made as for an arch except that the invert is assumed to be split longitudinally on its center line, and each end of the severed portion considered a fixed end. The arch then forms a complete lap with fixity at center of the invert. After the loads or shears are determined around the section, analysis may proceed as for any arch.

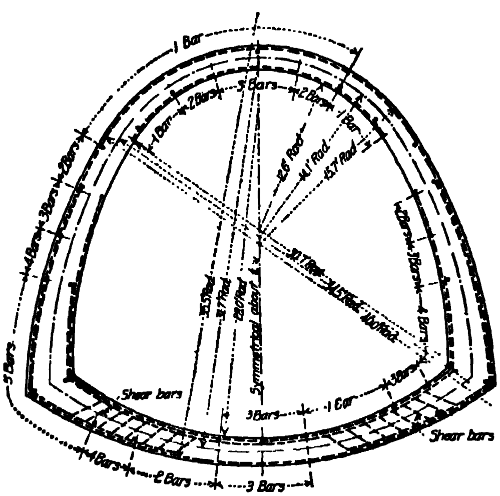


FIG. 41.

It will be found that the critical sections for moment are at the crown, the center of the invert, and along the junction of the invert with the side wall. The critical section for shear is usually near the junction of the invert with the side wall. Referring to Fig. 40, two analyses have been made. The first is for a case where the structure rests upon unyielding supports making virtually a fixed arch of the roof and walls. The pressure line or equilibrium polygon for this case will be seen to lie within the section of side wall and roof.

The second case provides for yielding supports, which call into play forces P_{16} to P_{19} inclusive. The thrusts will be seen to lie outside of the cross-section of the barrel at nearly all points, thus introducing moments throughout nearly all points along the ring.

It will be seen from a study of the force diagram at the right of Fig. 40, high shears are to be expected in the outer third of the invert span. If unit 16 of the arch ring were subdivided, it is clear that a high shear would be found in the invert near the inner face of the side wall. The cross-section in Fig. 41 shows the disposition of the steel to accommodate such shear. It will be noted that since the load on the invert is upward it is necessary to incline the shear reinforcement upward toward the center.

Conduits and sewers should have a minimum longitudinal reinforcement along the barrel of about 0.4 of one per cent. Additional longitudinal reinforcement should be added if for any reason flexural stresses may be set up through the action of the barrel as a hollow beam.

SECTION 10

CHIMNEYS

BY H. E. PULVER

DRAFT AND SIZE OF CHIMNEY

Before designing a large chimney the proper height and diameter must be selected. The height must be such as will give the required draft and the cross-sectional area must be large enough to permit the passage of the burnt gases.

The draft depends on the height of chimney, the temperature of the gases, the altitude or elevation of the chimney above sea level, the nature of the fuel, the furnace, and the flues between the furnace and the chimney. The cross-sectional area required depends on the kind and quantity of fuel to be burned, the draft available, and the friction losses in the chimney.

It is obviously impractical to produce any single formula for determining chimney sizes which will satisfactorily take all of the various factors into consideration and, consequently, the formulas used in selecting stack sizes are largely empirical.

1. Theoretical Draft.—Draft may be defined as the difference in pressure available for producing a flow of gases. If the gases in a chimney are heated, they will expand and occupy a larger volume than before and their weight per cubic foot will be less. Consequently, the unit pressure at the bottom of a chimney due to the column of heated gases will be less than the unit pressure exerted by a column of cold air outside the chimney. The difference between these two pressures will cause a flow of the gases up the stack. This flow will be continuous as long as the furnace is in operation.

The intensity of draft is usually measured in inches of water instead of pounds per square inch or pounds per square foot. The pressure of an inch of water is equal to a pressure of 5.193 lb. per sq. ft. assuming water to weigh 62.32 lb. per cu. ft.

Making no allowance for the difference in density between the air and the flue gases, and assuming the atmospheric pressure as 14.7 lb. per sq. in. and the average atmospheric temperature as 60 deg. F., the following formula may be used for finding the intensity of the draft.

$$D = KH$$

where

D = theoretical draft in inches of water.

H = height of chimney above grates in feet.

K = a factor which will vary according to the temperature of the gases in the chimney.

The following table gives the values of K for different temperatures of the chimney gases:

TABLE 1

Temperature of Chimney Gases (Deg. Fahr.)	Values of K
350	0.0053
400	0.0058
450	0.0063
500	0.0067
550	0.0071
600	0.0075
650	0.0078
700	0.0081
750	0.0084

2. Draft Losses.— In the ordinary power plant there are various draft losses due to leakage of air and to the resistances offered to the passage of the gases by the furnace, boiler, flues, and the interior of the chimney.

The loss due to leakage of air around boiler sections, flue joints, etc., is often considerable and difficult to estimate. Care should be taken to make and keep all joints as air tight as practical.

The draft loss in the furnace will usually vary from about 0.15 to 1.00 in. of water, depending on the kind and size of coal used and the amount burned per hour on each square foot of grate surface.

The draft loss caused by the boiler heating surface will vary largely according to the design of the boiler and the percentage of its capacity at which it is being operated. This loss will usually vary from about 0.25 to 1.00 in. of water and is usually less in good Babcock and Wilcox and good horizontal return tubular boilers than in Stirling and vertical tubular boilers.

The use of economizers in connection with boilers will cause a reduction of about 75 deg. F. or more in the flue gas temperatures. The loss of draft through the economizers will amount to about 0.3 in. of water. The installation of economizers frequently requires the use of a fan for increasing the draft.

The draft losses in round steel flues, having ample cross-sectional areas, is approximately 0.1 in. of water for each 100 ft. of flue length. Losses in square and rectangular flues are about 12 per cent more than for round flues. A loss of about 0.05 in. of water should be allowed for each right-angled turn between the boiler and the chimney. These figures should be doubled for brick or concrete flues. In designing flues, a cross-sectional area of about 35 sq. ft. should be provided for each 1,000 rated boiler hp.

Draft losses in chimneys due to friction may be computed by the following formula:

$$d = \frac{fW^2CH}{A^3}$$

where

d = loss of draft in inches of water.
W = weight of gases in pounds passing per second.
C = circumference of chimney in feet.
H = height of chimney in feet.
A = area of passage in chimney in square feet.
f = a sea level value depending on the temperature of the gases and the interior surface of the chimney, as given by the following table:

TABLE 2

f	TEMPERATURE OF GASES (DEG. FAHR.)	INTERIOR SURFACE O CHIMNEY
0.0011	350	steel
0.0015	600	steel
0.0015	350	brick
0.0020	600	brick

The available draft D_1 in a chimney is equal to the theoretical draft D minus the frictional loss d. Expressed as a formula and substituting values for D and d:

$$D_1 = D - d = KH - \frac{fW^2CH}{A^3}$$

3. Formulas for Height and Diameter of Chimneys.—From the formula for the available draft given in the preceding article, it is evident that the available draft depends on the diameter of the chimney as well as on the height. Hence, various combinations of heights and diameters could be selected which would give the same results. However, in studying the relation of costs to heights and diameters, it has been found that the chimney of minimum cost will usually have a diameter dependent on the boiler horsepower served and a height dependent on the available draft required.

Assuming 120 lb. of flue gas per hr. per boiler hp., which is the same as a consumption of 5 lb. of coal per hp. per hr. allowing 24 lb. of gas for 1 lb. of coal, and which provides for ordinary overload and the use of poor coal, this method gives the following formulas for sea-level conditions:

For a stack lined with masonry

$$\text{Diameter in inches} = 4.92 \times (\text{hp.})^{2/5}$$

where

hp. = the rated horsepower of the connected boilers.

The following table shows the diameters required for masonry lined stacks for boilers of various ratings when constructed at sea level.

TABLE 3

Rated boiler horsepower	Diameter of chimney (in.)	Rated boiler horsepower	Diameter of chimney (in.)
20	13	1,200	84
40	20	1,400	90
60	24	1,600	95
80	28	1,800	99
100	30	2,000	103
150	36	2,200	107
200	40	2,400	111
300	48	2,600	115
400	53	2,800	119
500	58	3,000	122
600	63	3,200	125
700	67	3,400	128
800	71	3,600	131
900	75	3,800	134
1,000	78	4,000	137

The diameters given by the formula and in Table 3 are those required for chimneys built at sea level. The diameter required at sea level must be increased for a chimney constructed at a higher altitude as explained in a following article.

When a large chimney serves a number of boilers equipped with mechanical stokers, the area calculated by the formula should be increased about one-third to allow for leakage of air through the settings of idle boilers and for irregular operating conditions.

Masonry-lined chimneys, whose diameters are found by the above formula, will give an available draft which bears a constant ratio to the theoretical draft. This ratio, allowing for the cooling of the gases in their passage up the chimney, is about 0.80, or

$$D_1 = 0.80\ KH$$

Solving for H

$$H = 1.25\frac{D_1}{K}$$

where

H = required height of chimney at sea level in feet above level of grates.
D_1 = available draft required in inches of water.
K = the value given in Table 1 and in the formula
D = KH in Art. 1.

The required height of chimney found by this formula must be increased for a chimney constructed at a higher altitude as explained in the following article on Correction of Chimney Sizes for Altitude.

These two formulas for the diameter and height of masonry-lined chimneys required at sea level are the ones used by the Babcock and Wilcox Company.

This company states that a convenient rule for large chimneys, 200 ft. high or over, is to provide 30 sq. ft. of cross-sectional area per 1,000 rated boiler hp.

Kent's formula for chimney sizes is:

$$\text{hp.} = 3.33(A - 0.6A^{1/2})H^{1/2}$$

when

hp. = rated boiler horsepower based on a coal consumption of 5 lb. per hr. per rated boiler hp.
A = area of chimney in square feet.
H = height of chimney in feet.
$A - 0.6A^{1/2}$ = effective area of chimney assuming that the frictional resistance in the chimney is equivalent to a layer of gas 2 in. thick around the inside circumference.

The Babcock and Wilcox Company strongly recommend that the chimney sizes given by Kent's formula be increased from 25 to 60 per cent for the low-grade bituminous coals of the Middle or Western states depending on the nature of the coal and the capacity desired.

Christie's formula for chimney sizes is:

$$\text{hp.} = 3.25A\sqrt{H}$$

where

hp. = rated boiler horsepower based on 4 lb. of coal burned per horsepower per hour.
A = cross-sectional area of chimney in square feet.
H = height of chimney in feet.

The M. W. Kellogg Company recommend that, for the Middle states and Western bituminous coal, the height as determined by Christie's formula be unchanged and that the areas be increased 25 per cent. Temperatures, flues, type of boilers, economizers, and other accessories may have a great influence on the proper size.

4. Correction of Chimney Sizes for Altitude.—As the altitude increases, the density or weight per unit volume of the air decreases. Consequently, as a certain weight of air for combustion is required per boiler horsepower, a larger volume of air will be required to produce the same results at higher altitudes than at sea level. If the areas of the boiler grates and flues are not changed, then the air at higher altitudes must pass through these grates and flues at a greater velocity in order to obtain the increase in volume required. This means that the draft must be greater than at sea level and, consequently, the chimney must be increased in height to obtain this increase in draft. For any given boiler horsepower and constant weight of gases, the mean velocity of the gases will be inversely proportional to the barometric pressure and the velocity head (or pressure), measured in column of external air, will be inversely proportional to the square of the barometric pressure. That means that the height at sea level must be multiplied by the square of the ratio of the barometer reading at sea level to that at the altitude given.

Frequently in designing a boiler for higher altitudes, the assumption is made that a certain fuel will require the same draft (measured in inches of water at the boiler damper) as at sea level. This means that the chimney height at sea level must be multiplied by the ratio of the barometer reading at sea level to that at the given altitude, and not according to the square of this ratio, in order to obtain the height necessary to give the required draft.

The Babcock and Wilcox Company says that the correct height probably falls between the values given by these two theories, as the flues are usually made larger for the boilers to be used in higher altitudes. Further, that in making capacity tests with coal fuel, no difference has been noted in the rates of combustion for a given draft suction, measured by a water column, at high and low altitudes. This indicates that the height of chimney at sea level should be multiplied by the direct ratio rather than the square of the ratio. Also if the direct ratio is used, the difference in capacity would not be more than 10 per cent at an altitude of 10,000 ft., assuming that the correct height lies between the values found by the two theories.

If the height of a chimney is increased, the friction loss in the chimney is increased by this added height. Consequently the diameter of the chimney must be increased to offset this added friction loss. This increase in diameter, in order to keep the total friction loss the same, is inversely as the two-fifths power of the barometric pressure. Hence, the diameter at sea level should be multiplied by the two-fifths power of the ratio of the sea level barometer reading to that at the given altitude.

The following table gives the altitude correction factors for chimney capacities. It is seen that altitude affects the height more than the diameter and that practically no increases in diameter are needed for altitudes less than about 3,000 ft. For very high altitudes, the increase in chimney height would increase the cost very greatly as well as making the proportions of height to diameter impractical. In such cases it is better to increase the grate areas so that the

required rate of combustion and the accompanying required draft will be lessened so that a shorter chimney will be satisfactory.

TABLE 4.—ALTITUDE CORRECTION FACTORS FOR CHIMNEYS

Altitude in feet above sea level	Normal barometer	R	$(R)^2$ Height factor	$(R)^{3/8}$ Diameter factor
0	30.00	1.000	1.000	1.000
1,000	28.88	1.039	1.079	1.015
2,000	27.80	1.079	1.164	1.030
3,000	26.76	1.121	1.257	1.047
4,000	25.76	1.165	1.356	1.063
5,000	24.79	1.210	1.464	1.079
6,000	23.87	1.257	1.580	1.096
7,000	22.97	1.306	1.706	1.113
8,000	22.11	1.357	1.841	1.130
9,000	21.28	1.410	1.988	1.147
10,000	20.49	1.464	2.144	1.165

$$R = \frac{\text{Sea level barometer reading}}{\text{Altitude barometer reading}}$$

To obtain correct height of chimney at any altitude, multiply height at sea level by height factor, R^2, for the altitude selected.

To obtain correct diameter of chimney at any altitude, multiply diameter at sea level by diameter factor, $(R)^{3/8}$, for the altitude selected.

Illustrative Problem.—Determine the proper height and diameter of a brick chimney to serve Babcock and Wilcox boilers rated at 2,000 hp. under the following conditions:

Boilers to operate at 50 per cent overload.
Altitude = 1,500 ft. above sea level.
Atmospheric temperature = 60 deg. F.
Flue gas temperature = 500 deg. F.
Grate surface = 400 sq. ft.
Combustion rate = 35 lb. per hr. per sq. ft. of grate surface.
Length of flues = 150 ft. with two right-angled turns.
Kind of flues = round steel.
Kind of coal = Illinois bituminous.

The available draft required at the base of the stack will equal the sum of the draft losses in the furnace, boiler, and flues.

Draft required it furnace for combustion of this coal at this rate = 0.40 in.
Boiler losses at 50 per cent overload = 0.40 in.
Flue losses $= (0.10)\left(\frac{150}{100}\right) + (2)(0.05)$ = 0.25 in.
Available draft required $= D_1$ = 1.05 in.

Substituting in formula $H = \dfrac{D_1}{0.8K}$

$(K = 0.0067$ for 500 deg. F.—See Table 1)

$$H = \frac{1.05}{(0.8)(0.0067)} = 196 \text{ ft.}$$

Altitude correction factor for height for an altitude of 1,500 ft. is 1.122 by interpolation in Table 4.

Required height = (1.122)(196) = 220 ft.

Required diameter of a brick chimney to serve boilers of 2,000 rated boiler hp. is 103 in. from Table 3.

Altitude correction factor for diameter for an altitude of 1,500 ft. is 1.023 by interpolation in Table 4.

Required diameter = (1.023)(103) = 105 in.

5. Chimneys for Oil Fuels.—The requirements for chimney sizes when oil fuel is used are quite different than when coal is burned, because some of the boiler losses are eliminated and the volume and temperature of the gas entering the chimney is less than with coal. This means that the cross-sectional area required for oil fuel is much less than that required for coal, as the volume of gas in the case of oil may be taken as approximately 60 per cent of that for coal. Also, as the draft requirements are less, the height of chimney required for oil fuel is less than that for coal.

In designing chimneys for oil fuel, care must be taken not to have an excess of draft as the admission of too much air reduces the efficiency fairly rapidly. If the stack is too high, some form of automatic control will give better results than ordinary hand control. This is especially true in the case of varying loads. Too little draft is also bad because if the draft is not enough to carry off the hot burnt gases, the action of the heat on the brickwork of the furnace will be very injurious. Consequently, great care should be used in designing chimneys where oil fuel is to be used.

The following table is taken from one calculated by C. R. Weymouth from actual test data. This table will ordinarily give satisfactory results.

TABLE 5.—CHIMNEY SIZES FOR OIL FUEL

Diameter (in.)	Height in feet above boiler room floor					
	80	90	100	120	140	160
	Nominal rated hour power					
33	161	206	233	270	306	315
36	208	253	295	331	363	387
39	251	303	343	399	488	467
42	295	359	403	474	521	557
48	399	486	551	645	713	760
54	519	634	720	847	933	1,000
60	657	800	913	1,073	1,193	1,280
66	813	993	1,133	1,333	1,480	1,593
72	980	1,206	1,373	1,620	1,807	1,940
84	1,373	1,587	1,933	2,293	2,560	2,767
96	1,833	2,260	2,587	3,087	3,453	3,740
108	2,367	2,920	3,347	4,000	4,483	4,867
120	3,060	3,660	4,207	5,040	5,660	6,160

Sizes given are good for 50 per cent overloads.

Sizes are based on centrally located chimneys with short direct flues and ordinary operating efficiencies.

6. Chimneys for Blast Furnace Gas.—Chimneys for blast furnace gas should be about the same diameter as those used for coal, as the slight increase in volume of gas is offset by the higher temperatures. A height of 130 ft. will produce a draft sufficient to care for 175 per cent of the boiler's rated capacity. Too much draft may result in improper mixtures of gas and air which may cause a pulsating action of the flame or, perhaps, explosions.

GENERAL CONSIDERATIONS IN CHIMNEY DESIGN

Large masonry chimneys are usually constructed either of brick or reinforced concrete, though some have been built of hollow clay tile filled with concrete and reinforced with steel rods. The brick may be of ordinary or of special types. For tall chimneys, radial brick are to be preferred to common brick as they permit the radial joints to be of the same thickness and, as they are larger than ordinary brick, they reduce the number of required joints about one-third. Chimney walls of ordinary brick should not be designed to carry any tension while those constructed of radial brick may carry a small amount. Reinforced concrete and reinforced hollow clay tile and concrete chimney walls are usually designed to take tensional stresses, depending on the character and amount of reinforcement used.

The shape of brick chimneys may be round, octagonal, or square, while reinforced concrete and clay tile chimneys are usually round. Square masonry chimneys are rarely ever built to great heights as they are seldom as economical as octagonal or round chimneys. Round chimneys are often more economical than octagonal ones. Large masonry chimneys usually taper from the top to the bottom, and are not of constant diameter. The batter of the walls usually is between $\frac{1}{30}$ and $\frac{1}{50}$.

In many instances, especially in the case of brick chimneys, the chimney is composed of a foundation, base, and column instead of a foundation and column only. When a base is used in connection with a brick chimney, the most economical height of the base is about one-fifth or one-sixth of the chimney height. Some designers prefer to have the breech opening above the base and use the base for a support for the breeching, while other designers extend the base section about 3 ft. above the top of the breech opening. The base section is usually square or octagonal in section and its walls have no batter.

Each chimney design should be treated as a separate problem and care and judgment should be used in selecting the proper type of chimney and in making the design. Various details of the design and construction of large chimneys are controlled by patents held by different companies who are usually very zealous in guarding against possible infringements. Most of these companies have had considerable experience and are reasonable in regard to fees charged and, consequently, it is frequently economical and advisable to make use of their services.

7. Forces Acting on Chimney.—The main forces acting on large masonry chimneys are: the weights of the chimney, lining, and foundation; the foundation reactions; the wind pressure; and the expansion and contraction in the chimney walls due to the heat of the gases.

8. Wind Pressures.—It has been generally accepted that wind pressure increases directly with the square of the velocity and that wind pressures are

greater at higher than at lower elevations. In studying the wind pressure on a chimney, the following three things should be considered: (1) The wind velocities in the section of the country where the chimney is to be built, (2) whether the chimney will be located in a high exposed place or in a comparatively low and sheltered spot, and (3) the height of the chimney.

The United States Weather Bureau has proposed the following formula for wind pressure:

$$p = 0.004 \frac{B}{30} V^2$$

where

p = pressure in pounds per square foot on a flat surface.
B = barometric pressure in inches.
V = velocity of wind in miles per hour.

For a barometric pressure of 30 in. the formula becomes

$$p = 0.0040 V^2 \qquad \text{(for square chimneys)}$$

Considering that the wind pressure on the side of an octagon is about 70 per cent of that on a side of a square and that the wind pressure on the side of a circle is about 60 per cent of that on the side of the square, this formula may be reduced to:

$$p = 0.0030 V^2 \qquad \text{(for an octagonal chimney)}$$
$$p = 0.0025 V^2 \qquad \text{(for a circular chimney)}$$

A projected area perpendicular to the direction of the wind is taken as the exposed surface in the case of a circular chimney, while a projected area at right angles to two opposite sides is taken as the exposed surface in the case of a square or octagonal chimney.

A wind velocity of 100 mi. per hr. is about the maximum in most localities. This velocity will give a pressure of 25 lb. per sq. ft. on the projected area of a circular chimney, which value is commonly used in the design of round chimneys. In localities where the wind velocities sometimes exceed 100 mi. per hrs., and especially if the chimney is to be in a high exposed place, wind velocities of more than 100 mi. per hr. should be used in determining allowable wind pressures. Also, in sections of the country where the maximum wind velocities are rarely severe, a wind velocity of about 90 mi. per hr. has been satisfactorily assumed, giving a value of 20 lb. per sq. ft. in the case of a circular chimney.

It has been found that wind velocities, and consequent wind pressures, increase with the distance above the earth's surface. Accordingly, it has been recommended that, in the design of high chimneys, the value found by the proper one of the preceding formulas be used for the first 300 ft. and that this value be increased about 2½ lb. per sq. ft. for each additional 100 ft. or fractional part. That is, if for a circular chimney 460 ft. high a wind pressure of 25 lb. per sq. ft. be used for the lower 300 ft., a pressure of 27½ lb. per sq. ft. should be used from 300 ft. to 400 ft., and a pressure of 30 lb. per sq. ft. from 400 to 460 ft.

9. Effect of Earthquake Shocks.—The usual effect of an earthquake shock is to cause the foundation to be moved quickly in a horizontal direction. The damaging effect is due to the rate of acceleration, the higher the acceleration the greater the effect.

J. G. Mingle, in his article on the "Design of Reinforced Concrete Chimneys" in the *Proceedings* of the American Concrete Institute, Vol. 14, p. 283, says that the earthquake acceleration may be taken from 7 to 9 ft. per sec. per sec. and that the force is applied instantaneously. In Japan, earthquake accelerations as high as 11 ft. per sec. per sec. have been noted.

The maximum stresses in a chimney will occur at a section about two-thirds the height of the chimney instead of at the base. If the chimney is considered as an inverted pendulum, this section would correspond to the center of percussion approximately.

The force due to an earthquake shock is applied at the base of the chimney and is given by the formula:

$$F_e = \frac{A_e}{32.2} W$$

where

F_e = force in pounds due to the shock.
32.2 = gravity acceleration in feet per sec. per sec.
W = weight of the chimney in pounds above the horizontal section considered.
A_e = acceleration in feet per sec. per sec. due to the quake.

The moment due to this force is given by the formula:

$$M_e = F_e y = \frac{A_e}{32.2} W \cdot y$$

where

M_e = moment in inch-pounds.
y = distance in inches of the center of gravity above the base of the chimney or above the section considered. For most chimneys, y is a little less than half the height above the section.

It seems impractical to try to reinforce brick chimneys against possible earthquake shocks, though reinforced-concrete chimneys may be so reinforced.

10. Lining.—Lining should be provided to protect the chimney walls from the heat of the burnt gases. This lining should extend from a section at least 2 ft. below the flue opening to a section at a varying distance below the top of the chimney, depending on the temperature of the gases. For ordinary boiler plants where the temperature does not exceed 800 deg. F., the lining should be about one-fifth the height, but not less than 20 or 30 ft. above the flue opening. For temperatures between 800 and 1,200 deg. F. the lining should be from one-half to two-thirds the chimney height. For temperatures between 1,200 and 2,000 deg. F., the lining should extend to the top of the chimney. The lining should be constructed independently of the shaft and should preferably rest directly on top of the footing. An air space of at least 2 in. should be left between the chimney inner wall and the lining, care being taken to keep this space free from mortar and dirt. When the lining does not extend to the top of the chimney, the chimney wall should be corbeled out about 2 in. above the top of the lining so as to prevent falling soot from collecting in the air space. Linings for masonry chimneys are nearly always constructed after the chimney shell is finished.

The lining material is usually fire brick about 4¼ in. thick, laid with a fire clay mortar. For the higher temperatures (above 2,000 deg. F.) a special high-grade radial cut firebrick should be used. A lining which is to be subjected to the

action of a destructive flue gas (such as a gas from a smelter, or still, etc.) should be of a material which is resistant to this particular gas as well as to temperature.

Some designers use a lining about 4¼ in. thick no matter what the height of lining, while others, in chimneys lined for more than 150 ft., use a 4¼-in. lining for the upper 150 ft. and a lining thickness of about 6½ in. for the bottom part. In some instances special firebrick, which give a lining less than 4¼ in. thick, have been used. Some companies which specialize in reinforced concrete chimneys prefer a reinforced concrete lining for all ordinary purposes.

11. Breech Opening.—Usually one breech opening is sufficient for a chimney, though it is sometimes necessary to have two or more openings. As a section through the breech opening is usually the weakest portion of the chimney, the breech opening should be carefully designed so that the chimney will not be unduly weakened at this place.

The area of the breech opening should be from 7 to 20 per cent more than the internal cross-sectional area at the top of the chimney. Whenever practical, the height of the breech opening should be from 1½ to 2 times its width. The maximum width of the opening should preferably not exceed two-thirds of the internal diameter for a round chimney. The opening should be lined on the reveals with a good refractory material.

In brick chimneys, the masonry above the breech opening should be supported by heavy I-beams resting on steel plates and with air spaces at each end to allow for expansion. A flat masonry arch should be constructed under the I-beams to protect them from the flue gases. The breech opening should be reinforced laterally by heavy tie rods and plates over the top and at the bottom. Steel bands, about ⅜ by 3 in., should be placed in the masonry above and below the opening. Frequently the brickwork is buttressed out around the breech opening to compensate for the reduction in area caused by the opening.

When it is necessary to provide a breech opening in a brick chimney wider than that ordinarily permitted in a round chimney, a base should be provided to take care of this opening. The base is usually square or octagonal in shape and should extend about 3 ft. above the top of the breech opening. Chimney bases are frequently constructed of common brick with straight vertical walls which are thicker and heavier than the walls of the chimney proper.

The M. W. Kellogg Company suggests the following rule for determining the maximum width of flue opening into chimney bases: Multiply the width of the chimney at the bottom by 33 per cent for round chimney bases, 42 per cent for octagonal chimney bases, and 50 per cent for square chimney bases. The width of the flue opening should be kept as narrow as possible in order to maintain the highest stability through the opening.

In reinforced concrete chimneys, the walls near the breech opening are thickened to make up for the concrete taken out of the opening. Ordinarily the wall thicknesses are increased from 1 to 3 in. The wall thickening should extend about 5 ft. above the top and an equal distance below the bottom of the breech opening. Extra reinforcement should be provided around the flue opening to more than make up for that removed. Whenever practical, vertical rods extending towards the opening should not be cut off but should be bent around the sides of the opening. The bends should not be sharp, and the rods should not all be bunched together. About three or more vertical rods, say of the same size as the

other vertical reinforcement, should be bent and placed on each side of the breech opening as extra reinforcement. Three or more sets of horizontal rings of rods should be placed both above and below the breech opening. These rods should preferably be of the same size as the vertical reinforcement. Sometimes a few extra horizontal rods, of a length sufficient to extend about 2 ft. beyond the opening, are placed over the top and under the bottom of the breech opening in addition to the horizontal rings. The temperature reinforcement should be increased from a section about 5 ft. below the opening to a section about 5 ft. above to aid in resisting the large temperature stresses that occur in this portion of the chimney. If a wire mesh is used about 2 in. from the outer surface, this mesh should be continued into the sides of the opening and then around the interior of the chimney about 2 in. from the surface. This extra mesh should also extend about 5 ft. above and below the opening. The mesh should be placed on the outside of the vertical rods and tied to them.

12. Baffle Plates.—When there are two or more flue openings into a chimney, baffle plates should be provided to keep the gases from any one flue from interfering with the proper operation of another flue. The baffle plates should extend from about 2 ft. below the bottom of the breech opening to a point about 5 ft. above the top of the opening so that the gases from the different flues will be moving along parallel lines when they come together. The baffle plates should be supported by I-beams and should be bonded to the lining. For chimney diameters of about 5 ft. or less, a 4-in. wall of firebrick makes a satisfactory baffle. For chimneys having larger diameters, the baffle wall should be about 6 or 8 in. thick, depending on conditions.

13. Clean-out Doors.—Clean-out doors of ample size should be provided at the base of each stack, preferably on the side opposite the breech opening. A standard cast-iron clean-out door, about 24 in. by 36 in. in size, and frame is usually satisfactory. In a brick chimney a flat masonry arch is usually constructed over the top of the clean-out door opening. In a reinforced concrete chimney, a little extra reinforcement is sometimes placed around the opening.

14. Ladder.—Permanent ladders should be provided for all large chimneys. In masonry chimneys, the ladders are frequently placed on the inside for the sake of appearance. Galvanized iron rungs about $\frac{3}{4}$ in. in diameter and bent in the shape of a "U" with hooked ends are satisfactory. These rungs should be spaced about 15 in. apart and securely anchored to the masonry.

15. Pulley and Cable.—Each chimney should be provided with a pulley securely attached to the top and a loop of wire cable extending the full height of the chimney. A bronze pulley with a $\frac{3}{16}$-in. galvanized wire cable is usually satisfactory.

16. Painter's Trolley and Track.—A painter's trolley and track are usually not provided for a masonry chimney unless it is desired to paint the outside of the chimney frequently, say for some such purpose as advertising.

17. Lightning Conductor.—Every masonry chimney should be provided with a lightning conductor. One point should be provided for about every 2 ft. of diameter and no chimney should have less than two points. Each point should consist of a platinum tipped copper rod about $\frac{3}{4}$ in. in diameter and from about 6 to 10 ft. in length, securely fastened to the top of the chimney. Some designers claim that the points should project 3 or 4 ft. above the top of the chimney,

while other designers say that a projection from 4 to 6 in. is enough if one point is used for every foot in diameter. The lower ends of the points should be fastened to a copper cable which passes entirely around the chimney. The down cable leading from the circular cable should be a ½-in. solid or seven-strand copper cable securely anchored to the chimney wall at intervals of about 6 or 8 ft. The lower end of this cable should be fastened to a copper plate buried in the ground.

Some constructors of reinforced concrete chimneys fasten the copper points to the reinforcing steel at the top of the chimney and claim that this method gives satisfactory results. In such a case, the reinforcing steel must be well groundedt

18. Architectural Treatment.—Many of the large masonry chimneys tha. have been constructed are far from pleasing to the eye. During later years there has been a great effort to improve the appearance by constructing the chimney with a taper and by providing some form of ornamentation. A chimney whose walls have no taper usually has the appearance of being top-heavy. The correct taper to use for any certain chimney depends upon conditions to be met.

The appearance of brick chimneys may be improved by the use of the taper, by the use of base courses, by the enlargement of the top, by paneling at the base and top, by using courses of stone or terra cotta at the base and head portions, and by the use of different colored brick. An effort should be made to make the ornamentation of the chimney conform to the general architectural style of the adjacent structure.

Care should be taken in the construction of reinforced concrete chimneys to make the surface smooth and uniform in appearance, to avoid humps, and other irregularities, to remove form marks, to avoid rough or pitted surfaces, and to avoid the construction lines which show where one day's work ends and another day's work begins. Fluting gives a pleasing appearance when properly done.

As a rule, advertising matter placed on chimney walls does not improve the appearance of the chimney. However, when permanently colored kiln-burnt brick are used for the lettering in brick chimneys, the effect is sometimes pleasing. A monogram or trade mark, of proper size and correctly located, placed on the walls of a concrete or brick chimney does not detract from the appearance.

If careful attention is given to the form, color, and ornamentation of a large chimney, it may result in a structure whose appearance is pleasing to the eye with but little additional cost.

LARGE BRICK CHIMNEYS

Large brick chimneys have been successfully constructed and used for many industries. Both common and radial brick have been used, though radial brick are to be preferred on account of their many advantages.

The shape of the chimney is usually circular or octagonal, with tapering walls. The chimney may be constructed in two parts, foundation and shaft, or three parts, foundation, base, and shaft. Usually the round tapered chimney is preferable as it offers less resistance to the wind than does the octagonal. The square shape on the other hand, offers more resistance to the wind than does the octagonal.

Square and octagonal brick chimneys require no special shapes of brick, except that a specially shaped brick should be provided for the octagonal corners. Radial brick are better than common brick for round chimneys.

19. The Brick.—When common building brick are used, they should be of good quality, straight, hard, well burned, well shaped, uniform in size and color, and have a specific gravity of at least 2.0 and a crushing strength of at least 4,000 lb. per sq. in. Sometimes special brick, such as face brick or brick made from selected clays, are used.

Radial brick should be of the best quality, molded from a suitable refractory clay, well burned, well shaped, free from chips, hard, sound ringing, acid and weather proof, and of a fairly even color. The brick should be made to conform closely with the circular and radial lines of the chimney in question, and their outside faces should be regular in size so that the brickwork will have a good general appearance. The per cent of absorption for 24 hours immersion, based on the dry weight of the brick, should be between 6 and 12 per cent (it is usually between 8 and 10 per cent). One cubic foot of radial brickwork should preferably weigh not less than 120 lb. per cu. ft. The total perforations should not be more than 25 per cent of the cross-sectional area of the brick and each perforation should not be so large that there is danger of filling the space with mortar. The crushing strength of radial brick should be about 6,000 lb. per sq. in. or more.

Some of the advantages of radial brick over ordinary common building brick are:

Uniform thickness of radial joints.

Fewer joints, due to larger sizes of brick.

Perforations giving dead air spaces which tend to reduce radiation.

Perforations permitting the mortar to form a better vertical anchorage and reducing the number of reinforcing bands needed.

Acid and weather-proof characteristics.

More highly refractory.

The M. W. Kellogg Company manufactures a corrugated perforated radial brick which, they claim, makes stronger mortar joints. The Heine Chimney Company uses a perforated, interlocking, radial brick laid in full mortar beds with shove joints. It is claimed that the interlocking feature and shove joints make the walls much stronger.

As mentioned in Art. **10**, the lining brick should be a firebrick of good quality such as will withstand the action of the flue gases.

20. The Mortar.—The mortar used in constructing brick chimneys should be a fairly rich Portland cement mortar, preferably with an addition of lime. The added lime makes the mortar smoother and easier to work. A mortar composed of 1 part Portland cement, ½ to 1 part lime, and 3 to 4 parts of sand is usually satisfactory. The sand should be clean, sharp and well graded with no particles over ¼ in. in size. It is well to choose a sand whose largest grains are not much more than half the thickness of the mortar joint used. However, there is a difference of opinion among companies and engineers as to the correct proportions and sizes of the mortar materials.

Professor C. C. Williams in his book on "Design of Masonry Structures and Foundations" advises the use of a mortar consisting of 1 part lime, 3 parts Portland cement, and 12 parts sand. The sand should be fairly coarse in character and screened to remove particles larger than ¼ in.

The M. W. Kellogg Company, and Wm. Summerhays and Sons both specify a mortar consisting of 1 part Portland cement, 2 parts fresh burnt lump lime mortar,

and 5 parts clean, sharp sand. The cement should be added to the sand and lime mortar as the mortar is required, and no mortar should be used after initial set has taken place. The cement should not be added while the lime is warm. The lime should be fresh burnt and thoroughly slaked before being mixed with the sand and cement. No old or air slaked lime should be used. The sand should be free from clay, loam, vegetable matter, and large pebbles, and should be washed and screened if necessary.

Mortar for a lining of firebrick should consist of a suitable mixture of cement, fireclay, and sand, about a 1:1:3 mix.

21. Allowable Unit Stresses.—In general, the allowable unit stresses in brick chimneys should be very conservative because of large possible damage to surrounding property due to a chimney failure.

In designing chimneys to be constructed of ordinary building brick, no tension should be allowed on any of the horizontal cross-sections under the maximum wind conditions considered. The allowable unit compression stresses depend on the strength of the brick, the strength of the mortar, and the care with which the brickwork is constructed. Most designers use allowable unit compression stresses ranging from 100 to 150 lb. per sq. in. for ordinary brick chimneys.

In designing radial brick chimneys, it has been customary to allow some unit tension on horizontal cross-sections under maximum wind loads. Various designers have used allowable unit tension values ranging from 0 to 35 lb. per sq. in., though an allowable unit tension stress of 20 or 25 lb. per sq. in. is not exceeded by many engineers in their chimney designs. It is also common to permit larger unit tension stresses near the base than near the top.

Allowable unit compression stresses, varying from about 150 to 300 lb. per sq. in., have been used in designing radial brick chimneys, though it is not thought advisable to use unit compression values in excess of 200 lb. per sq. in. unless it is certain that the brick, mortar, and workmanship will permit it.

Some designers allow greater unit compression stresses near the base than near the top of the chimney, while others permit no variation in the allowable unit stresses, and still others allow slightly greater unit compression stresses near the top than near the base.

The formulas which follow, taken from two different sources, are included to show the difference in ideas regarding allowable unit stresses in chimney design.

Professor G. Lang in the *Engineering Record* of July 27, 1901, gives the following maximum allowable unit stresses:

$$\text{Tension } S_t = 18.5 + 0.056L \text{ for single shell chimneys.}$$
$$S_t = 21.3 + 0.056L \text{ for chimneys having a complete lining.}$$
$$\text{Compression } S_c = 71 + 0.65L \text{ for single shell chimneys.}$$
$$S_c = 85 + 0.65L \text{ for chimneys having a complete lining.}$$

where S_c and S_t are stresses in pounds per square inch. and L is the distance from the top in feet.

Professor L. A. Harding in the 1921 edition of the Kidder's "Architects' and Builders' Handbook" says that the allowable unit stresses for radial brick chimneys should not exceed the following maximum values:

Maximum tension

Below 150 ft.	2 to $2\frac{1}{2}$ tons per sq. ft. (28 to 35 lb. per sq. in.)
From 150 to 200 ft.	1 to $1\frac{1}{2}$ tons per sq. ft. (14 to 21 lb. per sq. in.)
Above 200 ft.	0

Maximum compression

200 ft. and below	19 tons per sq. ft. (263 lb. per sq. in.)
Above 200 ft.	21 tons per sq. ft. (291 lb. per sq. in.)

22. Length of Sections.—The shaft of a brick chimney is constructed in sections which are usually of the same length. The length of section chosen may depend on custom, ordinance, judgment of the designer, or it may be fixed by the owner or contractor. A length of 20 ft. is common, though lengths of 15, 25, and 30 ft. have been used. A section length of 25 or 30 ft. is suitable for chimneys constructed of ordinary brick because the thickness of the walls must be increased by at least the width of a brick, practically 4 in., at a time. For radial brick chimneys, a section length of 20 ft. is good as the radial brick walls may be increased one or more inches at a time as the designer desires.

23. Batter of Chimney Walls.—Practically all large masonry chimneys are constructed with tapered walls because the taper improves the appearance of the chimney, permits the increases in wall thickness to be placed on the inside, and, by increasing the outer diameter as the base is approached, tends to reduce the unit stresses and make the construction more economical.

The batter of radial brick chimney walls usually varies from $\frac{1}{25}$ to $\frac{1}{45}$, being smaller for the higher chimneys and also smaller for the larger diameters. The American Chimney Corporation suggests a taper of about 4 ft. in 100 ft. (a wall batter of $\frac{1}{50}$), the M. W. Kellogg Company uses batters varying from about $\frac{1}{30}$ to $\frac{1}{60}$, and Professor C. C. Williams gives $\frac{1}{30}$ to $\frac{1}{36}$. The batter for chimney walls built of ordinary brick is usually between $\frac{1}{25}$ and $\frac{1}{50}$.

24. Thickness of Chimney Walls.—When the chimney is constructed of ordinary clay building brick, the thickness of the wall at the top of the chimney should not be less than the length of a brick, about 8 in. This thickness is satisfactory for chimneys having an inside top diameter of 8 ft. or less. For inside top diameters from 8 ft. to 18 ft., the thickness of the wall at the top should be 12 in., and this thickness should be increased to 16 in. or more for inside top diameters larger than 18 ft. Proceeding from the top down, the wall thickness is usually increased about 4 in. (half the length of a brick) for each succeeding section. This gives the following formula for the thickness of the wall at the bottom, which formula will usually give satisfactory results for round chimneys constructed of ordinary brick with vertical sections about 30 ft. in length, and with a maximum wind pressure of 25 lb. per sq. ft. on the vertical projected area:

$$t_{(\text{bottom})} = 4(n - 1) + t_{(\text{top})}$$

where

$t_{(\text{bottom})}$ = thickness of wall at base in inches.
$t_{(\text{top})}$ = thickness of wall at top in inches.
n = number of vertical sections in chimney.

For radial brick chimneys the thickness of the wall at the top is rarely taken at less than 7 in., and may be more than 7 in. when desirable. The American Chimney Corporation suggests the following rules for top thickness:

7 in. for top diameters up to 7 ft.
8 in. for top diameters from 7 ft. to 10 ft.
10 in. for top diameters more than 10 ft.

The following formula is suitable for approximately determining the thickness of the chimney wall at the top, though this formula gives values which may be a little large for radial brick chimneys whose inside top diameters are more than 20 ft.

$t_{(\text{top})} = 7 + \frac{1}{2}(d_{(\text{top})} - 7)$

$t_{(\text{top})}$ = thickness of chimney wall at top in inches, the minimum thickness being 7 in.

$d_{(\text{top})}$ = inside top diameter of chimney in feet.

The thickness of chimney wall required at the bottom of a radial brick shaft depends on the allowable unit stresses in tension and compression, the wind pressure assumed, the weight of the stack, the batter of the wall, whether or not the lining is carried by the chimney walls, and on the local building laws. Consequently it is impractical to derive an exact formula for wall thickness which would satisfactorily include all of the variables. The American Chimney Corporation suggests the following formula for radial brick chimneys of medium diameters and heights:

$$t_{(\text{bottom})} = \frac{H}{9} + 4$$

where

$t_{(\text{bottom})}$ = thickness of wall required at base in inches.

H = height of chimney shaft in feet.

The following formula also is satisfactory to use in approximately determining the thickness of wall required for radial brick chimneys of medium diameters and heights using the same notation as before:

$$t_{(\text{bottom})} = \frac{H}{10} + 5$$

These two formulas may be used to determine the approximate wall thickness required at any chosen distance below the top of the chimney. Then $t_{(\text{base})}$ would be wall thickness in inches required at a distance H feet from the top of the shaft.

When a square or octagonal base of common brick is provided, its height is usually made about ⅙ of the total height of the chimney. The wall thickness of this base when supporting a radial brick shaft (American Chimney Corporation) should not be less than $\left(\frac{H}{9} + 7\right)$ inches, where H is the height of the chimney in feet.

The various formulas given in this article for the thickness of chimney walls are empirical and approximate and, consequently, should only be used to deter-

mine approximate wall thicknesses. In designing a radial brick chimney it will often be found advisable to change slightly the approximate values so found.

25. Top Cap.—There have been several different forms of top caps used for brick chimneys. A sectional cast-iron cap is satisfactory, though reinforced concrete caps, terra cotta caps, cement mortar caps with wrought-iron retaining rings, and, in some instances, sheet-lead caps have been used. The choice of the kind of top cap to be used usually depends on the character of the flue gases and the opinion of the engineer or of the chimney construction company.

26. Reinforcing Rings.—In chimneys built of ordinary clay brick, wrought iron or steel bands should be placed in the wall at every change of wall thickness. One band should be placed at the top of the shaft and one or more bands above and below each flue opening. The purpose of the reinforcing rings is to aid the brickwork in resisting temperature stresses.

The reinforcing rings are usually omitted in the case of perforated radial brick chimneys as the brickwork in these chimneys is usually strong enough to resist ordinary temperature stresses. However, one band is usually placed near the top of the chimney and one or more bands placed both above and below each flue opening. In very large radial brick chimneys, and also when the lining is carried by the chimney walls, reinforcing rings are placed at each change in wall thickness.

Bands 3/8 by 3 in. are satisfactory for medium size chimneys, though the size of the bands may vary somewhat depending on the opinion of the designer. The bands are thoroughly embedded in the brickwork.

27. Design Formulas.

27*a*. Round Brick Chimney Shafts.—Structurally a round brick chimney shaft must be designed to resist:

(1) The overturning moment due to the wind.
(2) The horizontal shear due to the wind.
(3) The compressive stresses caused by the wind and the weight of the masonry.
(4) The tensile stresses whenever the unit tensile stress caused by the wind is greater than the unit compressive stress caused by the weight of the walls.
(5) The temperature stresses. In brick chimneys these stresses are taken care of by the masonry and the use of reinforcing rings when needed.

In addition to the formulas for determining the approximate wall thicknesses the following formulas are needed:

(1) For overturning moment due to wind.
(2) For weight of masonry in walls.
(3) For horizontal shear due to wind.
(4) For unit compression stress on a horizontal section caused by weight of walls.
(5) For unit tension or compression stress on a horizontal section caused by wind.
(6) For resultant unit stress on a horizontal section.

The formula for the overturning moment at any section due to the wind is

$$M = 2p_w h^2(D + 2D_t)$$

where

M = bending moment in inch-pounds.
p_w = unit wind pressure in pounds per square foot on the vertical projected area.
h = distance of section in feet from top of chimney.
D = outer diameter of section in feet.
D_t = outer diameter of top in feet.

For the moment at the bottom of the shaft, D would equal the diameter of the base in feet and h the height of the shaft in feet.

The derivation of this formula is as follows (refer to Fig. 1 and consider the projected area to consist of a parallelogram and a triangle): The moment of the wind pressure on the parallelogram about the section considered equals

$$12p_w hD_t \cdot \frac{h}{2} \text{ (in.-lb.)}$$

The moment of the wind pressure in the triangle about the section considered equals

$$12p_w h \frac{(D - D_t)}{2} \frac{h}{3} \text{ (in.-lb.)}$$

The total moment

$$M = 12p_w hD_t \frac{h}{2} + 12p_w h \frac{(D - D_t)}{2} \frac{h}{3}$$
$$= 12p_w h^2 \left(\frac{D_t}{2} + \frac{D}{6} - \frac{D_t}{6}\right)$$
$$= 12p_w h^2 \left(\frac{D}{6} + \frac{2D_t}{6}\right)$$
$$2p_w h^2(D + 2D_t)$$

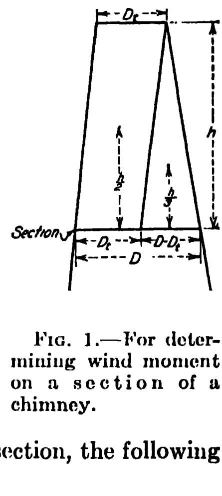


FIG. 1.—For determining wind moment on a section of a chimney.

In finding the weight of the shaft above any horizontal section, the following formula may be used:

$$W = \frac{\pi}{8} [(D^2 - d^2) + (D_t^2 - d_t^2)] wh$$

where

W = weight of shaft above the horizontal section in pounds.
D = outer diameter at section in feet.
d = inner diameter at section in feet.
D_t = outer diameter at top of shaft in feet.
d_t = inner diameter at top of shaft in feet.
w = weight of masonry in pounds per cubic foot.
h = distance of section from top of chimney in feet.

This formula will give only the approximate weight as it assumes a uniform variation of the inner diameter between the section considered and the top of the chimney.

A more accurate method of finding the weight of the shaft is to number the vertical sections (beginning at the top of the shaft), compute the weights (W_1, W_2, W_3, etc.) of each section separately, and then add the weights to obtain the total. The total weight of shaft above the bottom of any vertical section would be the sum of the weights of the vertical sections above.

The following formula may be used for finding the weight of any vertical section having a constant wall section. Suppose it is section 4 counting from the top of the stack, then

$$W_4 = \frac{\pi}{2} wh_4t_4[D_4 + D_3 - 2t_4]$$

where

W_4 = weight of section 4 in pounds.
w = weight of masonry in pounds per cubic foot.
h_4 = height of section in feet.
t_4 = thickness of wall in feet.
D_4 = outer diameter of bottom of section 4 (same as outer diameter at top of section 5).
D_3 = outer diameter of top of section 4 (same as outer diameter at bottom of section 3).

In other words, the weight of any section is equal to the average cross-sectional area multiplied by the height of the vertical section and the weight per cubic foot. of the masonry.

The total weight of shaft above bottom of section 4 would be $W_4 + W_3 + W_2 + W_1$, the values W_3, W_2, and W_1 being found in like manner by using the proper outer diameters and wall thicknesses for these sections.

The total weight of the shaft would be equal to the sum of the weights of the different vertical sections.

In chimneys where the lining is carried on corbels from the chimney walls, the lining weights must be included.

The unit shear on any horizontal section of the shaft is equal to the total wind pressure on the shaft above the section divided by the area of the section or

$$\text{Unit shear} = \frac{\frac{1}{2}(D + D_t)hp_w}{\pi t(D - t)} \cdot \frac{1}{144} = \frac{p_w h(D + D_t)}{288t(D - t)\pi}$$

where

D = outer diameter of the horizontal section in feet.
D_t = outer diameter of the top of shaft in feet.
h = distance from top of shaft to section in feet.
p_w = wind pressure in pounds per square foot.
t = thickness of wall in feet.
Unit shear = pounds per square inch.

The unit compressive stress in pounds per square inch on any horizontal section, due to the weight of the wall of the shaft, equals the total weight above the section divided by the cross-sectional area of the masonry A, or $\frac{W}{A}$, which equals

$$\frac{W}{144\pi t(D - t)} \text{ or } \frac{W}{36\pi(D^2 - d^2)}$$

depending on the expression used for the area, where

W = weight of shaft in pounds above horizontal section considered.
D = outer diameter of the horizontal section in feet.
d = inner diameter of the horizontal section in feet.
t = thickness of wall in feet.

The unit stress on any horizontal section due to wind moment will be tension on the windward side and compression on the leeward side of the shaft. Treating the shaft as a hollow cantilever beam, this unit stress will be equal to the wind moment divided by the section modulus—that is, the bending stress $= \frac{Mv}{I}$

The moment due to the wind in inch-pounds is

$$M = 2p_wh^2(D + 2D_t)$$

The section modulus, $\frac{I}{v}$, in in.³ for a hollow circular section is

$$\frac{I}{v} = \frac{\pi(D^4 - d^4)1{,}728}{32\ D} = \frac{54\pi(D^2 + d^2)(D^2 - d^2)}{D}$$

Or, the unit bending stess in pounds per square inch due to the wind equals

$$\frac{Mv}{I} = \frac{2p_wh^2(D + 2D_t)D}{54\pi(D^2 + d^2)(D^2 - d^2)} = \frac{p_wh^2D(D + 2D_t)}{27\pi(D^2 + d^2)(D^2 - d^2)}$$

In this formula, the letters used have the same meanings as in previous formulas in this article. If desired, $D - 2t$ may be substituted for d.

The resultant unit stress on any horizontal section is given by the formula

$$f = \frac{W}{A} \pm \frac{Mv}{I}$$

and will be compression on the leeward side of the shaft and compression or tension on the windward side depending on whether $\frac{W}{A}$ is larger or smaller than $\frac{Mv}{I}$

$$f_c = \frac{W}{A} + \frac{Mv}{I}$$

$$f_t = \frac{W}{A} - \frac{Mv}{I}$$

where f_c and f_t are the unit stresses on the leeward and windward sides of the shaft respectively, and W, A, M, and $\frac{I}{v}$ have the same meanings as before.

27b. Octagonal Brick Chimney Shafts.—The formulas for the octagonal shafts are practically the same as those for circular shafts with some changes.

The formula for bending moment in inch-pounds due to the wind is the same as before, except that D and D_t are the distances in feet between two outer parallel sides at the horizontal section considered and at the top of the shaft respectively.

$$M = 2p_wh^2(D + 2D_t)$$

The approximate formula for weight of shaft in pounds above any horizontal section becomes

$$W = 0.414[(D^2 - d^2) + (D_t^2 - d_t^2)]\ wh$$

where W, w and h have the same meanings as are given in the preceding article.

D and D_t are distances in feet between two outer parallel sides at the horizontal section considered and at the top of the shaft, and d and d_t are distances in feet between two inner parallel sides at the horizontal section considered and at the top of the shaft.

The formula for the weight in pounds of any vertical section of the shaft having a constant wall section becomes, say for the fourth section from the top,

$$W_4 = 1.656wh_4t_4(D_4 + D_3 - 2t_4)$$

where the notation is the same as in Art. **27a** except that D_4 and D_3 are the distances in feet between two parallel sides at the bottom and top respectively of the vertical section considered.

The unit shearing stress on any horizontal section becomes

$$\text{Unit shear} = \frac{(D + D_t)h\,p_w}{144 \times 1.656\,(D^2 - d^2)} = \frac{(D + D_t)h\,p_w}{144 \times 6.624\,t(D - t)}$$

where the letters have the same meanings as in Art. **27a**, with the exception of D, D_t, and d as previously noted.

The unit compressive stress in pounds per square inch on any horizontal section due to the weight of the shaft above the section is given by the formula

$$\frac{W}{144 \times 3.312t(D - t)} \text{ or } \frac{W}{144 \times 0.828(D^2 - d^2)}$$

where the letters have the same meanings as before.

The unit stress in lb. per sq. in. on a horizontal section, caused by the wind bending moment, equals $\frac{Mv}{I}$ where the moment in inch-pound equals

$$M = 2p_wh^2(D + 2D_t)$$

and the section modulus in in.³ equals

$$\frac{I}{v} = \frac{1{,}728 \times 0.109(D^2 + d^2)(D^2 - d^2)}{D}$$

where the letters have the same meanings as previously noted and where the neutral axis is taken parallel to two of the sides of the section and through the center of the section.

The formula for the resultant stress is the same as in Art. **27a**, and, as before,

$$f_c = \frac{W}{A} + \frac{Mt}{I}$$

$$f_t = \frac{W}{A} - \frac{Mv}{I}$$

27c. Brick Chimney Bases.—When a base of common brick is provided, it is usually made about one-fifth the height of the chimney and usually has a slightly larger diameter. Brick chimney bases are commonly octagonal or square in shape with vertical walls. A water table is provided at the top. The walls are usually made thicker than those required for an ordinary chimney shaft; no tension stress is permitted; and the allowable unit compression stress is kept comparatively small.

The wind moment at the bottom of the base may be said to consist of three parts: (1) The wind moment about the bottom of the shaft due to the wind pressure on the shaft; (2) the correction to be added to transfer this moment to the bottom of the base; and (3) the moment of the wind pressure on the base about a section at the bottom of the base.

The wind moment about the bottom of the shaft equals

$$M = 2p_wh^2(D + 2D_t)$$

The increment to be added, to transfer this moment to the bottom of the base is

$$M_1 = 6p_whh_b(D + D_t)$$

The moment of the wind pressure on the base about the bottom section is

$$M_2 = 6p_{wb}h_b^2D_b$$

Total moment about the section at the bottom of the base equals $M + M_1 + M_2$. If the base is fully protected from the wind by buildings or other structures, M_2 may be omitted.

In these three formulas

M, M_1, and M_2, are the moments in inch-pounds.
p_w = wind pressure on shaft projected area in pounds per square foot.
p_{wb} = wind pressure on base projected area in pounds per square foot.
h = height of shaft in feet.
h_b = height of base in feet.
D = outer diameter of bottom of shaft in feet.
D_t = outer diameter of top of shaft in feet.
D_b = outer diameter of base in feet.

The weight of the base W, in pounds is as follows:

Octagonal base

$$W_b = 0.828\, w_bh_b(D_b^2 - d_b^2)$$

Square base

$$W_b = w_bh_b(D_b^2 - d_b^2)$$

where

W_b = weight of base in pounds.
w_b = weight of masonry in pounds per cubic foot.
h_b = height of base in feet.
D_b = distance between two outer parallel sides in feet.
d_b = distance between two inner parallel sides in feet.

When the walls of the base are corbeled out at places, when the top of the base is arched over, or when the flue opening is in the base, corrections must be made to the weight as computed by the preceding formula.

Also, when the base supports the lining, the weight of the lining must be added. This weight is equal to

$W_l = \pi w_lh_lD_lt_l$ for round linings.
$W_l = 3.312\, w_lh_lD_lt_l$ for octagonal linings.

where

W_l = weight of lining in pounds.
w_l = weight of lining in pounds per cubic foot.
h_l = height of lining in feet.
D_l = average diameter of lining in feet.
t_l = wall thickness of lining in feet.

When the lining is constructed in sections having different thicknesses, the weight of each section should be found separately, and then the total weight of the lining found by adding the weights of the sections.

The unit compressive stress in pounds per square inch at the horizontal section at the bottom of the base is:

For an octagonal base

$$= \frac{\text{sum of weights of shaft, base, and lining}}{144 \times 3.312\, t_b(D_b - t_b)}$$

or

$$= \frac{\text{sum of weights of shaft, base, and lining}}{144 \times 0.828(D_b^2 - d_b^2)}$$

For a square base

$$= \frac{\text{sum of weights of shaft, base, and lining}}{144(D_b^2 - d_b^2)}$$

where the letters used have the same meanings as before.

The unit flexural stress due to the wind in pounds per square inch on the horizontal section at the bottom of the base is

For an octagonal base

$$= \frac{1{,}728(M + M_1 + M_2)r_b}{I_b}$$

where

$$\frac{I_b}{r_b} = \frac{1{,}728 \times 0.109(D_b^2 + d_b^2)(D_b^2 - d_b^2)}{D_b}$$

For a square base

$$= \frac{(M + M_1 + M_2)r_b}{I_b}$$

where

$$\frac{I_b}{r_b} = \frac{1{,}728(D_b^2 + d_b^2)(D_b^2 - d_b^2)}{6D_b}$$

and where the notation is the same as before.

The maximum unit compression stress on the horizontal section at the bottom of the base equals

$$f_c = \frac{W + W_b + W_l}{A_b} + \frac{(M + M_1 + M_2)r_b}{I_b}$$

where A_b = cross-sectional area of bottom of base in square inches.

The minimum unit compression stress on the horizontal section at the bottom of the base equals

$$f_t = \frac{W + W_b + W_l}{A_b} - \frac{(M + M_1 + M_2)r_b}{I_b}$$

where the first term must be equal to or greater than the second term so that there will be no tension on this section.

The unit shear on any horizontal section of the base will probably be the greatest at the bottom of the base provided that there is no breech opening in the base or that extra masonry has been added around the breech opening to compensate for that removed.

The total wind pressure equals that for the shaft plus that for the base or

$$\tfrac{1}{2}(D + D_t)hp_w + D_bh_bp_{wb}$$

The unit shear equals

$$\frac{\tfrac{1}{2}(D + D_t)hp_w + D_bh_bp_{wb}}{\text{cross-sectional area of the base}}$$

28. Design of Chimney Foundations not Reinforced.—Foundations for brick chimneys may be constructed of brick or stone masonry, or of plain or reinforced concrete. The design of a reinforced concrete foundation will be considered in a later paragraph.

The shape of a masonry foundation is usually of the form of a frustrum of a cone or of an eight-sided pyramid, and the sides are usually stepped. Square shaped foundations are rarely used because of the high pressures which may occur in the corners when the wind blows along the direction of a diagonal of the square. Round or octagonal shaped foundations are more economical. The side slope of the foundation should make an angle of less than 45 deg. with the vertical if no reinforcement is used. An angle of about 30 deg. with the vertical is usually satisfactory.

The width of the top of the foundation should be from 1½ to 3 ft. wider than the outer diameter of the bottom of the chimney. The width of the bottom of the foundation should be such that the allowable unit soil pressures will not be exceeded and also that there will be compression stress over all of the bottom of the foundation at all times. If the foundation is stepped, the offsets from one step to another should not be too large for good practice. The depth of the foundation varies from about 4 to 12 ft., depending on such conditions as the height and diameter of the chimney, the widths of the top and bottom of the foundation, and the opinion of the designer.

The most economical design under the most extreme conditions of loading would give practically a zero unit compressive stress on the windward edge increasing to the maximum allowable unit compressive stress at the opposite edge on the leeward side.

The weight of foundations may be taken as follows:

Plain concrete about 145 lb. per cu. ft.
Good building brick, about 125 lb. per cu. ft.
Stone masonry, about 5 lb. per cu. ft. less than that of the stone used, say approximately 140 lb. per cu. ft. for sandstone, and 160 lb. per cu. ft. for limestone, marble, and granite masonry.

The allowable unit soil pressure varies greatly with different kinds of soil. A value of 4,000 lb. per sq. ft. (2 tons per sq. ft.) should not be exceeded for average soil conditions. A conservative value for the allowable soil pressure should be selected as dangers due to possible uneven settlement of the foundations are very great.

Foundations are designed by the "cut and try" method. The dimensions are assumed and the maximum unit soil pressure is computed. If this pressure is just equal to or a little less than the allowable unit soil pressure, the design is satisfactory provided that there is compression on the entire under surface of the foundation. If the design is not satisfactory, other dimensions should be chosen and the computations made again. An experienced designer rarely has to make more than two or three trials.

29. Foundation Design Formulas.—Let

D_f = diameter of foundation base in feet.
D_u = diameter of foundation top in feet.
W_f = weight of foundation in pounds.

W_l = weight of lining in pounds.
W_s = weight of shaft in pounds or the weight of shaft and base if there is a base.
h_f = height of foundation in feet.
H = height of chimney above foundation in feet.
A_f = area of foundation base in square feet.
M = overturning moment of wind about top of foundation in inch-pounds (see Arts. **27a** and **27c**)
M_f = overturning moment of wind about base of foundation in inch-pounds.
I_f = moment of inertia of foundation base section in in.^{4}
v = distance from neutral axis to extreme fiber in inches.
w_f = weight per cubic foot of masonry in foundation.

$$W_f = \frac{\pi}{12} w_f h_f (D_f^2 + D_f D_u + D_u^2) = \text{for a frustrum of a cone.}$$

$$W_f = \frac{0.828}{3} w_f h_f (D_f^2 + D_f D_u + D_u^2) = \text{for a frustrum of an eight-sided pyramid.}$$

These formulas for W_f are approximately true when the foundation sides are "stepped." However, for accuracy, the weight of each section or "step" should be found and the sum of these weights taken for the total.

$$I_f = \frac{\pi D_f^4}{64} \times 144^2 = 1{,}018 D_f^4 \text{ (in.}^4 \text{ for a circle)}$$

$$I_f = 0.055\, D_f^4 \times 144^2 = 1{,}140 D_f^4 \text{ (in.}^4 \text{ for an octagon)}$$

$$v = 6D_f$$

$$M = 2\, p_w H^2 (D + 2D_t)$$

$$M_f = M + 6p_w H h_f (D + D_t)$$

where

p_w = wind pressure in pounds per square foot on projected area of chimney.
D = outer diameter of bottom of chimney in feet.
D_t = outer diameter of top of chimney in feet.

$$A_f = \frac{\pi D_f^2}{4} = 0.7854 D_f^2 \text{ in square feet for a circle.}$$

$$A_f = 0.828 D_f^2 \text{ in square feet for an octagon.}$$

The maximum unit compression stress on the base of the foundation in pounds per square inch for a circular base

$$= \frac{W_s + W_l + W_f}{144 \times 0.7854 D_f^2} + \frac{6D_f M_f}{1{,}018 D_f^4}$$

$$= \frac{W_s + W_l + W_f}{113 D_f^2} + \frac{M_f}{170 D_f^3} \text{ (in lb. per sq. in.)}$$

For an octagonal base, the maximum unit compression stress, assuming the wind to blow perpendicularly to a side,

$$= \frac{W_s + W_l + W_f}{144 \times 0.828 D_f^2} + \frac{6D_f M_f}{1{,}140 D_f^4}$$

$$= \frac{W_s + W_l + W_f}{119 D_f^2} + \frac{M_f}{190 D_f^3} \text{ (in lb. per sq. in.)}$$

The minimum unit compression stress on the base of the foundation for a circular base

$$= \frac{W_s + W_l + W_f}{113D_f^2} - \frac{M_f}{170D_f^3} \text{ (in lb. per sq. in.)}$$

For an octagonal base, the maximum unit compression stress, assuming the wind to blow perpendicularly to a side,

$$= \frac{W_s + W_l + W_f}{119D_f^2} - \frac{M_f}{190D_f^3} \text{ (in lb. per sq. in.)}$$

In these last two formulas, the first term must be equal to, or greater than, the last term or else there may be tension on the base.

The unit stresses obtained by the above four formulas should be multiplied by 144 to reduce the unit stresses to pounds per square foot.

30. Method of Procedure in Design of Brick Chimneys.—The general method of procedure in the design of a brick chimney is approximately as follows, having given the height above foundations, the inside top diameter, the elevation of the breech opening, the height of the lining, whether the chimney is to be round or octagonal in shape, and whether or not the chimney is to have a base section:

(*a*) Choose wall thickness at top of chimney, usually 7 in. or more.
(*b*) Choose number of vertical sections, usually about 30 ft. in length for ordinary brick and 20 ft. for perforated radial brick.
(*c*) Choose batter for walls, usually between $\frac{1}{36}$ and $\frac{1}{50}$. Sometimes the batter varies at different sections.
(*d*) Compute approximate wall thickness for each vertical section.
(*e*) It is usually of advantage to prepare a tabulation showing distance or number of section from the top, outer diameters, wall thicknesses, cross-sectional areas, masonry weights, wind moments, unit stresses, etc., in designing the chimney.
(*f*) Compute masonry weights and wind moments at base of upper (first) section and then compute maximum unit compression stress, minimum unit compression stress (or tension), and unit shearing stress on this horizontal base section.
(*g*) Repeat computations for each succeeding section.
(*h*) Design base section (if one has to be provided).
(*i*) Design breech opening.
(*j*) Design reinforcing rings.
(*k*) Design lining (and lining supports when lining is not built up from the foundation).
(*l*) Design top cap.
(*m*) Design ladder (usually placed inside).
(*n*) Design block and cable.
(*o*) Design lightning rod.
(*p*) Design clean-out door.
(*q*) Design foundation.
(*r*) Check design.
(*s*) Prepare general and detail drawings of chimney and foundations.

The general drawing should show the plan and elevation of the chimney and foundations with all such details as outer and inner dimensions, wall thicknesses, location of breech opening, clean-out door, typical cross-sections, etc. Detail drawings to a larger scale should often be provided for such details as breech opening reinforcement, ladder, lightning rods and fastenings, block and cable, top cap, lettering and trimmings to appear on chimney, etc.

31. Use of Diagrams and Tables as Aids in Designing Brick Chimneys.— The use of diagrams and tables in designing brick chimneys is a great time saver to an engineer. However, the engineer must know that the diagrams and tables that he is using apply to the chimney that is being designed.

TABLE 6.—INTERNAL TOP AND EXTERNAL BOTTOM DIAMETERS OF ROUND KELLOGG CHIMNEY SHAFTS

Height of chimney in feet	Internal diameter at top														
											8'	8'6"	9'	9'6'	10'
	External diameters in feet at bottom of column														
75	7.42	7.69	7.96	8.46	8.96	9.46	9.96								
80	7.80	8.04	8.27	8.70	9.13	9.58	10.02								
85	8.18	8.38	8.58	8.95	9.31	9.70	10.08								
90	8.57	8.73	8.88	9.18	9.48	9.81	10.13								
95	8.95	9.07	9.19	9.43	9.66	9.93	10.19								
100	9.33	9.42	9.50	9.67	9.83	10.04	10.25	10.75	11.25	11.75	12.25				
105	9.70	9.78	9.85	10.03	10.21	10.38	10.55	11.03	11.50	11.95	12.40				
110	10.06	10.13	10.20	10.40	10.60	10.73	10.85	11.30	11.75	12.15	12.55				
115	10.43	10.49	10.55	10.77	10.98	11.07	11.15	11.58	12.00	12.35	12.70				
120	10.79	10.85	10.90	11.14	11.37	11.41	11.45	11.85	12.25	12.55	12.85				
125	11.16	11.21	11.25	11.50	11.75	11.75	11.75	12.13	12.50	12.75	13.00	13.50	14.00	14.50	15.00
130			11.65	11.88	12.10	12.12	12.13	12.47	12.80	13.09	13.37	13.80	14.22	14.69	15.15
135			12.05	12.25	12.45	12.48	12.51	12.81	13.10	13.42	13.73	14.08	14.43	14.87	15.30
140			12.45	12.63	12.80	12.85	12.90	13.15	13.40	13.75	14.10	14.38	14.65	15.05	15.45
145			12.85	13.00	13.15	13.22	13.28	13.49	13.70	14.08	14.46	14.66	14.86	15.23	15.60
150			13.25	13.38	13.50	13.58	13.66	13.83	14.00	14.42	14.83	14.96	15.08	15.42	15.75
155			13.58	13.73	13.87	13.97	14.06	14.18	14.30	14.68	15.06	15.19	15.31	15.61	15.91
160			13.92	14.08	14.23	14.35	14.46	14.53	14.60	14.95	15.30	15.43	15.55	15.81	16.07
165			14.25	14.43	14.60	14.73	14.86	14.88	14.90	15.22	15.53	15.66	15.78	16.00	16.22
170			14.59	14.78	14.96	15.11	15.26	15.23	15.20	15.49	15.77	15.90	16.02	16.20	16.38
175			14.92	15.13	15.33	15.50	15.66	15.58	15.50	15.75	16.00	16.13	16.25	16.40	16.54
180									15.80	16.05	16.30	16.40	16.50	16.65	16.80
185									16.10	16.35	16.60	16.68	16.75	16.91	17.06
190									16.40	16.65	16.90	16.95	17.00	17.16	17.31
195									16.70	16.95	17.20	17.23	17.25	17.41	17.57
200									17.00	17.25	17.50	17.50	17.50	17.67	17.83
205													17.80	17.98	18.16
210													18.10	18.30	18.50
215													18.40	18.62	18.83
220													18.70	18.94	19.17
225													19.00	19.25	19.50

The tables included in this article are taken from some of the diagrams and tables used by the engineers of the M. W. Kellogg Company in designing their

perforated radial brick chimneys. It is very doubtful if these tables and diagrams should be used in designing other than Kellogg chimneys, though similar diagrams and tables could be prepared to aid in the design of common brick chimneys or other types of radial brick chimneys. If preferred, the data in Tables 6, 7, 8 and 9 may be expressed in the form of diagrams and curves for the convenience of the designer.

Table 6 shows the internal top diameters and external bottom diameters of round chimney shafts. It should be noted that chimneys of different heights have different "batters" for the outside of their walls; consequently, when using the table, the proper outside diameter for the required height should be chosen and no attention paid to the diameters for intermediate heights. The wall batter is a constant for any one chimney.

Table 7 shows the wall thicknesses of Kellogg radial brick chimneys and is self-explanatory. The length of each vertical section is taken as 20 ft. and the wall thickness varied to give a good design. The wind pressure is taken as 25 lb. per sq. ft. on the projected area and the masonry is assumed to weigh not less than 120 lb. per cu. ft. The measurements are taken from the chimney top down.

TABLE 7.—WALL THICKNESS OF KELLOGG CHIMNEYS

Distance from top of chimney (ft.)	Vertical section No.	Wall thickness (in.)
Top-20	1	7.50
20-40	2	9.00
40-60	3	11.75
60-80	4	13.25
80-100	5	15 25
100-120	6	16.75
120-140	7	18.25
140-160	8	20.25
160-180	9	22.25
180-200	10	24 00
200-220	11	26.00
220-230	12	28.00

The weights of Kellogg peforated radial brick chimneys of different diameters and heights are given in Table 8. This table gives the dead load due to the weight of the round shaft alone and does not include the weights of the lining, base, and foundations. The weights are given to the nearest 1,000 lb. These values are taken from the M. W. Kellogg Company's curves.

The width in feet of the bottom of the foundation required for Kellogg chimneys is given in Table 9. These foundations are octagonal in shape and the sides are stepped. The material is concrete of a 1:2½:5 mix of cement, sand, and stone or gravel which is assumed to weigh about 145 lb. per cu. ft. The values in the table are taken from curves used by the M. W. Kellogg Company. The

TABLE 8.—WEIGHT OF ROUND SHAFT

Height of shaft (ft.)	Internal top diameter in feet							
	3	4	5	6	7	8	9	10
	Weight of shaft in thousands of pounds							
75	125	140	165	195				
80	140	160	185	210				
90	180	195	220	245				
100	220	240	260	285	325	360		
110	265	285	310	335	375	415		
120	315	340	370	395	435	475		
125	315	370	400	430	470	510	550	600
130	.	405	435	465	505	545	590	635
140	...	475	510	510	580	620	655	715
150	.	555	590	620	660	705	750	800
160	.	635	680	715	750	805	845	895
170	.	725	775	820	850	910	950	1,000
175	...	770	825	875	900	960	1,005	1,055
180	...	.	..		960	1,020	1,065	1,110
190	...	...	...	. .	1,070	1,140	1,185	1,235
200	...	...	...	...	1,200	1,265	1,315	1,370
210	...	...	...	...	.....	.....	1,455	1,520
220	. .	...	...	...	.... .	. ..	1,610	1,685
225	...	...	...	...	.....	.....	1,690	1,775

maximum allowable compression on the soil has been taken as 2 tons per sq. ft. —about 28 lb. per sq. in.

Table 10 shows the normal total depth of foundation for Kellogg chimneys based on a bearing value for the soil of 2 tons per sq. ft.

A drawing of a typical Kellogg chimney, built round for the entire height and with no base, is shown in Fig. 2. Attention is called to the method of using I-beams for supporting the masonry over the flue opening and also to the method of enlarging the brickwork around the flue opening. Note the wrought-iron rings embedded in the masonry at the top and above and below the flue opening. In this chimney the lining is not built up from the foundation but rests on brickwork corbeled out from the chimney walls 2 ft below the bottom of the flue opening.

Figure 3 shows a typical Kellogg chimney with an octagonal base containing the flue opening. Note the different methods of constructing the base so that it will fit in with a building wall. When the base has two flue openings, the baffle plate is not placed perpendicular to the axis of the flue opening, but is constructed at an angle as shown. This arrangement of the baffle plate permits a better flow of the gases from the flue up the chimney. Figures 2 and 3 show

TABLE 9.—WIDTH OF BOTTOM OF OCTAGONAL FOUNDATION REQUIRED FOR PERFORATED RADIAL BRICK CHIMNEYS
(Based on a Soil Bearing Value of Two Tons per Square Foot)

Height of chimney (ft.)	Internal top diameter in feet							
	3	4	5	6	7	8	9	10
	Width of bottom slab of foundation in feet							
75	10.0	10.5	11.5	12.5				
80	10.5	11.0	12.0	13.0				
90	11.5	12.0	13.0	13.75				
100	12.5	13.0	14.0	14.75	15.5	16 0		
110	13.5	14.25	15.0	15.5	16.5	17 0		
120	14.5	15.25	16.0	16.5	17.5	18.0		
125	15.0	16.0	16.5	17 25	18.0	18.5	19.5	20.5
130	....	16.5	17.25	17.75	18.75	19.25	20.0	21.25
140	. .	17.75	18.5	18 75	19 75	20 25	21.0	22.25
150	...	19 0	19.75	20.0	21.0	21.50	22.25	23.50
160	....	20.5	21.0	21.5	22.5	23.0	23.5	24.75
170	....	22.0	22.5	23.0	23.75	24.25	25.0	26.25
175	....	22.5	23.0	23.75	24.5	25.0	25.75	27.0
180	....	.....	.....	.....	25.5	26.0	26.5	27.5
190	....	.....	.....	.....	27.0	27.5	28.25	29.25
200	....	.....	.....	.....	28.5	29.0	29.75	30.75
210	....	.....	.....	.....	.....	....	31.5	32.5
220	....	....	.....	.....	.....	.....	33.0	34.25
225	...	.. .	...	.... .	...	.....	34 0	35.0

TABLE 10.—NORMAL TOTAL DEPTH OF FOUNDATION
(Based on a Soil Bearing Value of Two Tons per Square Foot)

Size of chimney	Total depth of foundation (ft.)
75 ft. × 3 ft. up to and including 100 ft. × 6 ft.	4.50
100 ft. × 7 ft. up to and including 100 ft. × 8 ft.	5.00
125 ft. × 3 ft. up to and including 125 ft. × 8 ft.	5.00
125 ft. × 8.5 ft. up to and including 125 ft. × 10 ft.	6.00
150 ft. × 4 ft. up to and including 150 ft. × 8 ft.	6.00
150 ft. × 8.5 ft. up to and including 150 ft. × 10 ft.	6.50
175 ft. × 4 ft. up to and including 175 ft. × 9 ft.	6.75
175 ft. × 10 ft.	7.00
200 ft. × 7 ft. up to and including 200 ft. × 9 ft.	8.00
200 ft. × 10 ft.	9.00
225 ft. × 9 ft. up to and including 225 ft. × 10 ft.	10.00

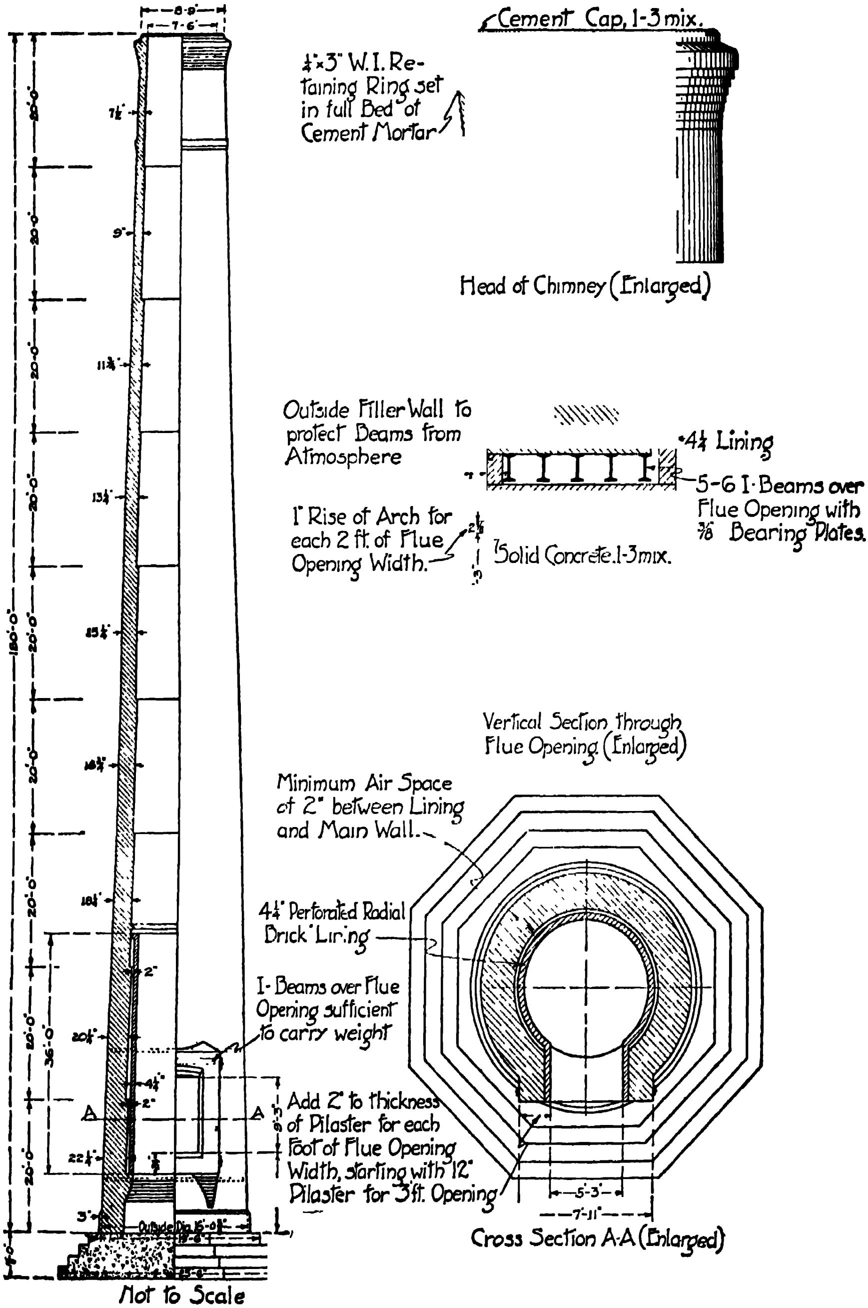


Fig. 2.—Typical Kellogg chimney built round for entire height.

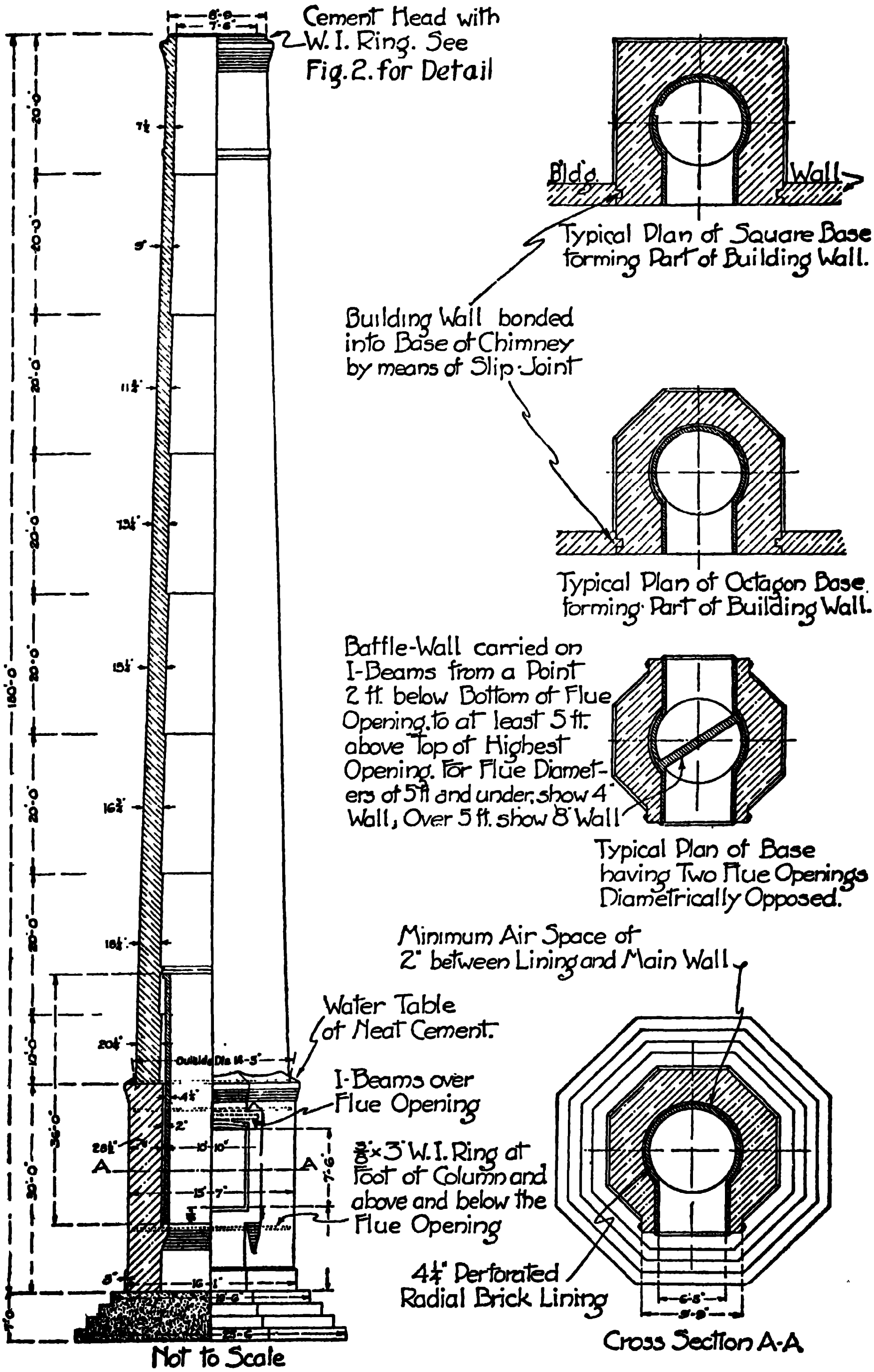


Fig. 3.—Typical Kellogg chimney with round shaft and octagonal base.

typical designs of radial brick chimneys constructed for industrial power plants where the flue gas temperatures are comparatively low.

32. Construction of Brick Chimneys.—Before beginning the actual construction of the chimney, space must be procured for the storage of the materials to be used. Dry storage must be provided for such materials as cement and lime which would be injured by exposure to the weather. An adequate supply of good water should be provided; a connection to the city water mains is usually satisfactory. One side of the chimney should be left open for the hoisting of materials.

The excavation for the foundation should be carefully made and the soil at the bottom left smooth and firm. All loose material should be removed and, when necessary, should be replaced by good soil firmly tamped in place. The foundation is usually octagonal or circular in shape with smooth or stepped sides. Concrete masonry is commonly used. Forms for the concrete are more easily constructed for the octagonal shape with stepped sides. The concrete materials should be carefully measured and thoroughly mixed before being deposited in the forms in layers not over 6 in. in thickness. Just enough water should be used in the concrete to make a workable mix as an excess of water reduces the strength. The concrete foundation should preferably be allowed to set from two to four weeks before the construction of the shaft starts. The foundation is often built by a local contractor about a month or so before the chimney contractor arrives to build the shaft.

In constructing an octagonal base of common brick, the workmanship is as important as the selection of the brick and the mortar. For the octagonal corners, flat-iron brick or brick moulded with 45-deg. corners should be used. All brick should be laid with a shoved joint and care taken to keep the outer faces of the walls smooth and vertical. Common bond is ordinarily used with header courses every third, fourth, or fifth course. Some engineers prefer English or Flemish bond. When the base is to form part of a side of the building, it should not be bonded to the building wall but should be constructed separately and a straight tongue and grooved joint provided.

In the construction of the shaft only skilled workmen should be used. The batter is commonly determined by a template having one side vertical with an attached plumb bob and the other side cut to the batter. A spirit level is placed on top of the template. The brick layers work on a platform supported by two or four horizontal cross timbers laid across the brick work. A horizontal timber resting on well-braced upright posts above the platform carries a pulley block or blocks for the hoisting of materials. If the diameter is small, the materials are hoisted on the outside, but they may be hoisted on the inside if the chimney diameter permits. The hoisting line passes over the pulleys and to the drum of a hoisting engine on the ground. The work is checked frequently by an inspector who makes measurements of the inside or outside diameter. A plumb line dropped from the platform at the center should hit the center of the base. The working platform must be raised from time to time. Wall bonding of radial brick should be made at every third course. Figure 4 shows the method of laying radial brick in the shaft.

In constructing the breech opening, care must be taken when placing the I-beams across the top of the opening to see that they rest firm and level on the

bearing plates and that space is left at their ends for expansion. There should be an outside filler wall to protect the I-beams from the weather and to improve the appearance of the shaft. The flat arch beneath the I-beams should be carefully constructed so that it will not settle or sag. The wrought-iron or steel reinforcing rings above and below the opening must be placed in the masonry so that they will assist the masonry in resisting the temperature stresses.

The head and cap of the chimney should be built so that there will be no cracking or disintegrating of the masonry at the top of the stack.

The lining is usually constructed after the shaft is completed. Care must be taken to keep the inside lining walls smooth and vertical and to keep the outer lining walls away from the inner walls of the shaft. Two inches is the minimum space allowed between the lining and shaft walls at any place, and many designers require 3 or 4 in. Mortar and pieces of brick should not be permitted to drop

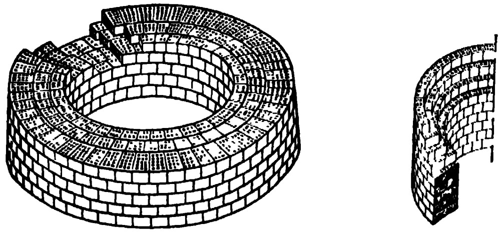


FIG. 4.—Method of bonding radial brickwork.

in this free space. The top of the lining must be corbeled out toward the walls, or the walls of the shaft corbeled out, over the lining, to prevent soot and ashes from falling into this free space. As the vertical expansion of the lining will be more than that of the shaft walls, provision for this must be made.

The ladder, block and cable, and lightning conductor should be properly constructed as it is expensive and often difficult to reconstruct these details.

When lettering and trimmings are called for, these should be carefully and accurately done or else the appearance of the chimney may be marred.

After the chimney is completed, the contractor is usually required to clean up the grounds and leave them in good condition.

It is customary for the chimney contractor to carry all liability and damage insurance when constructing the chimney and it is also customary for him to guarantee the chimney against all defects due to design and workmanship for a period of about five years.

33. Tacoma Smelting Company Stack.—This brick chimney, which is one of the highest in the world, was constructed by the Alphons Custodis Chimney Construction Co., New York City, for the Tacoma Smelting Co., at Tacoma, Wash The foundations, which were designed and constructed by the smelting company, presented some difficulties which were successfully overcome. The computed maximum pressure on the foundation base is 5 tons per sq. ft. due to the weight of the stack, lining, foundation, and the wind pressure for a wind velocity of 125 mi. per hr. The details of the foundation and shaft are shown in Fig. 5. It should be noted that the lining is constructed in about 25-ft. sections which are supported on corbels from the stack walls. For a more

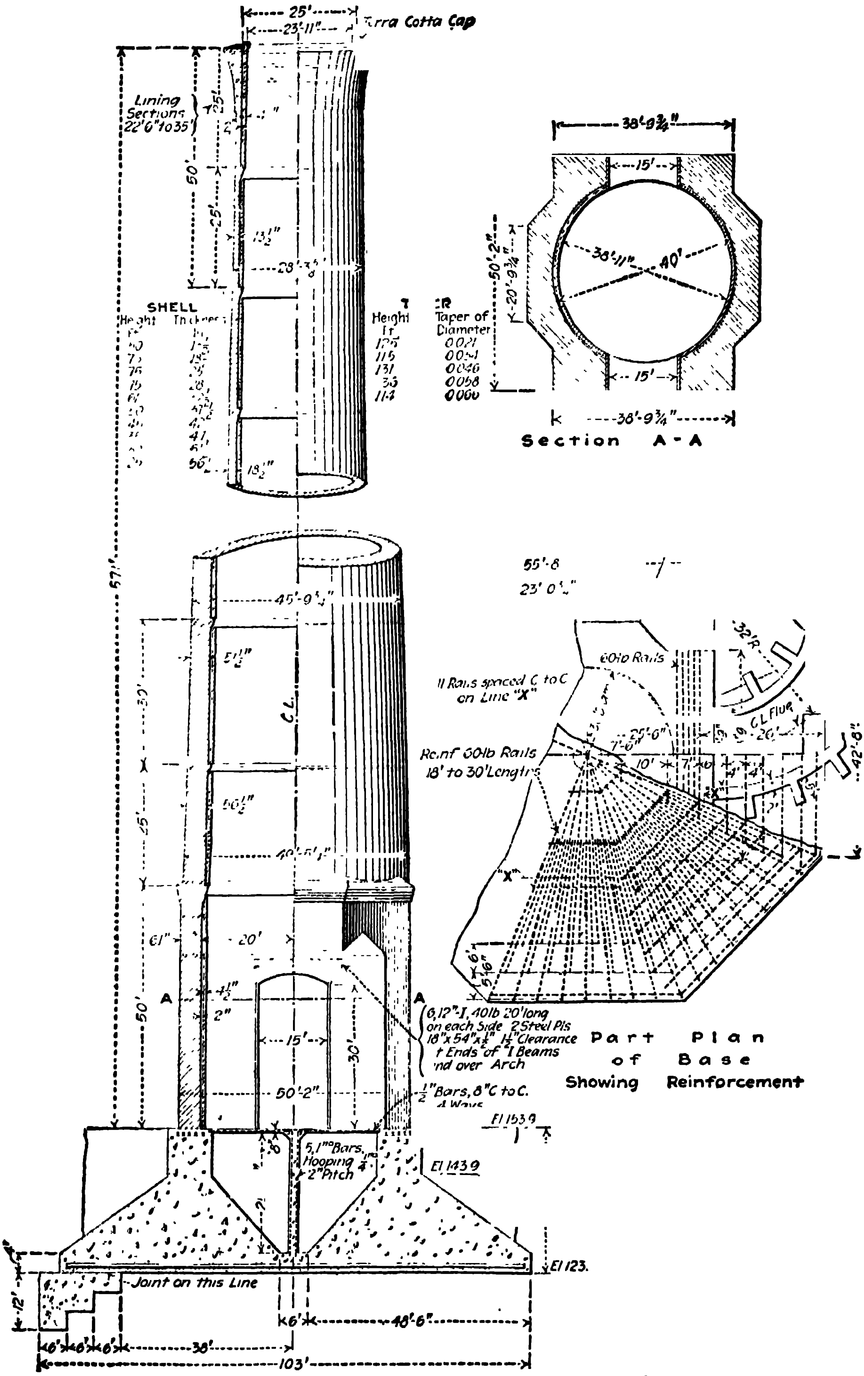


FIG. 5.—Base and shell details of Tacoma stack.

complete description of this chimney and especially of its foundations, see the *Engineering News-record* of April 4, 1918, Vol. 80, p. 644.

34. Braender Tire and Rubber Company Chimney.—This chimney is typical of the average radial brick chimney constructed for most industrial power plants.

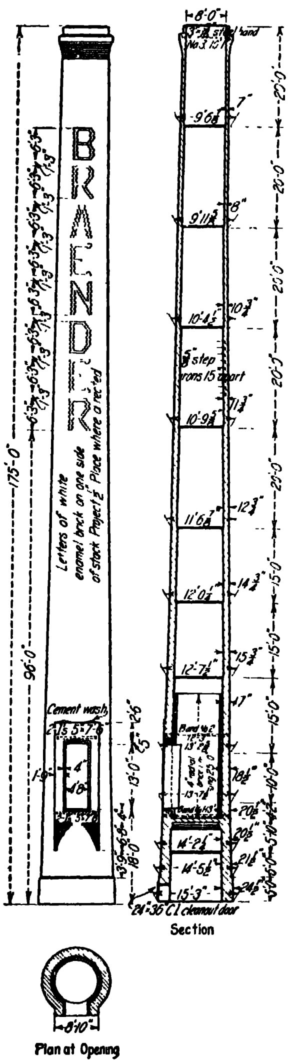


Fig. 6.—Elevation and section of a radial brick chimney built by the Heine Chimney Company.

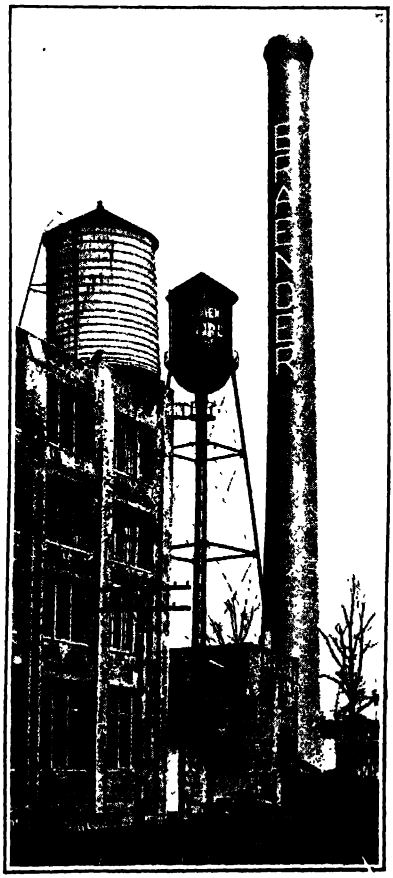


Fig. 7.—The Braender Tire and Rubber Company chimney completed.

Figure 6 shows the details of the shaft and Fig. 7 is a reproduction of a photograph taken after the chimney was completed. This chimney was designed and constructed by the Heine Chimney Company, Chicago, Ill. Note that lower part of the shaft is constructed in 15-ft. sections and the upper part in 20-ft.

sections and that the lining extends only a short distance up the shaft because of the low temperature assumed for the flue gases.

35. Design of a 180-Ft. Perforated Radial Brick Chimney.

35*a*. Statement of Problem.—Design a perforated radial brick chimney and foundation for the following conditions:

Height above foundations = 180 ft.
Internal top diameter = 7.5 ft.
Lining = 42 ft. high from a point 2 ft. below the bottom of the breech opening, to be of 4½-in. radial fire brick.
Size of breech opening = 5.00 ft. by 9.75 ft. and bottom of breech opening is to be 14 ft. above top of foundation.
Wind pressure = 25 lb. per sq. ft. on the vertical projection.
Brick for shaft = perforated radial brick.
Weight of brick masonry in shaft = 120 lb. per cu. ft.
Foundation = To be of plain concrete of 1:2½:5 mix. Octagonal shape with stepped sides.
Allowable soil pressure = Three tons per square foot compression and no tension.
Unit stresses = Allowable unit compression stress in perforated radial brick work is 200 lb. per sq. in. on the gross section.
Allowable unit tension stress in perforated radial brickwork is taken as ⅙ of the allowable unit compression stress, or 33⅓ lb. per sq. in.

35*b*. Design of Shaft.—The wall thickness for the top and upper section of the shaft will be taken as 7.5 in. making the external top diameter 8.75 ft.

The length of each vertical section will be taken as 20 ft., giving 9 sections for the shaft.

The batter will be taken as ¹⁄₆₀ approximately. Then the external diameter at the bottom of the shaft should be $8.75 + (2)(180)(\frac{1}{60}) = 15.95$ ft.

The approximate wall thicknesses for each vertical section will be, using formula (see Art. **24**)

$$\frac{H}{10} + 5$$

Section from top.......	3	4	5	6	7	8	9
Wall thickness (inches).	11	13	15	17	19	21	23

With perforated radial brick, wall thicknesses to the nearest quarter inch or half inch may be used. Consequently, stress computations may show that the wall thicknesses computed should be increased or decreased a little for each section.

The next step in the design is to prepare a form of tabulation for recording the partial and complete results of the computations for the various vertical sections. Figure 8 shows a suitable form.

Considering the first (upper) vertical section and using the formulas in Art. **27*a***, computations for wind pressure, weight, unit stresses, etc., are made as follows:

The area of the bottom of the section is given by the formula, $\pi t(D - t)$, and equals

$$(3.1416)\left(\frac{7.5}{12}\right)\left(9.55 - \frac{7.5}{12}\right) = 17.57 \text{ sq. ft.}$$

The section modulus of the bottom of the section is given by the formula

$$\frac{54\pi(D^2 + d^2)(D^2 - d^2)}{D} \text{ or } \frac{216A(D^2 + d^2)}{D}.$$

Substituting, the section modulus $= \frac{(216)(17.57)}{9.55}[(9.55)^2 + (8.30)^2] \quad 63,700 \text{ in.}^3$

The weight of the first section is given by the formula $W_1 = \frac{\pi}{8}[(D_1^2 - d_1^2) + (D_t^2 - d_t^2)]wh$

or $W_1 = \frac{\pi}{8}[(9.55^2 - 8.30^2) + (8.75^2 - 7.5^2)](120)(20) = 40,300$ lb. Perhaps the formula,

FIG. 8.—STRESS SHEET FOR A 180-FT. PERFORATED RADIAL BRICK CHIMNEY SHAFT

Internal Top Diameter 7.5 ft. Wind Pressure = 25 lb. per sq. ft. Weight of Brickwork = 120 lb. per cu. ft.

Section number	Distance of base of section from top (ft.)	Thickness of wall (in.)	External diameter at base of section (ft.)	Area of base of section (sq. ft.)	Section modulus of base of section (in.^{3})	Weight of shaft above base of section (lb.)	Wind pressure above base of section (lb.)	Wind moment about base of section (in.-lb.)	Unit horizontal shear (lb. per sq. in.)	Unit stress due to weight compression (lb. per sq. in.)	Unit stress due to wind moment tension or compression (lb. per sq. in.)	Maximum unit stress compression (lb. per sq. in.)	Minimum unit stress + compression − tension (lb. per sq. in.)
1	20	7.5	9.55	17.57	63,700	40,300	4,575	521,000	1.8	+16.0	± 8.0	+ 24.0	+ 8.0
2	40	9.0	10.35	22.65	87,600	92,400	9,550	2,148,000	2.9	+28.0	± 24.5	+ 52.5	+ 3.5
3	60	11.0	11.15	29.55	120,800	160,600	14,925	4,977,000	3.5	+38.0	± 41.0	+ 79.0	− 3.0
4	80	13.0	11.95	37.00	159,000	246,300	20,700	9,104,000	3.9	+46.0	± 57.0	+103.0	−11.0
5	100	15.0	12.75	45.25	205,000	351,300	26,875	14,625,000	4.1	+54.0	± 71.5	+125.5	−17.5
6	120	17.0	13.55	54.20	257,500	477,100	33,450	21,636,000	4.3	+61.0	± 84.0	+145.0	−23.0
7	140	19.0	14.35	63.65	316,500	625,100	40,425	30,233,000	4.4	+68.0	± 96.0	+164.0	−28.0
8	160	21.0	15.15	73.75	383,000	797,100	47,800	40,512,000	4.5	+75.0	±106.0	+181.0	−31.0
9	180	23.0	15.95	84.70	460,000	994,800	55,575	52,569,000	4.6	+81.5	±114.0	+195.5	−32.5

$W_1 = \frac{\pi}{2} wh_1 t_1 [D_1 + D_t - 2t_1]$, is a better formula to use, especially for the succeeding vertical sections. Substituting in this formula,

$$W_1 = \left(\frac{\pi}{2}\right)(120)(20)\left(\frac{7.5}{12}\right)\left[9.55 + 8.75 - \frac{(2)(7.5)}{12}\right] = 40{,}300 \text{ lb.}$$

The total wind pressure above the bottom of any section equals the average external diameter of the section times the height of the section times the wind pressure in pounds per square foot on the projected area. For the first section, the wind pressure equals

$$\frac{9.55 + 8.75}{2}(20)(25) = 4{,}575 \text{ lb.}$$

The formula for the bending moment due to the wind about any horizontal section is

$$M = 2p_w h^2(D + 2D_t)$$

For the first section, the bending moment of the wind about the base of the section equals

$$M = (50)(20)^2(9.55 + 16.50) = 521{,}000 \text{ in.-lb}$$

The unit shear in lb. per sq. in. at the base of any vertical section equals

$$\frac{\text{wind pressure above the base of section in pounds}}{144 \times \text{area of base in square feet}}$$

At the base of the first section the unit shear in pounds per square inch equals

$$\frac{4{,}575}{(144)(17.57)} = 1.8 \text{ lb. per sq. in.}$$

The unit shearing stress on any horizontal section is so small that the computations for shear could be neglected.

The unit compressive stress in pounds per square inch at any horizontal section due to the weight of the shaft above the section equals

$$\frac{\text{weight of shaft above horizontal section in pounds}}{144 \times \text{area of horizontal section in square feet}}$$

At the base of the first vertical section, the unit compressive stress due to the weight of the shaft above equals

$$\frac{40{,}300}{(144)(17.57)} = 16 \text{ lb. per sq. in.}$$

The unit stress in pounds per square inch on any horizontal section due to the bending moment of the wind equals

$$\frac{\text{wind moment about horizontal section in inch-pounds}}{\text{section modulus of horizontal section in inches}^3}$$

At the base of the first vertical section the unit stress caused by the bending moment of the wind equals

$$\frac{521{,}000}{63{,}700} = \pm 8 \text{ lb. per sq. in.}$$

This unit stress will be tension on the windward side and compression on the leeward side.

The maximum unit stress on any horizontal section will be compression and will equal the sum of the unit stresses due to weight of shaft and the wind bending moment. At the base of the first vertical section the maximum unit stress equals

$$16 + 8 = 24 \text{ lb. per sq. in., compression}$$

The minimum unit stress on any horizontal cross-section will equal the difference between the unit stress due to weight of shaft and the unit stress due to wind bending moment. If the unit stress due to weight of shaft is the greater, the minimum unit stress will be compression. If the unit stress due to the wind bending moment is the greater, the minimum unit stress will be tension.

The computations should be repeated for the horizontal base section of each succeeding vertical section and the values found placed on the design sheet.

On looking over the results on the design sheet it is seen that the allowable unit stresses in tension and compression are but a little greater than the computed stresses for the last (ninth) vertical section. The tabulation shows that the shaft walls in some vertical sections could be made thinner, but this would reduce the weigth of the stack and would increase the tension stresses in the lower part of the stack. This in turn would require an

increase in the thickness of the lower walls of the shaft and most of the gain would be lost. Further, the lower portions of the shaft are necessarily built first and have an opportunity to attain their strength before being subjected to the larger unit stresses. The upper portions of the shaft do not have such a good opportunity to age and obtain their strength and, consequently, many designers favor the reducing of the allowable unit stresses in the upper portions of the chimney. As a whole, the design of the shaft may be considered satisfactory.

35c. Design of Breech Opening.—As stated in Art. **35a**, the required size of the breech opening is 5 by 9.75 ft. giving an area about 10.3 per cent larger than the internal area of the top of the shaft. The brickwork around the flue opening should be corbeled out as shown in Fig. 2 to make up for the brickwork removed for the opening and also to provide a firm support for the end of the breeching. The inner surface of the breech opening should be lined with firebrick of the same character as those used in the lining of the shaft.

For reinforcement, four 6-in. 14.75-lb. I-beams about 7 ft. 3 in. long should be placed above the opening. These I-beams should rest on flat steel bearing plates 3/8 or 1/2 in. thick. A space of about 2 in. should be left free at the both ends of the I-beams to allow for expansion. A wrought-iron ring, 3/8 in. by 3 in. should be placed about 3 ft. below the opening and a similar ring about a foot above the I-beams above the opening.

A flat arch consisting of two courses of brick and having a rise of about 1/2 in. or 3/4 in. for each foot of flue opening width should be constructed at the top of the opening. Figure 2 shows details of a flue opening of this type.

35d. Design of Details.—The details in the design of a brick chimney usually include the reinforcing rings, the lining and lining supports, top cap, ladder, block and cable, lightning conductor, and clean-out door.

As this chimney is to be constructed of perforated radial brick, no reinforcing rings are required other than those at the top of the shaft and above and below the breech opening.

The lining is to be built of 4 1/4-in. radial firebrick and is to be 42 ft. high beginning at a point 2 ft. below the bottom of the breech opening or 12 ft. above the foundation. The outside diameter of the lining should be such that the minimum air space between the lining and the chimney walls will be 2 in. The shaft walls should be built out as shown in Fig. 2 to support the lining. The top of the lining should be built out so that it comes to within 2 in. of the shaft walls. About 2 in. above the top of the lining, two rows of brick in the shaft walls should be built out about 4 or 5 in. into the interior of the shaft so that there will be no danger of soot, etc. falling inside the air space. The inside diameter of the lining is 10.50 ft. at the bottom and 9.50 ft. at the top.

The top cap should be of 1 to 3 Portland cement mortar and should have a 3/8 by 3 -in. wrought-iron retaining ring set in a full bed of the mortar. Wrought iron is usually considered to offer a greater resistance to rust than steel does. The top section of the chimney should be belled out to give it a pleasing appearance.

The ladder is to be constructed on the inside of the chimney shaft. Galvanized iron rungs about 5/8 in. square or 3/4 in. in diameter bent in shape of a "U" are satisfactory. They should be spaced about 15 in. apart and should project from 4 to 6 in. from the wall. The rungs should have hooked ends and should be securely fastened in the masonry.

A 4- or 5-in. bronze pulley is to be securely fastened to the masonry at the outside of the top of the stack. A 5/16-in. galvanized wire cable, equal to twice the length of the chimney, is to be run through the pulley and the ends attached to a cleat near the base of the shaft.

The lightning rods shall consist of 4 points equally spaced around the top of the shaft. Each point shall consist of a platinum tipped 3/4-in. copper rod, 8 ft. long. The lower ends of the rods shall be securely fastened and soldered to a 1/2-in. copper cable extending completely around the chimney about 4 ft. from the top. This cable and the points should also be securely anchored to the masonry. The down cable shall be a 1/2-in. copper wire extending from the circular cable down to a copper plate buried in the ground below the foundations. This down cable shall be securely anchored to the outside of the chimney walls by brass anchors about 8 ft. apart which are sufficiently strong to support the weight of the cable.

A standard cast-iron clean-out door, 24 by 36 in. in size, with a cast-iron frame, should be placed at the base of the shaft about a foot above the foundations and preferably on the side opposite from the breech opening.

35e. Design of Foundations.—The foundations for this chimney are to be of plain concrete of a $1:2\frac{1}{2}:5$ mix. The shape is to be octagonal with stepped sides. The maximum pressure on the bottom of the foundation is to be limited to 3 tons per sq. ft. and there is to be no tension. Referring to Tables 9 and 10, a trial width of 25.5 ft. for the bottom slab and a depth of 7 ft. will be chosen. The width of the top slab of the foundation will be taken as 19.5 ft., and there will be 4 steps, 19.5, 21.5, 23.5, and 25.5 ft. in width respectively.

The weight of the shaft is 994,800 lb., from Fig. 8.

The weight of the lining will be assumed at 120 lb. per cu. ft. Since the average inside diameter is 10 ft., and the height is 42 ft., the weight of the lining equals

$$(120)(42)(\pi)\left(\frac{4.25}{12}\right)\left(10 + \frac{4.25}{12}\right)$$
$$= (5,610)(10.35) = 58,100 \text{ lb.}$$

The weight of the foundation, assuming the concrete to weigh 145 lb. per cu. ft., equals $(145)(\frac{2}{12})(0.828)[(25.5)^2 + (23.5)^2 + (21.5)^2 + (19.5)^2] = (210.1)(2.045) = 429,700$ lb.

The overturning moment, due to the wind, about the bottom of the foundation equals (see Fig. 8 for values)

$$52,569,000 + (55,575)(7)(12) = 57,237,000 \text{ in.-lb.}$$

The section modulus of the octagonal base of the foundation equals

$$(190)(25.5)^3 = 3,150,500 \text{ in.}^3$$

The area of the bottom of the foundation equals

$$(119)(25.5)^2 = 77,380 \text{ sq. in.}$$

The maximum and minimum unit stresses on the foundations are given by the formula $\frac{P}{A} \pm \frac{Mv}{I}$

$$P = 994,800 + 58,100 + 429,700 = 1,472,600 \text{ lb.}$$
$$A = 77,380 \text{ sq. in.}$$
$$\frac{I}{v} = 3,150,500 \text{ in.}^3$$
$$M = 57,237,000 \text{ in.-lb.}$$

$$\text{Maximum unit compressive stress} = \frac{1,472,600}{77,380} + \frac{57,237,000}{3,150,500}$$

$= 19.0 + 18.2 = 37.2$ lb. per sq. in., compression
which is equal to 2.68 tons per sq. ft. compression.

The minimum unit stress will equal
$19.0 - 18.2 = 0.8$ lb. per sq. in. compression
which is equal to 0.0576 tons per sq. ft. If the weight of the lining is omitted, which gives the most dangerous condition, the minimum unit stress will equal

$$\frac{1,414,500}{77,380} - 18.2 = 18.3 - 18.2$$
$$= 0.1 \text{ lb. per sq. in., compression.}$$

Hence there will be no tension on the base of the foundation.

Even though this size of foundation does not give the maximum unit stress permitted, it is as small as it can be made without having tension on some portion of the base.

35f. Drawings Required.—The drawings mentioned below are required for this design. An elevation as shown in Fig. 2 should be drawn to scale with all dimensions given. A satisfactory scale is ⅛ in. to 1 ft. Detailed drawings should show a plan of the foundation, a cross section through the breech opening, a section showing the location of I-beams over the breech opening, a partial section of the top showing the corbeling out of the top of the chimney, the cap and retaining ring, the lightning conductor, and the pulley block and cable. If necessary, other detailed drawings should be provided for the ladder rungs, lightning rod anchors, pulley block anchor, cast-iron clean-out door and frame, and all ornamentation appearing on the chimney. Except for the smaller details, a scale of ¼ in. or ⅛ in. to the foot is usually satisfactory. Figures 2, 3, 5, and 6 show in general what the drawings should include.

REINFORCED CONCRETE CHIMNEYS

The use of reinforced concrete for chimneys began during the first part of the twentieth century. The first chimneys of this kind were more or less experimental in character but, as more knowledge was obtained concerning the materials, design, and construction of reinforced concrete chimneys, it was found that chimneys of this type could be built which compared favorably with those of radial brick and steel.

The essential features of a reinforced concrete chimney are shaft, lining, and footing or foundation. The shaft is above the ground and is exposed to the action of the wind. The lining is usually constructed independently of the shaft and is inside the shaft with an air space of from 2 to 4 in. between the lining and the shaft walls. The lining is usually built from the top of the footing up. The footing is a reinforced concrete slab supporting the shaft and lining. The shaft must be securely connected to the footing so that stresses in the shaft may be safely transferred to the foundation.

At the present time, practically all of the reinforced-concrete chimneys constructed are of the tapered type (conical section). The taper improves the appearance of the chimney besides giving an increase in diameter at the base which is of advantage in the design. The wall thickness increases uniformly from the top to the base. Reinforced concrete chimneys in which the wall thickness is increased by offsets are rarely constructed now as it has been found that the temperature stresses frequently cause cracking at the place of offset.

Reinforced concrete chimneys have been safely constructed and used in almost every locality and for almost every purpose. Chimneys of this type have been constructed with heights up to 570 ft. and diameters up to 30 ft. or more. Chimneys of reinforced concrete may be reinforced for earthquake shocks, are usually lighter than brick chimneys of the same capacity, and have a longer "life" than steel chimneys.

36. Concrete Materials.—The cement should be Portland cement which passes the specifications of the American Society for Testing Materials. It should be stored until used in a suitable weather-tight building which will protect the cement from dampness.

The fine aggregate should be sand, crushed rock screenings, or air-cooled blast furnace slag screenings. The fine aggregate should be graded from fine to coarse and all should pass a ¼-in. sieve when dry. No injurious amounts of friable material, clay, loam, organic matter, or other impurities should be present in the aggregate. It is always advisable to screen and wash the fine aggregate.

The coarse aggregate should be well graded, clean, hard, and durable crushed rock, gravel, or air-cooled blast furnace slag. It should be free from all injurious matter. When necessary, the coarse aggregate should be washed and screened.

The water used in the concrete should be free from acid, oil, alkalies, or organic matter.

37. Reinforcement.—Reinforcing steel should pass the specifications of the American Society for Testing Materials for billet steel reinforcement. Cold drawn wire (or wire mesh) used for temperature and torsional reinforcement should have an ultimate tensile strength of from 75,000 to 85,000 lb. per sq. in.

The reinforcement should be free from paint, scale, excessive rust, etc., which would tend to reduce the bond between the steel and the concrete.

38. Design in General.—In general, the design of a reinforced concrete chimney should satisfactorily cover the following points:

(*a*) Unit stresses should be used for the concrete and the steel which are accepted as good practice by most engineers.
(*b*) Proper concrete materials for the concrete and reinforcing steel that passes the A.S.T.M. specifications should be chosen.
(*c*) A concrete mix should be selected that will give the required strength.
(*d*) The walls of the shaft should be battered so that the chimney will have a good appearance.
(*e*) The wall thickness should be as thin as practicable to help keep down the temperature stresses.
(*f*) Sufficient steel reinforcement should be provided to resist:
  Tension stresses due to wind.
  Diagonal tension stresses.
  Temperature stresses.
  Stresses caused by earthquakes (in some localities only).
(*g*) The concrete should be increased around the flue opening to compensate for that removed and extra reinforcement should be added.
(*h*) Extra reinforcement should be added where the inner wall of the shaft is corbeled out to project over top of lining.
(*i*) The lining should be of good materials and of sufficient height.
(*j*) An air space should be provided between lining and shaft.
(*k*) Good judgment must be used in estimating soil conditions and allowable bearing pressures.
(*l*) The footing should be made large enough and be reinforced sufficiently.
(*m*) The shaft must be properly connected to the foundation.

39. Batter of Walls.—Battering the walls of the chimney serves two purposes, namely to improve the appearance of the shaft and to increase the diameter as the base is approached. This increase in diameter permits the use of thinner walls without increasing the unit stresses.

The amount of batter depends on the designer's opinion. A wall batter of 1 in 75 or 1 in 80 is common, though batters varying from 1 in 50 to 1 in 100 have been used. A chimney with an outside top diameter of 10 ft., a height of 150 ft. and a batter of 1 in 75 would have an outside bottom diameter of 14 ft.

40. Thickness of Walls.—The thickness of the walls of the shaft is generally increased uniformly from the top to the base. This increase in thickness depends on the wind pressure, the diameter of the shaft, the batter of the shaft walls, and the unit stresses allowed. No simple formula has been deduced for determining the required increase in the thickness of the shaft walls and, consequently, the empirical rule of increasing the wall thickness 1 in. for every so many feet of height has been adopted.

An examination of chimneys already built shows that an increase in wall thickness of 1 in. for about every 20 or 25 ft. increase in height is about the average, though increases in wall thicknesses varying from 1 in. in 15 ft. to 1 in. in 30 ft. have been used. A wall thickness increase of 1 in. in 20 ft. seems fair

for a wall batter of 1 in 100, and 1 in. in 25 ft. for a wall batter of about 1 in 75. It should be remembered that these rules are only approximate and that designing experience and judgment are required in determining the proper wall batter and the correct increase in wall thickness needed to produce an efficient design in any particular problem.

The thickness of the shaft wall at the top depends on the inner diameter of the chimney and on the judgment of the designer. A thickness of less than 4 in. should not be used. The following values have been found satisfactory in many instances:

TABLE 11

Thickness of Wall at Top of Shaft	Inside Diameter of Shaft at Top
4 in.	Up to 8 ft.
5 in.	From 8 ft. to 12 ft
6 in.	From 12 ft. to 18 ft
7 in.	18 ft. and over

41. Allowable Unit Stresses.—There is some difference of opinion among engineers regarding the allowable unit stresses to be used for concrete and other conditions to be met when designing reinforced concrete chimneys.

Practically all agree that the tensile strength of the concrete should be neglected when computing tensile stresses in the reinforcing steel.

Some omit the compression strength of the steel while others include it when computing the compression stresses in the concrete. It is believed that most designers now favor the omission of the compression strength of the steel in this instance.

The maximum allowable tensile stress in the steel is usually taken as 16,000 lb. per sq. in., though this maximum value is not often attained when designing high reinforced concrete chimneys of comparatively small diameters. Slightly higher unit stresses are permitted for special steels of greater strength.

The maximum allowable compressive stress in the concrete (assuming a 2,000 lb. per sq. in. concrete at an age of 28 days) varies from 300 to 700 lb. per sq. in. though most engineers are now using the conservative values of 350 or 400 lb. per sq. in. A value of 350 lb. per sq. in. is recommended.

The ratio of the modulus of elasticity of steel to the modulus of elasticity of the concrete is usually taken as 15.

The maximum allowable unit shear in the concrete is usually taken as 40 lb. per sq. in. unless shearing (diagonal tension) reinforcement is provided. When such reinforcement is provided, values of 80, 100, 105, and 120 lb. per sq. in. have been used. A fair value is 100 lb. per sq. in.

In designing the foundation, the unit tensile stresses in the steel are taken at 16,000 lb. per sq. in., the unit compressive stresses in the concrete are taken as 450 lb. per sq. in. for bearing and 650 lb. per sq. in. in bending, the unit shearing stress is taken as 40 lb. per sq. in. when no shear reinforcement is provided, and as 100 lb. per sq. in. when shear reinforcement is provided, and as 120 lb. per sq. in. for punching shear.

The following allowable soil pressures are taken from the allowed bearing values on soils in tons per square foot as given by different city regulations:

TABLE 12.—ALLOWABLE SOIL PRESSURE

Kind of soil	Pressure (tons per sq. ft.)
Alluvial soil	0 5
Soft or wet clay	1 0
Dry clay; fine, clean dry sand, or clay and sand in layers	2 0
Firm dry loam, or hard dry clay	3 0
Compact coarse sand, gravel or natural earth	4 0
Coarse gravel, stratified stone and clay, well-cemented gravel and sand, or inferior rock	6 0 to 8 0
Good hard pan, or hard shale	8 0 to 10 0
Good hard native rock	10 0 to 20 0
Very hard bedrock, or rock under caisson	15 0 to 25 0

42. Design Specifications.—The following specifications for design are taken from the "Recommended Specifications for Design and Construction of Tapering Reinforced Concrete Chimneys" as given in the 1922 *Proceedings* of the American Concrete Institute. While many engineers take exception to some parts of these specifications, it is believed that these specifications include what is regarded as good practice among most engineers, and, consequently, these specifications will be followed in developing the design formulas in later paragraphs.

"*Design.*—In designing the chimney the following primary assumptions shall govern:

(1) A plane section before bending remains plane after bending.

(2) The thickness of the wall at any horizontal section shall be constant.

(3) The temperature and torsional stresses shall be properly provided for with reinforcement.

(4) The temperature and torsional reinforcement shall be spaced not more than 6 in. apart horizontally, and the distance from the face of the wall to the reinforcement shall not be more than 3 in.

(5) A wind pressure due to a wind having a velocity of 100 miles per hour for the entire height of the chimney.

(6) Weight of the reinforced concrete—150 lb. per cu ft.

(7) The tensile strength of concrete is to be neglected when computing for tensile stresses in the reinforcing steel.

(8) The compression strength of the steel is to be neglected when computing for the compression stresses in the concrete.

(9) Maximum allowable compression stress in concrete—350 lb. per sq. in.

(10) Maximum allowable tensile stress in open hearth steel—16,000 lb. per sq. in.

(11) Maximum allowable tensile stress in cold drawn steel—18,000 lb. per sq. in.

(12) Maximum allowable shear in concrete shall not exceed 40 lb. per sq. in., unless shear reinforcement is provided, in which case the shearing strength shall not exceed 100 lb. per sq. in.

(13) The full strength of the bars shall be assumed to be developed by the imbedment of the bar equal to forty times its diameter.

(14) The foundations shall be reinforced with steel bars to act as a cantilever footing."

43. Formulas for Weight of Shaft and Wind Moment.—As the shaft is a hollow frustum of a cone, its weight is given by the formula:

$$W = \frac{150h}{3}(A + \sqrt{AA_t} + A_t)$$

where

W = weight of shaft in pounds.
A = net area of base in square feet.
A_t = net area of top in square feet.
h = height in feet.
150 = weight of concrete in pounds per cubic foot.

Since

$$A = \frac{2\pi rt}{(12)(12)}$$

where

r = average radius of base wall in inches.
t = thickness of base wall in inches.

and

$$A_t = \frac{2\pi r_1 t_1}{(12)(12)}$$

where

r_1 = average radius of top wall in inches.
t_1 = thickness of top wall in inches.

substituting values for A and A_t, the formula for the weight of the shaft becomes.

$$\begin{aligned} W &= \frac{150h}{3}\left(\frac{2\pi rt}{144} + \frac{2\pi}{144}\sqrt{(rt)(r_1t_1)} + \frac{2\pi r_1 t_1}{144}\right) \\ &= \frac{(2)(150)(\pi h)}{(3)(144)}\left(rt + \sqrt{(rt)(r_1t_1)} + r_1t_1\right) \\ &= 2.182h\left(rt + \sqrt{(rt)(r_1t_1)} + r_1t_1\right) \end{aligned}$$

In Fig. 9, values of the expression $2.182\ (rt + \sqrt{(rt)(r_1t_1)} + r_1t_1)$ have been plotted for corresponding values of rt, which give the weights of chimneys 1 ft. high for various inside top diameters and top wall thicknesses. To obtain the weight of a chimney of any height, find the value rt and the corresponding weight of chimney 1 ft. high, and then multiply this weight by the height in feet.

If it is desired to express the weight in terms of the diameters, corresponding values may be substituted for A and A_t

$$A = \frac{\pi}{4}\,\frac{(D^2 - d^2)}{144}$$

$$A_t = \frac{\pi}{4}\,\frac{(D_t^2 - d_t^2)}{144}$$

where D = outside diameter of base in inches.
d = inside diameter of base in inches.
D_t = outside diameter of top in inches.
d_t = inside diameter of top in inches.

and the formula reduces to

$$W = 0.2727h\left(D^2 - d^2 + \sqrt{(D^2 - d^2)(D_t^2 - d_t^2)} + D_t^2 - d_t^2\right)$$

The bending moment due to the wind equals

$$M = 2p_w h^2\left(\frac{D}{12} + \frac{2D_t}{12}\right) = \frac{p_w}{6}h^2(D + 2D_t) \text{ (see Art. } \mathbf{27a})$$

where

M = wind moment in inch-pounds.
p_w = wind pressure in pounds per square foot.
h = height of shaft in feet.
D = outside diameter of base in inches.
D_t = outside diameter of top in inches.

Assuming a wind pressure of 25 lb. per sq. ft. on the vertical projected area of the shaft (see Art. **8**).

$$M = \tfrac{25}{6}(D + 2D_t)h^2$$

substituting $2r + t$ for D and $2r_1 + t_1$ for D_1

$$M = \tfrac{25}{6}(2r + t + 4r_1 + 2t_1)h^2$$

In Fig. 10, values of the expression $\frac{25}{6}(2r + t + 4r_1 + 2t_1)$ have been plotted for corresponding values of $2r + t$ or D, which gives the wind moments on chimneys 1 ft. high for various inside top diameters and top wall thicknesses. To obtain the wind moment on a chimney of any height, find the value of the wind moment on a corresponding chimney 1 ft. high and multiply that value by the square of the height in feet.

The eccentricity (distance from the center to the point where the resultant of the wind pressure and weight cuts the base) in inches is given by $e = \frac{M}{W}$. The ratio of $\frac{e}{r}$, which is used later in the design formulas for the shaft, is equal to $\frac{M}{Wr}$.

44. Shaft Design Formulas.

44a. Direct Stresses Due to Wind and Weight—No Tension on Horizontal Cross-section.—When there is no tension (or at least no appreciable tension—say less than about 30 lb. per sq. in.) on the horizontal cross-section of the shaft and the effect of the compression in the steel is neglected, the following formula will apply:

$$f_c = \frac{W}{A} \pm \frac{Mv}{I}$$

where

f_c = unit compressive stress in concrete in pounds per square inch. This stress is a maximum when the plus sign is used for the second term, and this maximum stress occurs at the extreme fiber on the leeward side of the shaft.

This stress is a minimum when the minus sign is used for the second term, and this minimum stress occurs at the extreme fiber on the windward side of the shaft.

W = weight of shaft above section in pounds.
A = cross-sectional area of base of shaft in square inches.

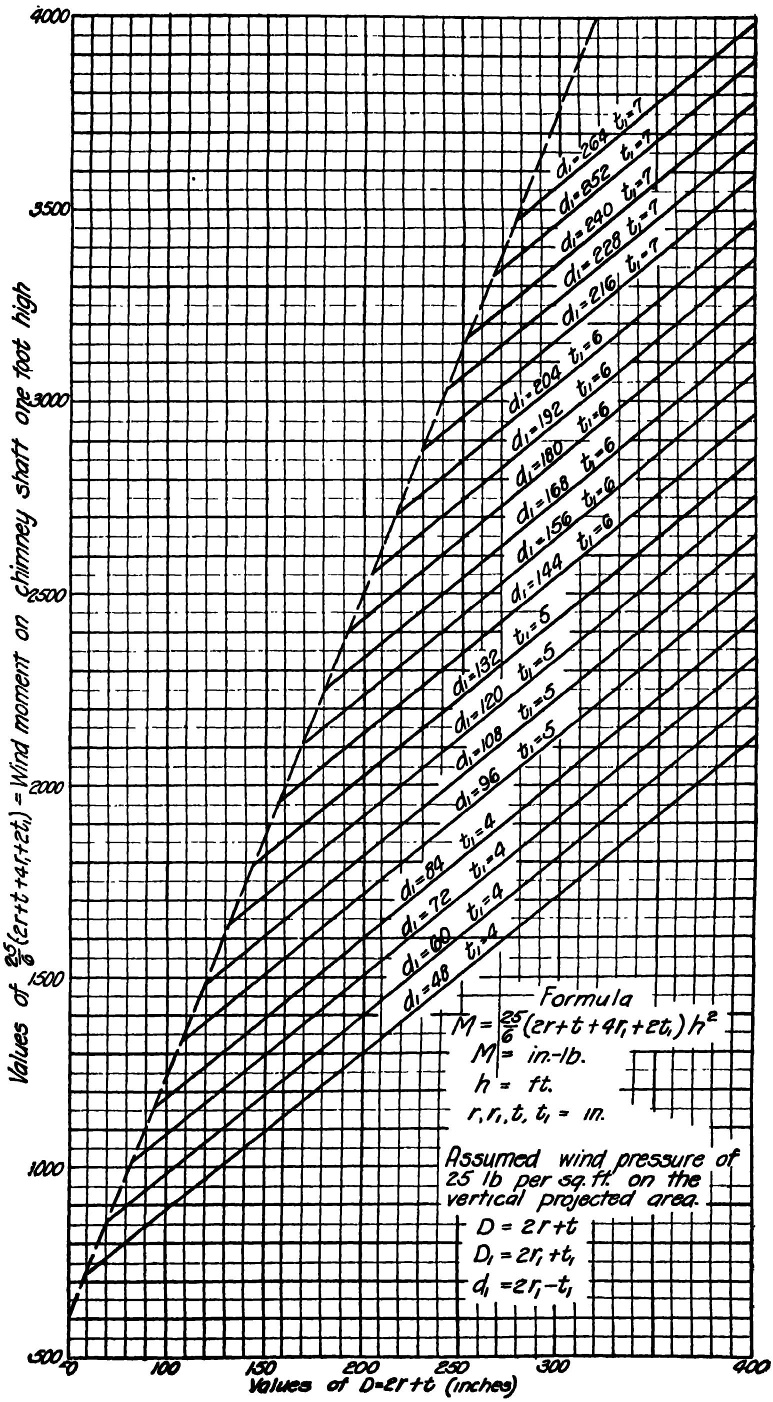


Fig. 10.—Wind moment on conical concrete chimneys one foot high.

M = wind moment in inch-pounds.

$\frac{I}{v}$ = section modulus of section in in.³

$$A = \frac{\pi}{4}(D^2 - d^2) = 2\pi rt = 6.2832rt$$

$$\frac{W}{A} = \frac{2.182}{6.2832}\left(\frac{h}{rt}\right)(rt + \sqrt{(rt)(r_1t_1)} + r_1t_1)$$

$$= 0.3472\frac{h}{rt}(rt + \sqrt{(rt)(r_1t_1)} + r_1t_1)$$

$$\frac{I}{v} = \frac{\pi}{32}\,\frac{(D^4 - d^4)}{D} = 0.982\frac{(D^2 + d^2)(D^2 - d^2)}{D}$$

It should be noted that $D = 2r + t$ and that $d = 2r - t$. These values for D and d may be used when convenient.

The maximum eccentricity permissible without causing tension on the section may be found by assuming the minimum unit stress equal to zero in the formula

$$f_c = \frac{W}{A} - \frac{Mv}{I}$$

and solving as follows:

$$0 = \frac{W}{A} - \frac{Mv}{I}$$

Transposing and substituting We for M

$$\frac{Wev}{I} = \frac{W}{A}$$

Cancelling W and substituting values for A and $\frac{I}{v}$

$$\frac{e \times 32D}{\pi(D^4 - d^4)} = \frac{4}{\pi(D^2 - d^2)}$$

$$\frac{e \times 8D}{D^2 + d^2} = 1$$

$$e = \frac{D^2 + d^2}{8D}$$

Substituting $2r + t$ for D and $2r - t$ for d

$$e = \frac{(2r + t)^2 + (2r - t)^2}{8(2r + t)}$$

In the average reinforced concrete chimney, t is quite small when compared to $2r$. Neglecting t, the above formula becomes

$$e = \frac{4r}{8} = \frac{r}{2} \quad \text{or} \quad \frac{e}{r} = \frac{1}{2}$$

That is, the resultant of the weight and wind pressure must cut the base within one of the middle quarters if there is to be no reversal of stress on any part of the base section. When the resultant cuts the base at the outer edge of one of the middle quarters, the maximum stress will equal twice the average while the minimum stress will be zero.

The formulas in this article are useful in designing the upper portions of reinforced concrete chimneys where there is no tension on the horizontal sections, and also in designing entire "gravity" concrete chimneys where there is no tension on any horizontal section.

In formula $f_c = \frac{W}{A} + \frac{Mv}{I}$, if values are substituted for A and $\frac{I}{v}$ and M, this formula becomes:

$$f_c = \frac{W}{2\pi rt} + \frac{We}{2\pi rt\left(\frac{D^2 + d^2}{8D}\right)}$$

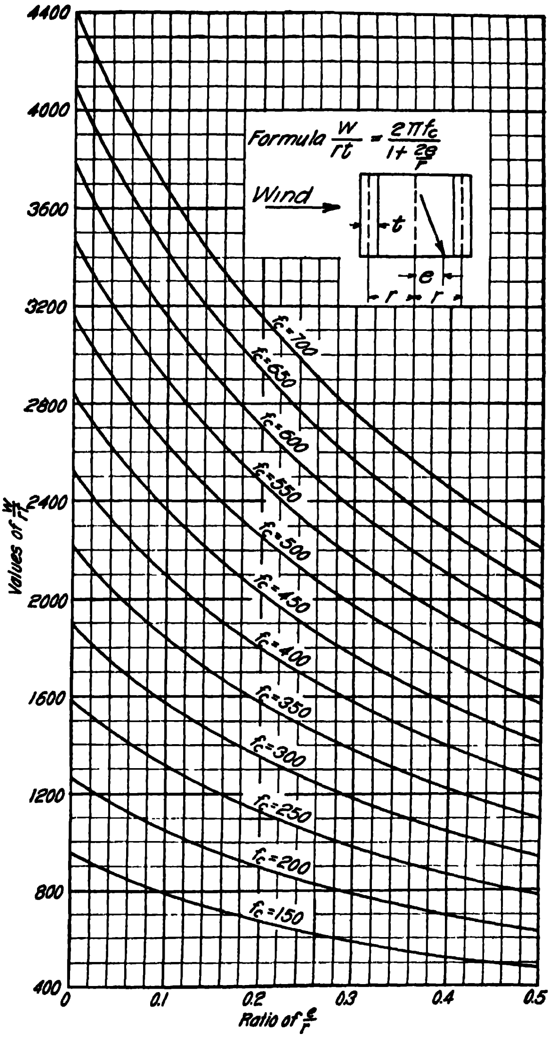


FIG. 11.—Direct stresses due to wind and weight in gravity section concrete chimneys.

As the expression $\frac{D^2 + d^2}{D}$ is approximately equal to $\frac{8r^2}{2r}$, or $4r$, this latter value may be substituted and then

$$f_c = \frac{W}{rt}\left[\frac{1}{2\pi} + \frac{2e}{2\pi r}\right] = \frac{W}{rt}\left[\frac{1}{2\pi}\left(1 + \frac{2e}{r}\right)\right]$$

This formula shows that there is a definite relation between f_c, $\frac{W}{rt}$, and $\frac{e}{r}$ as long as there is compression over the entire horizontal section. Curves may be plotted as in Fig. 11 showing this relation, and these curves may be used for the design and checking of design of concrete chimneys and portions of concrete chimneys having no reversal of stress on any horizontal section; that is where $\frac{e}{r}$ is 0.500 or less.

For plotting curves the following form of the formula is preferable:

$$\frac{W}{rt} = \frac{2\pi f_c}{1 + \frac{2e}{r}}$$

Illustrative Example.—Given a gravity section concrete chimney having a weight of 108,500 lb., a wall thickness of the base of 10 in., and an average radius of the base of 6 ft. 2 in. (74 in.), and being subjected to a wind moment of 3,857,000 in.-lb. What is the maximum unit fiber stress?

$$\frac{e}{r} = \frac{M}{Wr} = \frac{3,857,000}{108,500 \times 74} = 0.481$$

$$\frac{W}{rt} = \frac{108,500}{74 \times 10} = 1,467$$

Referring to the curves in Fig. 11, it is found that a unit stress of about 465 lb. per sq. in corresponds with the values computed for $\frac{e}{r}$ and $\frac{W}{rt}$.

It should be noted that the f_c found as above is the unit stress in the concrete at a distance r from the center of the section and is a little less than that in the extreme fiber. This difference usually is less than 5 per cent.

When designing gravity section reinforced concrete chimneys the procedure is a little more difficult. Usually the internal top diameter, wall thicknesses at top and base, wall batter (and consequently the base diameter), and the allowable unit compressive stress are known or assumed. Then the values for M, W, $\frac{e}{r}$ and $\frac{W}{rt}$, are computed, and f_c is found from the curves. If this value of f_c agrees closely with the allowable unit compressive stress, the design is satisfactory. If this value of f_c disagrees with the allowable unit stress, either the diameter or the thickness of wall at the base of the chimney shaft is changed and the computations made again. Changing the outside diameter of the base changes M, W, $\frac{e}{r}$ and $\frac{W}{rt}$, while changing the wall thickness at the base changes W, $\frac{e}{r}$, and $\frac{W}{rt}$. An experienced designer will usually obtain a satisfactory design after a few trials.

For the most efficient design, $\frac{e}{r} = 0.50$, and the unit stress caused by the weight is equal to that caused by the wind moment that is,

$$\frac{W}{A} = \frac{Mv}{I} \text{ and } Wr = 2M$$

Illustrative Problem.—Find r and t at the base of a gravity section reinforced concrete chimney having a height of 100 ft., an inside top diameter of 5 ft., and a wall thickness at the top of 4 in. Assume a wind pressure of 25 lb. per sq. ft. on the vertical projected area. Allowable unit compressive stress in the concrete = 350 lb. per sq. in.

With a gravity section chimney of this height the allowable unit compressive stress in the concrete will not be reached and the problem consists of finding an r and t consistent with good practice which will cause the ratio $\frac{e}{r}$ to be 0.50 or slightly less.

Assuming the wall thickness to increase 1½ in. for every 20 ft. of height, t (at the base) will equal 4 in. + 7.5 in. or 11.5 in.

Selecting a trial r of 90 in.

$$rt = 1{,}035 \qquad 2r + t = 191.5.$$

$W = (3{,}330)(100) = 333{,}000$ lb. (see Fig. 9)

$M = (1{,}370)(100)^2 = 13{,}700{,}000$ in.-lb. (see Fig. 10)

$$\frac{e}{r} = \frac{13{,}700{,}000}{(333{,}000)(90)} = 0.458$$

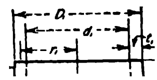
This value of $\frac{e}{r}$ is a little small, but it may be increased by decreasing t or r or both.

Decreasing r to 84 in., $rt = 966$ and $2r + t = 179.5$ in.

$W = (3{,}150)(100) = 315{,}000$ lb. (see Fig. 9)

$M = (1{,}320)(100)^2 = 13{,}200{,}000$ in.-lb. (see Fig. 10)

$$\frac{e}{r} = \frac{13{,}200{,}000}{(315{,}000)(84)} \quad 0.499$$



which is close enough.

Checking the unit stress in the concrete by means of Fig. 11,

$$\frac{W}{rt} = \frac{315{,}000}{966} = 312.6$$

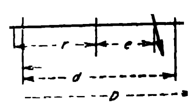
which shows that this unit stress is considerably less than 150 lb. per sq. in.

Checking by formula: $f_c = \frac{W}{rt}\frac{1}{2\pi}\left(1 + \frac{2e}{r}\right)$

$$f_c \quad \frac{(312.6)(2)}{6.2832} = 99.5 \text{ lb. per sq. in.}$$

Other combinations of r and t might also be selected which will give other solutions of the problem.

It should be noted that a gravity section concrete chimney requires a large weight, and a correspondingly large diameter or wall thickness, in order to have no tension on any horizontal cross-section of the shaft.



44b. Direct Stresses Due to Wind and Weight-Tension and Compression on Horizontal Cross-section.—In deriving formulas in this article, the shaft will be considered as a hollow conical cantilever reinforced concrete beam subjected to a load (the weight) coincident with the axis and a transverse load (wind pressure). Besides the ordinary assumptions made in deriving a beam formula for reinforced concrete, it will also be assumed that the concrete takes no tension, that the steel takes no compression, and that the steel reinforcement may be considered as equivalent to a thin circular steel shell whose radius at any section is equal to the average radius of the shaft.

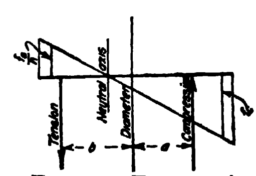


Fig. 12.—Forces acting on a conical chimney shaft due to weight and wind pressure.

Referring to Fig. 12, it is seen how the different forces act on the chimney shaft and how the combination of wind and weight causes a displacement of the neutral axis, which displacement may be to one side or the other of the center of the horizontal section. If the shaft was composed of one homogeneous

material capable of taking both tension and compression, the ordinary beam formula (weight divided by area plus or minus wind moment divided by section modulus) could be used. But, as steel and concrete are different materials and as the steel is assumed to take all of the tension and the concrete to take all of the compression, the old beam formula should not be used unless corrections are made in the section modulus to take care of the above assumptions and of the shifting of the neutral axis. Remember, that it is assumed that the steel takes all tension, which will be on one side of the neutral axis, and that the concrete takes all compression, which will be on the other side of the neutral axis, and also that the neutral axis will not coincide with the diameter. Using the old beam formula as it stands would mean that there would be compression over the whole sectional area due to the weight, that there would be compression on one side of the diameter due to wind moment, that there would be tension on the other side of the diameter due to wind moment, and that the neutral axis was considered as coinciding with the diameter of the section in computing the stresses due to wind moment.

The simplest method of securing the necessary formulas is to consider the shaft with the actions and reactions due to wind and weight, set up the fundamental equations, and then (keeping in mind the assumptions made) derive the needed formulas.

From Fig. 12 it is evident that

(Total compression in concrete) − (Total tension in steel) = (Weight of shaft)

and also that, taking moments about the center of the section,

Compression × distance to diameter + tension × distance to diameter = weight × eccentricity = wind moment.

Let

C = total compression resultant.
T = total tension resultant.
W = weight of shaft.
M = wind moment.
e = eccentricity.
a = distance of C from diameter.
b = distance of T from diameter.

Then the above equations become:

$$C - T = W$$
$$Ca + Tb = We = M$$

These two equations are the fundamental equations upon which the following design formulas are based.

From Fig. 13, the compression area equals $2rt\theta$ and the tension area equals $2prt(\pi - \theta)$, where

θ = angle in radians.
r = average radius in inches.
t = thickness of wall in inches.
p = ratio of total steel area to total concrete area.
(Usually expressed as a percentage.)

Hence, the compression and tension areas may be found for any value of the angle θ, or for any position of the neutral axis as the value of the angle θ locates the neutral axis.

The distance from the diameter to the center of gravity of the circular arc representing the compression area may be found by calculus to be $\frac{r \sin \theta}{\theta}$. Similarly the distance from the diameter to the center of gravity of the circular arc representing the steel area may be found to be $\frac{r \sin \theta}{\pi - \theta}$

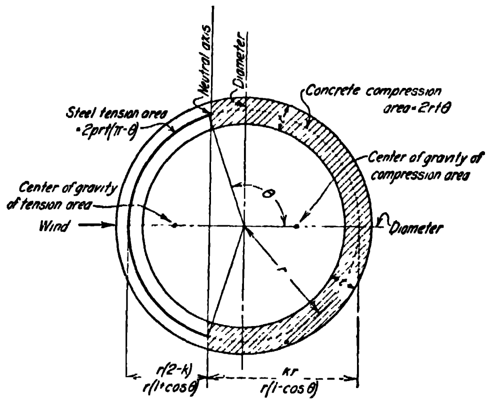


FIG. 13.—Compression and tension areas on a horizontal section of the shaft.

The distance from the neutral axis to the centers of gravity of these two circular arcs is:

$$r\left(\frac{\sin \theta}{\theta} - \cos \theta\right) \text{ for the compression area}$$

$$r\left(\frac{\sin \theta}{\pi - \theta} + \cos \theta\right) \text{ for the tension area}$$

Attention must be paid to the sign of $\cos \theta$ as it is minus when θ is between 90 and 180 deg.

The distance of the neutral axis from the compressive side of the shaft is given by:

$$kr = r(1 - \cos \theta)$$

where $k = (1 - \cos \theta)$

Assuming that the unit stresses increase uniformly from the neutral axis to the extreme fibers, the average unit stresses will occur at distances from the neutral axis equal to the distances of the centers of gravity of their respective arcs from the neutral axis. Then, by similar triangles, it may be shown that the average unit compressive stress equals

$$f_{c(\text{ave.})} = f_c \frac{(\sin \theta - \theta \cos \theta)}{\theta(1 - \cos \theta)}$$

and that the average unit tension equals

$$f_{s(\text{ave.})} = f_s \frac{[\sin \theta + (\pi - \theta) \cos \theta]}{(\pi - \theta)(1 + \cos \theta)}$$

The total compressive stress, C, will equal the average unit compressive stress times the compression area, or

$$C = f_c \frac{(\sin \theta - \theta \cos \theta)}{\theta(1 - \cos \theta)} \times 2\theta rt.$$

reducing,

$$C = 2f_c rt \left(\frac{\sin \theta - \theta \cos \theta}{1 - \cos \theta} \right)$$

In like manner, the total tension stress, T, will equal

$$T = f_s \frac{[\sin \theta + (\pi - \theta) \cos \theta]}{(\pi - \theta)(1 + \cos \theta)} \times 2(\pi - \theta)\, prt$$

reducing,

$$T = 2f_s prt \left(\frac{\sin \theta + (\pi - \theta) \cos \theta}{1 + \cos \theta} \right)$$

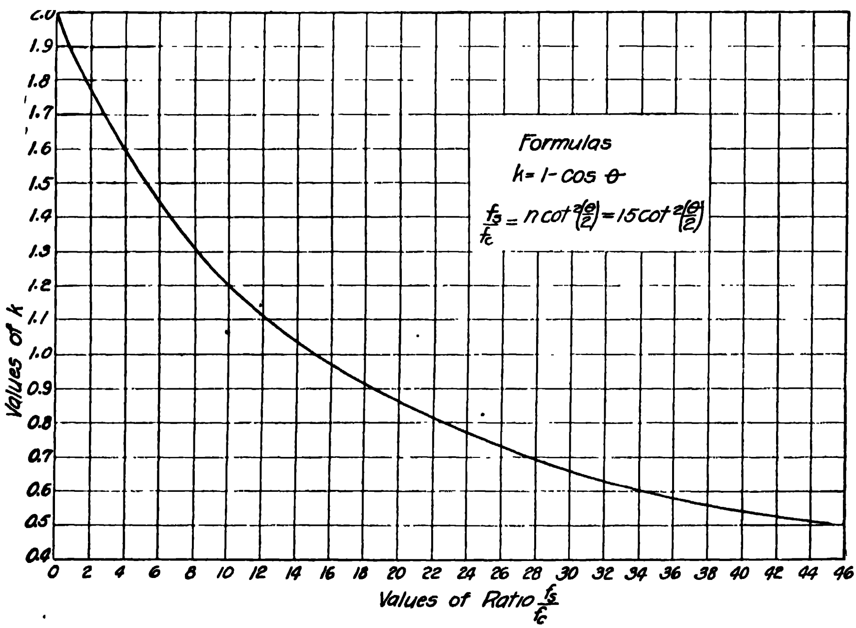


FIG. 14.—Relation of ratio $\frac{f_s}{f_c}$ to position of neutral axis.

There is a certain definite relation between f_s and f_c for any one position of the neutral axis (or for the corresponding value of θ).

Referring to Fig. 12

$$\frac{\frac{f_s}{n}}{1 + \cos \theta} = \frac{f_c}{1 - \cos \theta}$$

$$\frac{f_s}{f_c} = n \left(\frac{1 + \cos \theta}{1 - \cos \theta}\right) = n \cot^2 \left(\frac{\theta}{2}\right)$$

Assuming $n = 15$

$$\frac{f_s}{f_c} = 15 \left(\frac{1 + \cos \theta}{1 - \cos \theta}\right) = 15 \cot^2 \left(\frac{\theta}{2}\right)$$

(See Fig. 14 for a curve showing the relation between the ratio $\frac{f_s}{f_c}$ and the position of the neutral axis.) Substituting the value for f_s in the equation for T, that equation becomes

$$T = 30f_cprt\left(\frac{\sin\theta + (\pi - \theta)\cos\theta}{1 - \cos\theta}\right)$$

The distance from the diameter to the point at which the resultant of the total compressive stress acts may, by means of calculus, be shown to be

$$a = \frac{r}{2}\left[\frac{\theta - \frac{1}{2}\sin 2\theta}{\sin\theta - \theta\cos\theta}\right]$$

In like manner, the distance from the diameter that the resultant of the total tensile stress acts may be shown to be

$$b = \frac{r}{2}\left[\frac{(\pi - \theta) + \frac{1}{2}\sin 2\theta}{\sin\theta + (\pi - \theta)\cos\theta}\right]$$

The moment of the total compressive stress about the diameter equals

$$Ca = 2f_crt\left(\frac{\sin\theta - \theta\cos\theta}{1 - \cos\theta}\right)\frac{r}{2}\left(\frac{\theta - \frac{1}{2}\sin 2\theta}{\sin\theta - \theta\cos\theta}\right)$$
$$= f_cr^2t\left(\frac{\theta - \frac{1}{2}\sin 2\theta}{1 - \cos\theta}\right)$$

The moment of the total tension stress about the diameter equals

$$Tb = 30f_cprt\left(\frac{\sin\theta + (\pi - \theta)\cos\theta}{1 - \cos\theta}\right)\frac{r}{2}\left(\frac{(\pi - \theta) + \frac{1}{2}\sin 2\theta)}{\sin\theta + (\pi - \theta)\cos\theta}\right)$$
$$= 15f_cpr^2t\left(\frac{(\pi - \theta) + \frac{1}{2}\sin 2\theta}{1 - \cos\theta}\right)$$

Returning to the two fundamental equations and substituting values

$$W = C - T = 2f_crt\left(\frac{\sin\theta - \theta\cos\theta}{1 - \cos\theta}\right) - 30f_cprt\left(\frac{\sin\theta + (\pi - \theta)\cos\theta}{1 - \cos\theta}\right)$$

$$M = We = Ca + Tb = f_cr^2t\left(\frac{\theta - \frac{1}{2}\sin 2\theta}{1 - \cos\theta}\right) + 15f_cpr^2t\left(\frac{(\pi - \theta) + \frac{1}{2}\sin 2\theta}{1 - \cos\theta}\right)$$

Values for θ, k, C, Ca, T, Tb, and $\frac{f_s}{f_c}$ may be computed for different values of θ and tabulated. The angle θ will probably never be less than 60 deg. (compression area equal to one-third of the total area) in a rational design, and will be equal to 180 deg. when the unit tension stress (f_s) is zero. When computing values, careful attention must be paid to the algebraic signs of the trigonometric functions.

These two fundamental equations, in their present forms, are not convenient for use in the design of conical reinforced concrete shafts. However, by plotting a series of curves for some of the quantities, it is possible to reduce the laborious part of the computations for the design very greatly.

In the equations for C, T, Ca, and Tb, let

$$k_c = 2\left(\frac{\sin\theta - \theta\cos\theta}{1 - \cos\theta}\right)$$

$$k_t = 30\left(\frac{\sin\theta + (\pi - \theta)\cos\theta}{1 - \cos\theta}\right)$$

$$K_c = \left(\frac{\theta - \frac{1}{2}\sin 2\theta}{1 - \cos\theta}\right)$$

$$K_t = 15\left(\frac{(\pi - \theta) + \frac{1}{2}\sin 2\theta}{1 - \cos\theta}\right)$$

Then the fundamental equations become

$$W = k_c f_c rt - k_t p f_c rt$$

$$M = We = K_c f_c tr^2 + K_t p f_c tr^2$$

Dividing the last equation by the one preceding and reducing

$$\frac{e}{r} = \frac{K_c + pK_t}{k_c - pk_t}$$

Now, as all of the values of k_c, k_t, K_c, and K_t are functions of θ and as $k = 1 - \cos\theta$, a set of curves may be plotted showing the relation of $\frac{e}{r}$, k, and p by assuming values for p, using the tabulated values for k_c, k_t, K_c, and K_t, and computing corresponding values of $\frac{e}{r}$ (see Fig. 15). From these curves, and knowing $\frac{e}{r}$ and p, the position of the neutral axis can be readily determined.

The second fundamental (moment) equation may be written in the form:

$$\frac{W}{rt} = \frac{I}{e} f_c(K_c + K_t p)$$

For any position of the neutral axis there are certain definite values of K_c, $K_t p$, and $\frac{e}{r}$. Also, $\frac{e}{r}$ depends on p as well as k, as is shown in Fig. 15. Assuming values for f_c, p, and k, values may be found from the tables and curves already prepared for $\frac{e}{r}$ and $(K_c + K_t p)$. Then, by substituting in the equation, corresponding values of $\frac{W}{rt}$ may be computed. From the results of these various computations, curves may be plotted which will show the relation between $\frac{W}{rt}$, $\frac{e}{r}$, f_c, and p. Figures 16 to 25 inclusive show these relations for values of f_c equal to 200, 250, 300, 350, 400, 450, 500, 550, and 600 lb. per sq. in. and for percentages of reinforcing steel equal to 0, ¼, ½, 1, 1½, 2, 2½, 3, 3½, and 4 per cent. The curves for zero percentage of steel are really limiting values as some reinforcing steel should be used whenever the ratio $\frac{e}{r}$ exceeds 0.500, if the concrete is to take no tension.

During the derivation of the formulas, f_c has been taken as the unit compressive stress in the concrete at a distance r from the center of the shaft. The maximum

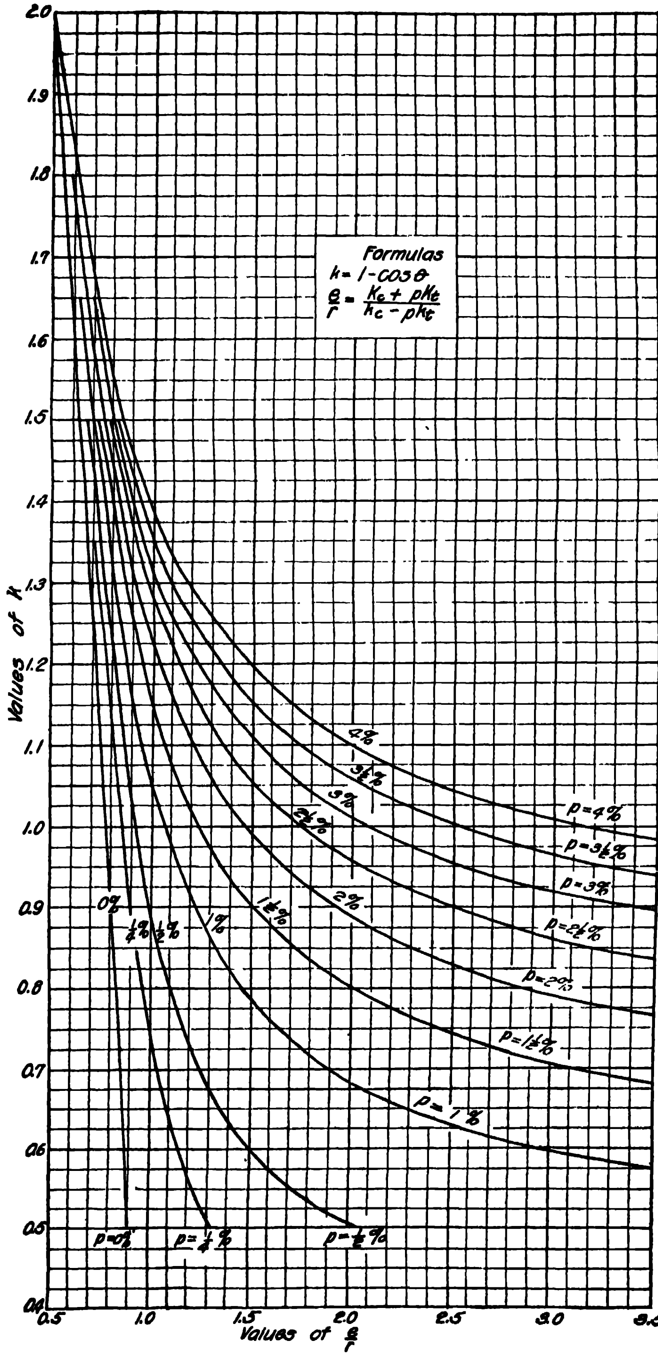


FIG. 15.—Relation of ratio $\frac{e}{r}$ to k for different values of p.

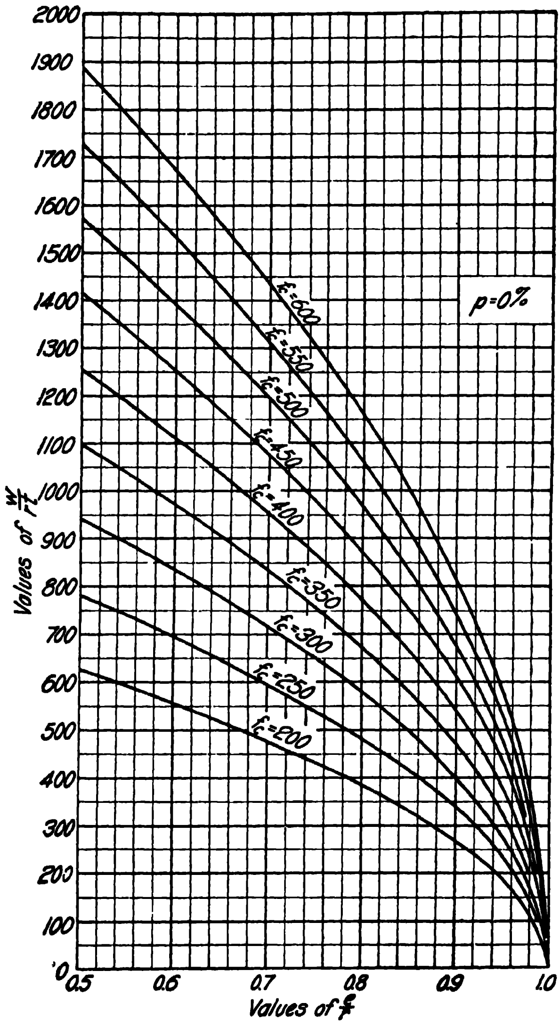


FIG. 16.—Relation between $\frac{W}{rt}$ and $\frac{e}{r}$ for p of 0%.

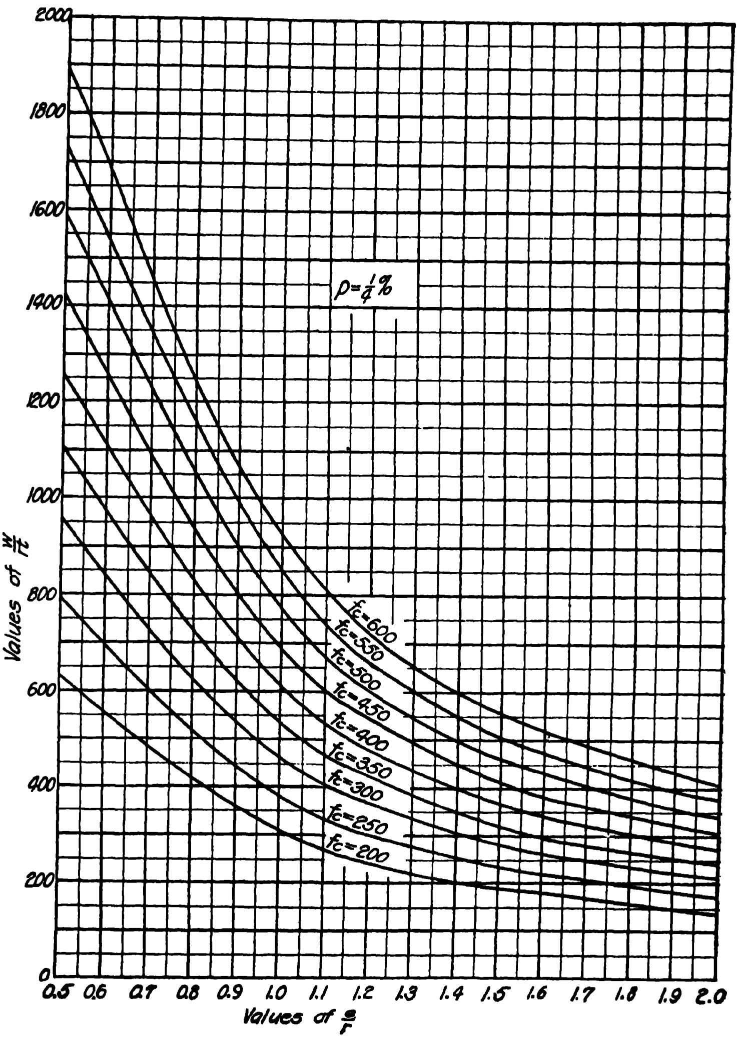


FIG. 17.—Relation between $\frac{W}{rt}$ and $\frac{e}{r}$ for p of $\frac{1}{4}$%.

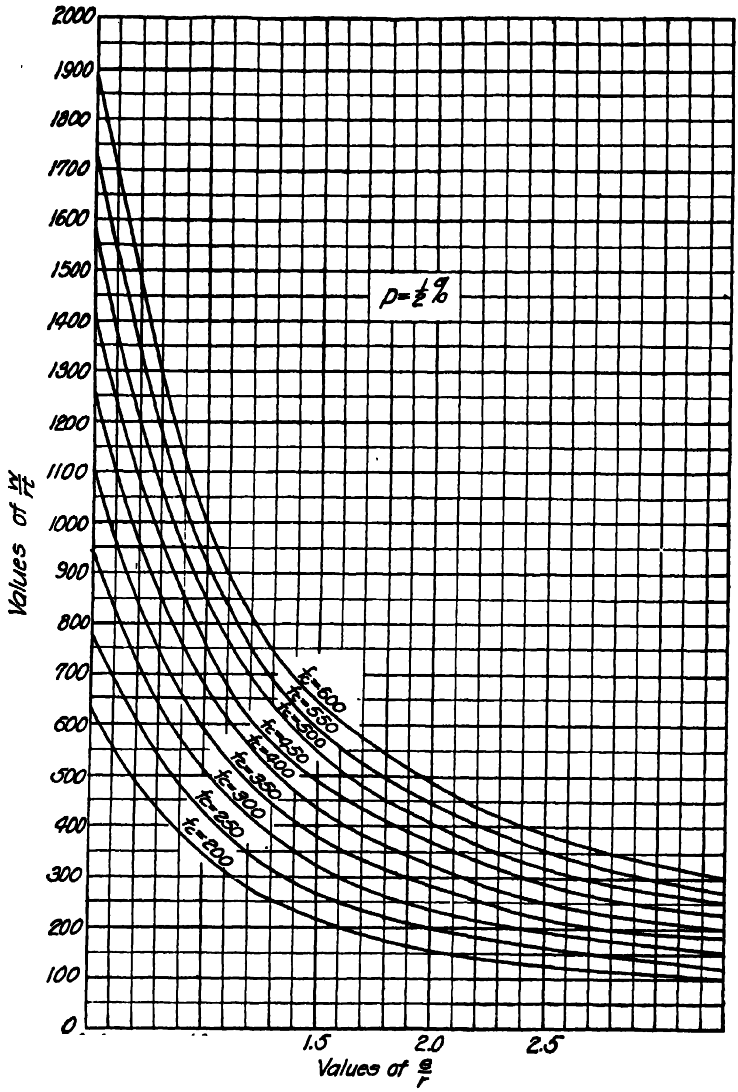


FIG. 18.—Relation between $\frac{W}{rt}$ and $\frac{e}{r}$ for p of ½%.

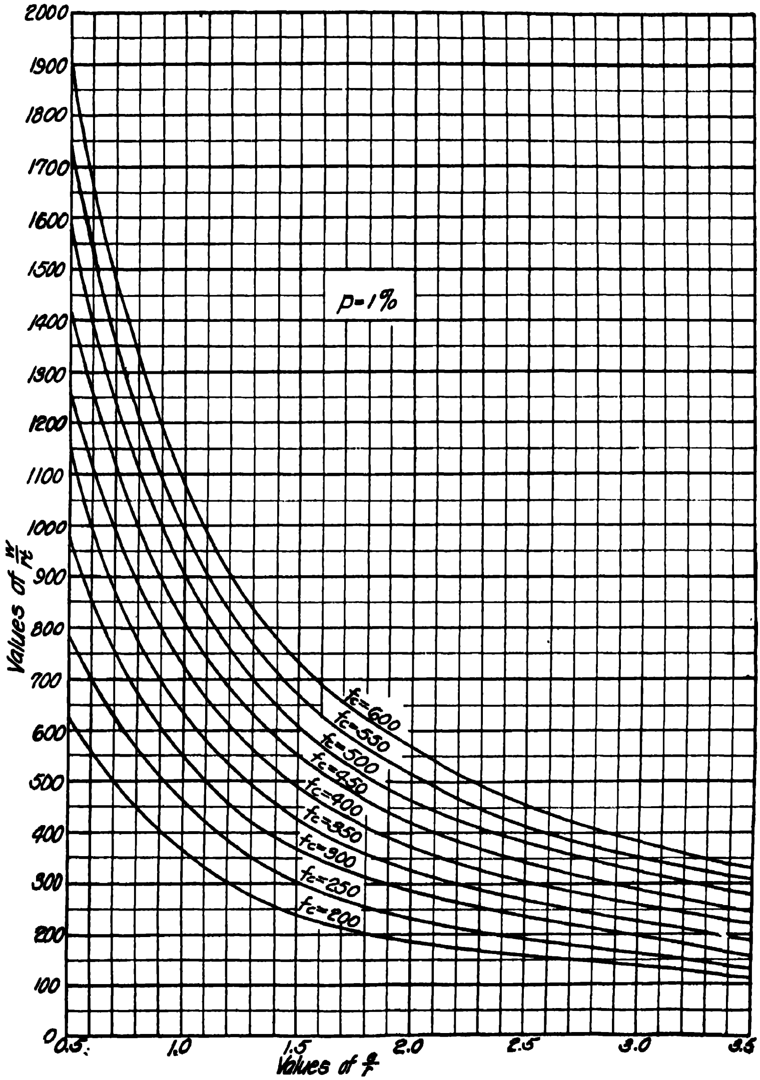


Fig. 19.—Relation of $\frac{W}{rt}$ to $\frac{e}{r}$ for p of 1%.

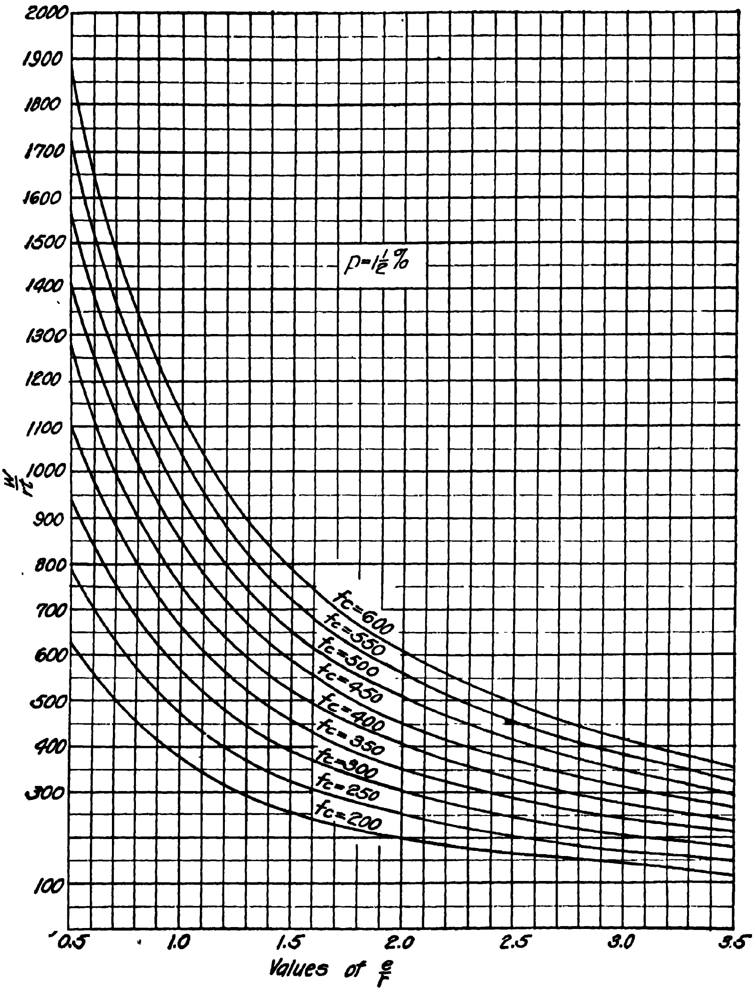


FIG. 20.—Relation of $\frac{W}{rt}$ to $\frac{e}{r}$ for p of $1\frac{1}{2}$ %.

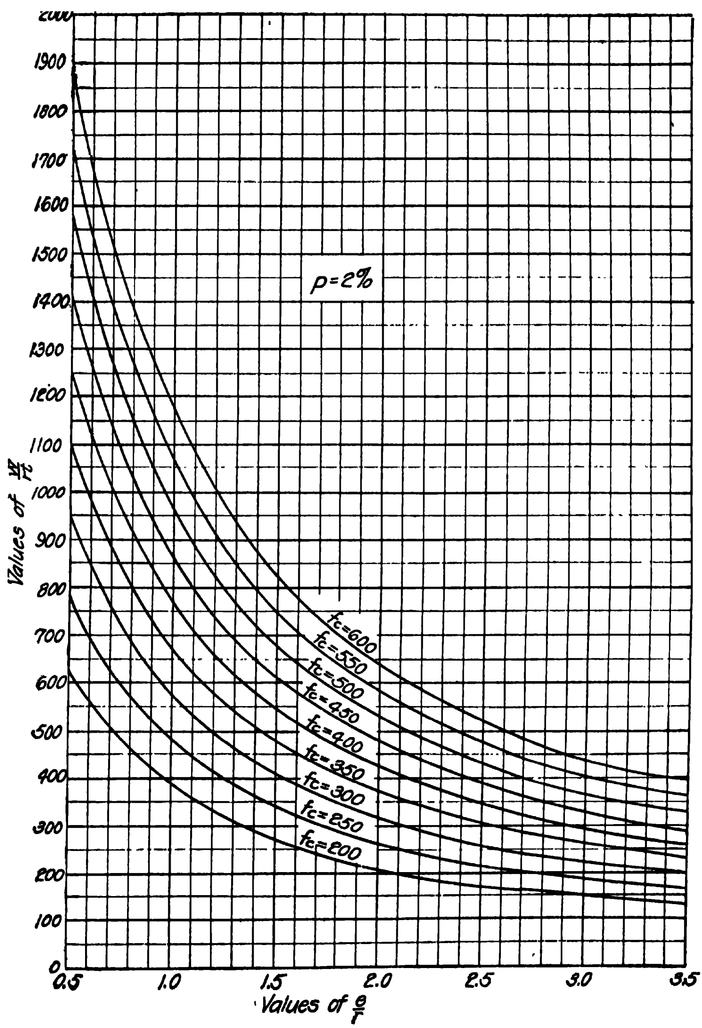


FIG 21.—Relation between $\frac{W}{rt}$ and $\frac{e}{r}$ for p of 2 %.

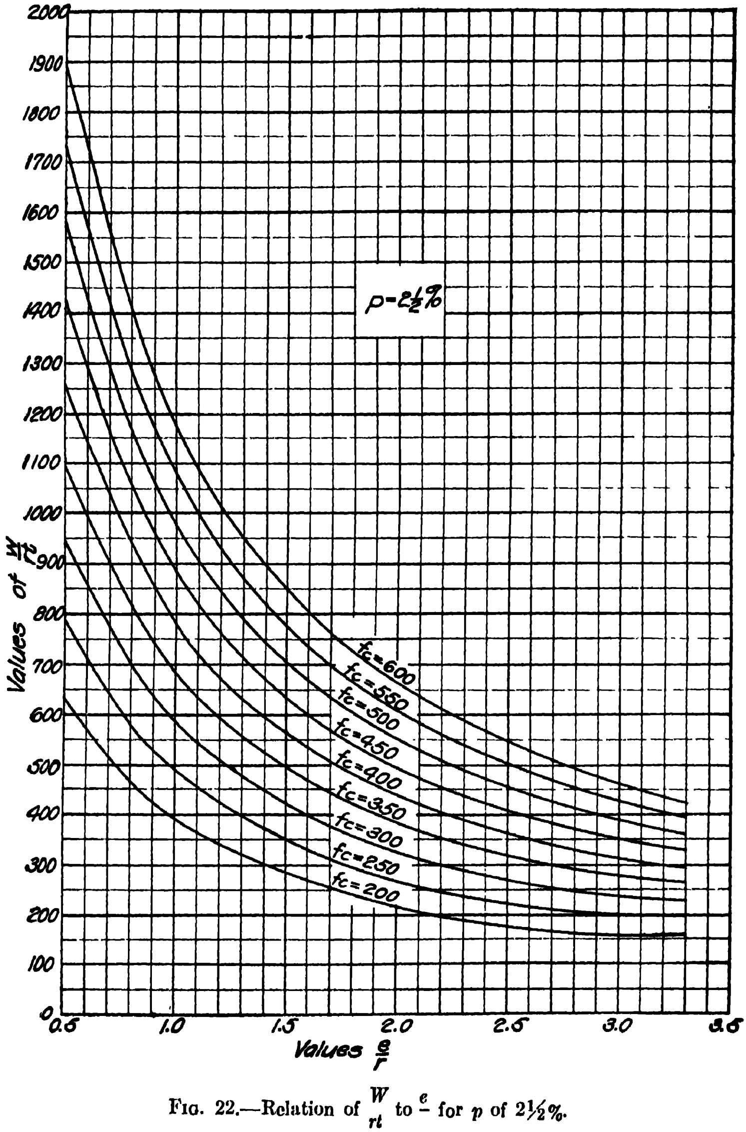


FIG. 22.—Relation of $\frac{W}{rt}$ to $\frac{e}{r}$ for p of $2\frac{1}{2}\%$.

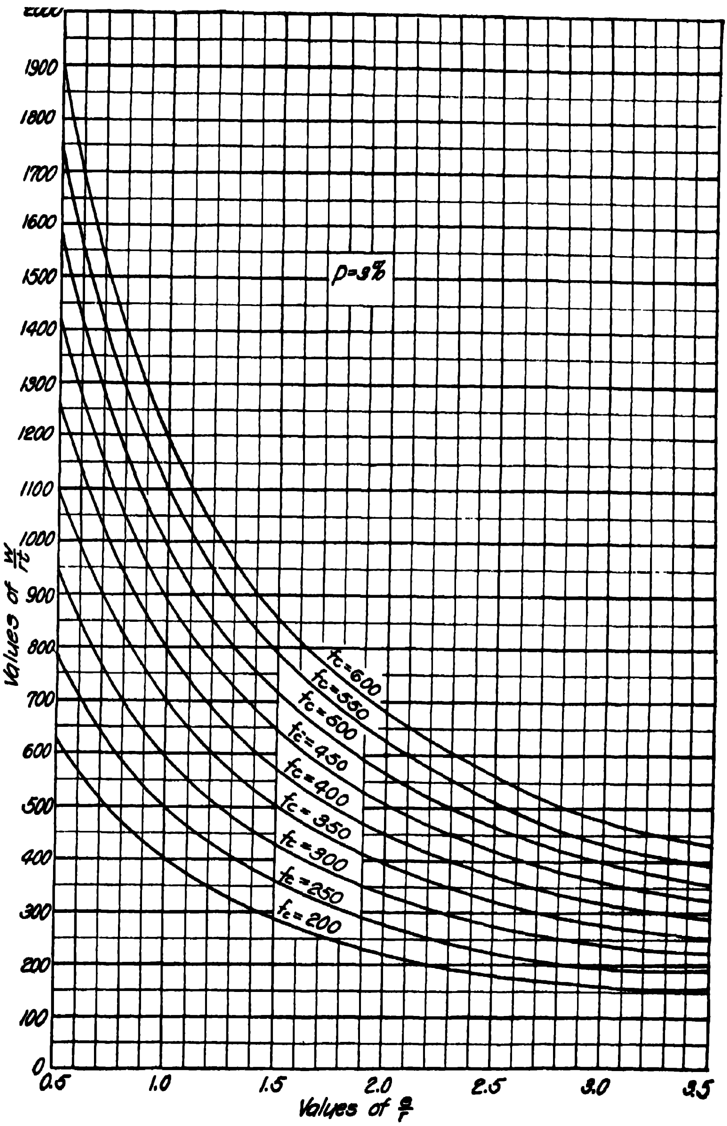


FIG. 23.—Relation of $\frac{W}{rt}$ to $\frac{e}{r}$ for p of 3%.

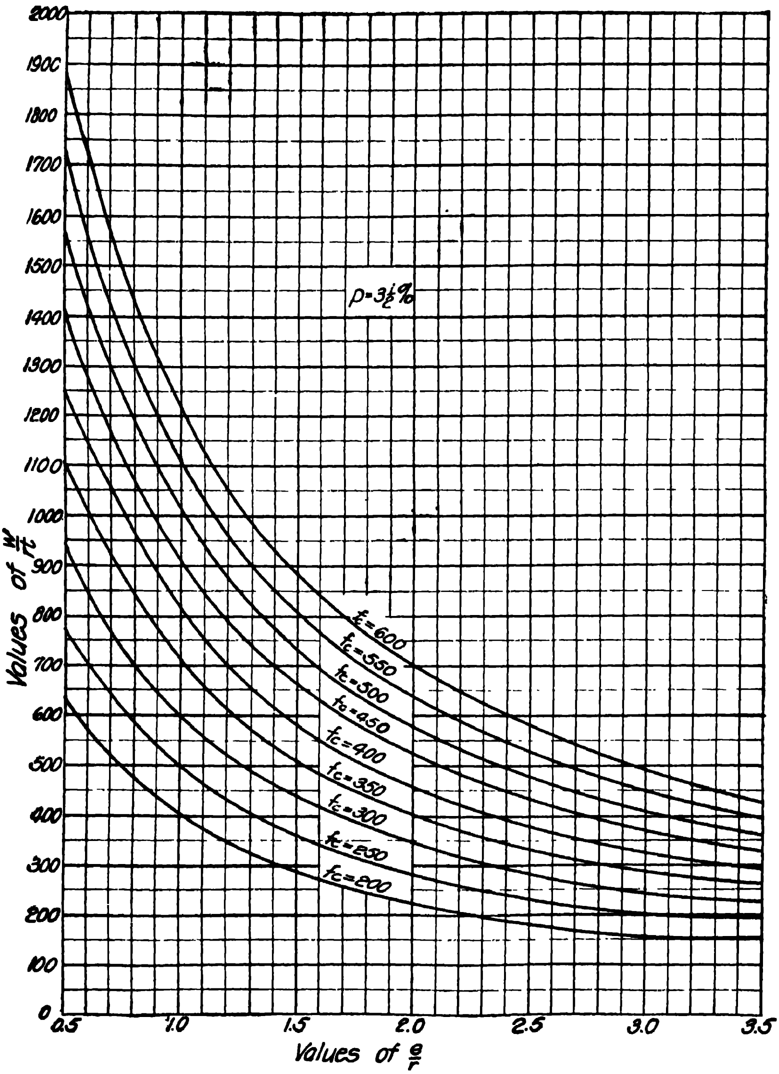


FIG. 24.—Relation of $\frac{W}{rt}$ to $\frac{e}{r}$ for p of $3\frac{1}{2}$

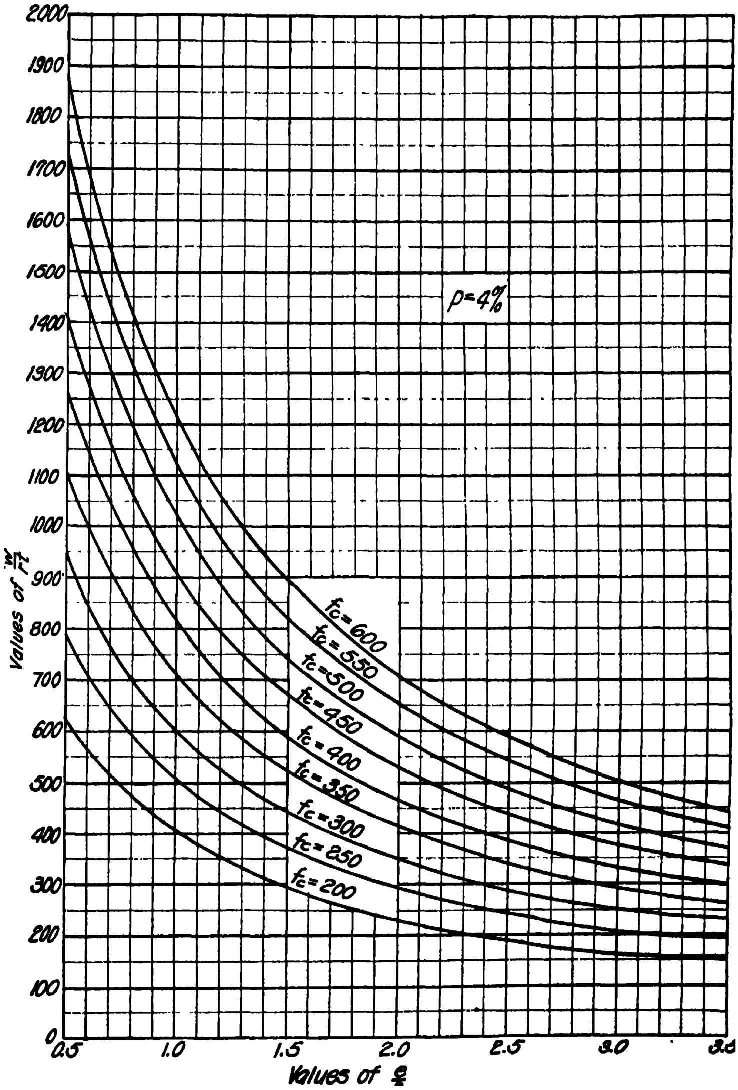


FIG. 25. Relation between $\frac{W}{rt}$ and $\frac{c}{r}$ for p of 4 %.

unit compressive stress in the concrete is in the extreme fiber which is equal to a distance $r + \frac{t}{2}$ from the center of the shaft. The relation between $f_{c(\max.)}$ and f_c may be deduced from Fig. 12. By means of similar triangles it is evident that

$$\frac{f_{c(\max.)}}{f_c} = \frac{r(1 - \cos\theta) + \frac{t}{2}}{r(1 - \cos\theta)}$$

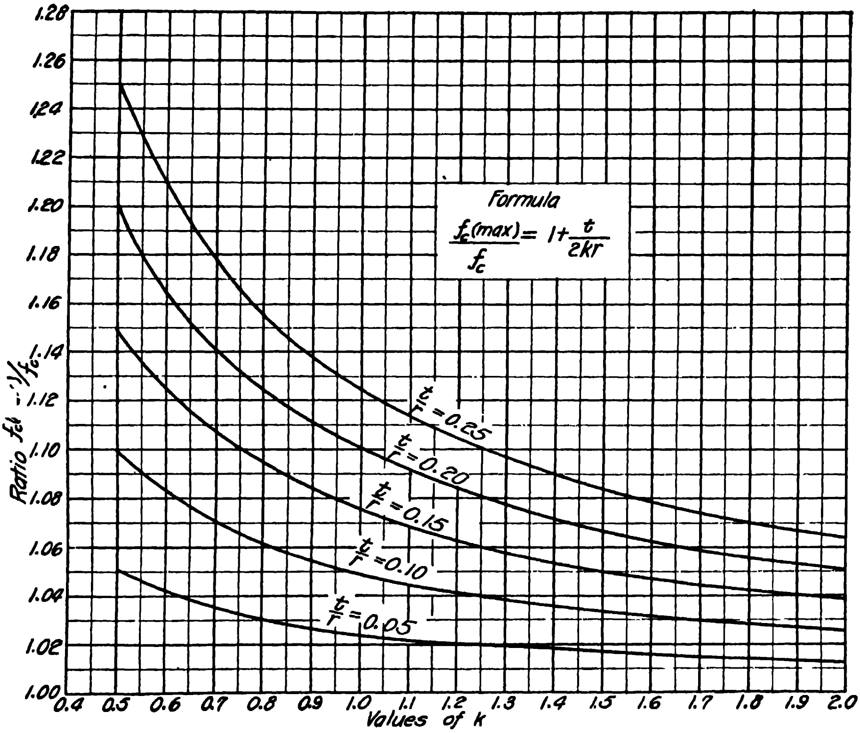


FIG. 26.—Relation between k and ratio of $f_{c(\max.)}$ to f_c.

As $k = 1 - \cos\theta$,

$$\frac{f_{c(\max.)}}{f_c} = \frac{rk + \frac{t}{2}}{rk} = 1 + \frac{t}{2rk}$$

$$f_{c(\max.)} = f_c\left(1 + \frac{t}{2rk}\right)$$

Assuming values for $\frac{t}{r}$ and k, values for the ratio $\frac{f_{c(\max.)}}{f_c}$ may be easily obtained. Plotting these values as in Fig. 26 gives curves from which the ratio $\frac{f_{c(\max.)}}{f_c}$ may be found for any particular design. Then $f_{c(\max.)}$ may be computed.

The method of procedure in the design of a conical reinforced concrete chimney shaft, as far as the direct stresses caused by the wind and weight are concerned, is as follows:

Have given d_1, t_1, h, and allowable $f_{c(\max.)}$.
Assume r and t.
Obtain W from curves of Fig. 9.
Compute $\frac{W}{rt}$.
Obtain M from curves of Fig. 10.
Compute ratio $\frac{e}{r}$.
Obtain p from curves in Figs. 16 to 25 inclusive, making an allowance of about 20 lb. per sq. in. for the difference between $f_{c(\max.)}$ and f_c.
Obtain k from Fig. 15.
Compute value of $\frac{t}{r}$.
Obtain $f_{c(\max.)}$ from curves of Fig. 26. If this does not agree closely with assumed value, change f_c a little and obtain new values for p, k, and $f_{c(\max.)}$.
Obtain f_s from Fig. 14.
Select number and size of reinforcing rods.

Illustrative Example.—Given a conical reinforced concrete shaft 160 ft. high. Inside top diameter equals 5 ft. and top wall thickness equals 4 in. The outside diameter at the base (wall batter 1/100) equals 8.87 ft. (106.5 in.), and the wall thickness at the base equals (increasing 1 in. for every 20 ft.) 4 + 8 or 12 in. The percentage of vertical reinforcement steel is 1½ per cent. Find the maximum unit stresses in the concrete and the steel due to wind and weight, assuming a wind pressure of 25 lb. per sq. ft. of vertical projected area.

$r = 47.25$ in.	$t = 12$ in.	$D = 106.5$ in.
$r_1 = 32$ in.	$t_1 = 4$ in.	$D_1 = 68$ in.

$rt = 567$

$W = 2{,}110 \times 160 = 337{,}600$ lb. (See Fig. 9)

$M = 1{,}010 \times 160^2 = 25{,}856{,}000$ in.-lb. (See Fig. 10)

$$\frac{W}{rt} = \frac{337{,}600}{567} = 595.5$$

$$\frac{e}{r} = \frac{M}{Wr} = \frac{25{,}856{,}000}{(337{,}600)(47.25)} = 1.62$$

$p = 1\frac{1}{2}$ per cent $= 0.015$

$f_c = 485$ lb. per sq. in. (From Fig. 20)

$k = 0.877$ (From Fig. 15)

$$\frac{t}{r} = \frac{12}{47.25} = 0.254$$

$f_{c(\max.)} = (485)(1.144) = 555$ lb. per sq. in. (See Fig. 26)

$f_s = (485)(19.4) = 9{,}410$ lb. per sq. in. (See Fig. 14)

44c. Stress Due to Diagonal Tension.—The stresses due to diagonal tension or shear in a conical reinforced concrete shaft may be found by the following formula for a wind pressure of 25 lb. per sq. ft.

$$v = \left(\frac{D + D_1}{3rt}\right)h$$

where

v = unit shearing stress in pounds per square inch.
D = outside diameter of base (or section considered) in inches.
D_1 = outside diameter of top of shaft in inches.
h = height of shaft above base (or above section considered) in feet.
r = average radius of section in inches.
t = thickness of wall in inches at section considered.

This formula is readily derived from the shear formula

$$v = \frac{V}{bjd}$$

where

Total vertical shear $V = \frac{p_w}{24}(D + D_1)h = \frac{25}{24}(D + D_1)h$ assuming a wind pressure of 25 lb. per sq. ft.

Breadth $b = 2t$

$jd = 1.56r$ (average value)

The value for jd may vary from $1.5708r$ to $1.5000r$ depending on the location of the neutral axis. A fair average value for most positions of the neutral axis is $1.56r$.

$$v = \frac{p_w(D + D_1)h}{24 \times 2 \times 1.56rt} = p_w\frac{(D + D_1)h}{75rt}$$

$$= \frac{(D + D_1)h}{3rt} \text{ for a wind pressure of 25 lb. per sq. ft.}$$

The proportion of steel required is

$$p = \frac{p_w(D + D_1)h}{75rtf_s}$$

where

f_s = unit tensile stress in steel in pounds per square inch.

For a wind pressure of 25 lb. per sq. ft., the proportion of steel required is

$$p = \frac{(D + D_1)h}{3rtf_s}$$

The amount of steel or steel area in square inches required in a wall section 1 ft. high is

$$A_s = \frac{(D + D_1)h}{3rtf_s}(12t) = \frac{4(D + D_1)h}{rf_s}$$

The value of f_s is usually taken as 16,000 or 18,000 lb. per sq. in. depending on whether round rods of open hearth steel or wire mesh of cold drawn steel wire is used.

On account of the possibility of small cracks due to temperature stresses, it is common to provide enough steel to take all of the diagonal tension (shear) stresses. Wire mesh or closely spaced small rods are better than larger rods spaced farther apart. About 6 in. is thought to be the maximum spacing desirable.

44*d*. Horizontal Temperature Stresses.—In deriving formulas for the horizontal temperature stress reinforcement it will be assumed that the horizontal temperature drop through a concrete wall is a straight line, and also that there is an abrupt drop in temperature in passing from the hot gases to the wall and another abrupt drop in passing from the wall to the outer air. These assumptions have been verified by experimental data. It is thought that the difference in temperature between the inside and outside of the chimney wall will not be more than 50 or 60 per cent of the difference in temperature between the hot gases in the shaft and the outside air.

When a concrete chimney is in use, the inside is hotter than the outside and, consequently, the greater expansion of the hotter concrete will tend to cause

both horizontal and vertical cracks in the outside of the shaft walls. These cracks may be kept small and localized by proper systems of horizontal and vertical reinforcement.

As the concrete in the inner part of the shaft wall expands more than that in the outer part, the result will be to cause compression in the inner part of the wall and tension in the outer, and the total compression, C, will equal the total tension, T. But, as the concrete is assumed to carry no tension, steel must be provided to carry this tension.

Somewhere between the compression and the tension there will be a plane of zero stress, or a neutral surface, as is illustrated in Fig. 27. The actual variation in unit compression stress, f_c, is a curved line due to the curvature of the shaft wall. The variation of the curved line from a straight line depends on the ratio of $\frac{r_1 + \Delta r_1}{r_1}$, as Δr_1 varies from zero to kt'. This ratio is very nearly equal to unity in most chimneys and, consequently, the curve tends to approach a straight line.

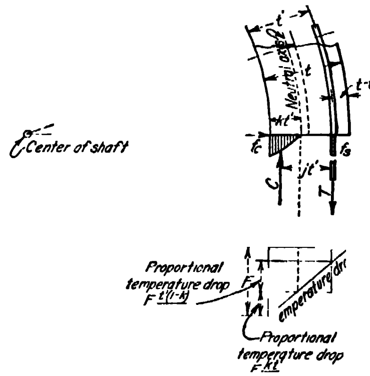


FIG. 27.—Temperature stresses in a horizontal section of a concrete chimney shaft.

Assuming a straight line variation of stress and a constant modulus of elasticity, E_c, in compression for the concrete for the range of temperatures ordinarily encountered, it is seen that a very short section of the wall is like a reinforced concrete beam of depth t' to the steel.

The unit compression stress in the concrete at the inner surface is

$$f_c = 0.000006E_cF\frac{kt'}{t}$$

and, assuming a value of 2,000,000 lb. per sq. in. for E_c,

$$f_c = 12F\,kt'$$

The unit tensile stress in the steel is

$$f_s = 0.000006\,E_sF\frac{t'(1-k)}{t}$$

Substituting 30,000,000 lb. per sq. in. for E_s,

$$f_s = 180F\frac{t'(1-k)}{t}$$

where

f_c = unit compression stress in concrete in pounds per square inch.
f_s = unit tensile stress in steel in pounds per square inch.
0.000006 = coefficient of expansion for 1 deg. F., assumed the same for concrete and steel.
F = temperature drop across wall in degrees Fahrenheit.
t' = distance in inches from inner wall surface to reinforcing steel.
kt' = distance from inner wall surface to neutral surface.
t = thickness of wall in inches.

The total compression equals

$$C = \frac{12Fkt'}{t} \times \frac{kt'}{2}$$

assuming a straight line variation of the compression stress.

The total tension equals

$$T = \frac{180Ft'(1 - k)pt'}{t}$$

where p is the proportion of steel. (Note that the percentage of steel for horizontal temperature reinforcement is based on t' and not on t.)

Equating $C = T$ and solving,

$$k^2 = 30(1 - k)p$$

As $30 = 2n$, this equation is the same as that for k in the straight line beam formulas. Figure 28 shows the relation between values of k and corresponding proportion of steel.

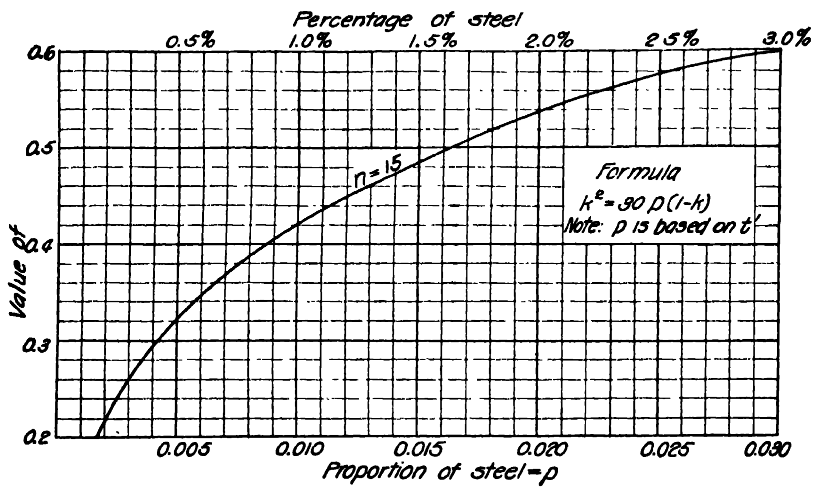


FIG. 28.—Relation between k and corresponding proportion of steel.

In designing a reinforced concrete chimney, a lining is usually provided when the temperature of the flue gases is 800 deg. F. or more. Assuming that the temperature of the gases is 700 deg. F. at the inner wall of the shaft near the flue opening and that the drop in the shaft wall is 50 per cent of this, provision would have to be made for the stresses caused by the drop of 350 deg. across the wall. The temperature drop across the wall will usually be found to decrease as the distance up the stack away from the flue opening increases due to the decrease in temperature of the gases. When the chimney is lined, the temperature drop across the shaft wall is much less as there are drops at the inner and outer surfaces of the lining and shaft wall and also across the lining, the air space, and the shaft wall. Probably a drop of 150 to 250 deg. would be about right for flue gas temperature of 1,200 deg. F. or less.

44e. Vertical Temperature Stresses.—Considering a temperature drop in a vertical section of the shaft as shown in Fig. 29, it is evident that, approximately,

$$f_c = 0.000006E_cF\frac{kt'}{t} = 12F\frac{kt'}{t}$$

and

$$f_s = 0.000006E_sF\frac{t'(1-k)}{t} = 180F\frac{t'(1-k)}{t}$$

Now, if the proportion of steel, p, is based on the distance t' instead of t, the relation of k, p, and n will be the same as that in a simple reinforced concrete beam using the straight line formula. Hence, assuming n as 15, Fig. 28 may be used for locating k for varying proportions of p.

44f. Stresses Due to Earthquakes.—Reinforced concrete chimney shafts may be reinforced against possible earthquake shocks by the addition of reinforcement. As the unit stress in the vertical steel is usually quite a little less than 16,000 lb. per sq. in., it is doubtful in most instances if extra reinforcing steel will be required except at the most dangerous section. This section is approximately at two-thirds the height, though the extra reinforcement may extend from about one-half to three-quarters of the height.

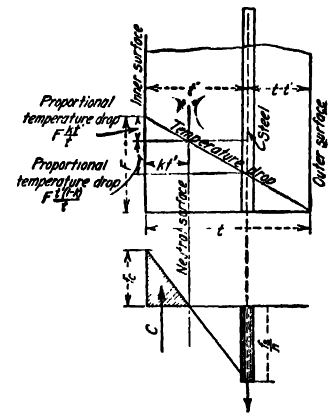


Fig. 29.—Temperature stresses in a vertical section of a concrete chimney shaft.

To obtain the amount of steel necessary and the unit stresses in the concrete and steel, the earthquake moment may be computed by the formula given in Art. **9** and the percentage of steel and the unit stresses in steel and concrete may be determined as outlined in Arts. **44a** and **44b**. The sum of the unit stresses due to weight, wind moment, and earthquake moment should not exceed about 600 or 700 lb. per sq. in. compression in the concrete and 18,000 or 20,000 lb. per sq. in. tension in the steel. These higher values are permissible due to the probable rare occurrence of severe earthquake shocks even in localities where earthquake shocks are common.

45. Reinforced Concrete Chimney Linings.—A reinforced concrete chimney should be provided with an independent lining built up from the base to a height equal to one-third or more of the height of the chimney, according to the opinion of most engineers. Present practice seems to require a lining from one-third to one-half the chimney height for initial flue gas temperatures of about 800 deg. F. or less, and a lining height equal to about three-fourths that of the chimney for initial flue gas temperatures of 800 to 1,200 deg. F., and chimneys should be fully lined for initial flue gas temperatures over 1,200 deg. F. The presence of the lining keeps the hot gases away from the shaft wall and also lowers the temperature drop across the shaft wall which is a decided advantage as this decreases the temperature stresses.

For the lower temperatures, a reinforced concrete lining is usually satisfactory. Such linings are often from 4 to 6 in. in thickness and reinforced with hooping

and vertical steel. The hooping reinforcement may vary from 1/6 to 1/2 per cent and the vertical reinforcement from 1/2 to 2 per cent depending an conditions and the opinion of the designer. There seems to be no generally accepted method for the design of reinforced concrete linings.

For higher temperatures, a firebrick lining is usually required. See Art. **10** on Linings for further discussion.

46. Foundation Design Formulas.

46*a*. Compression Over All of Base.—Foundations for reinforced concrete chimneys are usually flat slabs—round, square, or octagonal in shape. A round or octagonal shape is to be preferred to that of a square because of the probable high unit stresses in the corners of the square. The slab is made thick enough so that the unit stresses in the concrete do not exceed the permitted values. The reinforcement at the bottom of the slab is usually placed in four layers. Practically no reinforcement is needed in the top of the slab, except in cases where there is no wind and the inner diameter of the base of the chimney is greater than twice the projection of the slab from the shaft wall. The slab must be large enough so that the pressures due to the weight and the wind are safely transmitted to the earth, and also so that the chimney will not overturn when subjected to high wind pressures. Chimneys are often placed on yielding soil, and in these cases the foundations must be carefully designed.

In designing foundations where all of the base is subjected to compression the following formulas may be used for determining the compressive stresses on the soil in pounds per square inch:

For round foundations:

Maximum unit stress (lb. per sq. in.) $= \dfrac{W_s + W_l + W_f}{0.7854D_f^2} + \dfrac{M_f}{0.098D_f^3}$

Minimum unit stress (lb. per sq. in.) $= \dfrac{W_s + W_l + W_f}{0.7854D_f^2} - \dfrac{M_f}{0.098D_f^3}$

Maximum eccentricity (inches) $= 0.125D_f = 0.250R$

where W_s, W_l, and W_f are the weights in pounds of the shaft, lining, and foundation respectively; D_f is the diameter in inches of the base of the foundation; R is half of D_f; and M_f is the wind moment in inch-pounds about the foundation base.

When the resultant of the weights and wind cuts the base at a distance 0.125 D_f from the center, the minimum unit stress is zero and occurs on the windward edge.

For octagonal foundations with the wind blowing perpendicular to a side:

Maximum unit stress (lb. per sq. in.) $\dfrac{W_s + W_l + W_f}{0.828D_f^2} + \dfrac{M_f}{0.109D_f^3}$

Minimum unit stress (lb. per sq. in.) $\dfrac{W_s + W_l + W_f}{0.828D_f^2} - \dfrac{M_f}{0.109D_f^3}$

Maximum eccentricity (inches) $0.132D_f$

where D_f is distance in inches between two parallel sides.

For octagonal foundations with the wind blowing along a diagonal:

$$\text{Maximum unit stress (lb. per sq. in.)} = \frac{W_s + W_l + W_f}{0.828D_f^2} + \frac{M_f}{0.102D_f^3}$$

$$\text{Minimum unit stress (lb. per sq. in.)} = \frac{W_s + W_l + W_f}{0.828D_f^2} - \frac{M_f}{0.102D_f^3}$$

$$\text{Maximum eccentricity (inches)} = 0.122D_f$$

where D_f is distance in inches between two parallel sides. The diagonal of an octagon equals 1.082 D_f.

For square foundations with the wind blowing perpendicular to a side:

$$\text{Maximum unit stress (lb. per sq. in.)} = \frac{W_s + W_l + W_f}{D_f^2} + \frac{6M_f}{D_f^3}$$

$$\text{Minimum unit stress (lb. per sq. in.)} = \frac{W_s + W_l + W_f}{D_f^2} + \frac{6M_f}{D_f^3}$$

$$\text{Maximum eccentricity (inches)} = 0.167D_f$$

where D_f is distance in inches between two parallel sides of the square.

For square foundations with the wind blowing along a diagonal:

$$\text{Maximum unit stress (lb. per sq. in.)} = \frac{W_s + W_l + W_f}{D_f^2} + \frac{M_f}{0.118D_f^3}$$

$$\text{Minimum unit stress (lb. per sq. in.)} = \frac{W_s + W_l + W_f}{D_f^2} - \frac{M_f}{0.118D_f^3}$$

$$\text{Maximum eccentricity (inches)} = 0.118D_f$$

where D_f is the side of the square in inches.

The diagonal of a square $= 1.414D_f$.

After the dimensions of the foundation have been selected so that the allowable soil pressure will not be exceeded and so that there will be compression over all of the base, the foundations may be designed for moment and shear according to the methods outlined in Art. 24, Sec. 5. The maximum positive moment in the bottom of the foundation slab will occur when the wind is blowing, while the maximum negative moment at the top of the foundation slab will occur with no wind. The slab should be investigated in regard to punching shear. Stirrups should be provided for diagonal tension when needed.

Great care must be taken to secure a good bond between the foundation slab and the shaft so that they will act as a single monolithic structure. The vertical steel in the shaft is usually carried down and hooked under the lower foundation reinforcement.

46b. Compression over Part of Base.—When the eccentricity is greater than that which will just cause zero stress on the windward side of the base of the foundation, there would tend to be tension near this side. However, as there cannot be any tension between the foundation and the earth, part of the foundation will be considered to be under no stress and the other part under compression stress. Knowing that the unit compression stress increases uniformly from zero to a maximum value at the leeward edge, that the average unit compression stress occurs at the center of gravity of the compression area, that the resultant of all of the compressive stress equals the weight of the shaft, lining,

and foundation, and that this stress resultant cuts the base at the same point that the resultant of the weights and wind moment does, it is possible, by assuming

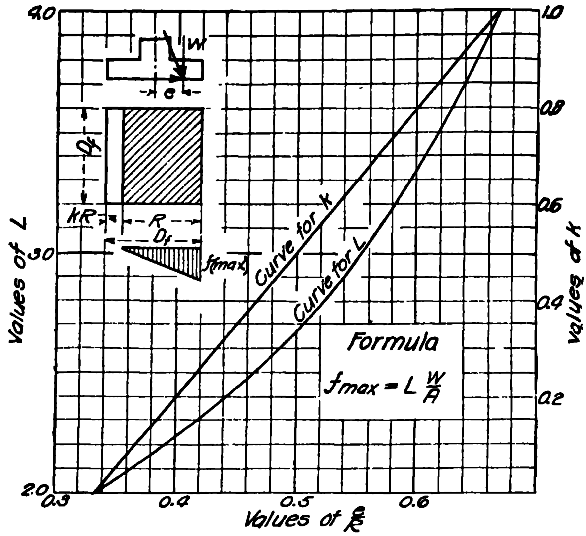


FIG. 30.—Curves for k and L for square base with wind perpendicular to side.

different compression areas, to compute values and plot curves so that the maximum stress on the leeward side may be easily found. The computations are fairly

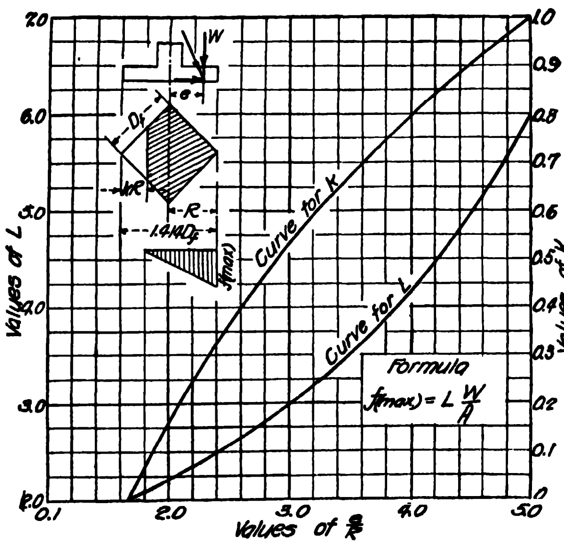


FIG. 31.—Curves for k and L for square base with wind along diagonal.

simple in the case of a square foundation with the wind perpendicular to a side and a little more difficult in the cases of a circle and of a square with the wind blowing

along a diagonal. For an octagonal foundation, the results are approximately the same as those of a circle having a diameter equal to the average of the diagonal of the octagon and the perpendicular distance between two parallel sides.

Curves illustrating the results of these computations have been plotted in Figs. 30, 31, and 32. The maximum unit compressive stress on the base of the foundation for any eccentricity, e, is given by the formula

$$f(\text{max.}) = L\frac{W}{A}$$

where

$f(\text{max.})$ = maximum unit compressive stress in pounds per square inch.
L = value taken from curve.
$W = (W_s + W_l + W_f)$ in pounds.
A = total area of foundation base in square inches.

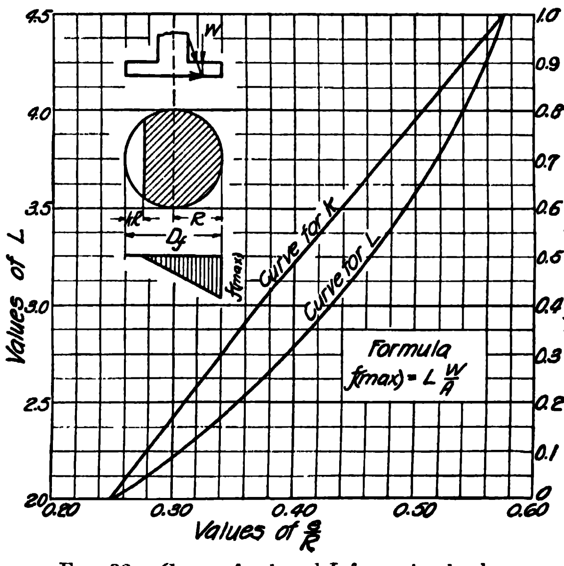


FIG. 32.—Curves for k and L for a circular base.

After the correct size of the foundation has been selected, the foundation may be designed for moment and shear by the methods given in the preceding article and in Art. 24, Sec. 5. Diagonal tension reinforcement should be provided when needed. The allowable unit stress in punching shear should not be exceeded.

47. Method of Procedure in Design of Reinforced Concrete Chimneys.—The general procedure in the design of a conical reinforced concrete chimney is approximately as follows, having given the height above foundations, the inside top diameter, elevation of breech opening, height of lining.

(*a*) Select wall thickness at top, wall batter, and rate of increase in wall thickness of shaft.
(*b*) Compute weights and wind moments at base section and at other sections of the shaft spaced about 20 ft. apart.
(*c*) Design vertical steel reinforcement required for wind and weight stresses at those sections of the shaft for which the weight and wind moments were

computed. Sometimes it will be found advisable to select a different wall batter and a different rate of increase in wall thickness from those chosen in (*a*).

(*d*) Design diagonal tension and shear reinforcement for shaft.
(*e*) Design horizontal and vertical temperature reinforcement for shaft.
(*f*) Design breech opening.
(*g*) Design lining.
(*h*) Design minor details such as clean-out door, lightning conductor, ladder, block and cable, ornamental top, etc.
(*i*) Design foundation.
(*j*) Check design.
(*k*) Prepare general and detail drawings of the chimney and foundation.

48. Construction of Reinforced Concrete Chimneys.—The forms used in the construction of conical reinforced concrete chimneys may be of wood or of steel but they must be rigid, tight, easily removable, and so designed that the proper wall batter and wall thickness will be obtained. Practically all of the types of suitable forms for chimney construction are controlled by patents.

The proportions for the concrete mix for the shaft are frequently 1:2:3, 1:3, or 1:4, with 1:5 as the limit. The proportion of fine aggregate should not be more than twice that of the cement. While the concrete should be as dense as practicable, enough fine aggregate should be present to form smooth surfaces next to the molds. The proportions for the foundation are usually a little leaner than for the shaft and may be 1:2:4, 1:2½:5, 1:5, or 1:6, with a 1:8 mix as the limit. The proportion of fine aggregate in the foundation should not be more than three times that of the cement.

Before placing the concrete, the reinforcement should be correctly placed and fastened so that it will not be displaced by the pouring of the concrete. The concrete should be poured as soon as possible after mixing and worked and compacted in the molds by spading and rodding so as to secure a good dense concrete. The operation of concreting should be as continuous as possible. In case there is an interruption in the process of concreting care must be taken to remove laitance, etc., and to secure a good bond between the old and new concrete. All steel reinforcement splices should be equal to forty or fifty times the diameter of the rods. Not more than half of the vertical tension reinforcement should be spliced at any horizontal section.

After the foundation is completed, an inside tower scaffold is usually constructed for the workmen and to assist in hoisting the concrete. One set of forms is carefully put in place on the foundation, the reinforcement placed and tied, and then concrete pouring is begun. When this set of forms is full, another set of forms is placed above and the concreting continued. When the second set of forms is about full, the first set is removed and placed above the second. The inside tower scaffold is built higher from time to time as required. An outside scaffold may be provided if it is necessary to paint or wash the outside surface. After the shaft is completed, it is usually washed down with a cement wash. The concrete is usually hoisted in a bucket attached to a cable operated by a hoisting engine.

The lining is usually constructed after the shaft is complete, care being taken to keep the air space free from debris.

After the chimney is complete the contractor removes his plant and cleans up the grounds.

If possible, the chimney should be permitted to "season" for at least a month before being used.

49. Description of a Medium-sized Conical Reinforced Concrete Chimney.—Figure 33 shows a view of the reinforced concrete chimney built recently by the

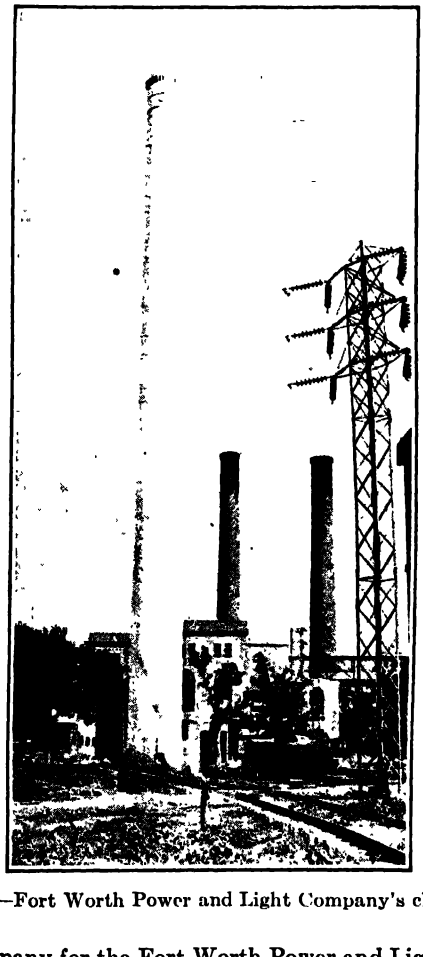


Fig. 33.—Fort Worth Power and Light Company's chimney.

Heine Chimney Company for the Fort Worth Power and Light Company. This chimney is 328 ft. high with an inside top diameter of 14 ft. The details of the design are shown in Fig. 34.

50. Fluted Reinforced Concrete Chimneys.—Placing vertical ribbing on the outer surface of the shaft tends to improve the appearance of the ordinary coniform concrete chimney. Figure 35 is a view of a "Conirib" concrete chimney built by the Polk, Genung, Polk Company. With the exception of the ribbing or fluting, the general design is the same as that for a chimney with a smooth conical surface.

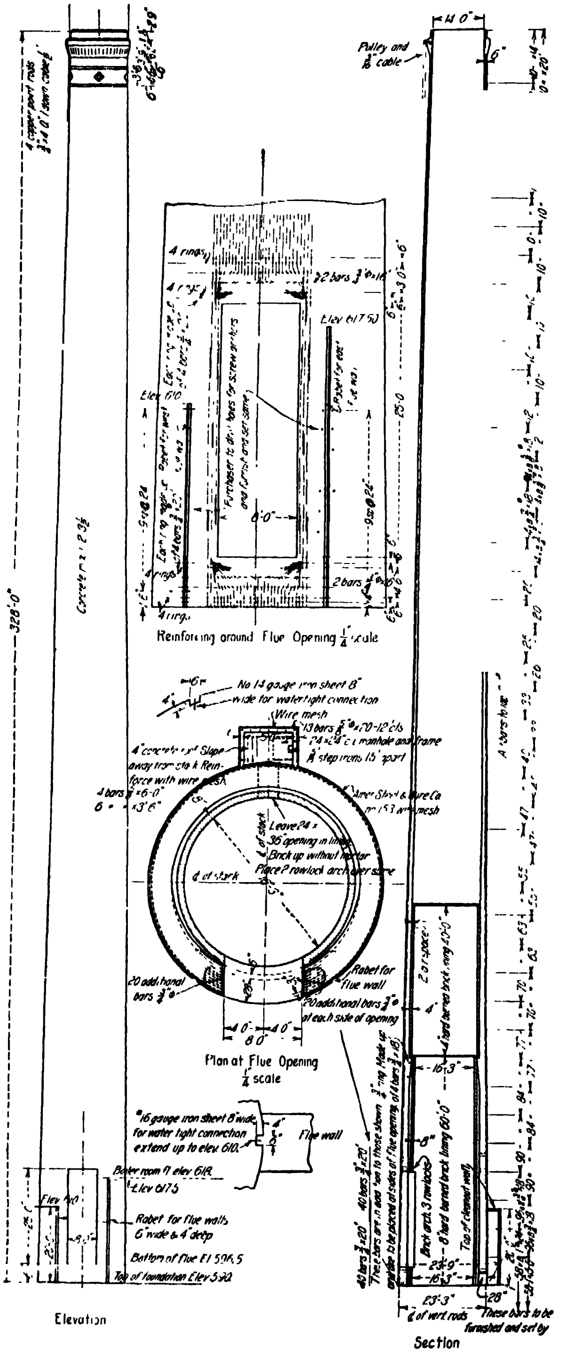


FIG. 34.—Details of Forth Worth Power and Light Company's chimney.

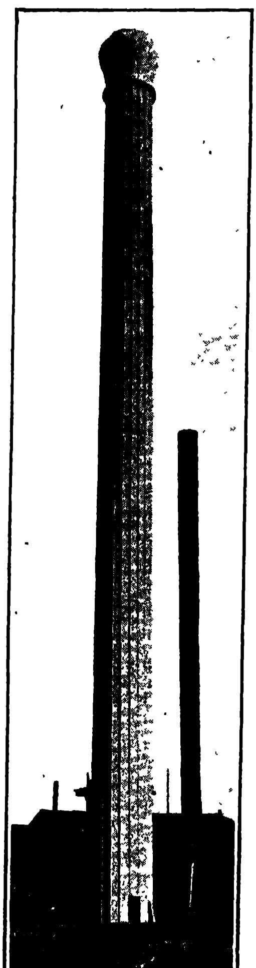
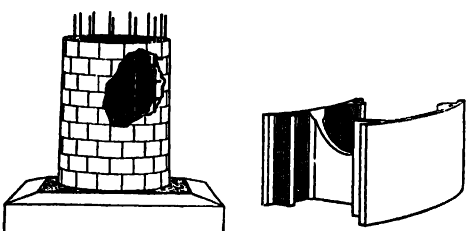


FIG. 35.—Conirib reinforced concrete chimney.

51. Reinforced Concrete and Tile Chimney.—This type of chimney is built by the Weiderholt Construction Company and consists of hollow hard burned clay tile of patented design, filled with concrete reinforced with both vertical and horizontal steel. The hollow tile form the outer and inner surfaces of the shaft and act as a mold for the concrete. The web of the hollow tile is constructed so as to permit the placing of horizontal as well as vertical reinforcement and also to permit the concrete to bind all the way around the shaft.

After the foundation of reinforced concrete has been constructed with the vertical reinforcement for the shaft in place, the tile for the first course are slipped over the steel rods and arranged in a circular course on the top of the foundation. The tile are then filled with a Portland cement mortar of a 1 :3 mix deposited and tamped in three layers. After the first course is filled, each succeeding course of tile is arranged and filled in the same manner. Twisted or deformed steel rods, bent in circular shape, are placed between the second and third layers of concrete in each course just outside the vertical reinforcement and in contact with it. Both the vertical and horizontal reinforcement are overlapped sufficiently to develop the required tensile strength. An interior scaffold is used for construction purposes.



(Enlarged view of tile.)

FIG. 36.—Manner of construction of a reinforced concrete and hollow clay tile chimney.

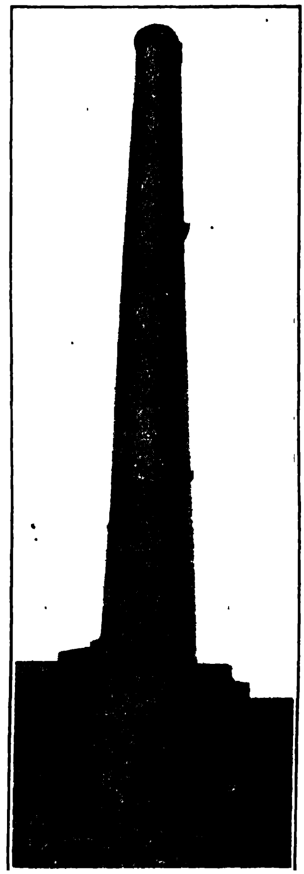


FIG. 37.—Weiderholt reinforced concrete and hollow clay tile chimney.

The shape of the tile and the manner of construction are shown in Fig. 36, while Fig. 37 shows a completed chimney.

52. Design of a 160-Ft. Conical Reinforced Concrete Chimney.

52a. Statement of Problem.—The problem is to design a conical reinforced concrete chimney for the following conditions:

Height above foundations = 160 ft.

Internal top diameter = 7 ft.

Lining = 100 ft. high constructed of 4¼ radial firebrick from the foundation up.

Size of breech opening = 4 ft. 9 in. by 9 ft.; the bottom of the breech opening is to be 12 ft. above top of the foundation.

Wind pressure = 25 lb. per sq. ft. on the vertical projected area.

Allowable soil pressure = 2 tons per sq. ft.

Unit stresses in shaft = As given in Art. 42. No provision is to be made for possible earthquake stresses.

Unit stresses in foundation = As given in Art. 41.

Flue gas temperatures = Maximum of 1000 deg. F. at entrance to shaft.

52b. Design of Shaft for Weight and Wind Moment Stresses.—The thickness of the wall at the top will be taken as 4 in. and the batter of the wall as 1 ft. in 80 ft., giving a base diameter of 140 in. The increase in wall thickness will be taken as 1 in. in 20 ft., giving a thickness of 12 in. at the base of shaft. The base section will be designed first. From data given and assumptions made above:

$D_1 = 92$ in. $d_1 = 84$ in. $r_1 = 44$ in. $t_1 = 4$ in. $r_1t_1 = 176$ sq. in.

$D = 140$ in. $d = 116$ in. $r = 64$ in. $t = 12$ in. $rt = 768$ sq. in.

$W_s = (2,850)(160) = 456,000$ lb. (See Fig. 9.)

$M = (1,340)(160)^2 = 34,300,000$ in.-lb. (See Fig. 10.)

$$\frac{e}{r} = \frac{M}{r \times W_s} = \frac{34,300,000}{(64)(456,000)} = 1.175$$

$$\frac{W}{rt} = \frac{456,000}{768} = 591$$

From Fig. 23, $f_c = 335$ lb. per sq. in. for $p = 3$ per cent of steel.

Select 3 per cent of steel as $f_{c(\max)}$ is probably 15 or 20 lb. per sq. in. more than f_c.

$$k = 1.225 \qquad \text{(From Fig. 15.)}$$

$$\frac{t}{r} = \frac{12}{64} = 0.1875$$

$f_{c(\max.)} = (335)(1.075) = 360$ lb. per sq. in. (See Fig. 26.)

$f_s = (335)(9.6) = 3,215$ lb. per sq. in. (See Fig. 14.)

The steel area required = (0.03)(2)(64) (12) = 144.7 sq. in.

For this area there can be used

185 — 1-in. round rods,
146 — 1⅛-in. round rods,
118 — 1¼-in. round rods,
or 98 — 1⅜-in. round rods,

which should be equally spaced around the circumference.

In like manner, computations may be made for sections 20 ft. apart and the results tabulated as in Table 13.

An examination of the results indicates that a little increase in the wall thicknesses at the 140 and 160-ft. sections would materially reduce the amount of steel required at these places.

52c. Diagonal Tension Reinforcement for Shaft.—Using the formula given in Art. 44c, the amount of diagonal tension reinforcement per foot of height in wall may be readily computed. Unit tensile stress in steel equals 18,000 lb. per sq. in. if wire mesh is used.

$$\text{Required steel area} = \frac{4(D + D_1)h}{18,000r} = \frac{(D + D_1)h}{4,500\,r}$$

Values for D, D_1, and r may be obtained from Table 13.

Amount of steel required in vertical cross-section of wall 1 ft. high at the base is

$$\frac{(140 + 92)160}{(4,500)(64)} = 0.129 \text{ sq. in.}$$

Similar computations may be made at height intervals of 20 ft. and the results tabulated as in Table 14. Enough steel is provided to carry all of the shear stress.

Table 13.—Stress Sheet for a 160-ft. Conical Reinforced Concrete Chimney Shaft for Stresses Due to Weight and Wind Moment

$h = 160$ ft. $D_1 = 92$ in. $d_1 = 84$ in. $r_1 = 44$ in. $t_1 = 4$ in. $r_1t_1 = 176$ in.^{2}

Distance from top (ft.)	D (in.)	d (in.)	r (in.)	t (in.)	rt (in.^{2})	W_s (Fig. 9) (lb.)	M (Fig. 10) (in.-lb.)	$\frac{e}{r}$	$\frac{W}{rt}$	f_c (Figs. 16–25) $\left(\frac{\text{lb.}}{\text{in.}^2}\right)$	p (Figs. 16–25)	k (Fig. 15)	$\frac{t}{r}$	f_c(max.) (Fig. 26) $\left(\frac{\text{lb.}}{\text{in.}^2}\right)$	f_s (Fig. 14) $\left(\frac{\text{lb.}}{\text{in.}^2}\right)$	Steel area (sq. in.)	Reinforcement rods required (No. and size)
0	92	84	44	4	176	0	0	......	0	0	0.0025	.....	0.091	0	....	2.76	14—½ in. φ
20*	98	88	46½	5	232	27,000	468,000	0.373	116	32	0.0025	.....	0.108	34	....	3.65	19—½ in. φ
40	104	92	49	6	294	61,600	1,920,000	0.636	209	80	0.0025	1.6	0.123	83	330	4.62	24—½ in. φ
60	110	96	51½	7	360	103,200	4,395,000	0.827	287	150	0.0025	1.04	0.136	160	2,100	5.62	29—½ in. φ
80	116	100	54	8	432	153,600	7,970,000	0.961	355	225	0.0025	0.79	0.148	246	5,175	6.78	35—½ in. φ
100	122	104	56½	9	508	215,000	12,700,000	1.046	423	300	0.0025	0.685	0.159	335	8,550	7.98	41—½ in. φ or 26—⅝ in. φ
120	128	108	59	10	590	285,600	18,650,000	1.107	484	312	0.0080	0.89	0.170	343	5,930	29.7	68—¾ in. φ or 50—⅞ in. φ or 38—1 in. φ
140	134	112	61½	11	676	364,000	25,870,000	1.156	538	320	0.0200	1.12	0.179	346	3,775	85.0	108—1 in. φ or 86—1⅛ in. φ or 69—1¼ in. φ
160	140	116	64	12	768	456,000	34,300,000	1.175	594	335	0.0300	1.225	0.188	360	3,215	144.7	185—1 in. φ or 146—1⅛ in. φ or 118—1¼ in. φ or 98—1⅜ in. φ

* The 20-ft. section in a "gravity" section.

TABLE 14.—STEEL AREA REQUIRED FOR DIAGONAL TENSION IN VERTICAL CROSS-SECTION OF SHAFT WALL 1 FT. HIGH

Distance from top of shaft (feet)	20	40	60	80	100	120	140	160
Required steel area (square inches)	0.018	0.036	0.053	0.069	0.085	0.100	0.115	0.129

The diagonal tension reinforcement will be combined with the horizontal temperature reinforcement.

52*d*. Design of Horizontal Temperature Reinforcement for Shaft.—In selecting the horizontal temperature reinforcement, use is made of the formulas given in Art. **44*d*.** and the curve of Fig. 28. As the horizontal temperature reinforcement should not be placed more than 3 in. from the outer wall, and as it should also be placed just outside of and tied to the vertical reinforcement to aid in holding it in place during the pouring, the values of t', as given in Table 15, will be assumed. Wire mesh will be used having an allowable unit tensile stress of 18,000 lb. per sq. in. The following procedure is used in the design. Knowing or assuming F, t', t and f_s,

$$\text{Solve for } k \text{ in formula, } f_s = \frac{180Ft'(1-k)}{t}$$

Obtain p from Fig. 28.

Compute steel area required for a vertical wall section 1 ft. high, noting that p in Fig. 28 is based on t'.

$$\text{Compute } f_c \text{ in formula, } f_c = \frac{12Fkt'}{t}$$

The temperature drop, F, across the shaft wall near the flue opening probably will not be more than 200 deg. F. as there will also be drops at both surfaces of the lining, across the lining, across the air space, and at both surfaces of the shaft wall. Assume F as 200 deg. The temperature drop across the shaft wall near the top of the lining will be less than 200 deg., say about 165 deg.

TABLE 15.—HORIZONTAL TEMPERATURE REINFORCEMENT

Distance from top of shaft (ft.)	Assumed tempera-ture drop across wall in deg. Fahr. F	Thick-ness of wall (in.) t	Distance from inner wall sur-face to tempera-ture steel (in.) t'	Location of neutral axis k	Propor-tion of steel (Fig. 28) p	Required steel area for wall section 1 ft. high (sq. in.)	Unit stress in concrete $\left(\frac{\text{lb.}}{\text{in.}^2}\right)$ f_c	Steel area required for wall section 1 ft. high For diagonal tension and horizontal temperature (sq. in.)
0	165	4	2.75	0.119	0.00054	0.018	162	0.018
20	170	5	3.50	0.159	0.00100	0.042	227	0.060
40	175	6	4.25	0.193	0.00155	0.079	287	0.115
60	180	7	5.00	0.222	0.0022	0.132	344	0.185
80	185	8	5.75	0.248	0.0027	0.186	395	0.254
100	190	9	6.50	0.271	0.0033	0.211	447	0.296
120	195	10	7.25	0.293	0.0040	0.348	497	0.448
140	200	11	8.00	0.312	0.0047	0.451	547	0.566
160	200	12	9.00	0.333	0.0055	0.594	600	0.723

Table 15 gives the tabulated results for the horizontal temperature reinforcement. About three extra horizontal ½-in. rods should be placed near the top of the shaft, and a like number of ¾-in. rods just above the top of the lining, when the lining does not extend to the top of the shaft.

If desired, small rods may be used instead of the wire mesh provided that enough steel is used so that the unit stress will not exceed 16,000 lb. per sq. in.

52*e*. Vertical Temperature Reinforcement for Shaft.—If the effect of the vertical steel used for wind moment reinforcement is neglected, the amount of wire mesh needed would be the same as that required for the horizontal temperature reinforcement as given in Table 15.

If the diagonal tension and horizontal temperature reinforcement is in the form of horizontal rods, the vertical reinforcement for wind moment may be assumed to carry vertical temperature tensile stresses up to the value of 16,000 lb. per sq. in. minus the value of f_s given in Table 13. This assumption will often give large vertical temperature stresses in the concrete. Also, at some sections, extra vertical rods may have to be added in order to keep the combined unit tensile stresses within bounds.

The use of a wire mesh for diagonal tension and both horizontal and vertical temperature stresses is thought to give the most satisfactory results.

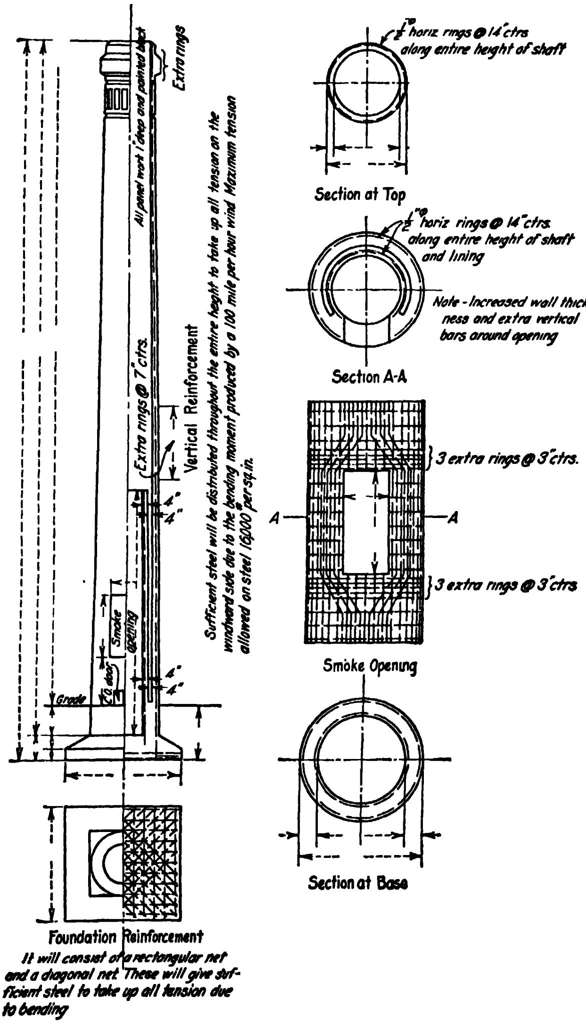


FIG. 38.—General design of a Weber Coniform chimney.

52*f*. The Breech Opening.—The breech opening is to be 4 ft. 9 in. by 9 ft. and the bottom is to be 12 ft. above, and the top 21 ft. above, the top of the foundation. The outside diameters at the top and bottom of the breech opening are 133.7 in. and 136.4 in. and the wall thicknesses are 10.95 in. and 11.4 in. respectively.

The amount that the wall thickness should be increased to compensate for the concrete removed is about 13.5 per cent at the top and 13.3 per cent at the bottom of the opening.

Increasing the wall thickness of the shaft 1.5 in. from a section 5 ft. above the top of the breech opening to the base of the shaft will be satisfactory. This increase in wall thickness above the top of the opening should not be abrupt, but should be gradual and should extend over at least 1 ft. The vertical steel at the opening should not be cut off but should be bent around the opening, and the bends should not be sharp. The rods should not be bunched together. Three sets of three horizontal rings of rods of the same size of the vertical reinforcement should be placed both above and below the breech opening. About four extra rods, each about 4 ft. longer than the width of the opening, should also be provided both above and below the opening. About 10 or 12 extra vertical rods, each about 4 ft. longer than the length of the opening, should be provided on each side of the flue opening. The temperature reinforcement should be bent around the vertical rods at the side of the opening and then should extend about 2 ft. along the inside shaft wall and about 2 or 3 in. from the surface. It is advisable to extend this extra mesh about 5 ft. above and below the opening. Figures 34 and 38 show how the reinforcement is placed around the flue opening.

52g. Lining and Minor Details.—The lining is to be constructed of 4½-in. radial firebrick laid in a cement fire clay mortar. The lining begins at the foundation and is 100 ft. high with an internal diameter of 7 ft. The minimum air space should be 2 in. About 3 horizontal rings of rods should be placed in the shaft just above the top of the lining as mentioned in Art. **52d**. See Arts. **10, 20,** and **35d** for further information.

Among the minor details are the clean-out door, lightning conductor, ladder, pulley and cable, and architectural treatment.

If a ladder is desired, it may be placed either on the outside or on the inside, but preferably on the inside. See Arts. **14** and **35d** for further information.

A 4-in. bronze pulley with a $\frac{3}{16}$-in. galvanized wire cable should be provided.

A clean-out door, 24 × 36 in. in size, should be located in a suitable position near the base on the side of the shaft opposite from the breech opening.

The lightning conductor should be designed and constructed as outlined in Arts. **17** and **35d**. If the lightning rods at the top of the stack are to be attached to the vertical reinforcement, this reinforcement must be well grounded.

The architectural treatment usually consists of building out the top of the stack and painting a monogram or other design on the shaft near the top. When the top of the stack is corbeled out, some horizontal rings of rods should be added for reinforcement.

Figures 34 and 38 give a good idea regarding the design of details.

52h. Design of Foundations.—The foundation will be octagonal in section and will be built of reinforced concrete.

$$\text{Weight of shaft} = 456{,}000 \text{ lb.} \qquad \text{(From Table 13.)}$$

$$\text{Weight of lining} = (120)\left(\frac{88.5}{12}\right)(\pi)(100) = 27{,}800 \text{ lb.}$$

assuming that the lining weighs about 120 lb. per cu. ft.

The depth required for punching shear is

$$\frac{(12)(350)}{(2)(120)} = 17.5 \text{ in.}$$

Assuming a depth of 36 in. and a distance between parallel sides of 25.5 ft. or 306 in., gives an area of 77,500 sq. in. or about 538 sq. ft. and a weight of 242,000 lb.

Total weight $W = 725{,}800$ lb.

The wind moment about the base of the foundation is

$$34{,}300{,}000 + \frac{140 + 92}{(2)(12)}(160)(25)(36) = 35{,}700{,}000 \text{ in.-lb.}$$

$$\frac{W}{A} = \frac{725{,}800}{77{,}500} = 9.37 \text{ lb. per sq. in.}$$

It is seen that the ordinary formula $\left(f = \frac{W}{A} \pm \frac{Mv}{I}\right)$ does not apply and that all of the base of the foundation will not be under compression.

Using the curve in Fig. 32 and obtaining the equivalent radius of the circle

$$\frac{e}{R} = \frac{68.1}{153 \times 1.04} = 0.4275 \quad L = 2.97 \qquad \text{(From Fig. 32.)}$$

Using formula $f_{(max.)} = L\dfrac{W}{A}$

$f_{(max.)} = (2.97)(9.37) = 27.8$ lb. per sq. in.

which is close enough to that allowed—that is 2 tons per sq. ft. or 27.78 lb. per sq. in.

$$k = 0.565 \qquad \text{(From Fig. 32.)}$$

$$(2 - k)R = (1.435)(153)(1.08) = 237.5 \text{ in. measured along a diagonal.}$$

Considering a strip 1 in. wide, the resultant upward pressure at the end equals the earth pressure less the weight of footing or $27.8 - 3.1 = 24.7$ lb. per sq. in.

At the stack, the resultant upward pressure equals $18.1 - 3.1$ or 15.0 lb. per sq. in.

The bending moment of this cantilever strip is

$$M = (15)(83)\left(\frac{83}{2}\right) + \left(\frac{9.7}{2}\right)(83)\left(\frac{2}{3}\right) = 73{,}900 \text{ in.-lb.}$$

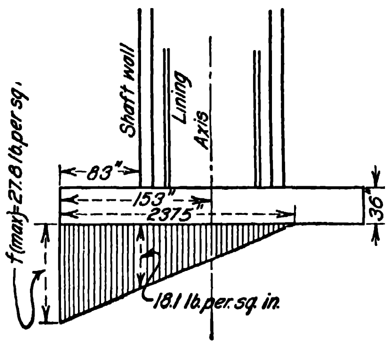


Fig. 39.—Foundation stresses.

For $f_s = 16{,}000$, $f_c = 650$, $n = 15$ and $d = 30$ in.

Use $p = 0.6$ per cent giving $f_s = 16{,}000$ lb. per sq. in. about,

and $f_c = 510$ lb. per sq. in. about.

Use 4 layers of reinforcement.

$$\text{Steel area for each layer} = \frac{(306)(30)(0.006)}{4} = 13.77 \text{ sq. in.}$$

Use 23—⅞-in. round rods,
18—1-in. round rods, or
14—1⅛-in. round rods in each layer.

The maximum unit shear occurs at the edge of the stack and is, considering a depth of 30 in.,

$$v = \frac{8}{7}\cdot\frac{V}{bd} = \frac{(8)(83)(24.7 + 15)}{(7)(2)(30)} = 62.7 \text{ lb. per sq. in.}$$

Stirrups must be provided to take the excess shear over 40 lb. per sq. in.

The chimney is not large enough to have any upward bending stresses at the top center of the foundation.

The vertical reinforcement in the shaft should pass into the foundation concrete and be hooked under the foundation reinforcement.

52i. Drawings Required.—The drawings usually required are a general plan and elevation drawn to scale and showing walls, linings, reinforcement, foundation, etc., with all necessary dimensions. Detail drawings to a larger scale are often required for the breech opening, clean-out door, ladder, architectural treatment, etc. Figures 34 and 38 give a good idea of what is needed in regard to drawings, though they are not quite complete in all details.

SECTION 11

CONCRETE DETAILING

By Walter W. Clifford

Concrete detailing lacks the standardization of steel detailing. It is, however, none the less important. In fact, it should be considered *more* important, for three reasons: (1) Because, while rigid steel members must come together in the field or be replaced by ones which will, almost any concrete job may, by pointing up the voids, be made to look all right when it is finished; (2) steel erectors belong to a skilled trade, while concrete work is done largely by unskilled labor and the foreman who thinks that the reinforcement matters but little, because it will all be covered by concrete, is not yet extinct; and (3) compared with the number of good steel detailers, few good concrete detailers are available. Practical field and drafting room experience are necessary for the first-class concrete detailer and as yet there are comparatively few drafting rooms where first-class concrete detailing experience is obtainable.

In concrete detailing two things must be considered: (1) the outlines of concrete which give the necessary information for formwork, and (2) reinforcement details to be used in the bending shed to fabricate the steel, and in the field to place it.

1. Outlines.—Outlines, or outside dimensions of concrete, are invariably given by the architect or engineer designing the work. For structural outlines the common rules of drafting apply and further suggestions are given in following articles. Outlines and reinforcement can usually be taken care of on the same drawing. But where the outlines are very complicated, separate outline and reinforcement drawings avoid confusion, and save time in the drafting room as well as in the field. In such cases outline drawings give all information for forms, and the outlines as represented by forms being defined, reinforcement is then located from them.

Outlines of ornamental work are worthy of considerable study. They must result from close cooperation between the designing architect and the constructing engineer if satisfactory work is to be had. By means of glue and plaster forms, almost any results may be obtained, but this entails considerable expense, and where concrete is used for ornamental work, cost is usually an important factor.

Around entrances and similar locations subject to close inspection anything but the simplest of ornament should be of cast stone or precast concrete.

For base and belt courses and also for cornices, very satisfactory "cast in place" mouldings can be made if the designer will consider the practical as well as the esthetic. He must remember that a fair design well built will be more effective than a better design poorly executed.

It goes without saying that concrete is not suited to fine, small scale work; minute indentations or projections are almost sure to be injured when wood forms are stripped. The precision of finish of concrete edges is not sufficient to stand small breaks. Nothing less than ⅞ in. should be used. Small breaks should be designed to fit stock thickness of dressed lumber to save form cost.

A knowledge of the manner in which the forms are to be constructed is necessary for the design of concrete mouldings. Forms of solid pieces, as shown in Fig. 1, are usually cheaper than those built up of lagging on templates, as

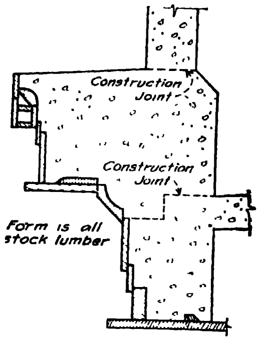


FIG. 1.—Good cornice form.

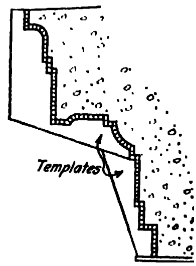


FIG. 2.—Expensive form.

shown in Fig. 2. The former type is also more easily made up in panels for repeated use than is the latter.

Satisfactory joints between individual pieces of form cannot be made on curved surfaces or at the tangent points. They should be made at angles. This means that reverse curves are difficult propositions unless they are small enough to be cut from one piece.

The moulding shown in Fig. 3 is practical. If it is so large that it needs to be made in two pieces, as shown in Fig. 4, it is difficult to construct. The form

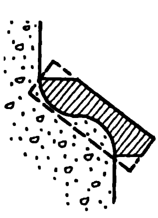


FIG. 3.

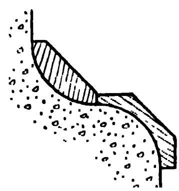


FIG. 4.

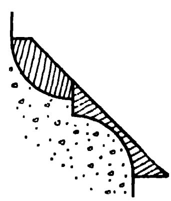


FIG.

shown in Fig. 4 is bad, for the joint must be trued after the two pieces are fastened together, if a satisfactory result is to be obtained. It should be made from lagging as shown in Fig. 2, or of plaster. If a break is introduced, as shown in Fig. 5, it again becomes practical.

The shape of a curve does not affect expense if any great length is to be used. Special curves entail only the use of the knives on the planer. Constant radius curves are carried in stock and are obtained more readily and quickly. The architect can easily determine the relative cost of concrete mouldings by realiz-

ing that the complimentary moulding of wood is required as a form—with the previously mentioned limitation in joints.

Plaster forms are financially practical for ordinary work when they have a constant section which can be struck with a single moving template.

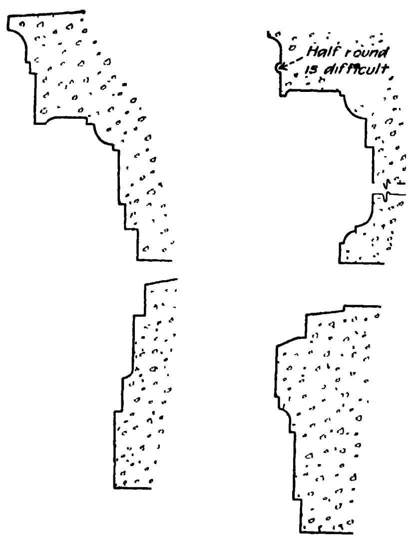


Fig. 6.

Figure 6 shows some practical types of mouldings and cornices which have been built.

2. Reinforcement Details of the Architect.—The architect's concrete drawings are more simple than those of the engineer or contractor who give complete information for the bending shop. The old time architect who locates the columns and specifies that the floors shall carry a certain load and conform to the city building laws, as well as the architect who imagines that a beam section designed more or less correctly for shear, together with the steel area at the center, constitutes concrete design, have very nearly passed into history. Under the common business arrangement with clients, the architect is, however, quite justified in expressing his design in the simplest possible manner and passing on to the contractor the actual bending details.

The practice of allowing competitive designs for reinforcement is bad, especially so in places where building laws are lax or lacking. Under competitive

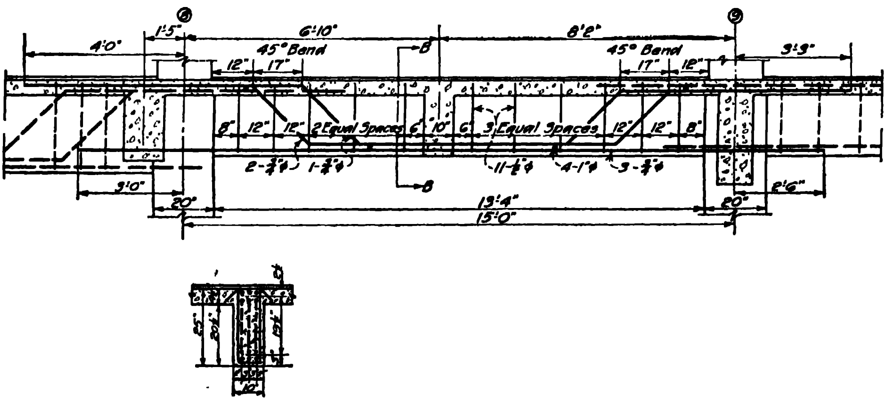


Fig. 7.—Architect's beam detail—no schedule.

designs the capable organizations are subject to ruinous competition by ignorant or unscrupulous reinforcement dealers, because of the absolute lack of standards such as are almost universal in structural steel. Unless the architect is fully competent to handle concrete design he should employ the services of an experienced engineer.

The information which the architectural office must give, is, in general: Size and location of all main reinforcement together with the angle and location of all cambers and bends; also the size, shape and location or spacing of auxiliary rods such as stirrups, hoops and spacers. The architect must remember that if he is to justify himself as a designer of his work he must at least give such information that details can be made in only one way and then he must check bending details to see that they are properly made.

Much of the necessary information can be covered by notes on drawings or in specifications such as:

All cambers shall be 45 deg. unless otherwise noted.

One-inch round spacer bars not over 50 times the diameter of the supported rods apart shall be used to separate all layers of beam rods.

Short column rods shall be 6 in. shorter than the distance floor to floor.

Long rods shall be 50 diameters longer than the distance floor to floor.

All columns are to be concentric, except those on the A, C, 1, and 10 lines, which are to be flush on the outside face or faces.

All hooks are to be 9 diameters unless otherwise noted.

Chairs or supports for reinforcement may be covered by note or in specifications in the following manner:

Chairs of an approved type shall be used to support all slab steel. At least one chair shall be used to each 15 sq. ft. of floor.

A typical beam detail from a first-class architect's office is shown in Fig. 7.

3. Reinforcement Details of the Engineer or Contractor.—Detailing by the contractor is analogous to steel shop drawing. Assembly drawings should be made on which each piece of reinforcement is given a mark, with the place it is to occupy in the form definitely indicated. Complete schedules should be given and also bending diagrams. A number of engineers, whose business arrangements with clients permit it, detail the concrete fully and schedule the reinforcement. This is the most satisfactory method, for the designer of concrete should be entirely responsible for the details, since concrete details are an even more essential part of the design than with other forms of construction, owing to the combination of two materials and the impossibility of satisfactorily inspecting completed work.

4. Drafting Room Organization.—In the smaller architectural offices where design drawings only are made, the concrete squad will usually consist of the engineer and a draftsman or two to trace his drawings or make drawings from his sketches. The virtue of such an organization is its simplicity. It hinges entirely on the engineer.

In the larger engineering and architectural offices where design and detail drawings are combined, organization is more complex. One or more concrete squads will be included in the drafting force. Six to twelve men are commonly grouped in one concrete squad, the number varying with the skill of the squad chief and the quality of the draftsmen.

A typical concrete squad may consist of a squad chief, two designers, five detailers, three tracers and a checker.

The squad chief must have an excellent working knowledge of concrete. Above all he must be an executive and this means he must have teaching ability. The more he knows about design the better.

The designers must be familiar with the theory and practice of design and able to execute work along the lines laid down by the engineer, with accuracy, speed and neatness. They should have initiative.

Detailers are good draftsmen with practical concrete experience. More often than not they do not have a technical education. A good squad chief can make a concrete detailer out of any good draftsman familiar with building work in general.

Tracers need only the qualifications needed by tracers in any other form of drafting work.

Checkers need the qualifications already mentioned for designers. A good checker is, however, of an entirely different temperament from the designer. His natural mental assumption is that a thing is wrong until it is proved correct. It is as natural for him to patiently look for the flaws in work as for the designer to look for the way out of a difficulty.

In many offices where complete records are kept, each squad is provided with a clerk, preferably a girl, to keep the necessary time and cost records and take care of reference prints. By taking from the squad chief all the routine clerical work which so often hampers the minor executive and leaving him free to devote all his time to work on the board, a clerk is a good investment.

Detailing by reinforcing companies is usually done from the design drawings of the architect and consists of not too elaborate setting plans with necessary reinforcing schedules. These are ordinarily made by or under the direction of the salesman and are then submitted to the architect for approval after the manner of steel shop drawings.

5. Drafting Procedure.—Considering the large drafting room the procedure on a concrete job will ordinarily be somewhat as follows:

The size, type of construction (beam and girder or flat slab), loading and column locations etc. will ordinarily be determined by the engineer before the job reaches the drafting room. The engineer will often furnish also the design of the typical cross-section.

For the drafting room there remains the design of the many secondary members and the detailing of all parts. In the division of parts of a large job between squads it is the practice in concrete to divide the work as a whole—that is to give separate buildings or sections of the work to the different squads rather than have one squad on framing plans, one on beam details, one on column details, etc. Since reinforcement from one member often goes through other members, as beam rods through columns, this is the more satisfactory method of dividing work and it has the added advantage of producing more versatile draftsmen.

The squad chief or chief draftsman in consultation with the engineer will first make a list of required drawings, bearing in mind how they will fit on standard size sheets which are used in most large offices. For a single moderate size building such a list may be as follows:

Footing plan.
Footings details (sheets 1 and 2).
Basement wall details (sheets 1 and 2), a key plan being included on each sheet.
Interior column details (sheets 1 and 2).

Exterior column details (sheets 1 and 2).
First floor framing plan
First floor beam details (sheets 1 and 2)
First floor spandrel details.
etc. to.
Roof framing plan.
Roof beam details.
Cornice details.
Pent house details.

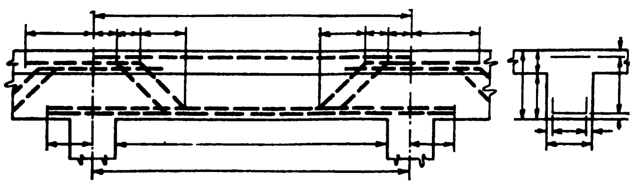
It is to be understood that reinforcement schedules as needed are to accompany each drawing.

A schedule of dates when each of these drawings must be in the field (after consulting the construction schedule) will then be made.

After these schedules are made the squad chief will turn the information over to the designers who will make designs and design sketches for all members.

Routine concrete design computations are more complex than those for steel, moreover stress assumptions are not so well standardized. Unless computations are systematized therefore trouble ensues.

The first use of design sheets is to give information to the detailer, and this necessitates habitual and good methods of stating results. But for the benefit of checkers and engineering executives—or those who have to explain to executives—all the assumptions of stress and loading should be equally well indicated. Furthermore, a year later a second structure may be desired with the same stresses or loading as a previous one, or information may be required as to the ability of a certain structure to withstand loading conditions different from those assumed in the original design. In such cases haphazard methods of computation become expensive.



Figs. 8 and 9.

Printed forms for design, such as are in use in the larger engineering offices, may seem like an undue expense for the smaller office, but a consideration of the relative cost of printing and writing the many often repeated letters, words and equality signs in concrete beam computations, for example, will demonstrate their real economy. In even a moderate sized office a detailer will use the design sheets of several designers and it is obvious that a form with a definite fixed place for all data and results will save time and mistakes on the part of the detailer, and omissions on the part of the designer.

Beams are particularly amenable to design forms. These forms should contain sketches for convenient use, such as shown in Figs. 8 and 9. On each designing form, the general information is given at the top of the sheet and the unit stresses and loads below. Then follows computation of the total loads. The middle of the page—which should be quadrilled in the interest of neatness—is

for the computation proper. In this part all equations showing loads used in computing moment, steel area, flange width, shear, etc., should appear, but arithmetical computations which should be done on a slide rule merely confuse the sheet and should not appear.

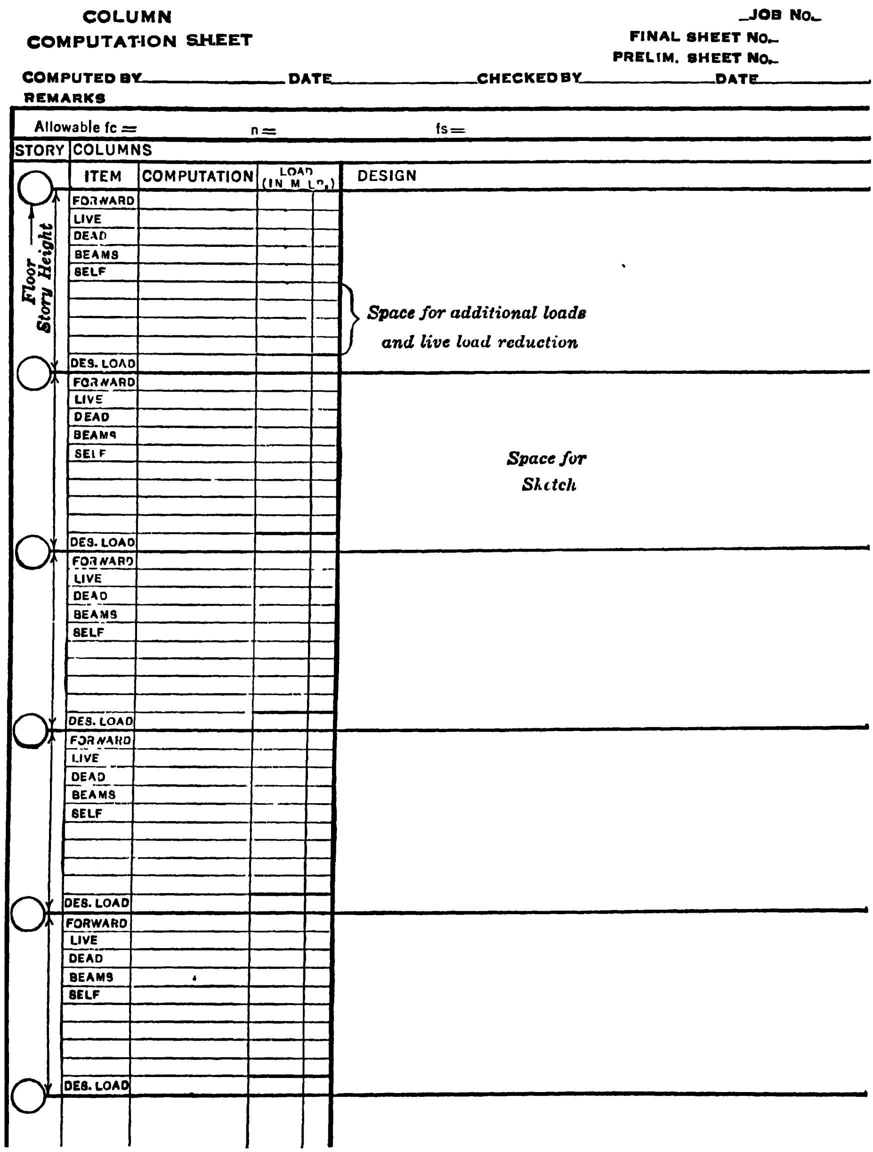


Fig. 10.

A good column design form is shown in Fig. 10.

There will always remain some special members which are not easily amenable to standard forms. Footings are the largest group in this class. A few offices

have standard forms for footing design, with a heading similar to that on the beam sheet and a sketch for data as shown in Fig. 11, but owing to the wide variation in size and shape, they are only worth while where much similar work is done. For special members a plain quadrilled sheet with a heading for general information and unit stresses, similar to that shown on the beam sheets, is well adapted. A few typical cases neatly worked out and included in office data books are helpful.

The adoption of definite tables or curves for design as an office standard makes for efficiency. Such tables and diagrams as are given in the volume on "Structural Members and Connections" are suitable.

Design sheets should be of the most convenient size for filing, which is usually letter size.

The detailers will take the information from the designers and make the actual drawings under the constant supervision and advice of the squad chief.

In detailing a concrete structure, structural steel practice rather than architectural practice should be followed in the choice of views to be made—that is, beams, columns, slabs and walls should be considered as units, and should be given an identifying mark on assembly drawings and then detailed individually. In other words, the structure should be taken to pieces and the individual parts detailed as separate members. Detailing by means of numerous general sections showing some members in cross-section and others in elevation produces complicated, incomplete and unsatisfactory drawings.

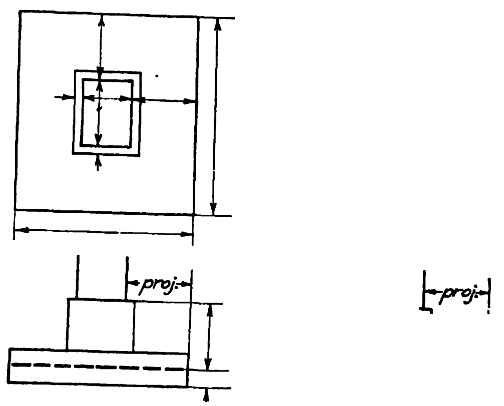


Fig. 11.

Detailed views should be taken as sections rather than as *Detail at A* because the section automatically orients the view.

Sections should all be taken looking North and West or according to some other standard to avoid frequent change of mental viewpoint in reading the drawing.

Sectional views should be arranged in the order in which they are taken in the general view and then lettered or numbered in sequence.

In making the drawing the detailer should endeavor to foresee all needed sections so that the original drawing may be well arranged, but it is seldom that improved arrangement can not be made in tracing.

Very often nowadays pencil drawings are made on transparent paper so that prints may be sent to the field more quickly. However, many changes are often made on concrete drawings between the time they are started and the time that the work is completed and few transparent papers stand erasure well. In addition the surface of the paper becomes grimed so that pencil prints are apt to be a considerable bugbear to the field men. At least all the framing plans and all important or complicated details should therefore be inked on cloth.

This inking of drawings on cloth or paper will be done by tracers.

When drawings are traced, the squad chief has a chance to rearrange them and see that sections and subordinate details are logically arranged with reference to section letters used and the points where they are taken. A few minutes' study of the best arrangement of a drawing before tracing, saves much future time in consulting the drawing both in the field and in the office.

Individual sections or details should always be separated by a space 1 or 2 in. wide. Crowded views that tooth into each other are an unnecessary waste of time and temper.

When original drawings are made on transparent paper, checking prints are often taken before tracing so that time may be saved by carrying on checking and tracing operations simultaneously.

Checking on the original drawing may safely be done when drawings are simple and are on cloth, otherwise the expense of a print for checking is well worth while.

The checker will verify the correctness and completeness of all given information. In addition it is very important that he check the concrete drawings against those of the mechanical, electrical and other squads for interferences. There is a temptation to run pipes and ducts through concrete members rather indiscriminately and checkers must be careful to see that such equipment as necessarily intersects concrete members is so located as to be safe from a design standpoint and practical from the construction viewpoint.

Checking on prints is done with colored pencil and a standard use of marking is helpful. The following is a successfully used standard in a large drafting room:

Black is to be used by checkers for lines or information to be added.
Red is to be used by checkers for lines or information which is wrong or to be taken out.
Yellow is to be used by checkers for lines or information which is correct.
White is to be used by draftsmen to check off corrections made.
Blue is to be used on tracings in case it is necessary to check thereon.

In connection with the use of yellow pencil it should be noted that, on complicated reinforcement details, check marks are confusing because of the closeness of lines. Tracing lines, checked and found correct with a yellow pencil, is a quick and clear method. The yellow is not heavy enough to obliterate checked information.

After checking, all changes should be back checked by the detailer who made the drawing, before final revision is made, and of course the revision is back checked by the checker.

6. General Rules.—Conventions in concrete detailing have been developed in the larger offices and play an important part in concrete drawing. They simplify the drawing operation and are much more easily read than the most beautiful drawings.

6a. Scale.—Framing plans are usually 1/8 or 1/4 in. = 1 ft. according to the size and complexity of the plan. Details are most commonly 1/2 in. = 1 ft. One-fourth inch = 1 ft. is sometimes used, but except in the simplest work it is expensive of time. When in doubt use the larger scale. Larger details than 1/2 in. scale are not needed except in the case of ornamental outlines or unusually small details.

6b. Conventions.—Reinforcement details should be primarily diagrams. Clear indication of the way rods are to be placed is vastly more important than true orthographic projection. For example, the rods shown over a support in actual projection in Fig. 12 may actually be as shown in diagram in Fig. 13 or Fig. 14. They should always be diagrammed correctly, as shown in one of the latter figures. The section or notes will indicate that the rods are at the same elevation and proper scheduling will bring them there.

FIG. 12.

FIG. 13.

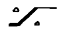


FIG. 14.

Full heavy lines are used for reinforcement in the details in this chapter (except for flat slabs) and this is the most satisfactory convention. Dash lines as sometimes used are slower to draw and often lead to confusion at points where two or more rods meet. Sufficient distinction between reinforcement rods and the lighter outlines on the one hand and the heavier beam lines shown on $\frac{1}{8}$-in. scale framing plans are easily made by weight alone. Figure 15 shows a satisfactory standard.

Dimension and witness lines

Concrete outlines, ties, and stirrups

Reinforcement except as above

Steel beams on concrete framing plans

Concrete beams on $\frac{1}{8}$" scale framing plans

FIG. 15.

Dash lines of the same weight as other reinforcement are effectively used, however, to show rods detailed in another member but projecting into the member being detailed or tying with some reinforcement of the latter member. This is illustrated in Figs. 7 and 24. Further conventions are mentioned under the descriptions of various classes of details.

Sections of concrete can be most easily indicated by shading on the back of the cloth or paper with a soft pencil. This is a much quicker process than the conventional sign which is necessarily used for illustrations in this chapter, and is fully as effective.

6c. Dimensions and Listing.—Most important in dimensioning concrete details and listing the reinforcement is to have a standard, or habit, and to stick to it. On plans the marks of the column lines, the bay sizes, overall dimensions, etc., should be in the same corresponding positions on all similar views. Listing of bands of slab rods should be in logical sequence.

On each of a group of similar details all the information should be on corresponding places on the detail. Considering beam details, for example, proceed as follows: Give the locations of intersecting beams in a line of dimensions above the elevation; the clear span and support width in the first line of dimensions below the elevation; the span center to center of supports below this (see

Fig. 24); the stirrup spacing near the center of the elevation; the cambered or bent steel list just below the left end; the straight steel below the right end; and the stirrups and spacers under the center of the beam, etc. Consistency of this kind is essential for good details. The location of the information, so long as it is clearly given, is of less importance than the consistency in placing it in a given location.

Rods usually appear in more than one view. They will, of course, be listed in one view only, and be noted in the others if necessary for identification. It is important for good detailing that they be listed in the best place. Ordinarily, this is in the view in which the rods appear in projection as a straight line. Whenever a structure is detailed in parts, however, rods which run into two parts should always be listed with the part which will be poured first. For example, in a tunnel, angle rods from the floor into walls should be listed in the floor detail.

6*d*. Key Plans.—The use of small key plans on detail sheets should be more common than it is. A $\frac{1}{16}$ in. = 1 ft. or even smaller skeleton assembly plan placed directly over the title, with the members detailed on that sheet shown heavy and marked, and the balance of the plan shown very lightly, gives at a glance information that may otherwise take some time to obtain. A similar key plan should also be used with large complicated mat footings, and is also very helpful in detailing long tunnels or retaining walls and flat slabs (see Figs. 21 and 30).

6*e*. Beam and Column Marks.—The common method of giving columns and beams consecutive numbers is open to grave objections. In the course of changes which most plans undergo, No. 99 is very likely to land between No. 13 and No. 14 and is then difficult to locate.

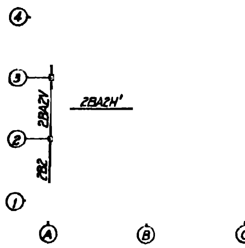


Fig. 16.

The coordinate system (see Fig. 16) while it may at first seem complicated is actually simple and quickly learned. In this system column center lines vertical on the plan are lettered and horizontal column center lines are numbered. Columns are then easily located by their designation as $A1$, $C11$, etc. being at the intersection of the A and 1 lines and the intersection of the C and 11 lines, respectively. $A1$ is obviously a corner column. All A columns and all 1 columns will usually be exterior columns. The other outside lines will usually be remembered by all the men working on a building before drawings are far along as will also any other special lines such as those adjacent to elevator wells or locating construction joints.

In the case of repetition of similar buildings, standard letters for certain parts may be used—as, for example, in a typical steam-generating station: A for the outside of the electrical bay, B the line between the electrical bay and the turbine room, C the line of the fire wall between the turbine and boiler rooms, D and E the rear and front boiler columns of the first line of boilers, F and G the front and rear of the other line of boilers, and H the outside wall. With no electrical bay, the B line becomes an exterior line. With only one row of boilers F becomes an outside line. Extra wall columns in the turbine room, or boiler or ash pit columns may be Bx, By, $Bz - Fx$, etc. With a

system like this every one in an organization places column *C*4 in the fire wall without need of consulting any plan.

Beams are given the location mark of the column in the lower left-hand corner of the bay in which they occur and are marked *V* if vertical and *H* if horizontal—as *C*2*H* or *D*3*V*. 2*B* (second floor beam), *RB* (roof beam), etc. are used as a prefix to these marks. Intermediate beams are given a prime, as 2*BA*2*V'* or 2*BB*3*H'*.

In the case of two or more identical beams on the same floor, typical numbers are given instead of location marks; odd numbers for beams horizontal on the plan and even numbers for vertical beam, as 2*B*1, 2*B*2, etc.

The coordinate system can be adapted to practically any structure and is an immense time saver.

6*f*. Specifications and Notes.—The limits of information given on drawings and in specifications vary greatly in different offices. The criterions are (1) clearness and (2) economy. A good general rule is that all information which is general—form requirements, method of mixing and placing, quality and size of aggregates, etc.—should be taken care of in the specifications, and requirements that differ for various subdivisions should be placed on the drawings. This is on the assumption that the information can be expressed in few words, for hand lettering is expensive. Under this latter clause falls the mix, which is commonly noted on each drawing.

All general notes such as specification items, grades, cross-references, etc., should be grouped over or beside the title for easy reference and not scattered around over the drawing.

7. Framing Plans.—Framing plans are the first super-structure drawings of buildings. All framing plans, as well as all detail plans, should be similarly oriented, otherwise confusion is almost sure to arise.

The columns shown on framing plans should be those below the floor. They are ordinarily shown in full lines despite the fact that they are invisible.

On ⅛ in. = 1 ft. scale plans, concrete beams are usually shown as single full lines (Fig. 16). Single lines representing beams are made heavier than those showing reinforcement (see Fig. 15). Where the supporting beams are steel, a dash line of the same weight is used (see Fig. 17). On ¼ in. = 1 ft. scale drawings, beams are usually shown with two dotted lines with the distance between these lines approximately the beam width (see Fig. 18).

Framing plans should show: (1) The dimensions of bays each way; (2) beam marks; (3) the location of all beams from the nearest column center line (on single line framing plans, beams only a few inches off center should be shown slightly out of scale for clearness); (4) size of all beams (in case of sloping floors note the grade from which the depth of the beam is taken—or make a general note "all beam depths are given below the rough slab grade"); and (5) the size of the columns below the floor. In addition the framing plan is usually the slab detail plan.

The general notes on a framing plan will give the grade of all floors shown, noting carefully *rough* or *finished* grade; any specification data; and any cross-reference notes such as *West end of second floor shown on Sheet H*79, *All beams on this sheet similar to corresponding beams on 2nd floor, For details of beams see sheets F*21, *F*22, etc.

On large buildings, it is often worth while to make on each framing plan an index table for detail sheets in such form as the following:

Beam	Detail on Sheet
1*B*1	*H*42
1*B*2	*H*42
1*B*3	*H*43
1*BA*6*V*	*F*44
1*BJ*2*H*	*F*36

Columns	Detail on Sheet
*A*1	*H* 22
*A*2	*H* 22
*B*1	*H* 23
*B*2	*H* 23

Slab reinforcing details are considered in Art. 9.

8. Surface Plans.—Complicated floors require two plans to show without confusion all the required information. This follows from the fact that openings not only have to be dimensioned themselves, but they also complicate greatly the details of the slab reinforcing. In such a case a surface plan, so-called, is made as an outline plan (see Fig. 19). On the surface plan, openings and pedestals are located and dimensioned, surface slope (if any) is shown, and beams are marked, sized and located.

The second plan then becomes simply a reinforcing plan showing, but not dimensioning outline features except that it is often a convenience in working out slab reinforcing details to have on the reinforcing sheet the distance of beams off column center lines.

Occasionally power station or factory floors are so complicated as to need in addition to the above-mentioned plans a bolt location plan. The name of this plan is sufficiently descriptive of its purpose and scope. Anchor bolts are considered in Art. **12.**

9. Slabs and Walls.—Slabs and walls are similar in detail and vary only in position. They have in general main reinforcement perpendicular to a system of beams, and spacers at right angles to the main rods. The main steel may be cambered to give negative reinforcement (Fig. 18) or the so-called loose-rod system of separate bars to take care of negative moment may be used (see Fig. 17). This latter system is commonly used for slabs 6 in. and under since cambers less than 4 in. are not practical. In walls, vertical rods are placed otside (nearer the face) wherever possible. This is better for placing concrete.

9*a*. Listing.—Steel in plan, or elevation if in walls, is best indicated by considering bands consisting of rows of evenly spaced identical bars. The outside bars of the band are shown and the band listed as shown in Figs. 17, 18 and 23. In architectural detailing the bands may be similarly shown and simply listed as $\frac{5}{8}$ ϕ 6 in. *c*. to *c*.

A diagram of two adjacent rods will be noted in Fig. 18 in the center of the bays. This is an advantage in working out the detail and will save separate sections to a large extent.

To differentiate clearly between steel in top and bottom, or far and near side, a good method is to add to the listing *f.s.* or *t.s.*—thus 29-5/8 in. ϕ-*A*42-6 in. *c.*

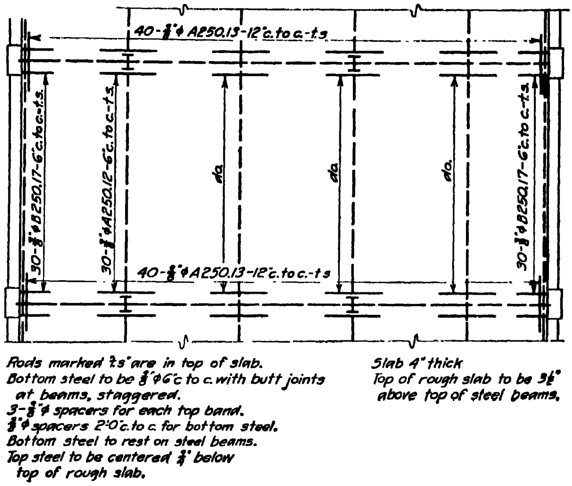


FIG. 17.

to *c.-t.s.* Then use as a general note:—*All rods marked t.s. are in the top of the slab, all other rods are bottom or cambered steel* or *All rods marked f.s. are in the far side, all other rods are in the near side.*

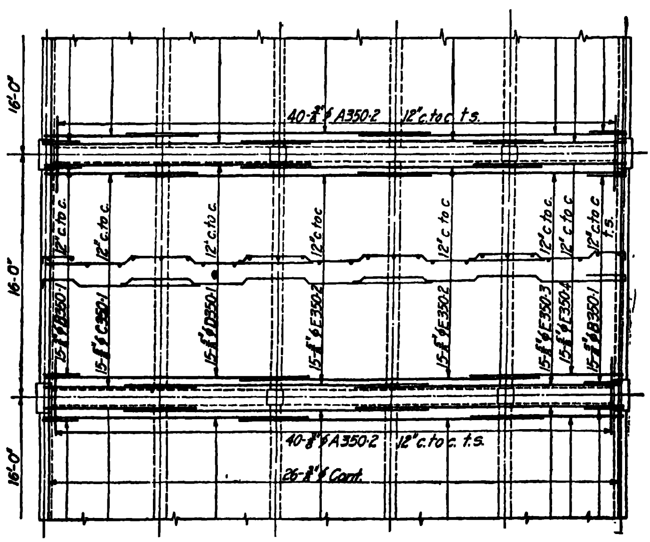


FIG. 18.—Slab detail.

In listing bands, the type, and spacing are obviously needed for setting the steel on the floors. The size should also be given because rods are ordinarily stored by sizes on the job, and this informaton is, therefore, helpful in finding them.

Schedules are ordinarily not used in setting, and if used, cross-reference between plan and schedule is a nuisance.

Ambiguity in rod listing must be avoided. Notation as shown in Fig. 20 is common with the inexperienced draftsmen. In such a case there is doubt

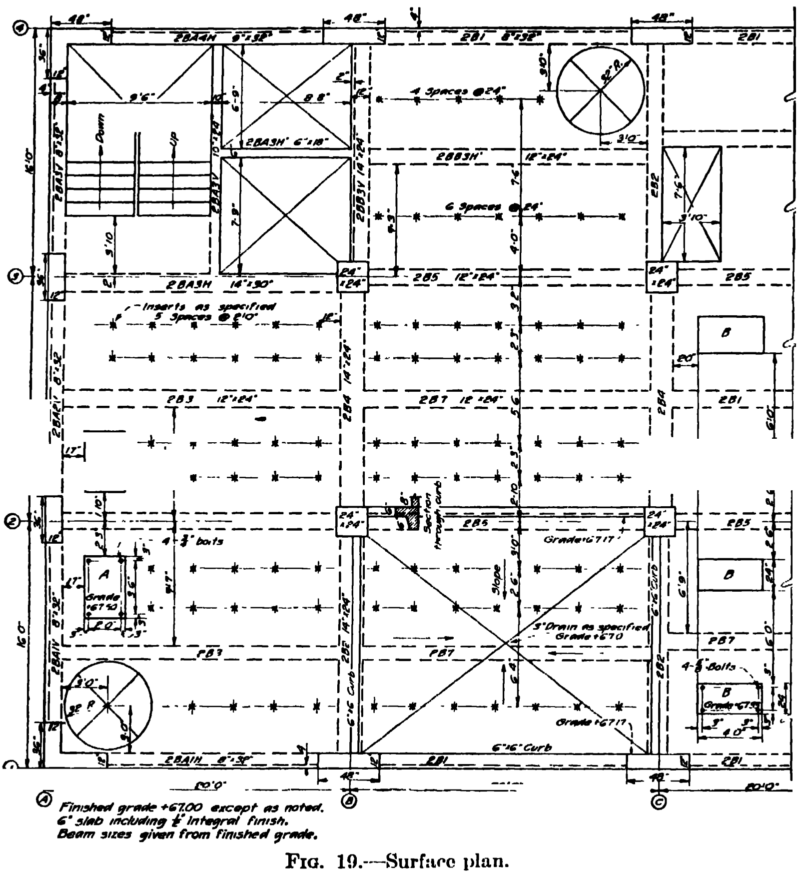


FIG. 19.—Surface plan.

whether two bent and two straight rods are called for or only two rods total. In the case shown each rod should be listed separately.

9b. Spacers.—Spacers are very commonly ⅜-in. rounds 2 ft. on centers, for ordinary slabs. In walls a size smaller than the main reinforcement is commonly used with a maximum of ¾ in. and a minimum of ⅜ in. and with spacing 18 in. to 3 ft. They are ordinarily random length for the smaller rods, scheduled as total length and cut on the floor. They may be covered by a note, or indicated in the diagram (see Fig. 18). The larger spacers (⅝ or ¾ in.) should be listed and typed in bands like main reinforcement.

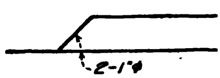


FIG. 20.

9c. Rod Spacing.—Rod spacing in slabs is limited in the Joint Committee's report to 2½ times the slab thickness and the minimum should

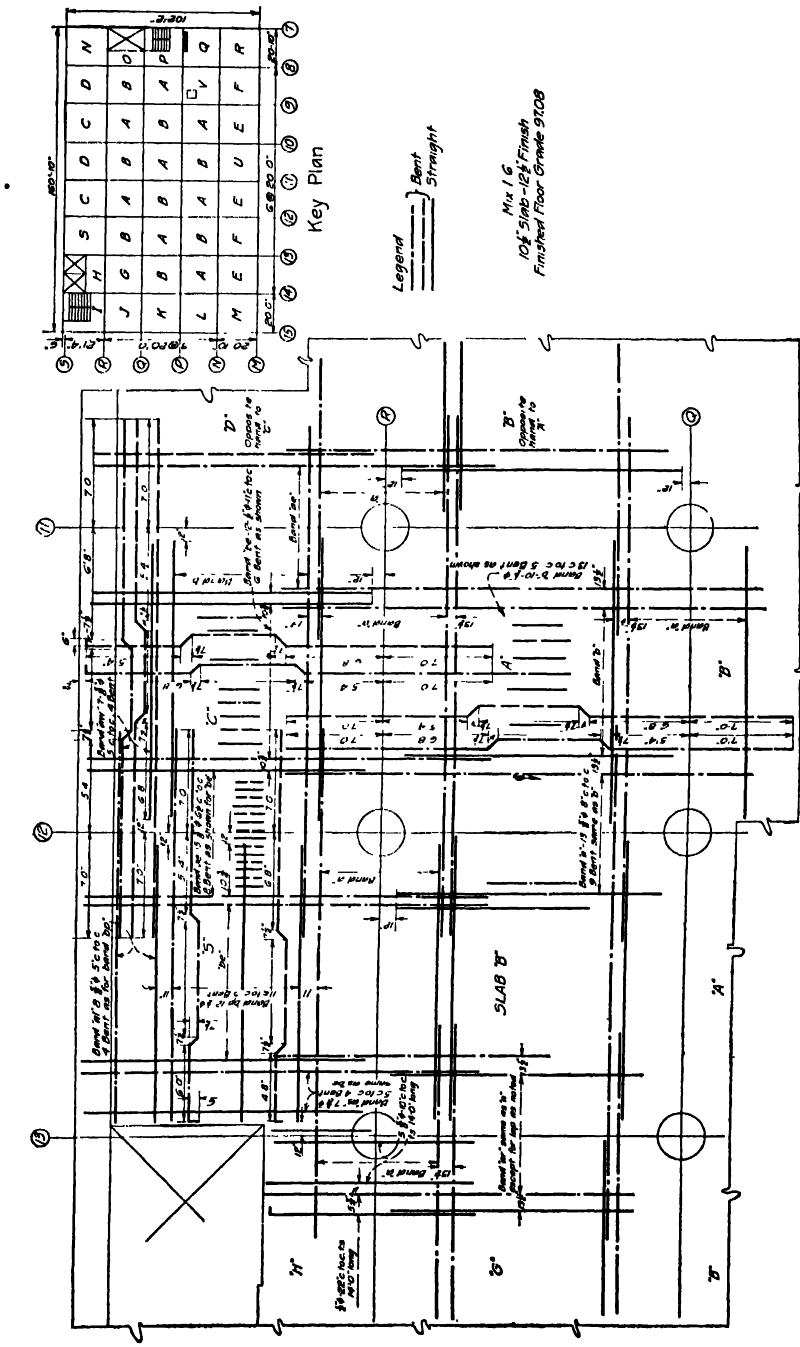


FIG. 21.—Part of a detail of two-way flat slab.

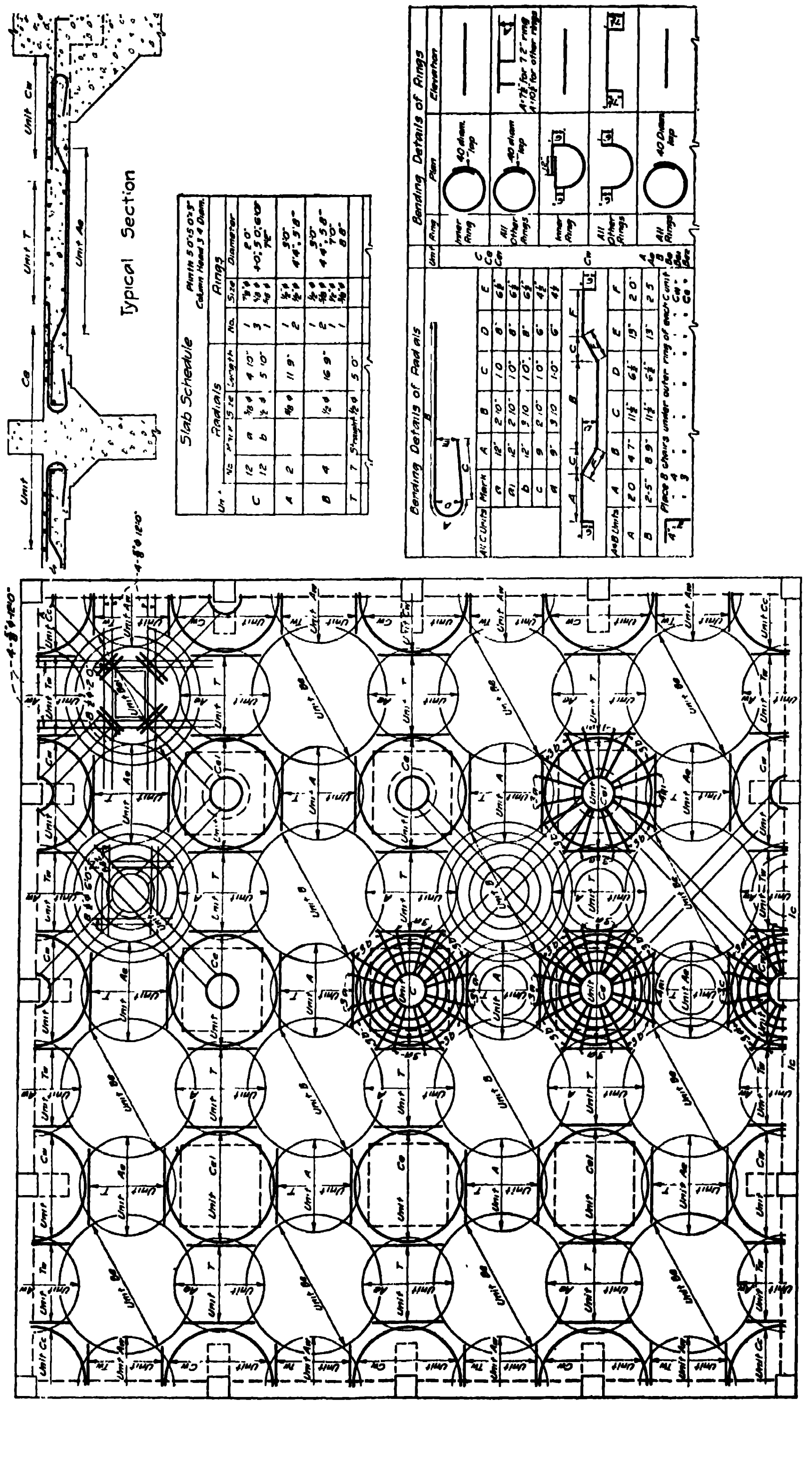


FIG. 22.—Slab detail, S.M.I. flat slab system.

be as in beams. Common practice for ordinary work is 1 to 1½ times the slab thickness.

9*d*. **Sections.**—In addition to slab plans and wall elevations, sufficient sections must be given to clearly indicate the location of all steel (see Fig. 23).

9*e*. **Flat Slabs.**—Flat slab construction is detailed like other slabs, except that typical bands may well be listed *Band A*, etc., and described in detail only once, or the schedule may indicate the makeup of the various bands (see Fig. 21). This is sometimes possible with beam-and-slab construction.

The S.M.I. flat-slab system makes use of units of spider type over columns and in the center of bays. On reinforcement plans of this system each unit is completely shown once and elsewhere simply a circle is shown (the outside ring) and marked *Unit C*, etc. Where separate units are used for positive and

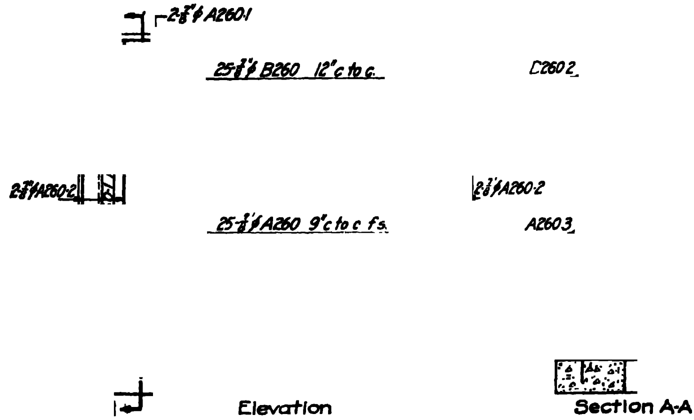


FIG. 23.— Wall detail.

negative reinforcement, different weights of lines may be used for top and bottom steel. This helps greatly in the clearness of the drawings. Figure 22 shows an excellent example of much information clearly and economically given in comparatively small space.

10. Beams.—Beam details will constitute a large part of the drawings on a beam and girder building. Great care in method and standards of beam detailing is therefore worth while. Owing to field fabrication, printed templates for beam details such as are used in most larger structural steel shops have met with little favor in concrete work and picture diagrams to scale are accepted practice.

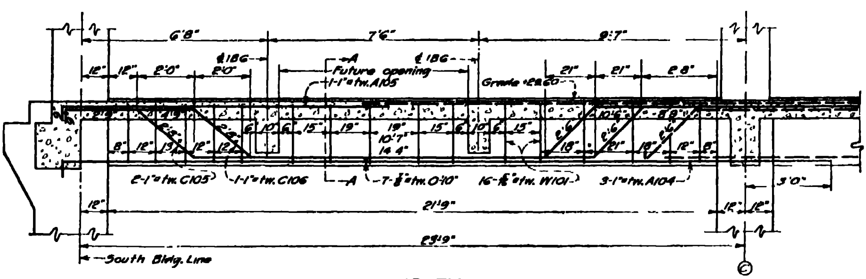
Some conventions are used primarily in beams. The dash line is used in the section to indicate cambers in elevation. In the elevation it is used to indicate rods belonging to another detail. A somewhat lighter line is used for stirrups than for main steel. The open circle at the top of the camber is used for a horizontal rod in elevation while the solid circle is used for the rods cut by the section.

10*a*. **Rod Spacing.**—Rod spacing in beams must be determined by theoretical considerations. In addition to this the clear distance between rods should be not less than twice the largest aggregate size. Rods are often used in two layers, very seldom more than two. Layers of beam rods are usually separated 1 in. by short spacer bars. The distance between these spacers depends

on the size of the main steel. Fifty times the diameter of the main steel is reasonable. There should be at least two spacers under each rod of the top layer.

10*b*. Connections.—The intersection of beam, girder, and column steel over the column head must be carefully studied. With a beam centered on a column, careless detailing often shows a rod in the center of the column and one in the center of the beam. Small rods (½ in. or less) are easily offset, but this is not the case with larger rods. Beam and girder intersections must also be detailed with care to see that interference is not caused by rods at the same grade.

10*c*. Inflection Points.—Locating cambers so that there is sufficient steel, both negative and positive, for extreme variations of inflection point with all loading conditions is a point which requires careful study for an economical



MARK	BEAMS REQD.	BARS FOR ONE BEAM				
		TYPE	NO.	SIZE	LENGTH	
1Ba7V	1	C105	2	1"□tw.	31'9"	
		C106	1	1"□tw.	31'9"	
		A104	3	1"□tw.	28'9"	
		A105	1	1"□tw.	13'9"	
		W101	16	⅜"□tw.	10'0"	
			7	⅜"□tw.	0'10"	

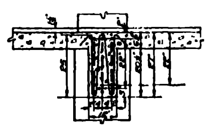


FIG. 24.—Complete beam detail—schedule on same sheet.

detail. The use of two sizes—such as 3-¾ in. and 3-⅞ in. for main steel in continuous beams with the smaller rods cambered each side of the support and the loop rods continuous over supports—often gives an economical detail. For other conditions, supplementary straight negative bars offer the best solution. This part of the detail, particularly in the case of typical beams, should receive careful attention.

10*d*. Stirrups.—Except in very large and important members stirrups are left to the detailer to space. Girders will ordinarily have constant stirrup spacing, of course, as will upstanding spandrels where the principal function of the stirrup is to tie in the slab. For uniformly loaded beams, after a careful layout of stirrups in a typical beam, an experienced detailer will locate stirrups satisfactorily from a determination of the theoretical spacing at the supports. It is good practice to place stirrups 4 or 6 in. from the face of all intersecting beams. The first stirrup is located by many engineers about ¼ to ⅓ of the depth of the beam from the face of the support since diagonal tension cracks seldom start at the support. In very wide beams where stirrups of more than four legs would be needed it is better from a practical standpoint to use several U's or W's, as shown in Fig. 25. Rods larger than ⅝ in. should not be

used as stirrups, unless absolutely necessary, on account of the difficulty of bending. In special spandrels such as cornices, stirrups or binders are needed as spacers and ties more than for shear (see Fig. 24*a*).

10*e*. Bond.—Bond is seldom an important item in beam and slab design. Most properly designed beam reinforcement is sufficient for bond. In beams continuous over supports, part of the main reinforcement is usually cambered. The balance is continued across the support as compression steel in T-beams, and this use determines the lap rather than bond (see right-hand support, Fig. 24). At end supports straight steel is often hooked. It is good practice to hook the ends of tension rods at all end supports. The ends of stirrups usually need hooks for bond and it is good practice to hook all of them.

10*f*. Loop Rods.—Loop bars (as shown in Figs. 7 and 24) to catch the stirrup hooks where there is no main steel in the top of the beam are not required from design considerations although they may be used for negative

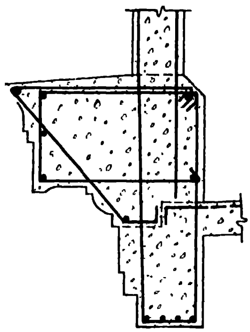


Fig. 24*a*

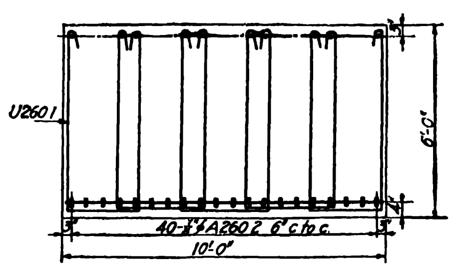


Fig. 25.

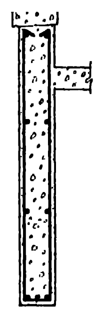


Fig. 25*a*.

steel if continuous over supports, but they will repay their cost in keeping the stirrups in place during pouring. They are a necessity for beam cages fabricated outside the forms and can be used with cross joists to support the beam steel in the form.

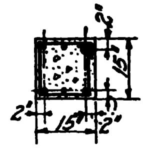
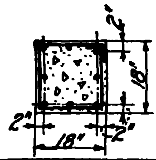
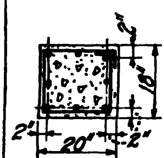
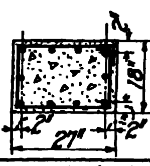
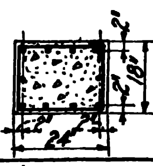
Deep narrow beams usually have intermediate horizontal rods for expansion, as shown in Fig. 25*a*.

11. Columns.—An office standard for column detailing is shown in Fig. 27. In the architectural type of detailing, a column schedule as shown in Fig. 26 is satisfactory. Main steel may be listed as long rods and short rods, and notes added. In the case of columns having complications, such as brackets, elevations should be drawn, as shown in Fig. 27.

11*a*. Rod Spacing.—The rod spacing of the main rods usually takes care of itself with standard percentages of steel and commercial rod sizes. The maximum spacing of vertical rods allowed by good practice is about 10 or 12 in. In the case of large columns with high percentages of steel it is difficult to get all the bars that are required in one band. The largest rod easily available in most localities is 1¼ in. In large columns these should be spaced at least 4 in. apart, and, where spiral hooping is used, at least 6 in. Where too many rods are required for this spacing, two rows of rods should be used or some of the rods should be placed in the form of a cross inside the core.

11*b*. Spirals and Hoops.—Spiral hooping for columns is expressed in percentage of volume of hooping to volume of core per unit of length.

Hooping has great possibility of irregularity when the core is of large diameter. In order to ship flat, two vertical ties only are used, and this leads to deformation in handling. One inch cover will do on 12 to 16-in. columns but, on 3-ft. cores or larger, at least 3 in. of cover should be allowed irrespective of fire risk. Hoops are limited by the Joint Committee's report to a maximum spacing of 12 in., or 16 times the diameter of the longitudinal bars. Light rods suffice for this hooping,

COLUMN SCHEDULE					
		Col. Nos. A1, A2		A5, A7, A9	
		Steel	Section	Steel	Section
18'-0"	3rd Story	6-7/8"φ A450-5 16-1/2"φ O450-3 12" c. to c.	Section same as 2nd Story	6-3/4"φ A450-6 16-1/2"φ O450-3	
20'-0"	2nd Story	6-7/8"φ A450-3 2-7/8"φ A450-4 18-1/2"φ O450-3 12" c. to c.		Steel same as A1	Section same as A1
20'-0"	1st Story	6-1"φ C450-2 2-1"φ A450-2 18-1/2"φ O450-2 12" c. to c.		Steel same as A1	Section same as A1.
24'-5"	Basement	8-1"φ C450-1 2-1"φ A450-1 20-1/2"φ O450-1 12" c. to c.		Steel same as A1	

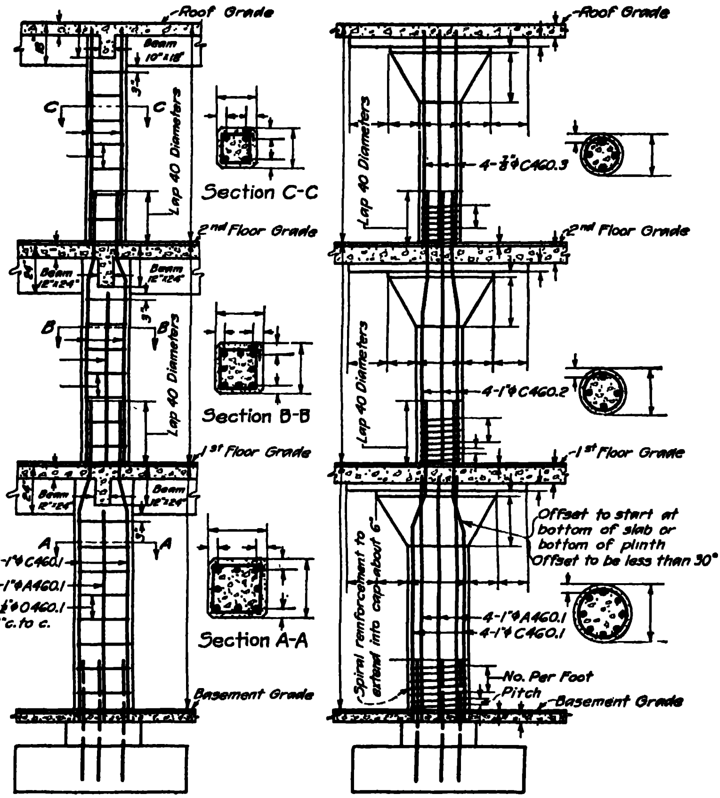
All splice rods to be lapped 40 diameters.

Fig. 26.

¼ to ½ in. being the common sizes; ⅜ in. round the most used. Corner and other special columns of irregular outlines need special attention to ties, to assure rigidity of vertical steel during pouring. The number and spacing of vertical rods may also have to be considered in making a rigid cage. Several special cases are shown in Fig. 28. Some engineers also add extra ties in rectangular columns over 24 in., as shown in this figure.

11*c*. Splices.—Horizontal joints in columns ordinarily occur at the bottom of the deepest girder, at the rough floor grade, and, in some cases, at the top of upstanding spandrel beams. The top of the rough floor is usually a splice point (except in wall columns with upstanding spandrels where the rods

should be spliced at the top of the spandrel) and good practice requires rods, to the number of those in the upper section, run up from the lower section, the distance



Notes
Make all details ¼" Scale
Dimension details as shown in sketch

FIG. 27.—Column details.

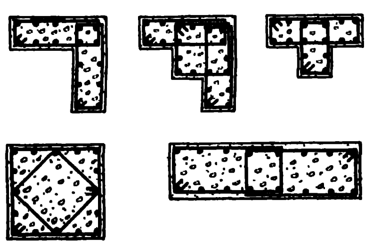


FIG. 28.

required for bond. These rods should preferably be so located that the rods in the upper section can be wired directly to them. In the case of large rods some engineers require rods to be faced and held in a sleeve. It is very difficult,

however, to so place and hold faced rods for the direct transfer of load. Where offsets are required in extended rods on account of change of column sections, they should be at least 6 in. below the splice, and offsets should not be by slopes of more than 30 deg. with the vertical.

12. Footings.—Footings are usually simple in detail. Owing to field requirements stirrups are practically never used in footings and cambered rods are used very seldom except in continuous footings. Two elevations or a plan and one elevation suffice for most footings. Figure 29 shows an unusually complete footing detail and also illustrates a schedule on the sheet with the detail.

Stubs for column steel, equivalent to the reinforcement of the lowest column section, are detailed with the footing. They should be 2 times the bond distance

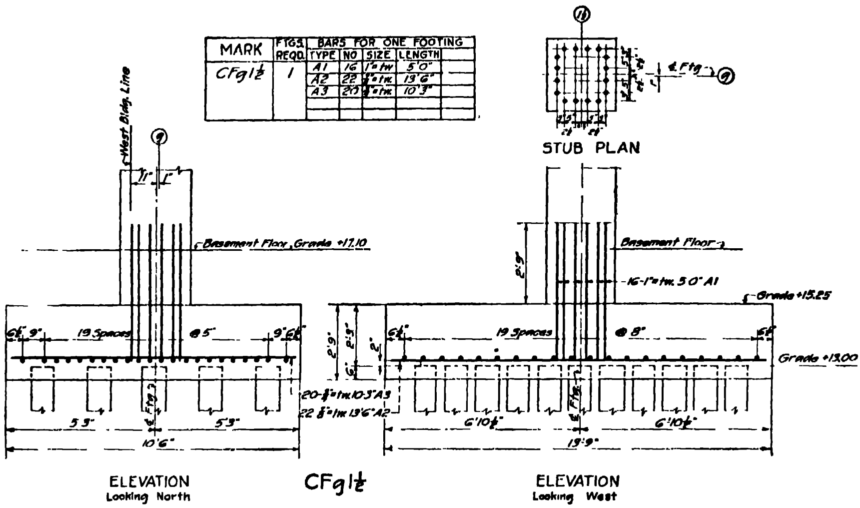


FIG. 29.

in length and project ½ this length. Small bearing plates for column rods are occasionally used in place of stubs.

Anchor bolts needed in foundations for machines or steel columns should be detailed with the footings. Sometimes they are anchored by a right-angle hook (which should be at least nine diameters in length) or a 180-deg. hook, and sometimes with a nut and washer. They are set in a pipe sleeve to allow adjustment. It is usually a convenience to have noted on foundation drawings who is to furnish anchor bolts.

13. Pits and Tunnels.—Complicated or large pits and tunnels should be separated into component parts for detailing. Floor slabs, walls, and roof slab should be given marks on an assembly or key plan, and then detailed individually with a plan or elevation and one or more sections. Small pits in floors can be detailed with the floor slab by the use of two sections, an exception to the general rule.

Long tunnels such as are often used for power house intake and discharge tunnels can be clearly and economically detailed by the use of a small scale key plan defining the extent over which certain typical cross-sections are to be used.

This method is also the general method of detailing sewer or aqueduct sections (see Fig. 30).

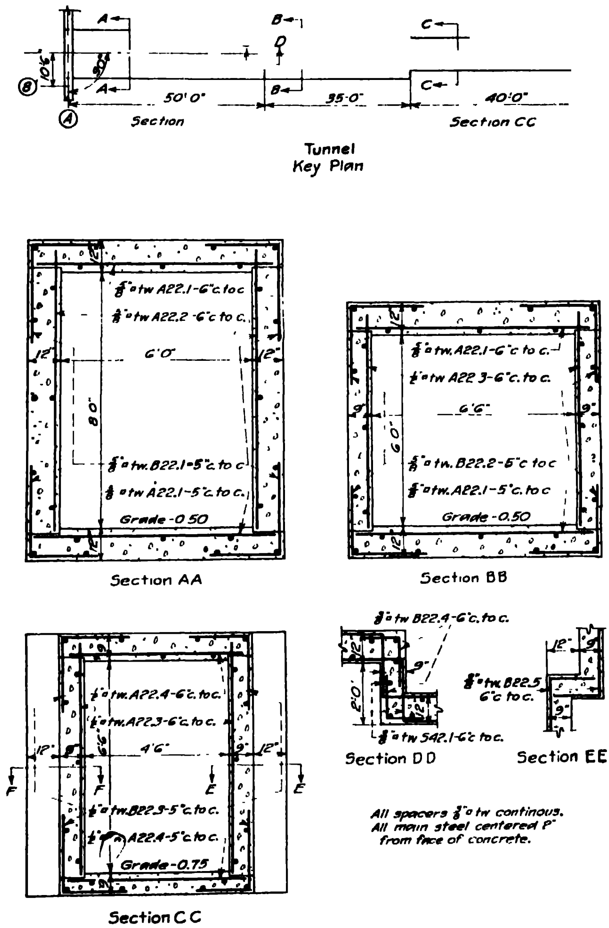


Fig. 30.

14. Turbine Foundations.—The designers of large condensing turbines and condensers leave little room for concrete supports. Clearance outlines need careful study and skilled draftsmanship. Spans are short, loads heavy, and stiffness requirements are so severe that bond and shear problems are difficult. Haunched beams and hooked rods are usually required. Foundations for the largest units should be broken up like a small building and the members detailed individually.

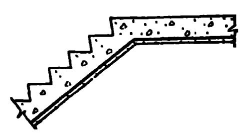


Fig. 31.

15. Retaining Walls.—Retaining walls of uniform section may be detailed by the method suggested for long tunnels. Counterfort or buttress walls should have a key plan giving a mark to wall panels, buttress or counterforts and footings. These walls should then be detailed as units.

16. Stairs.—Stairs with intermediate landings, as are usually required, make difficult details. One common mistake occurring frequently in stairs is the use

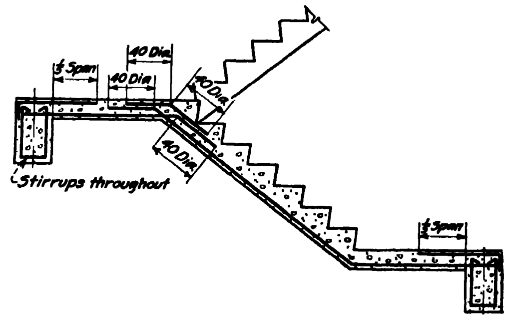


FIG. 32.—Stair reinforcement detail.

of a rod continuous around the inside of an angle, as shown in Fig. 31. This is bad since the resultant of the tension in the two runs of the rod acts only

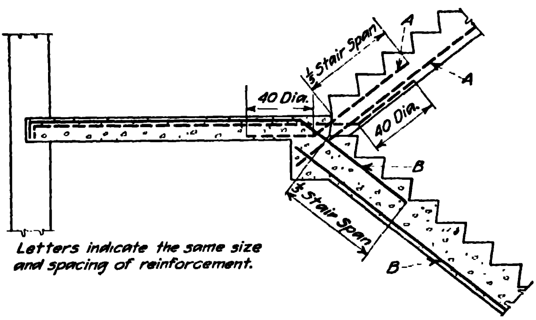


FIG. 33.—Stair reinforcement detail.

against the fireproofing. Three standard stair details are shown Figs. 32, 33 and 34.

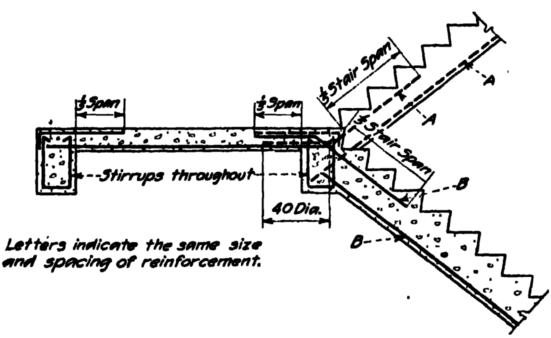


FIG. 34.—Stair reinforcement detail.

17. Arches, Dams, etc.—Large hydraulic structures such as arches and dams usually have simple reinforcement which is easily shown. Outlines may be

complicated curves but their detailing is simply a matter of defining and locating a locus curve and tying in sections to this curve sufficiently close to give required precision to the surface. Such structures are usually covered completely by the designing engineer and require little of the draftsman.

18. Construction Joints in Miscellaneous Structures.—Construction joints should be included in some details. For example, tunnels are usually poured in three parts: floor, walls and roof. If the walls are subject to pressure, it is important that they have bearing on floor and roof. Details such as those shown in Fig. 35 should be designed for shear.

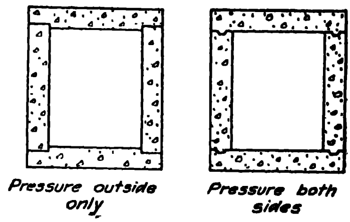


Fig. 35.

19. Spacers in Miscellaneous Structures.—Spacers in miscellaneous members need more attention than is often given them. In addition to their theoretical use for temperature, or to distribute loads, they have the important function of holding the main steel rigidly in place during the pouring of the concrete. Some practical thought of how the steel is to be placed and held, is necessary in locating spacers. For example, bands of *L*-shaped rods need three spacers at least, one in the angle and one near each end, if the band is to be held rigid.

20. Rod Splices in Miscellaneous Structures.—Construction joints must also be considered in reinforcement detailing. It is bad practice to have rods extend through a construction joint with only a small part of their length imbedded in the first pouring. This is especially bad in the case of vertical rods. They are difficult to support and very likely to be bent out of shape. As far as possible, where rods would project 6 ft., or more than half their length beyond a joint, they should extend only the bond distance and should then be spliced by other rods starting at the joint. Figure 36 shows a typical illustration of this.

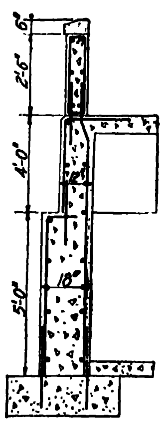


Fig. 36.

In some cases vertical rods cannot conveniently be extended up through the joint, and special short rods called stubs are used in such cases as noted in Art. **12**. Vertical rods should always start at a construction joint when possible, so that they may be set directly on the old concrete when placed (see Fig. 36). Design factors sometimes overrule the foregoing—for example, high walls often require vertical steel from top to bottom while one or more construction joints are necessary. Care must be used in all such cases to conform to design requirements and at the same time make placing as simple as possible.

21. Reinforcement Cover.—The cover over reinforcing rods—as, for example, under slab or beam rods or outside of column rods—serves to protect them from fire and weather and also to develop bond on the entire surface of the rod. Too little cover means danger from fire or sometimes moisture; too much in beams and slabs means cracks in the concrete below. A ½-in. clear cover for slabs 4 in. thick, with rods not over ½ in., and a small fire risk, is the minimum. A 1-in. clear cover is about the maximum used for ordinary slabs. For beams and girders

1½ to 3 in. is used according to the importance of member and the fire risk. In columns, from 1 to 4 in. is used.

22. Shop Bending.—Every concrete detailer should be familiar with reinforcement in place in the forms, and as far as possible with the process of bending and placing. With odd-shaped rods, bending difficulties should receive careful consideration. Radius bends larger than 4 in. are difficult and expensive to obtain. Small bends are made around pipe sleeves or blocks. An exception to this is spirals, and circles such as are used in the S. M. I. flat slab system. Special machines in well equipped yards take care of these economically. It should be remembered that on large rods a precision on offsets closer than 1 in. is difficult to obtain. Details should not, therefore, be made which require such precision. Angles in rods, except parallel offsets, cannot be made with great precision, and accurately bent rods will spring in handling unless very heavy compared to their length. Details, therefore, in which a slight variation in the angle of the rod would cause trouble should not be made. For example, Fig. 37 is bad. The detail should be as shown in Fig. 38. Cambers, in slab rods (⅝ in. or under) may be as many as four within reason. With larger rods, as used in beams, not more than two cambers should be used in a single rod.

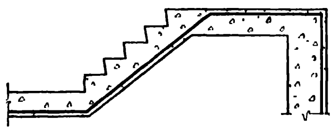


FIG. 37

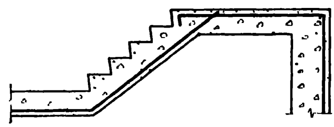


FIG. 38.

23. Field Assembly.—Bending may be done in the contractor's yard or on the job. In either case the bent rods tagged with type numbers are stored, usually by sizes, in racks, or, if space is available, on the ground opposite the place where they are to be used.

Column steel is usually assembled on horses and placed as a unit. Beam steel may be handled in this way but where beams intersect over the columns at least part of them must be assembled in the forms. Beam rods hooked into spiraled columns should be avoided on account of the difficulty of placing. When beam steel is assembled in the form, loop rods and stirrups are first placed, then the bottom steel and finally any loose negative bars.

In slabs, assembly by units is generally impracticable except in some types of flat-slab construction. Spacers are laid down preferably on suitable chairs, and the main reinforcement is placed on them and wired.

In wall reinforcement, vertical rods are usually placed first and then the horizontal rods tied to these. In slab and wall reinforcement, deformed rods are held more rigidly in place by wiring than plain rounds, which have a tendency to slip through the ties.

24. Rod Sizes.—In the choice of rods there are a few points to be considered. In the first place, rods of ¾ to 1-in. diameter have base price—that is, the lowest price per pound—and other things being equal, are the cheapest. One-sixteenth-inch sizes with the possible exception of $^5\!/_{16}$ in. square are not commercial sizes. Three-eighths to 1¼ in. are the readily available sizes. Good detailing limits the sizes in a single unit to two adjacent sizes and limits as far as possible the number

of sizes on the whole job, to avoid confusion. Squares and rounds should not be used together.

25. Schedules.—Rod schedules are sometimes made as a table on the drawing itself, but best practice is a separate sheet which is commonly about 12 × 21 in. This size is easily handled in the yard.

Type numbers must be considered in connection with rod schedules. Letters for various types are convenient. Those shown in Fig. 39 are in successful use.

The individual rods are given separate numbers and care is necessary to avoid duplication of numbers. The use of the number of the sheet on which the detail of the rod occurs, as part of the type number, is open to the objection of giving

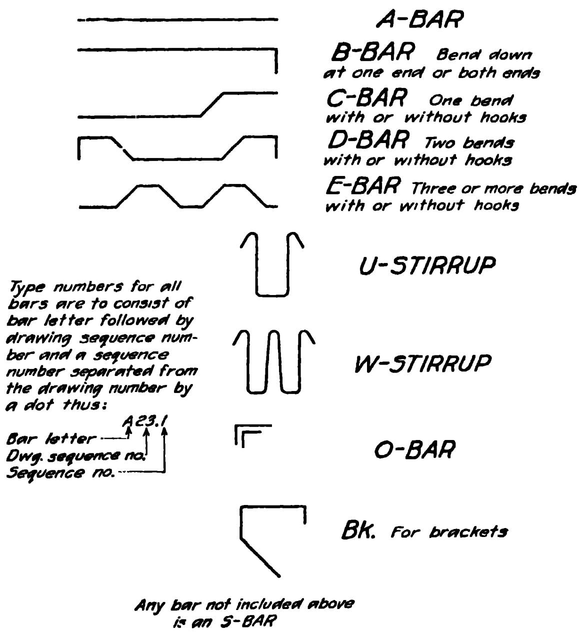


FIG. 39.– Bar types.

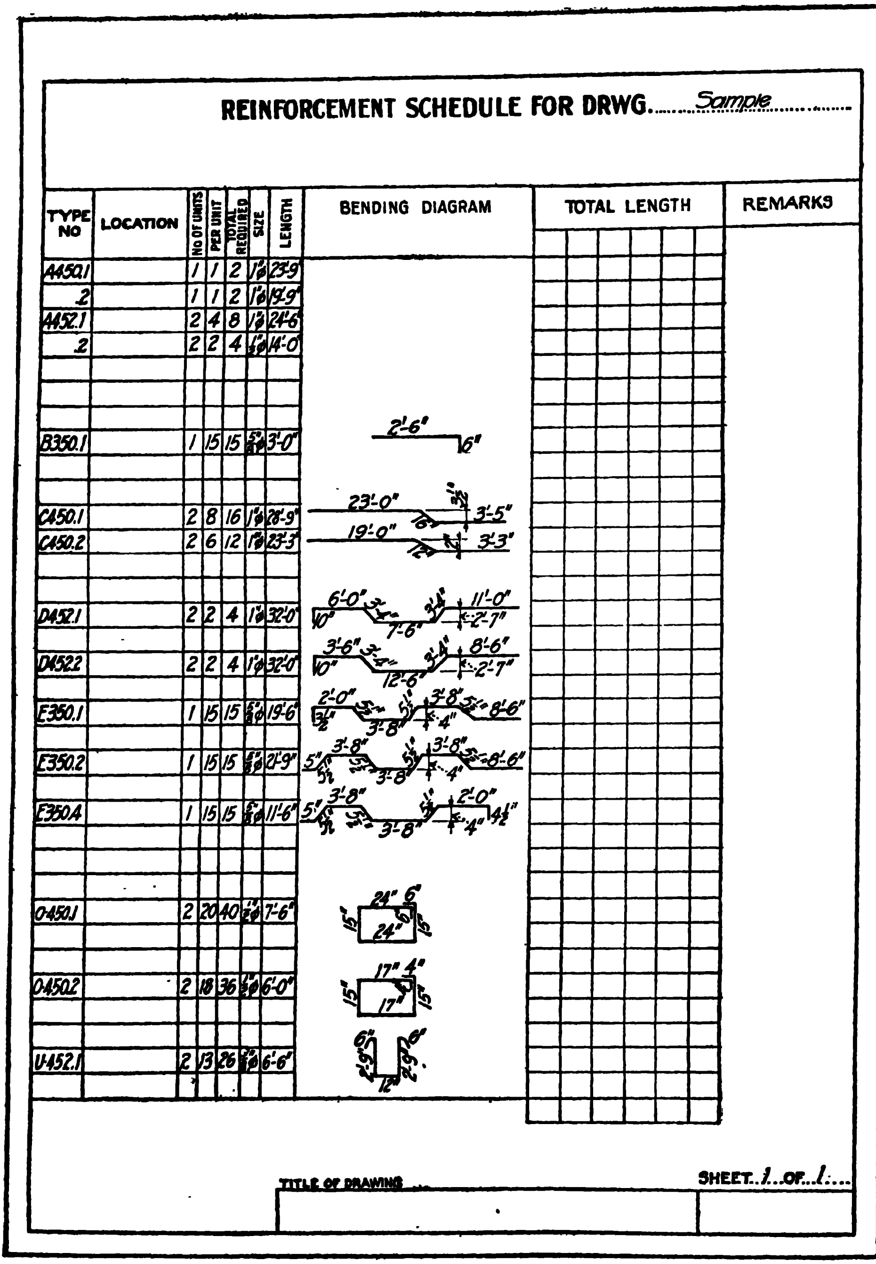
a long number, but it automatically avoids duplication. This is illustrated on the schedules given.

Two types of schedule are shown (Figs. 40 and 41). The first giving a sketch (not to scale) of each type is particularly suited to irregular reinforcement or to inexperienced bending gangs and is the simpler in the field. For ordinary building work or any other work with a reasonable limit to the number of types, a schedule like Fig. 41 is more economical of time in the office.

The space at the right in each schedule shown is for estimate, weight, shipping and cutting notes, etc.

Blank schedules can be printed on bond paper to advantage.

Schedules include, of course, the lengths of bar in each run—that is, the distance between angles and the total length. In getting the total length of rods it is customary to make all rods even ¼ or ⅛ ft., allowing the odd inch or two at one end. In the case of bent rods it is necessary in computing length to allow for bends.



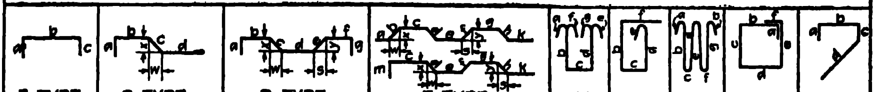
REINFORCEMENT SCHEDULE FOR DRWG. Sample

TYPE NO	LOCATION	No OF UNITS	PER UNIT	TOTAL REQUIRED	SIZE	LENGTH	BENDING DIAGRAM	TOTAL LENGTH	REMARKS
A450.1		1	1	2	1"φ	23'-9"			
.2		1	1	2	1"φ	19'-9"			
A452.1		2	4	8	1"φ	24'-6"			
.2		2	2	4	½"φ	14'-0"			
B350.1		1	15	15	⅝"φ	3'-0"	2'-6" 6"		
C450.1		2	8	16	1"φ	28'-9"	23'-0" 3'-5"		
C450.2		2	6	12	1"φ	23'-3"	19'-0" 3'-3"		
D452.1		2	2	4	1"φ	32'-0"	10" 6'-0" 7'-6" 11'-0" 2'-7"		
D452.2		2	2	4	1"φ	32'-0"	10" 3'-6" 12'-6" 8'-6" 2'-7"		
E350.1		1	15	15	⅝"φ	19'-6"	2'-0" 3'-8" 3'-8" 4" 8'-6"		
E350.2		1	15	15	⅝"φ	21'-9"	5" 3'-8" 3'-8" 3'-8" 4" 8'-6"		
E350.4		1	15	15	⅝"φ	11'-6"	5" 3'-8" 3'-8" 2'-0" 4" 4½"		
O-450.1		2	20	40	½"φ	7'-6"	24" 6" 15" 24" 15"		
O-450.2		2	18	36	½"φ	6'-0"	17" 4" 15" 17" 15"		
U-452.1		2	13	26	⅜"φ	6'-6"	6" 2'-9" 12" 2'-9" 6"		

TITLE OF DRAWING ..... SHEET 1 OF 1

FIG. 40.

REINFORCEMENT SCHEDULE FOR DRAWING____



TYPE NO.	NO. OF UNITS	PER UNIT	TOTAL REQUIRED	SIZE	LENGTH	BENDING DIMENSIONS														
						a	b	c	d	e	f	g	h	k	m	s	w	x	y	SPECIAL BENDING
A450.1	1	1	2	1"φ	23'9"															
.2	1	1	2	1"φ	19'9"															
A452.1	2	4	8	1"φ	24'6"															
.2	2	2	4	½"φ	14'0"															
B350.1	1	15	15	⅝"φ	3'0"	6"	2'6"													
C450.1	2	8	16	1"φ	28'9"		23'0"	16"	3'5"									3½"		
.2	2	6	12	1"φ	23'5"		19'0"	12"	3'3"									2"		
D452.1	2	2	4	1"φ	32'0"	10"	6'0"	3'4"	7'6"	3'4"	11'0"							2'7"	2'7"	
.2	2	2	4	1"φ	32'0"	10"	3'6"	3'4"	12'6"	3'4"	8'6"							2'7"	2'7"	
E350.1	1	15	15	⅝"φ	19'6"			2'0"	5½"	3'8"	5½"	3'8"	5½"	8'6"	3½"			4"	4"	
.2	1	15	15	⅝"φ	21'9"	5"	5½"	3'8"	5½"	3'8"	5½"	3'8"	5½"	8'6"				4"	4"	
.4	1	15	15	⅝"φ	11'6"			2'0"	5½"	3'8"	5½"	3'8"	5½"	5"	4½"					
O450.1	2	20	40	¼"φ	7'6"	6"	24"	15"	24"	15"	6"									
.2	2	18	36	¼"φ	6'0"	4"	17"	15"	17"	15"	4"									
U452.1	2	13	26	⅜"φ	6'6"	6"	2'9"	12"	2'9"	6"										
3452.1	1	3	3	¼"φ	7'0"		See	Sketch												

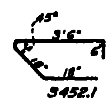


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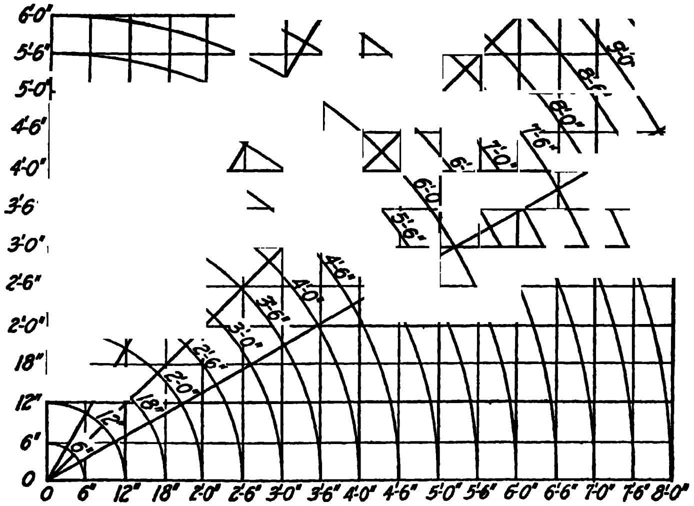
FIG. 41.

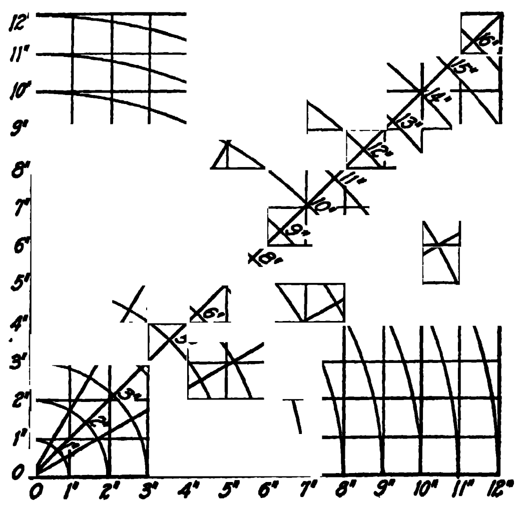
One inch for a 90-deg. bend is usually safe. The dimension of one hook on rods with many bends like hoops or stirrups may be left off, but the total length should



Use for Beam Rods

Fig. 42.

be sufficient to insure a satisfactory hook. The curves in Figs. 42 and 43 are convenient for finding camber lengths. At the intersection of the vertical line for the



Use for Slab Rods

Fig. 43.

camber height, with the horizontal line for the horizontal projection of the camber, read the slope lengths with the arcs as a scale. For 30 or 45-deg. cambers the slope

distance can be read at the intersection of either height or distance with the corresponding slope line.

26. The Concrete Detailer.—The capable concrete detailer must have knowledge and ability equal to, or greater than, that of the ordinary steel detailer. He must know enough of design to know where tension exists and to locate camber point and stirrups in ordinary work. He must be sufficiently familiar with practice to allow sufficient fireproofing; to detail separators, spacers, stubs and hooks; and to make complete and practical bending schedules.

He must know that concrete reinforcement details are primarily diagrams. Precise scale is secondary to clear picturing.

He must list rods once, and once only and know where to list them.

Lastly, of course, he must do his work in a reasonable time. The following figures for 27 × 37-in. drawings of ¼ in. = 1-ft. plans and ½ in. = 1-ft. details are averages for one complicated powerhouse building. They should be bettered in simple building work. The time includes squad foreman, but no one above, and includes detail design, drawing, checking and tracing.

Hours per drawing:

Design	20 (includes checking of design)
Detailing	40
Tracing	20
Checking	15 (of drawing only)
Revision	3

SECTION 12

ESTIMATING CONCRETE COSTS

ESTIMATING UNIT COSTS

By Leslie H. Allen and C. M. Chuckrow

There are four principal items that enter into the cost of reinforced concrete construction that should be separately considered and priced: (1) The concrete, (2) temporary formwork or centering, (3) steel reinforcement, and (4) finishing of the exposed surfaces. The cost of concrete work cannot be accurately estimated unless these four items are each analyzed separately.

The costs given in this chapter are based on the cost of concrete materials in the larger cities and the rates paid to labor in these cities (namely: Carpenters \$1 per hour, laborers 50 cents per hour, carpenter foreman \$10 per day). It is difficult to compute labor costs by estimating only the time of mechanics and laborers, as on any construction job there is a great deal of supplementary labor engaged beyond that spent directly on the operations of the job. The costs given, therefore, include these and the figures given can readily be adjusted, where labor rates differ, by varying the unit cost given in proportion to the labor rates paid in the locality.

1. Estimating Unit Cost of Concrete.—In analyzing the cost of the first item, "concrete," it is necessary to consider: (1) The materials—cement, sand, stone and water; (2) the labor of unloading, mixing and placing of these materials; and (3) the contractor's plant and necessary tools.

1a. **Materials**—*Cost of Cement.*—Cement in carload lots is figured at \$2.50 per bbl. f.o.b. the job. If the cement is delivered in paper bags, there is an extra charge of 10 cts. per bbl. If delivered in cotton sacks, the extra charge is 40 cts., but this 40 cts. is refunded by the mills if the sacks are returned in good condition. Most contractors prefer to buy cement in cotton sacks and return the sacks when empty.

The cost of testing cement runs between 2 and 4 cts. per bbl. Contractors and engineers usually arrange for the whole shipment of cement being sampled and tested by a commercial testing laboratory. The usual charge per carload is about \$5 or \$6. The quality of cement today is so uniform that tests on every shipment are no longer deemed essential by many purchasers.

It usually costs about 8 cts. per bbl. to unload cement and place it in temporary storehouses. If there is no railroad track to the site of the construction job, the cost of teaming must also be added and at a distance of one mile this would cost about 10 cts. per bbl.

In addition, the contractor must figure the cost of handling and returning empty sacks, the freight on same, and the loss of a few damaged or torn ones. It is usually safe to estimate these items as averaging 3 or 4 cts. per bbl.

A tabulation of these costs, therefore, will show the cost of cement per bbl. ready for use in the mixer as follows:

Cement f.o.b. cars at point of destination	$2.50	
Cotton sacks	0.40	
Total cost of cement at job	$2.90	per bbl.
Deduct for empty sacks	0.40	
	$2.50	
Add cost of testing	0.03	
Cost of unloading	0.08	
Cost of teaming, if any	0.10	
Freight and loss on empty sacks	0 03	
Total net cost of cement ready for use at the mixer	$2.74	per bbl.

The quotations given by the cement companies always include freight and sacks. To arrive at the net cost it is necessary to deduct for the sacks and add for the supplementary items as shown above.

Cost of Sand.—It usually costs about 80 cts. per cu. yd. to dig sand and load on teams or cars. If it has to be screened or washed, this will add from 10 to 25 cts. to the cost. Teaming or freight will vary according to the length of haul but will usually bring the cost of the sand ready for use up to $1.75 per cu. yd., delivered to the job. If it comes by rail, there should be added 25 cts. per cubic yard for unloading from the railroad cars, and, if it comes by team, the cost of receiving it and trimming stock piles will usually average about 10 cts.

Cost of Coarse Aggregate.—Crushed stone will usually cost from $1.35 to $1.60 per ton at the crusher. Teaming or freight on a short haul may run from 50 to 75 cts. per ton so that the cost of crushed stone, delivered to the job, generally varies between $1.85 and $2.30 per ton. If the stone has to be unloaded from railroad cars at the job, the labor of doing so will cost about 35 cts. per ton.

If screened gravel of suitable size and quality is available for use, it can generally be obtained for about $2 per cu. yd., a considerable saving on the price of crushed stone. In comparing the price of gravel and crushed stone, 1 cu. ft. of crushed stone may be considered as weighing 100 lb.

In large bridges, dams and other structures built in places that are difficult of access, the cost of teaming materials may be much higher than those given. It is often necessary to figure on the cost of a crushing plant for the supply of stone. The cost of installing and operating a small temporary plant is considerably greater than that of running a large permanent plant and the contractor, therefore, prefers to buy his crushed stone if possible rather than crush his own.

1b. Labor.—The labor cost of mixing and placing concrete varies considerably according to the conditions of the job and the nature of the work. Large masses of concrete such as in heavy foundations and retaining walls or dams can be placed with much less labor than the concrete in the slender columns

of a lofty building or in thin floor slabs, arched ribs of bridges, etc. The cost of mixing, however, should not vary greatly.

Mixing.—On a job where the plant is laid out with a view to maximum labor efficiency, the cost of mixing concrete should be between $1 and $1.25 per cu. yd. This will cover the operations of loading wheelbarrows with sand and stone and discharging them into the mixer, bringing cement from the shed and placing in mixer, and the engineer's time running the mixer and discharging it. On large jobs it often pays to build sand and stone bins and measuring hoppers so that the mixer can be charged automatically at a very considerable saving in labor. Against this labor saving, however, must be figured the cost of installing the bins and the conveyors for filling them and in the long run it is usually found that this expense is very little less than the labor saving effected, the principal advantage of such plant installation being the additional speed that it is possible to gain on the job with fewer men to control.

Placing.—The cost of placing concrete will cover the labor of receiving the concrete from the discharge on the mixer in wheelbarrows, chutes or cars and transporting it to its place in the building, spreading, spading and screening it in place in the forms. On average work in a factory building, this will cost about 90 cts. per cu. yd. Columns and thin walls will require more labor as more spading is needed to get a smooth surface on the exposed faces and the cost may be as much as 50 per cent more. Large masses of concrete, such as dams and thick retaining walls will cost considerably less, especially if the distributing plant is well laid out and the equipment is good.

For approximate purposes the following schedule of labor costs for mixing and placing concrete will be of service:

Concrete in footings	$2.00 per cu. yd.
Concrete in floor slabs 4½ in. thick or less	2.00 per cu. yd.
Concrete in floor slabs 5 in. thick or more	1.50 per cu. yd.
Concrete in columns and thin walls	2.50 per cu. yd.
Concrete in walls 18 in. thick or more	1.60 per cu. yd.
Concrete in dams and heavy retaining walls	1.25 per cu. yd.

In massive concrete work such as dams it is usual to allow the use of large stones to reduce the cost of construction. These stones should be placed not less than 6 in. apart and at least 12 in. away from the face of the work. From 20 to 40 per cent of the volume of a massive pier can be composed of large stone used in this way. The cost of placing these stone—or "plums" as they are often called—should not exceed $1.25 per cu. yd.

1c. Plant and Tools.—One of the most difficult items to estimate on a construction job is the cost of tools, plant and supplies. This will vary a good deal according to the size of the work. The tools and plant that are used for mixing and placing the concrete are properly a part of the cost of the concrete and should not be ignored when estimates on the construction are being made. It is very rarely that this item is estimated high enough, one of the most common sources of loss on a job being the excessive cost of tools and supplies needed to keep the job running. Shovels and wheelbarrows wear out. Mixers require frequent repairs. The erection of mixer, tower and chutes is an expensive item

and these items require constant attention all of which should appear in the cost of the concrete work.

Although it might seem that there should be very little difference between the cost of plant required for a job of 3,000 cu. yd. of concrete and a job containing 6,000 cu. yd., it is found in actual practice that there is a fairly close relation between the yardage and the cost of plant. Therefore, up-to-date contractors are dividing the total cost of plant, tools and supplies by the yardage of concrete in order to get a unit figure to add to their price.

Assuming a building containing 5,000 cu. yd. of concrete, the cost of plant, etc., will run about as follows:

Building mixing platform, hoisting tower, setting up mixers, engines, hoist platforms and bins, etc. would cost about $2,500. Depreciation and repairs on mixers and other machinery would be about $1,500. (Some contractors figure a daily rental on such items to cover this cost.) Loss or depreciation of shovels, wheelbarrows, picks and other small tools, rope, etc. will be about $1,500. Power expense, whether electric or coal is used, $1,000; the temporary buildings such as cement shed, tool house, shanties, etc., about $1,000. This gives a total of $7,500, which is equal to $1.50 per cu. yd. on the cost of the concrete. Jobs below 3,000 cu. yd. may run up to $2.00 per cu. yd. and jobs of from 8,000 to 10,000 cu. yd. may run as low as $1.20.

1*d*. Summary.—In estimating the quantity of materials required for a yard of concrete it should be borne in mind that conditions on construction work are very different from those of the technical laboratory and there must always be a certain amount of waste of material. Tables are given in engineers' handbooks showing the amount of cement, sand and stone required in accordance with the percentage of voids in the aggregate. The estimator cannot foretell what this percentage of voids will be in the material used on the job as it does not pay to screen and regrade aggregates accurately on a big construction job and the product of a gravel bank may vary considerably from day to day. It is usual, therefore, to allow ½ cu. yd. of sand for every cubic yard of concrete and 9⁄10 cu. yd. of crushed stone. (Crushed stone is usually sold by weight, and 1 cu. ft of crushed stone will weigh about 100 lb.) This allowance will take care of a certain amount lost at the bottom of the stock pile.

In figuring cement, allowance should be made not only for the cement placed in the mixer but for the small amounts used in patching voids and rubbing down, etc. It is best to allow 2 bbl., of cement per cu. yd. for 1:1½:3 mix; 1⅔ bbl. for 1:2:4 mix; 1.4 bbl. for 1:2½:5 mix; and 1.2 bbl. for 1:3:6 mix.

To summarize, therefore, the cost of 1 cu. yd. of concrete on a job containing 5,000 cu. yd. of reinforced concrete work in floors, columns, walls and foundations, may be estimated as follows:

1⅔ bbl. cement	**$2.74**	**$5.48**
½ cu. yd. of sand	**1.75**	**0.87**
1¼ tons stone	**2.00**	**2.50**
Labor		**2.00**
Plant, power and supplies		**1.50**
Liability insurance on payroll		**0.10**
Total		**$12.45 per cu. yd.**

On a large typical bridge job the following example is given: Abutments and piers—1:2½:5 mix:

1.4 bbl. cement	$2.74	$3.84
½ cu. yd. of sand	1.75	0.88
1¼ tons crushed stone	2.00	2.50
Labor		1.25
Plant, power and supplies		1.30
Liability insurance on payroll		0.06
		$9.83 per cu. yd.

Cyclopean concrete for abutments and piers—1:2½:5 mix with 25 per cent large stone:

7½ cu. yd. of concrete	$9.83	$73.73
2½ cu. yd. labor placing large stone (assuming stone from excavation is used)	1.50	3.75
Cost of 10 cu. yd. of concrete and rock in place		$77.48
Average cost		$ 7.75 per cu. yd.

If the large stone is to be purchased for the purpose, the cost of this should be added

2. Estimating Unit Cost of Temporary Formwork.

2a. **Considerations Involved.**—Forms for building work should be measured by the square foot of surface—all surfaces that touch the concrete being measured—and the price will vary according to the labor required in erecting, bracing and stripping. The item to be priced is the operation of supporting wet concrete and the material used (that is, the lumber, nails and oil) does not vary in the same way as the labor involved. Formwork is not usually shown on drawings of the structure that the estimator figures from, and it is very seldom that two competing contractors use the same amount of lumber or the same methods in matters of detail in constructing forms.

Contractors, therefore, are making a general practice of estimating labor on formwork by the square foot and of computing the cost of lumber, nails and oil on the same square foot basis. This procedure gives more accurate results than where an attempt is made to estimate in detail the amount of lumber required.

The labor required for different kinds of formwork varies very greatly and it is usually advisable, therefore, to figure separately formwork for columns, walls, footings, slab floors, and beam and girder floors.

2b. **Materials.**—A good average for general use or for estimating cost of lumber, nails, oil and wire and other form material for a simple factory building will be between $4 and $5.50 per 100 sq. ft., $4.50 being the customary figure. Each contractor, however, will be guided by his own experience in these matters as these costs vary considerably according to the efficiency of the job organization and method of form design used.

On bridge formwork and other complicated structures, figures as high as $5.50 to $6 are sometimes necessary.

2c. **Labor**—*Floor Forms.*—The labor on floor forms in the building includes the operations of unloading lumber, making panels, setting up and bracing posts, laying joists and girts on top of the posts, and placing the floor panels on them, and stripping the formwork.

The labor of making form panels costs about 4½ cts. per sq. ft. Assuming that these are used three times, 1½ cts. per sq. ft. on the whole area to be formed would be the figure to use in estimating. The cost of labor in erecting studs and bracing would average about 5 cts. per sq. ft. and the cost of putting on joists and laying on panels about 5 cts. more, giving the entire labor cost for form making and erection at 11½ cts. per sq. ft. The cost of stripping will run about 1½ to 2 cts., giving a total labor cost of 13 or 13½ cts. per sq. ft. On a small or irregular building, costs will be very much higher, but for the large factory these costs are sometimes bettered. If the height from the floor to the ceiling is more than 12 ft., the studs will be longer and need extra bracing. The extra cost will depend upon the increase in height and may run from 2 to 5 cts. per sq. ft.

Beam Forms.—In like manner the cost of forms for beams is arrived at by computing first the cost for making beam bottoms and beam sides, then of erecting them on the temporary framing that supports the floor slab and then of stripping them. The cost of making up will be about 6 cts. per sq. ft. However, it is not safe to figure on using these forms more than twice, giving an average cost of 3 cts. per sq. ft. Erection of these forms costs about 10 cts. and stripping 2 cts. There will be a total cost of 15 cts. per sq. ft. for the beam and girder forms, assuming that the total cost of the posts has already been estimated for the slab forms. For beams that are isolated with no slabs, the cost of posts and bracing should be added. If beams and girders have haunches at the ends, 50 cts. should be added for each haunch.

Column Forms.—The labor connected with column forms consists of the operations of making panels, erecting pane s and placing, bolting and stripping. The cost of making up the panels will be about 6 cts. per sq. ft. It is generally safe to assume they can be used three times, giving a cost of 2 cts. per sq. ft. of formwork. Erecting, plumbing and bolting will be about 14 cts. and stripping 2½ cts. per sq. ft. giving a total labor cost of 18½ cts. per sq. ft. on columns of average height.

2d. Summary.—The cost per square foot of formwork on a simple reinforced concrete building is summarized as follows:

Forms to floor slabs:

Lumber, nails, oil, etc.	$0.045
Labor making panels	0.015
Labor erecting studs and bracing	0.05
Labor laying panels	0.05
Labor stripping	0.015
Insurance	0.006
Total	$0.181

Forms to beams and girders:

Lumber, nails, oil, etc.	$0.045
Labor making	0.03
Labor erecting	0.10
Labor stripping	0.02
Insurance	0.007
Total	$0.202

Forms to columns:

Lumber, nails, oil, etc.	$0.045
Labor making panels	0.02
Labor erecting, plumbing and bolting	0.14
Labor stripping	0.025
Insurance	0.01
Total	$0.24

Forms to footings:

Lumber, nails, oil, etc.	$0.045
Labor making and erecting	0.10
Labor stripping	0.015
Insurance	0.005
Total	$0.165

Forms to walls:

Lumber, nails, oil, etc.	$0.045
Labor making	0.03
Labor erecting and plumbing	0.11
Labor stripping	0.015
Insurance	0.005
Total	$0.205

3. Estimating Unit Cost of Steel Reinforcement.—The cost of steel reinforcement varies considerably from time to time and must be verified from quotations from steel companies on every job. On small jobs it is customary to buy steel cut to exact lengths out of local warehouse stock, but on large jobs, requiring more than 40 tons, it is well to order steel from mill delivery, provided that the building has time enough to wait for it.

Assuming that steel from mill shipment in carload lots costs at the present time $2.25 per 100 lb. base and that the freight rate on a 500-mile haul is about 35 cts., we have a total of $2.60 per 100 lb. for steel bars f.o.b. cars at job.

When steel is bought from mill shipment, it is the general practice to buy sufficient steel for footings and foundations from warehouse stock in order to get the job started, and under ordinary conditions, the mill shipments should arrive in time for use in the superstructure. Steel bought from warehouse is usually bought cut to exact lengths, and the cost is usually from 40 to 60 cts. per 100 lb. above mill prices. In times of great scarcity this spread may be considerably larger.

The cost of steel bars varies according to the size of the bar. Bars ¾ in. in diameter or larger are sold at the base price. Smaller sizes take a higher rate as follows:

⅝ in.	base plus 5 cts.
½ in.	base plus 10 cts.
⅜ in.	base plus 25 cts.
¼ in.	base plus 50 cts.

This differential is an important factor in design as well as in estimating. For example, assume a floor 200 by 100 ft., having a slab 6 in. thick and an area

of steel per square foot of 0.462 sq. in. The total weight of steel required (allowing for laps) would be 33,301 lb.

Cost of ½-in. square bars
  6½ in. on centers, 33,301 lb. @ $2.69 per 100 lb. = $895.80
Cost of ⅜-in. round bars
  3 in. on centers, 33,301 lb. @ $2.84 per 100 lb. = $945.75

Thus there is a difference of $49.95, or 25 cts. per 100 sq. ft. in favor of the ½-in. bars in addition to the saving in labor required in placing a smaller number of bars.

The cost of unloading steel and piling it on the job is about $1.50 per ton, and if it has to be teamed from the freight yards to the site of the work, the cost of doing this will be $1 per ton and upward.

The cost of bending and placing steel in a building will vary according to the amount of work that is done. Thus, placing steel bars ⅝ in. in diameter in a floor slab will cost about $13 per ton. If the bars have to be bent up at the end $6 per ton should be added. Bending and placing steel bars and stirrups in beams will cost from $22 to $24 per ton. Wiring up and placing steel bars in columns and placing hoops around them will cost from $24 to $28 per ton. Placing ½ and ⅜-in. steel in walls will cost from $20 to $30 per ton.

These prices are sufficient to include the cost of wire, tools for bending, etc. Twenty-five dollars per ton is a good average price for labor on steel reinforcement all through the job.

The cost of steel reinforcement in a reinforced concrete building may be estimated as follows:

Steel from mill @ $2.75 (ave.) per 100 lb.	$55.00 per ton
Unloading, teaming and piling	2.50
Labor, bending and placing	25.00
Workmen's compensation and liability insurance	2.50
Total	$85.00 per ton

Extra allowance should be made for steel from stock.

4. Estimating Unit Cost of Surface Finish.—One hundred square feet of granolithic finish laid 1 in. thick in the proportion of one of cement, one of sand, and one of fine crushed stone will require 1 bbl. of cement, 4 cu. ft. of sand and 4 cu. ft. of crushed stone. This may, therefore, be estimated as follows:

1 bbl. cement @ $2.74 per bbl.	$2.74
4 cu. ft. of sand @ $1.75 per cu. yd.	0.26
400 lb. fine crushed stone @ $2.75 per ton	0.55
Labor mixing and placing	1.50
Finishers' time trowelling surface	4.00
Insurance	0.25
Total cost per 100 sq. ft.	$9.30

If the surface of the concrete has to be cleaned off with acid or sand blasting, this will cost from 2 to 3 cts. per sq. ft. additional.

The exterior surfaces of concrete columns and beams are frequently rubbed smooth with carborundum stone, using a little water and cement. The cost of this work, including hanging swing stages for the finishers, will be from 4 to 6 cts. per sq. ft.

For ornamental effect, external surfaces are sometimes picked with a pointed tool or Crandall hammer. The cost of this runs between 6 and 15 cts. per sq. ft., according to quality of work desired.

ESTIMATING QUANTITIES

By Leslie H. Allen

5. Systematic Procedure Advisable.—The operation of estimating quantities is that of calculating (from plans supplied) quantities of labor and material which go to make up the completed building. This is usually called *taking off* or *scaling*. It should be done quite independently of the pricing or the arithmetical work of extending the quantities to obtain the totals of the quantities of work. The secret of accurate, speedy taking off is to be found in a systematic way of going about the work. No printed forms, tables, or special rules for taking off will insure against error; the surest way of making an accurate estimate is to have a good system to work on and a clear and easily followed way of setting down the items. A good method is to use plain squared paper 8 by 10½ in., ruled in nine columns. In the first column is placed a description of the items measured; the next four columns are for the number, length, width, and height of the members of the building; the next two are for arithmetical calculations and totals; and the last two for unit and total price. It is very important to keep length, breadth, and height in the same order in every item. Each can then be readily identified. In taking off a reinforced-concrete building, start with the structural members in the order in which they are built; that is first take concrete footings, then columns, then floor slabs, then beams and girders, then curtain walls and partitions, then cornice, and then stairs and landings. Take all the concrete first, one item at a time and complete it. When all the concrete is taken off, proceed to take off the forms, and after that take off the reinforcement, and then the finish to the surfaces. After that take off excavation, windows and doors, plastering, roofing and other incidental items necessary to complete the cost of the building.

In putting down the dimensions, it is well to put a note identifying each item, thus:

Concrete columns:

Basement,	(mark A)	$5 \times 1\frac{1}{2} \times 1\frac{1}{2} \times 10$
	(mark B)	$4 \times 1\frac{1}{2} \times 1\frac{2}{3} \times 10$
	(mark C)	$7 \times 1\frac{1}{2} \times 1\frac{1}{3} \times 10$
First floor,	(mark A)	$5 \times 1\frac{1}{3} \times 1\frac{1}{3} \times 10$
	(mark B)	$4 \times 1\frac{1}{3} \times 1\frac{1}{2} \times 10$
	(mark C)	$7 \times 1\frac{1}{3} \times 1\frac{1}{8} \times 10$

It takes a little more time to do this, but it is well worth the labor, and any item can be readily identified afterward; if an item is left out in error, the omission

can be more easily detected. It is good practice to put all dimensions in feet and fractions. Some estimators work in feet and inches and some in feet and decimals. There seems to be less chance for error in using fractions, but this is a matter of individual judgment.

6. Rules for Measurement of Concrete Work.—The following rules should govern the measurement of concrete work:

All concrete should be measured by the cubic foot or cubic yard, and in all cases forms should be measured separately. All concrete should be measured net as placed or poured in the structure or building; an excess measurement of concrete should never be taken to pay for the cost of forms or extra labor in placing. All openings and voids in concrete should be deducted, but no deduction should be made for steel reinforcement, I-beams, bolts, etc. embedded in the concrete, unless they have a sectional area of more than 1 sq. ft. No deduction should be made for chamfered, beveled, or splayed angles to columns, beams, and other work.

For beams and girders it is usual to show on the plan the depth of concrete from the top of the slab. Thus, if the quantity of concrete for the slab has been taken right across the floor, it will be necessary to measure only the extra concrete below the slab in taking off beam and girder quantities. For example, in a floor 6 in. thick having 12 by 30-in. girders, the concrete to take off for the girders should be considered as 1 ft. wide by 2 ft. deep, since the other 6 in. is measured with the slab.

Each class of concrete having a different proportion of cement, sand, or aggregate should be measured and described separately. Concrete in the different members of a building or structure should be measured and described separately according to the accessibility, location, or purpose of the work; concrete in floor slabs should be measured and priced separately from columns or walls, and so on. Concrete with large stones and rocks embedded in same (cyclopean masonry) should be measured as one item and described according to the richness of the mix and the percentage of rock in same. Concrete in stairs should be measured by the lineal foot of tread and it is usual to include surface finish with same in this case, as it forms such a small item in the cost.

7. Estimating Amount of Formwork.—Forms should be measured in square feet, taking the area of the surface of the concrete which is actually touched by the forms or falsework. Forms should in all cases be measured and described as a separate item and never included with the concrete. No deduction should be made in measurement of surface of concrete supported by forms because of forms being taken down and re-used 2 or 3 times in the course of construction.

It is not necessary to measure struts, posts, bracing, bolts, wire ties, oiling, cleaning, and repairing forms, as these should be covered by the price put on the square foot measurements.

Forms to the different parts of a building should be measured and described separately according to their nature; that is, forms to floor slabs, walls, columns, footings, etc., should be separated from each other. No allowance need be made for angle fillets or bevels to beams and columns, etc., but curved moldings should be measured and described separately.

No deduction in measurement of forms should be made for openings having an area of less than 25 sq. ft. as the labor in forming same is often greater than

the cost of the omitted area. No deduction is usually made in floor forms for heads of columns or in column and girder forms for ends of girders, cross beams, etc.

The correct measurement of column forms is the girth of the four sides, or circumference, multiplied by the height from the floor surface to the under side of floor slab above. Forms to octagonal, hexagonal, and circular columns should be measured and priced separately from forms to square columns. Caps and bases to columns and other ornamental work should be enumerated and fully described by sketches in the estimate.

The correct measurement of beam forms is the net length between columns multiplied by the sum of the breadth (b) and twice the depth below the slab (d), except for beams at edge of floor or around openings, which shall have the thickness of the floor (t) added to the sum of the breadth and twice the depth (see Fig. 1).

Fig. 1.

Wall forms should be measured for both sides of concrete walls.

Forms to the upper side of sloping slabs such as saw-tooth roofs should be measured whenever the slope of such slab with the horizontal exceeds an angle of 25 deg.

Moldings in formwork should be measured by the linear foot. Forms to circular work should always be measured separately from forms to straight work.

No measurement or allowance should be made for construction joints in slabs, beams, etc., to stop the day's concreting, but construction joints in dams and other large masses of concrete should be measured by the square foot as they occur.

Forms to cornices should be measured by the linear foot and the girth stated. Plain forms to back of cornice should be measured separately. Forms to window sills, copings, and similar work should be measured by the linear foot. Forms to the underside of stairs should be measured by the superficial foot, and forms to the front edge by the linear foot. Forms to the ends of steps should be measured by number.

8. Estimating Amount of Steel.—Reinforcing bars should be measured by the linear foot and reduced to weight in pounds for pricing. The net weight placed in the building should be taken and no allowance made for waste and cutting, or wire ties and spacers, etc., but laps should be allowed for as called for by the plans or by the necessities of the design. Deformed bars should be measured separately from plain.

The cost of bending and placing in columns and beams is greater than in slabs, but as the difference is not great it is not usual to make any distinctions but to take off the whole of the steel together, except in special cases.

Pipe sleeves, turnbuckles, clamps, threaded ends, nuts, forgings, and other special items should be measured separately by number and size, and allowed for in addition to the weight. Wire cloth, expanded metal, and other steel fabrics sold in sheets are measured and described by the square foot. The size of mesh and weight per square foot of steel will govern the price, and should be stated. All laps should be measured and allowed for.

9. Estimating Amount of Surface Finish.—Finish of concrete surfaces should be measured by the square foot. Finish should always be measured and de-

scribed separately. Allowance should be made for going over concrete work after removal of forms, and patching up voids and stone pockets, removing fins, etc., as this is part of the labor incidental to placing the concrete and the cost will depend upon the care used in spading the concrete into the forms.

Granolithic finish should be measured by the square foot and should include all labor and materials for the thickness specified. Finish laid integral with the slab should be measured separately from finish laid after the slab has set. No allowance should be made for protection of finish with sawdust, sand, or covering it to protect from weather. Grooved surfaces, gutters, curbing, etc., should be measured separately from plain granolithic and should be measured by the square foot or linear foot, as the case may require.

Putting on cement wash, rubbing with carborundum, scrubbing with wire brushes, tooling, and picking, are other surface labors that should each be separately measured and priced. The price should include the use of swing stages, tools, and materials required.

APPENDIX A

STANDARD SPECIFICATIONS FOR PORTLAND CEMENT[1]

These specifications were approved Jan. 15, 1921, as "Tentative American Standard" by the American Engineering Standards Committee.

1. Definition.—Portland cement is the product obtained by finely pulverizing clinker produced by calcining to incipient fusion, an intimate and properly proportioned mixture of argillaceous and calcareous materials, with no additions subsequent to calcination excepting water and calcined or uncalcined gypsum.

I. Chemical Properties

2. Chemical Limits.—The following limits shall not be exceeded:

Loss on ignition, per cent	4.00
Insoluble residue, per cent	0.85
Sulphuric anhydride (SO_3), per cent	2.00
Magnesia (MgO), per cent	5.00

II. Physical Properties

3. Specific Gravity.—The specific gravity of cement shall be not less than 3.10 (3.07 for white Portland cement). Should the test of cement as received fall below this requirement a second test may be made upon an ignited sample. The specific gravity test will not be made unless specifically ordered.

4. Fineness.—The residue on a standard No. 200 sieve shall not exceed 22 per cent by weight.

5. Soundness.—A pat of neat cement shall remain firm and hard, and show no signs of distortion, cracking, checking, or disintegration in the steam test for soundness.

6. Time of Setting.—The cement shall not develop initial set in less than 45 min. when the Vicat needle is used or 60 min. when the Gillmore needle is used. Final set shall be attained within 10 hr.

7. Tensile Strength.—The average tensile strength in pounds per square inch of not less than three standard mortar briquettes composed of 1 part cement and 3 parts standard sand, by weight, shall be equal to or higher than the following:

Age at test, days	Storage of briquettes	Tensile strength, lb. per sq. in.
7	1 day in moist air, 6 days in water	200
28	1 day in moist air, 27 days in water	300

8. The average tensile strength of standard mortar at 28 days shall be higher than the strength at 7 days.

III. Packages, Marking and Storage

9. Packages and Marking.—The cement shall be delivered in suitable bags or barrels with the brand and name of the manufacturer plainly marked thereon, unless shipped in bulk. A bag shall contain 94 lb. net. A barrel shall contain 376 lb. net.

10. Storage.—The cement shall be stored in such a manner as to permit easy access for proper inspection and identification of each shipment, and in a suitable weather-tight building which will protect the cement from dampness.

IV. Inspection

11. Inspection.—Every facility shall be provided the purchaser for careful sampling and inspection at either the mill or at the site of the work, as may be specified by the purchaser. At least 10 days from the time of sampling shall be allowed for the completion of the 7-day test, and at least 31 days shall be allowed for the completion of the 28-day test. The cement shall be tested in accordance with the methods hereinafter prescribed. The 28-day test shall be waived only when specifically so ordered.

[1] These specifications were adopted by letter ballot of the American Society for Testing Materials on Sept. 1, 1916, and became effective Jan. 1, 1917.

V. Rejection

12. Rejection.—The cement may be rejected if it fails to meet any of the requirements of these specifications.

13. Cement shall not be rejected on account of failure to meet the fineness requirement if upon retest after drying at 100 deg. C. for 1 hr. it meets this requirement.

14. Cement failing to meet the test for soundness in steam may be accepted if it passes a retest using a new sample at any time within 28 days thereafter.

15. Packages varying more than 5 per cent from the specified weight may be rejected: and if the average weight of packages in any shipment, as shown by weighing 50 packages taken at random, is less than that specified, the entire shipment may be rejected.

APPENDIX B

STANDARD METHOD OF TEST FOR ORGANIC IMPURITIES IN SANDS FOR CONCRETE

1. *Scope.*—The test herein specified is an approximate test for the presence of injurious organic compounds in natural sands for cement mortar or concrete. The principal value of the test is in furnishing a warning that further tests of the sand are necessary before they be used in concrete. Sands which produce a color in the sodium hydroxide solution darker than the standard color should be subjected to strength tests in mortar or concrete before use.

2. (*a*) *Sample.*—A representative test sample of sand of about 1 lb. shall be obtained by quartering or by the use of a sampler.

(*b*) *Procedure.*—A 12-oz. graduated glass prescription bottle shall be filled to the 4½-oz. mark with the sand to be tested.

(*c*) A 3-per cent solution of sodium hydroxide (NaOH) in water shall be added until the volume of sand and liquid after shaking gives a total volume of 7 liquid ounces.

(*d*) The bottle shall be stoppered and shaken thoroughly and then allowed to stand for 24 hours.

(*e*) A standard color solution shall be prepared by adding 2.5 cc. of a 2-per cent solution of tannic acid in 10-per cent alcohol to 22.5 cc. of a 3-per cent sodium hydroxide solution. This shall be placed in a 12-oz. prescription bottle, stoppered and allowed to stand for 24 hours, then 25 cc. of water added.

(*f*) *Color Value.*—The color of the clear liquid above the sand shall be compared with the standard color solution prepared as in paragraph (*e*) or with a glass of color similar to the standard solution.

3. Solutions darker in color than the standard color have a "color value" higher than 250 parts per million in terms of tannic acid.

APPENDIX C

METHOD OF TEST FOR QUANTITY OF CLAY AND SILT IN SAND

(American Society for Testing Materials)

1. *Scope.*—This test covers the determination of the quantity of clay and silt in natural sand to be used in highway construction.

2. *Treatment of Sample.*—The sample as received shall be moistened and thoroughly mixed, then dried to constant weight at a temperature between 100 and 110 deg. C. (212 and 230 deg. F.)

3. *Apparatus.*—The pan or vessel to be used in the determination shall be substantially 9 in. (22.9 cm.) in diameter by not less than 4 in. (10.2 cm.) deep. It shall have vertical sides and be provided with a pouring lip.

4. *Procedure.*—A representative portion of the dry material weighing 500 g., shall be selected from the sample and placed in the pan which has been dried and accurately weighed. Sufficient water shall be poured into the pan to cover the sand (about 225 cc.) and agitated vigorously for 15 seconds. After it has settled for 15 sec the water shall be poured off into a tared evaporating dish, care being taken not to pour off any sand. This is repeated until the wash water is clear, a glass rod being used to stir the material for the last few washings. The pan and washed sand shall be dried to constant weight in an oven at a temperature between 100 and 110 deg. C. (212 and 230 deg. F.), weighed and the net weight of sand determined.

5. *Percentage of Clay.*—The percentage of clay and silt shall be calculated from the formula:

$$\text{Percentage of Clay and Silt} = \frac{\text{Original weight} - \text{weight after washing}}{\text{Original weight}} \times 100.$$

6. *Check Determination.*—For a check on the results, the wash water shall be evaporated to dryness and the residue weighed:

$$\text{Percentage of Clay and Silt} = \frac{\text{Weight of residue}}{\text{Original weight}} \times 100.$$

APPENDIX D

METHOD OF TEST FOR UNIT WEIGHT OF AGGREGATE FOR CONCRETE

(American Society for Testing Materials)

1. The unit weight of fine, coarse, or mixed aggregates for concrete shall be determined by the following method:

2. (a) *Apparatus.*—The apparatus required consists of a cylindrical metal measure, a tamping rod, and a scale or balance, sensitive to 0.5 per cent of the weight of the sample to be weighed.

(b) *Measures.*—The measure shall be of metal, preferably machined to accurate dimensions on the inside, cylindrical in form, water-tight, and of sufficient rigidity to retain its form under rough usage, with top and bottom true and even, and preferably provided with handles.

The measure shall be 1/10, 1/2 or 1-cu. ft. capacity, depending on the maximum diameter of the coarsest particles in the aggregate, and shall be of the following dimensions:

Capacity (cu. ft.)	Inside diameter (in.)	Inside height (in)	Minimum thickness of metal, U. S. Gage	Diameter of largest particles of aggregate (in.)
1/10	6.00	6.10	No. 11	Under 1/2
1/2	10 00	11.00	No. 8	Under 1 1/2
	14.00	11.23	No. 5	Over 1 1/2

(c) *Tamping Rod.*—The tamping rod shall be a straight metal rod 5/8 in. in diameter and 18 in. long with one end tapered for a distance of 1 in. to a blunt bullet-shape point.

3. *Calibrating the Measure.*—The measure shall be calibrated by accurately determining the weight of water at 16.7 deg. C. (62 deg. F.) required to fill it. The factor for any unit shall be obtained by dividing the unit weight of water at 16.7 deg. C.) (62 deg. F.)[1] by the weight of water at 16.7 deg. C. (62 deg. F.) required to fill the measure.

4. The sample of aggregate shall be room dry and thoroughly mixed.

5. (a) *Method.*—The measure shall be filled one-third full and the top levelled off with the fingers. The mass shall be tamped with the pointed end of the tamping rod twenty-five times, evenly distributed over the surface. The measure shall be filled two-thirds full and again tamped twenty-five times as before. The measure shall then be filled to overflowing, tamped twenty-five times, and the surplus aggregate struck off, using the tamping rod as a straight edge.

In tamping the first layer the rod should not be permitted to forcibly strike the bottom of the measure. In tamping the second and final layers, only enough force to cause the tamping rod to penetrate the last layer of aggregate placed in the measure should be used. No effort should be made to fill holes left by the rod when the aggregate is damp.

(b) The net weight of the aggregate in the measure shall be determined. The unit weight of the aggregate shall then be obtained by multiplying the net weight of the aggregate by the factor found as described in Sec. 3.

6. *Accuracy.*—Results with the same sample should check within 1 per cent.

[1] The unit weight of water at 16.7 deg. C. (62 deg. F.) is 62.355 lb. per cu. ft.

APPENDIX E

SPECIFICATIONS FOR CONCRETE REINFORCEMENT BARS

STANDARD SPECIFICATIONS FOR BILLET-STEEL CONCRETE REINFORCEMENT BARS

(American Society for Testing Materials)

1. (*a*) These specifications cover three classes of billet-steel concrete reinforcement bars, namely: plain, deformed, and cold-twisted.

(*b*) Plain and deformed bars are of three grades, namely; structural-steel, intermediate, and hard.

2. (*a*) The structural-steel grade shall be used unless otherwise specified.

(*b*) If desired, cold-twisted bars may be purchased on the basis of tests of the hot-rolled bars before twisting in which case such tests shall govern and shall conform to the requirements specified for plain bars of structural steel grade.

3. *Manufacture.*—(*a*) The steel may be made by the Bessemer or open-hearth process.

(*b*) The bars shall be rolled from new billets. No re-rolled material will be accepted.

4. Cold-twisted bars shall be twisted cold with one complete twist in a length not over 12 times the thickness of the bar.

5. *Chemical Properties and Tests.*—The steel shall conform to the following requirements as to chemical composition:

Phosphorus, Bessemer	not over 0.10 per cent
Open-hearth	not over 0.05 per cent

6. An analysis of each melt of steel shall be made by the manufacturer to determine the percentages of carbon, manganese, phosphorus and sulphur. This analysis shall be made from a test ingot taken during the pouring of the melt. The chemical composition thus determined shall be reported to the purchaser or his representative, and shall conform to the requirements specified in Sec. 5.

7. Analyses may be made by the purchaser from finished bars representing each melt of open-hearth steel, and each melt, or lot of 10 tons, of Bessemer steel. The phosphorus content thus determined shall not exceed that specified in Sec. 5 by more than 25 per cent.

8. *Physical Properties and Tests.*—(*a*) The bars shall conform to the following requirements as to tensile properties:

Properties considered	Plain bars: Structural-steel grade	Plain bars: Intermediate grade	Plain bars: Hard grade	Deformed bars: Structural-steel grade	Deformed bars: Intermediate grade	Deformed bars: Hard grade	Cold-twisted bars
Tensile strength, lb. per sq. in	55,000 to 70,000	70,000 to 85,000	80,000 min.	55,000 to 70,000	70,000 to 85,000	80,000 min.	Recorded only
Yield point, min., lb. per sq. in	33,000	40,000	50,000	33,000	40,000	50,000	55,000
Elongation in 8 in. min. per cent[1]	1,400,000 / Tens. str.	1,300,000 / Tens. str.	1,200,000 / Tens. str.	1,250,000 / Tens. str.	1,125,000 / Tens. str.	1,000,000 / Tens. str.	

[1] See Sec. 9.

(*b*) The yield point shall be determined by the drop of the beam of the testing machine.

9. (*a*) For plain and deformed bars over ¾ in. in thickness or diameter, a deduction of 1 from the percentages of elongation specified in Sec. 8(*a*) shall be made for each increase of ⅛ in. in thickness or diameter above ¾ in.

(*b*) For plain and deformed bars under $\frac{7}{16}$ in. in thickness or diameter, a deduction of 1 from the percentages of elongation specified in Sec. 8(*a*) shall be made for each decrease of $\frac{1}{16}$ in. in thickness or diameter below $\frac{7}{16}$ in.

10. The test specimen shall bend cold around a pin without cracking on the outside of the bent portion, as follows:

Thickness or diameter of bar	Plain bars: Structural-steel grade	Plain bars: Intermediate grade	Plain bars: Hard grade	Deformed bars: Structural-steel grade	Deformed bars: Intermediate grade	Deformed bars: Hard grade	Cold-twisted bars
Under $\frac{3}{4}$ in	180 deg. $d = t$	180 deg. $d = 2t$	180 deg. $d = 3t$	180 deg. $d = t$	180 deg. $d = 3t$	180 deg. $d = 4t$	180 deg $d = 2t$
$\frac{3}{4}$ in. or over. .	180 deg. $d = t$	90 deg. $d = 2t$	90 deg. $d = 3t$	180 deg. $d = 2t$	90 deg. $d = 3t$	90 deg. $d = 4t$	180 deg. $d = 3t$

d = diameter of pin about which the specimen is bent.
t = thickness or diameter of specimen.

11. (*a*) Tension and bend test specimens for plain and deformed bars shall be taken from the finished bars and shall be of the full thickness or diameter of bars as rolled; except that the specimens for deformed bars may be machined for a length of at least 9 in., if deemed necessary by the manufacturer to obtain uniform cross-section.

(*b*) Tension and bend test specimens for cold-twisted bars shall be taken from the finished bars, without further treatment; except as specified in Sec. 2(*b*).

12. (*a*) One tension and one bend test shall be made from each melt of open-hearth steel, and from each melt, or lot of 10 tons, of Bessemer steel, except that if material from one melt differs $\frac{3}{8}$ in. or more in thickness or diameter, one tension and one bend shall be made from both the thickest and the thinnest material rolled.

(*b*) If any test specimen shows defective machining or develops flaws, it may be discarded and another specimen substituted.

(*c*) If the percentage of elongation of any tension test specimen is less than that specified in Sec. 8(*a*) and any part of the fracture is outside the middle third of the gage length, as indicated by scribe scratches marked on the specimen before testing, a retest shall be allowed.

13. *Permissible Variations in Weight.*—The weight of any lot of bars shall not vary more than 5 per cent from the theoretical weight of that lot.

14. *Finish.*—The finished bars shall be free from injurious defects and shall have a workmanlike finish.

15. *Inspection and Rejection.*—The inspector representing the purchaser shall have free entry, at all times while work on the contract of the purchaser is being performed, to all parts of the manufacturer's works which concern the manufacture of the bars ordered. The manufacturer shall afford the inspector, free of cost, all reasonable facilities to satisfy him that the bars are being furnished in accordance with these specifications. All tests (except check analyses) and inspection shall be made at the place of manufacture prior to shipment, unless otherwise specified, and shall be so conducted as not to interfere unnecessarily with the operation of the works.

16. (*a*) Unless otherwise specified, any rejection based on tests made in accordance with Sec. 7 shall be reported within 5 working days from the receipt of the samples.

(*b*) Bars which show injurious defects subsequent to their acceptance at the manufacturer's works shall be rejected, and the manufacturer shall be notified.

17. Samples tested in accordance with Sec. 7, which represent rejected bars, shall be preserved for 2 weeks from the date of the test report. In case of dissatisfaction with the results of the tests, the manufacturer may make claim for a rehearing within that time.

STANDARD SPECIFICATIONS FOR RAIL-STEEL CONCRETE REINFORCEMENT BARS

(American Society for Testing Materials)

1. The specifications cover three classes of rail-steel concrete reinforcement bars, namely: plain, deformed, and hot-twisted.

2. *Manufacture.*—The bars shall be rolled from standard section Tee rails.

3. Hot-twisted bars shall have one complete twist in a length not over 12 times the thickness of the bar.

Properties considered	Plain bars	Deformed and hot-twisted bars
Tensile strength, lb. per sq. in.	80,000	80,000
Yield point, lb. per sq. in.	50,000	50,000
	1,200,000	1,000,000
Elongation in 8 in., per cent[1]	Tens. str.	Tens. str.

[1] See Sec. 5.

4. *Physical Properties and Tests*—(*a*) The bars shall conform to the following minimum requirements as to tensile properties:

(*b*) The yield point shall be determined by the drop of the beam of the testing machine.

5. (*a*) For bars over $\frac{3}{4}$ in. in thickness or diameter, a deduction of 1 from the percentages of elongation specified in Sec. 4(*a*) shall be made for each increase of $\frac{1}{8}$ in. in thickness or diameter above $\frac{3}{4}$ in.

(*b*) For bars under $\frac{7}{16}$ in. in thickness or diameter, a deduction of 1 from the percentages of elongation specified in Sec. 4(*a*) shall be made for each decrease of $\frac{1}{16}$ in. in thickness or diameter below $\frac{7}{16}$ in.

6. The test specimen shall bend cold around a pin without cracking on the outside of the bent portion as follows:

Thickness of diameter of bar	Plain bars	Deformed and hot-twisted bars
Under $\frac{3}{4}$ in.	180 deg. $d = 3t$	180 deg. $d = 4t$
$\frac{3}{4}$ in. or over	90 deg. $d = 3t$	90 deg. $d = 4t$

7. (*a*) Tension and bend test specimens for plain and deformed bars shall be taken from the finished bars, and shall be of the full thickness or diameter of bars as rolled; except that the specimens for deformed bars may be machined for a length of at least 9 in., if deemed necessary by the manufacturer to obtain uniform cross-section.

(*b*) Tension and bend test specimens for hot-twisted bars shall be taken from the finished bars, without further treatment.

8. (*a*) One tension and one bend test shall be made from each lot of 10 tons or less of each size of bar rolled from rails varying not more than 10 lb. per yd. in nominal weight.

(*b*) If any test specimen shows defective machining or develops flaws, it may be discarded and another specimen substituted.

(*c*) If the percentage of elongation of any tension test specimen is less than that specified in Sec. 4(*a*) and any part of the fracture is outside the middle third of the gage length, as indicated by scribe scratches marked on the specimen before testing, a retest shall be allowed.

9. *Permissible Variations in Weight.*—The weight of any lot of bars shall not vary more than 5 per cent from the theoretical weight of that lot.

10. *Finish.*—The finished bars shall be free from injurious defects and shall have a workmanlike finish.

11. *Inspection and Rejection.*—The inspector representing the purchaser shall have free entry, at all times while work on the contract of the purchaser is being performed, to all parts of the manufacturer's works which concern the manufacture of the bars ordered. The manufacturer shall afford the inspector, free of cost, all reasonable facilities to satisfy him that the bars are being furnished in accordance with these specifications. All tests and inspection shall be made at the place of manufacture prior to shipment, unless otherwise specified, and shall be so conducted as not to interefere unnecessarily with the operation of the works.

12. Bars which show injurious defects subsequent to their acceptance at the manufacturer's works will be rejected, and the manufacturer shall be notified.

APPENDIX F

PROGRESS REPORT OF NEW JOINT COMMITTEE ON STANDARD SPECIFICATIONS FOR CONCRETE AND REINFORCED CONCRETE

(Submitting Tentative Specifications for Concrete and Reinforced Concrete)

PREFACE

The Joint Committee on Standard Specifications for Concrete and Reinforced Concrete consists of five representatives from each of the following:

American Society of Civil Engineers,
American Society for Testing Materials,
American Railway Engineering Association,
American Concrete Institute,
Portland Cement Association.

This Committee is the successor of the Joint Committee on Concrete and Reinforced Concrete which was organized in Atlantic City, N. J., June 17, 1904, and was formed by the union of special committees appointed in 1903 and 1904 by the above-named organizations, except the American Concrete Institute which was added by invitation of the Joint Committee in 1915. The previous Committee presented progress reports in 1909 and 1912 and adopted a final report to its constituent organizations on July 1, 1916. It was the purpose of that Committee to prepare a Recommended Practice for Concrete and Reinforced Concrete. Its final report stated:

"The report is not a specification but may be used as a basis for specifications."

The present Joint Committee is charged with the preparation of Specifications for Concrete and Reinforced Concrete and in preparing these specifications is using as a basis the report of the former Joint Committee with such modifications as are necessary to make its recommendations agree with current practice, and such new data as mark advances in the art.

The initiative in bringing about the present Joint Committee was taken by the Committee on Reinforced Concrete of the American Society for Testing Materials on June 27, 1917, when the committee voted to request the Executive Committee of the Society to invite the Member-Societies of the previous Joint Committee to cooperate in the formation of a new Joint Committee. The Executive Committee approved this request on April 25, 1919, and an invitation was issued to each of the above-named organizations by the Executive Committee on behalf of the American Society for Testing Materials, to appoint five members on a Joint Committee on Specifications for Reinforced Concrete. The last of these organizations accepted the invitation on November 22, 1919. On January 21, 1920, a call for an organizing meeting on February 11, 1920, was sent by the Executive Committee of that Society to each of the twenty-five representatives of cooperating organizations, together with a list of members of the Joint Committee, and an outline of organization that had been previously submitted by the American Society for Testing Materials to and approved by the cooperating organizations.

The Rules of Organization of the Joint Committee which were submitted to, and approved by, each of its constituent organizations, provide that,

"The initial report of the Joint Committee shall be considered by each of the five organizations as a tentative report submitted for criticism and discussion limited to not less than six months nor more than one year. Such discussions shall then be referred to the Joint Committee for consideration in revising its report." (Article IX, Sec. 2.)

The Joint Committee in submitting these Tentative Specifications for Concrete and Reinforced Concrete in accordance with the above requirement, wishes it clearly understood that it reserves the right to make such changes as may be found desirable, after a further study of the available data. While not prepared to submit a final report at this time the Committee is of the opinion that the specifications are in such shape as to make it desirable to issue them tentatively for the purpose of facilitating the final submission of Standard Specifications for Concrete and Reinforced Concrete.

The Joint Committee earnestly requests that every facility be provided by its constituent organizations for the fullest consideration of these Tentative Specifications in order that it may be in a position, as a result of their thorough discussion, to reflect in the final specifications the best current practice.

The Joint Committee further calls attention to the fact that it has undertaken to prepare specifications covering the fundamentals to be observed in the general use of concrete and reinforced concrete; no attempt has been made to cover the details involved in the use of these materials in special structures.

While the sections relating to design deal primarily with building construction, nevertheless the principles involved are in general applicable to structures of other types. It is expected that in using these specifications the necessary supplemental requirements will be added covering details.

TENTATIVE SPECIFICATIONS FOR CONCRETE AND REINFORCED CONCRETE

I. General Instructions

1. General Instructions.—These specifications are not complete; they cover the general conditions affecting the use of concrete and reinforced concrete. To complete them it will be necessary for the engineer to

(*a*) Provide the detail specifications covering the work in particular in which the concrete and reinforced concrete are to be used.

(*b*) Insert in Sec. 4 the strengths required for the several classes of concrete specified, based either upon preliminary tests or upon the values given in Table IV.

(*c*) Insert in Sec. 14 the sizes of aggregates required.

(*d*) Strike out one of the titles of the specifications in Sec. 20.

(*e*) Strike out one of the titles of the specifications in Sec. 24.

(*f*) Strike out one of the words "volume" or "weight" in Sec. 27

(*g*) Strike out two of the three Secs. 28 and fill in the necessary blanks for the proportions.

(*h*) Insert in Sec. 29 the slumps required.

(*i*) Strike out the method or methods inapplicable to the work, in Sec. 50.

(*j*) Strike out one of the two Secs. 97.

II. Definitions

2. Definitions.—The following definitions give the meaning of certain terms as used in these specifications:

Acid Proofing.—Treatment of a concrete surface to resist the action of acid solutions.

Aggregate.—Inert material which is mixed with Portland cement and water to produce concrete; in general aggregate consists of sand, pebbles, gravel, crushed stone or gravel, or similar materials. (See *Fine Aggregate, Coarse Aggregate.*)

Approved.—Meeting the approval of, or specifically authorized by, the Engineer.

Buttressed Retaining Wall.—A reinforced concrete wall having a vertical stem and a horizontal base, with brackets on the side opposite the pressure face uniting the vertical section with the toe of the base.

Cantilever Retaining Wall.—A reinforced concrete wall having a vertical stem and a horizontal base, each of which resists by cantilever action the pressure to which it is subjected.

Cellular Retaining Wall.—A reinforced concrete wall with a horizontal base, longitudinal vertical sections, and a series of transverse walls, dividing the space between the longitudinal walls into cells which are filled with earth, or other suitable material. If the top of the cells is covered by a floor slab, the front longitudinal wall and the filling may be omitted.

Coarse Aggregate.—Aggregate retained on a No. 4 sieve and of a maximum size generally not larger than 3 in. (See *Aggregate, Fine Aggregate.*)

Column.—A vertical compression member whose length exceeds three times its least horizontal dimension.

Column Capital.—An enlargement of the upper end of a reinforced concrete column built monolithic with the column and flat slab to increase the moment of inertia of the column and the shearing resistance of the slab at sections where high bending moment or high shear may occur.

Column Strip.—A portion of a panel of a flat slab which has a uniform width equal to one-fourth of the panel length on a line perpendicular to the direction of the strip, and whose outer edge lies on the edge of the panel. (See *Middle Strip.*)

Concrete.—A mixture of Portland cement, fine aggregate, coarse aggregate and water. (See *Mortar.*)

Consistency.—A general term used to designate the relative plasticity of freshly mixed mortar and concrete.

Counterforted Retaining Wall.—A reinforced concrete wall having a vertical stem and a horizontal base with brackets on the pressure face uniting the vertical section with the heel of the base.

Crusher-run Stone.—Unscreened crushed stone. (See *Stone Screenings.*)

Cyclopean Concrete.—Concrete in which stones larger than one-man size are individually embedded.

Dead Load.—The weight of the structure plus fixed loads and forces.

Deformed Bar.—Reinforcement bar with shoulders, lugs or projections formed integrally from the body of the bar during rolling.

Diagonal Direction.—A direction parallel or approximately parallel to the diagonal of the panel.

Dropped Panel.—The structural portion of a flat slab which is thickened throughout an area surrounding the column capital.

Effective Area of Concrete.—The area of a section of the concrete which lies between the tension reinforcement and the compression surface of the beam or slab.

Effective Area of Reinforcement.—The area obtained by multiplying the right cross-sectional area of the metal reinforcement by the cosine of the angle between the direction of the reinforcement bars or wires, and the direction for which the effectiveness of the reinforcement is to be determined.

Engineer.—The engineer in responsible charge of design and construction.

Fine Aggregate.—Aggregate passing through a No. 4 sieve. (See *Aggregate, Coarse Aggregate.*)

Flat Slab.—A flat concrete floor or roof plate having reinforcement bars extending in two or more directions and having no beams or girders to carry the load to the supporting columns.

Footing.—A structural unit used to distribute wall or column loads to the supporting material, either directly or through piles.

Gravel.—Loose material containing particles larger than sand, resulting from natural crushing and erosion of rocks. (See *Sand.*)

Laitance.—The extremely fine particles which separate from freshly deposited mortar or concrete and collect on the top surface.

Live Load.—Loads and forces which are variable.

Membrane Waterproofing.—A coating reinforced by fabric, felt, or similar toughening material applied to structures to prevent contact of moisture.

Middle Strip.—The portion of a panel of a flat slab which extends in a direction parallel to a side of the panel, whose width is one-half the panel length on a line at right angles to the direction of the strip and whose center line lies on the center line of the panel. (*See Column Strip.*)

Mortar.—A mixture of Portland cement, fine aggregate and water. (See *Concrete.*)

Negative Reinforcement.—Reinforcement so placed as to take stress due to negative bending moment.

Oilproofing.—Treatment of a concrete surface to resist the action of mineral, animal, or vegetable oils.

One-man Stone.—Stone larger than coarse aggregate and not exceeding 100 lb. in weight. (See *Rubble Concrete.*)

Panel Length.—The distance between centers of two columns of a panel, in either rectangular direction.

Pedestal or Pier.—A vertical compression member whose length does not exceed three times its least horizontal dimension.

Pedestal Footing.—A member supporting a column, in which the projection from the face of the column on all sides is less than one-half the depth.

Plain Concrete.—Concrete without metal reinforcement.

Positive Reinforcement.—Reinforcement so placed as to take stress due to positive bending moment.

Portland Cement.—The product obtained by finely pulverizing clinker produced by calcining to incipient fusion an intimate and properly proportioned mixture of argillaceous and calcareous materials, with no additions subsequent to calcination excepting water and calcined or uncalcined gypsum.

Principal Design Section.—The vertical sections in a flat slab on which the moments in the rectangular directions are critical. (See Sec. 146.)

Ratio of Reinforcement.—The ratio of the effective area of the reinforcement cut by a section of a beam or slab to the effective area of the concrete cut by that section.

Rectangular Direction.—A direction parallel to a side of the panel.

Reinforced Concrete.—Concrete in which metal is embedded in such a manner that the two materials act together in resisting stress.

Rubble Aggregate.—Stone or gravel larger than coarse aggregate and not larger than one-man stone. (See *One-man Stone.*)

Rubble Concrete.—Concrete in which pieces of rubble aggregate are individually embedded. (See *Rubble Aggregate.*)

Sand.—Loose material consisting of small grains (commonly quartz) resulting from the natural disintegration of rocks. (See *Gravel.*)

Screen.—A metal plate with closely spaced circular perforations. (See *Sieve.*)

Sieve.—Woven wire cloth with square openings. (See *Screen.*)

Slump.—The shortening of a standard test mass of concrete used as a measure of workability.

Standard Sand.—Natural sand mined at Ottawa, Ill., screened to pass a No. 20 sieve and retained on a No. 30 sieve, used as the fine aggregate in standard strength tests of Portland cement.

Stone Screenings.—Unscreened crushed stone passing through a No. 4 sieve. (See *Crusher-Run Stone.*)

Tremie.—A water-tight pipe of suitable dimensions, generally used in a vertical position, for depositing concrete under water.

Wall Beam.—A reinforced concrete beam which extends from column to column along the outer edge of a wall panel.

III. Quality of Concrete

3. Quality.—The quality of concrete shall be expressed in terms of workability as determined by the slump test and of the compressive strength at 28 days as determined by concrete tests of the materials to be used as specified in Sec. 28. The proportions required to produce concrete having the strength specified in Sec. 4 shall be determined in advance of the mixing of the concrete.

4. Strength.—The concrete shall develop under the conditions specified in Sec. 3, for the various parts of the work, the following strengths:[1]

.... lb. per sq. in.
.... lb. per sq. in.
.... lb. per sq. in.
.... lb per sq. in.

5. Tests of Field Specimens.—Field concrete test specimens shall be made, stored and tested in accordance with the Standard Methods of Making and Storing Specimens of Concrete in the Field (Serial Designation: C 31-21) of the American Society for Testing Materials.

IV. Materials

A. Portland Cement

6. Portland Cement.—Portland cement shall conform to the Standard Specifications and Tests for Portland Cement (Serial Designation: C 9-21) of the American Society for Testing Materials[2] and subsequent revisions thereof.

B. Fine Aggregate

7. General Requirements.—Fine aggregate shall consist of sand, stone screenings or other inert materials with similar characteristics, or a combination thereof, having clean, hard, strong, durable, uncoated grains and free from injurious amounts of dust, lumps, soft or flaky particles, shale, alkali, organic matter, loam or other deleterious substances.

8. Grading.—Fine aggregate shall range in size from fine to coarse, preferably within the following limits:

Passing through No. 4 sieve.............................. not less than 95 per cent
Passing through No. 50 sieve .... not more than 30 per cent
Weight removed by decantation.......................... not more than 3 per cent

9. Sieve Analysis.—The sieves and method of making sieve analysis shall conform to the Tentative Method of Test for Sieve Analysis of Aggregates for Concrete (Serial Designation: C 41-21 T) of the American Society for Testing Materials.

10. Decantation Test.—The decantation test shall be made in accordance with the Standard Method of Test for Quantity of Clay and Silt in Sand for Highway Construction (Serial Designation: D 74-21) of the American Society for Testing Materials.

11. Mortar Strength Test.—Fine aggregate shall preferably be of such a quality that mortar briquettes, cylinders or prisms, consisting of one part by weight of Portland cement and three parts by weight of fine aggregate,[3] mixed and tested in accordance with the methods described in the Standard Specifications and Tests for Portland Cement will show a tensile or compressive strength at ages of 7 and 28 days not less than that of 1 : 3 standard Ottawa sand mortar of the same plasticity made with the same cement. However, fine aggregate which fails to meet this requirement may be used, provided the proportions of cement, fine aggregate, coarse aggregate and water are such as to produce concrete of the strength specified.[4] Concrete tests shall be made in accordance with the Tentative Methods of Making Compression Tests of Concrete (Serial Designation: C 39-21 T) of the American Society for Testing Materials.

12. Organic Impurities in Sand.—Natural sand which shows a color darker than the standard color when tested in accordance with the Tentative Method of Test for Organic Impurities in Sands for Concrete (Serial Designation: C 40-21 T) of the American Society for Testing Materials shall not be used, unless the concrete made with the materials and in the proportions to be used on the work is shown by tests to be of the required strength.

C. Coarse Aggregate

13. General Requirements.—Coarse aggregate shall consist of crushed stone, gravel, or other approved inert materials with similar characteristics, or combinations thereof, having clean, hard, strong, durable, uncoated particles free from injurious amounts of soft, friable, thin, elongated or laminated pieces, alkali, organic or other deleterious matter.

[1] The engineer should insert the strengths required for the several classes of concrete specified, based either upon preliminary tests or upon the values given in Table IV.

[2] These specifications are also a standard of the following organizations: American Engineering Standards Committee, United States Government, American Railway Engineering Association, American Concrete Institute, and the Portland Cement Association.

[3] In testing aggregate, care should be exercised to avoid the removal of any coating on the grains which may affect the strength. Natural sand should not be dried before being made into mortar, but should contain natural moisture. The quantity of water contained may be determined on a separate sample and the weight of the sand used in the test corrected for the moisture content.

[4] Table IV furnishes a guide in determining the proportions of materials required to produce a concrete of a given strength, using aggregates of various sizes and concrete of different consistencies.

14. Grading.—Coarse aggregate shall range in size from fine to coarse within the following limits:[1]

Passing —[2] in. sieve (maximum size)	not more than 95 per cent
Passing —[2] in. sieve (intermediate size)	—[2] to —[2] per cent
Passing No. 4 sieve	not more than 15 per cent
Passing No. 8 sieve	not more than 5 per cent

15. Sieve Sizes.—The test for size and grading of aggregate shall be made in accordance with the Tentative Method of Test for Sieve Analysis of Aggregates for Concrete.

D. Rubble and Cyclopean Aggregate

16. Rubble Aggregate.—Rubble aggregate shall consist of clean, hard, durable stone larger than coarse aggregate and not larger than one-man stone.

17. Cyclopean Aggregate.—Cyclopean aggregate shall consist of clean, hard, durable stone, free from fissures and planes of cleavage and larger than one-man stone.

E. Storage of Aggregate

18. Aggregate Storage.—Aggregate shall be so stored on platforms or otherwise as to avoid the inclusion of foreign materials. Before using, frost, ice and lumps of frozen materials shall be removed.

F. Water

19. General Requirements.—Water for concrete shall be clean and free from oil, acid, alkali organic matter, or other deleterious substance.

G. Metal Reinforcement

20. Quality.—Metal reinforcement shall be of a quality and character meeting the requirements of the Standard Specifications[3] for Billet-steel Concrete Reinforcement Bars (Serial Designation: A 15-14) of the American Society for Testing Materials, Standard Specifications[3] for Rail-Steel Concrete Reinforcement Bars (Serial Designation: A 16-14) of the American Society for Testing Materials except that the provision for machining deformed bars before testing shall be eliminated.

21. Wire.—Wire for concrete reinforcement shall conform to the requirements of the Tentative Specifications for Cold-drawn Steel Wire for Concrete Reinforcement (Serial Designation: A 82-21 T) of the American Society for Testing Materials.

22. Standard Sizes of Bars.—Reinforcement bars shall conform to the areas and equivalent sizes shown in Table I.

[1] Where several suitable aggregates are available, a thorough investigation of the relative economy of each for producing concrete of the desired strength is advisable, especially for work of considerable magnitude.

[2] The engineer should insert in these blanks the sizes of aggregates required. The size and grading to be used will be governed by local conditions. The limitation on size and grading is intended to secure uniformity of aggregate. The following table indicates desirable gradings for coarse aggregate for certain maximum sizes:

Maximum size of aggregate, in.	Per cent by weight passing through standard sieves with square openings					Per cent passing, not more than	
	3 in.	2 in.	1½ in.	1 in.	¾ in.	No. 4 sieve	No. 8 sieve
3	100	...	40–75	...	...	15	5
2	...	100	...	40–75	...	15	5
1½	...	...	100	...	40–75	15	5
1	...	...	...	100	...	15	5
¾	...	...	...	...	100	15	5

[3] The engineer should strike out one of these titles. The Committee recommends as preferred material for reinforcement that meeting the requirements of the Standard Specifications for Billet-steel Concrete Reinforcement Bars of intermediate grade (except as noted under Section 20), made by the open-hearth process.

TABLE I.—SIZES AND AREAS OF REINFORCEMENT BARS

Size of bar, in.	Area, sq. in.	
	Round	Square
3/8	0.110	
1/2	0.196	0.250
5/8	0.307	
3/4	0.442	
7/8	0.601	
1	0.785	1.000
1 1/8	.....	1.266
1 1/4	.....	1.563

The areas of deformed bars shall be determined by the minimum cross-section thereof.

23. Deformed Bars.—An approved deformed bar shall be one that will develop a bond strength at least 25 per cent greater than that of a plain round bar of equivalent cross-sectional area.[1]

24. Structural Shapes.—Structural steel shapes used for reinforcement shall conform to the requirements of the Standard Specifications[2] for Structural Steel for Bridges (Serial Designation: A 7–21) of the American Society for Testing Materials, Standard Specifications[2] for Structural Steel for Buildings (Serial Designation: A 9–21) of the American Society for Testing Materials.

25. Cast Iron.—The quality of cast iron used in composite columns shall conform to the requirements of the Standard Specifications for Cast-iron Pipe and Special Castings (Serial Designation: A 44–04) of the American Society for Testing Materials.

V. PROPORTIONING AND MIXING CONCRETE

A. Proportioning.

26. Unit of Measure.—The unit of measure shall be the cubic foot. Ninety-four pounds (one bag or 1/4 bbl.) of Portland cement shall be considered as one cubic foot.

27. Method of Measuring.—Each of the constituent materials shall be measured separately by volume[3] weight.[3] The method of measurement shall be such as to secure the specified proportions in each batch. If volume measurement is used, the fine aggregate and the coarse aggregate shall be measured loose as thrown into the measuring device. The water shall be measured by an automatic device that will insure the same quantity in successive batches.

28.[4] Proportions.—The proportions of cement, water and aggregate shall be such as to produce concrete of the strength and quality specified in Secs. 3 and 4. The proportions shall be 1 part of Portland cement, —[5] parts of fine aggregate, and —[5] parts of coarse aggregate as determined by the engineer from concrete tests of the materials to be used. The tests shall be made in accordance with the Tentative Methods of Making Compression Tests of Concrete. The quantity of water used shall be such as to produce concrete of the consistency required by the particular class of work and shall be as specified in Sec. 29. In case the grading of the supply of available aggregate varies from that upon which the proportions were based, such aggregate may be used, provided the new proportions, as determined by the engineer, are such as to produce concrete of the required strength and quality.

28.[4] Proportions.—The contractor shall use materials, so proportioned and mixed, as to produce concrete of the required workability and strength.[6] Frequent compression tests of the concrete used in the work will be made by the engineer, and in case of failure to meet the specified strength, the contractor shall make such changes in the materials, proportions, or mixing, as may be necessary to secure concrete of the required strength. Concrete tests shall be made in accordance with the Standard Methods of Making and Storing Specimens of Concrete in the Field and the Tentative Methods of Making Compression Tests of Concrete.

[1] The Committee has under consideration a specification for deformed bars but is not prepared at this time to make more definite recommendations.

[2] The engineer should strike out one of these titles.

[3] The engineer should strike out one of these terms.

[4] The engineer should indicate his choice of the method of proportioning to be used by striking out two of the Sections numbered 28.

[5] The engineer should fill in these blanks.

[6] The use of this method should be accompanied by a clause in the contract which indicates the rpocedure to be followed in case tests show that concrete of the specified strength has not been obtained.

28.[1] **Proportions.**—The proportions shall be 1 part of Portland cement, —[2] parts of fine aggregate, and —[2] parts of coarse aggregate.. The proportions of materials shall be selected from Table IV. In case the grading of the supply of available aggregate varies from that upon which the proportions were based, such aggregate may be used, provided the new proportions, as determined by the engineer, are such as to produce concrete of the required strength and quality.

B. Consistency

29. Consistency.—The engineer shall determine and specify the consistency of the concrete for various portions of the work based on tests of the materials to be used. The consistency of the concrete shall be measured by the slump test in the manner described in the Tentative Specifications for Workability of Concrete for Concrete Pavements (Serial Designation: D 62–20 T) of the American Society for Testing Materials. The slump for different types of concrete shall not be greater than indicated in Table II.

The consistency shall be checked from time to time during the progress of the work.

TABLE II.—WORKABILITY OF CONCRETE

Type of concrete	Maximum slump, in.
1. Mass concrete	
2. Reinforced concrete:	
(a) Thin vertical sections and columns	
(b) Heavy sections	
(c) Thin confined horizontal sections	
3. Roads and pavements:	
(a) Hand finished	
(b) Machine finished	
4. Mortar for floor finish	

[a] The engineer should insert the slumps required, based on tests called for in this section. The slump test requirement is intended to insure concrete mixed with the minimum quantity of water required to produce a plastic mixture. The following table indicates the maximum slump desirable for the various types of concrete, based on average aggregates and proportions:

Type of concrete	Maximum slump, in.
1. Mass concrete	
2. Reinforced concrete:	
(a) Thin vertical sections and columns	
(b) Heavy sections	
(c) Thin confined horizontal sections	
3. Roads and pavements:	
(a) Hand finished	
(b) Machine finished	
4. Mortar for floor finish	

C. Mixing

30. Machine Mixing.—Mixing, unless otherwise authorized by the engineer, shall be done in a batch mixer of approved type, which will insure a uniform distribution of the materials throughout the mass, so that the mixture is uniform in color and homogeneous. The mixer shall be equipped with suitable charging hopper, water storage, and a water-measuring device controlled from a case which can be kept locked and so constructed that the water can be discharged only while the mixer is being charged. It shall also be equipped with an attachment for automatically locking the discharge lever until the batch has been mixed the required time after all materials are in the mixer. The entire contents of the drum shall be discharged before recharging. The mixer shall be cleaned at frequent intervals while in use.

31. Time of Mixing.—The mixing of each batch shall continue not less than 1½ minutes after all the materials are in the mixer, during which time the mixer shall rotate at a peripheral speed of about 200

[1] The engineer should indicate his choice of the method of proportioning to be used by striking out two of the sections numbered 28.

[2] The engineer should fill in these blanks.

ft. per minute. The volume of the mixed material per batch shall not exceed the manufacturer's rated capacity of the mixer.

32. Hand Mixing.—When hand mixing is authorized by the engineer it shall be done on a water-tight platform. The materials shall be turned at least six times after the water is added and until the batch is homogeneous in appearance and color.

33. Retempering.—The retempering of concrete or mortar which has partially hardened, that is, remixing with or without additional cement, aggregate or water, shall not be permitted.

VI. Depositing Concrete

A. Depositing in Air

34. General.—Before beginning a run of concrete, hardened concrete and foreign materials shall be removed from the inner surfaces of mixing and conveying equipment.

35. Approval.—Before depositing concrete, debris shall be removed from the space to be occupied by the concrete; forms shall be thoroughly wetted (except in freezing weather) or oiled. Reinforcement shall be thoroughly secured in position and approved by the engineer.

36. Handling.—Concrete shall be handled from the mixer to the place of final deposit as rapidly as practicable by methods which shall prevent the separation or loss of the ingredients. It shall be deposited in the forms as nearly as practicable in its final position to avoid rehandling. It shall be deposited in approximately uniform horizontal layers; the piling up of the concrete in the forms in such manner as to permit the escape of the mortar from the coarse aggregate will not be permitted. Forms for walls or other thin section of considerable height, shall be provided with openings, or other devices that will permit the concrete to be placed in a manner that will avoid accumulations of hardened concrete on forms or metal reinforcement. Under no circumstances shall concrete that has partially hardened be deposited in the work.

37. Spouting.—When concrete is conveyed by spouting, the plant shall be of such size and design as to insure a practically continuous flow in the spout. The angle of the spout with the horizontal shall be such as to allow the concrete to flow without separation of the ingredients.[1] The spout shall be thoroughly flushed with water before and after each run. The delivery from the spout shall be as close as possible to the point of deposit. When operation must be intermittent, the spout shall discharge into a hopper.

38. Compacting.—Concrete, during and immediately after depositing, shall be thoroughly compacted by means of rods or forks. For thin walls or inaccessible portions of the forms where rodding or forking is impracticable, the concrete shall be assisted into place by tapping or hammering the forms. The concrete shall be thoroughly worked around the reinforcement, and around embedded fixtures, into the corners of the forms.

39. Removal of Water.—Water shall be removed from excavations before concrete is deposited unless otherwise directed by the engineer. A continuous flow of water into the excavation shall be diverted through proper side drains to a sump, or by other approved methods which will avoid washing the freshly deposited concrete.

40. Protection.—Exposed surfaces of concrete subjected to premature drying shall be kept thoroughly wetted for a period of at least 7 days.

41. Cold Weather.—Concrete mixed and deposited during freezing weather shall have a temperature of not less than 50 deg. F. nor more than 100 deg. F. Suitable means shall be provided for maintaining a temperature of at least 50 deg. F. for not less than 72 hours after placing, or until the concrete has thoroughly hardened. The methods of heating the materials and protecting the concrete shall be approved by the engineer. Salt, chemicals or other foreign materials shall not be used to prevent freezing.

42. Depositing Continuously.—Concrete shall be deposited continuously and as rapidly as practicable and until the unit of operation, as approved by the engineer, is completed. Construction joints at points not provided for in the plans shall be made in accordance with the provisions in Sec. 69.

43. Bonding.—The surface of the hardened concrete shall be roughened and thoroughly cleaned of foreign matter and laitance, and saturated with water and forms retightened before depositing concrete. An excess of mortar on vertical or inclined surfaces shall be secured by thoroughly rodding or forking the freshly deposited concrete to remove the coarse aggregate from contact with the hardened concrete.

B. Rubble and Cyclopean Concrete

44. Rubble Concrete.—Rubble aggregate shall be thoroughly embedded in the concrete. The individual stones shall not be closer to any surface or adjacent stone than the maximum size of the coarse aggregate plus 1 in. Each successive layer of concrete shall be keyed in accordance with the provision in Sec. 69.

45. Cyclopean Concrete.—Cyclopean aggregate shall be thoroughly embedded in the concrete; no stone shall be closer to a finished surface than 1 ft., nor closer than 6 in., to any adjacent stone. Stratified stone shall be laid on its natural bed.

[1] An angle of about 27 deg., or one vertical to two horizontal, is good practice. Spouting through a vertical pipe is satisfactory when the flow is continuous; when it is unchecked and discontinuous it is highly objectionable unless the flow is broken by baffles.

C. Depositing Under Water[1]

46. General.—The methods, equipment, and materials to be used shall be submitted to and approved by the engineer before the work is started. Concrete shall be deposited by a method that will prevent the washing of the cement from the mixture, minimize the formation of laitance and avoid flow of water until the concrete has fully hardened. Concrete shall be placed so as to minimize segregation of materials. Hand mixing will not be permitted. Concrete shall not be placed in water at temperatures below 35 deg. F.

47. Proportions.—Concrete deposited under water shall consist of not less than 1 part of Portland cement to 6 parts of fine and coarse aggregate, measured separately.

48. Cofferdams.—Cofferdams shall be sufficiently tight to prevent flow of water through the space in which concrete is to be deposited. Pumping will not be permitted while concrete is being deposited, nor until it has fully hardened.

49. Depositing Continuously.—Concrete shall be deposited continuously, keeping the top surface as nearly level as possible, until it is brought above water or to the required height. The work shall be carried on with sufficient rapidity to insure bonding of the successive layers.

50. Method.—The following method[2] shall be used for depositing concrete under water:

(*a*) *Tremie.*—The tremie shall be water-tight and sufficiently large to permit a free flow of concrete. It shall be kept filled[3] at all times during depositing. The concrete shall be discharged and spread by raising the tremie in such manner as to maintain as nearly as practicable a uniform flow and avoid dropping the concrete through water. If the charge is lost during depositing the tremie shall be withdrawn and refilled.

(*b*) *Drop-bottom Bucket.*—The bucket shall be of a type that cannot be dumped until it rests on the surface upon which the concrete is to be deposited. The bottom doors when tripped shall open freely downward and outward. The top of the bucket shall be open. The bucket shall be completely filled, and slowly lowered to avoid back-wash. When discharged, the bucket shall be withdrawn slowly until clear of the concrete.

(*c*) *Bags.*—Bags of jute or other coarse cloth shall be filled about two-thirds full of concrete and carefully placed by hand in a header-and-stretcher system so that the whole mass is interlocked.

51. Laitance.—The concrete shall be disturbed as little as possible while it is being deposited, in order to avoid the formation of laitance. Laitance shall be removed.

VII. FORMS

52. General.—Forms shall conform to the shape, lines and dimensions of the concrete as called for on the plans. Lumber used in forms for exposed surfaces shall be dressed to a uniform thickness, and shall be free from loose knots or other defects. Joints in forms shall be horizontal or vertical. For unexposed surfaces and rough work, undressed lumber may be used. Lumber once used in forms shall have nails withdrawn, and surfaces to be in contact with concrete thoroughly cleaned, before being used again.

53. Design.—Forms shall be substantial and sufficiently tight to prevent leakage of mortar; they shall be properly braced or tied together so as to maintain position and shape. If adequate foundation for shores cannot be secured, trussed supports shall be provided.

54. Workmanship.—Bolts and rods shall preferably be used for internal ties; they shall be so arranged that when the forms are removed no metal shall be within 1 in. of any surface. Wire ties will be permitted only on light and unimportant work; they shall not be used through surfaces where discoloration would be objectionable. Shores supporting successive stories shall be placed directly over those below, or so designed that the lead will be transmitted directly to them. Forms shall be set to line and grade and so constructed and fastened as to produce true lines. Special care shall be used to prevent bulging.

55. Moldings.—Unless otherwise specified, suitable moldings or bevels shall be placed in the angles of forms to round or bevel the edges of the concrete.

56. Oiling.—The inside of forms shall be coated with non-staining mineral oil, or other approved material, or thoroughly wetted (except in freezing weather). Where oil is used, it shall be applied before the reinforcement is placed.

[1] Concrete should not be deposited under water if practicable to deposit in air. There is always uncertainty as to the results obtained from placing concrete under water; where conditions permit, the additional expense and delay of avoiding this method will be warranted. It is especially important that the aggregate be free from loam and other material which may cause laitance. Washed aggregates are preferable. Coarse aggregate consisting of washed gravel of a somewhat smaller size than used in open-air concrete work will give best results. Concrete should never be deposited under water without experienced supervision. Many failures, especially of structures in sea water, can be traced directly to ignorance of proper methods or lack of expert supervision.

[2] The engineer should strike out the method or methods inapplicable to the work.

[3] The tremie may be filled by one of the following methods. (1) Place the lower end in a box partly filled with concrete, so as to seal the bottom, then lower into position; (2) plug the tremie with cloth sacks or other material, which will be forced down as the tube is filled with concrete; (3) plug up the end of the tremie with cloth sacks filled with concrete.

57. Inspection of Forms.—Temporary openings shall be provided at the base of column and wall forms, and other places where necessary to facilitate cleaning and inspection immediately before depositing concrete.

58. Removal of Forms.—Forms shall not be disturbed until the concrete has adequately hardened, nor shall the permanent shores be removed until the structure has attained its full design strength[1] and all excess construction load has been removed. Wall and column forms shall be left in place until the concrete has hardened sufficiently to sustain its own weight and the construction loads likely to come upon it. Forms other than wall or column forms shall be left in place until the concrete has hardened sufficiently to carry the full load which it must sustain, unless removed in sections and each section of the structure is immediately re-shored.

VIII. Details of Construction

A. Metal Reinforcement

59. Cleaning.—Metal reinforcement, before being positioned, shall be thoroughly cleaned of mill and rust scale, and of coatings of any character that will destroy or reduce the bond. Reinforcement appreciably reduced in section shall be rejected. Reinforcement shall be re-inspected and when necessary cleaned, where there is delay in depositing concrete.

60. Bending.—Reinforcement shall be carefully formed to the dimensions indicated on the plans or called for in the specifications. The radius of bends shall be 4 or more times the least diameter of the reinforcement bar.

61. Straightening.—Metal reinforcement shall not be bent or straightened in a manner that will injure the material. Bars with kinks or sharp bends shall not be used.

62. Placing.—Metal reinforcement shall be accurately positioned, and secured against displacement by using annealed iron wire of not less than No. 18 gage or suitable clips at intersections, and shall be supported by concrete or metal chairs, or spacers, or by metal hangers. Parallel bars shall not be placed closer in the clear than 1½ times the diameter of round bars or 1½ times the diagonal of square bars; if the ends of bars are hooked as specified in Sec. 130 the clear spacing may be made equal to the diameter of round bars or to the diagonal of square bars, but in no case shall the spacing between bars be less than 1 in., nor less than 1¼ times the maximum size of the coarse aggregate.

63. Splicing.—Splices of tension reinforcement at points of maximum stress shall be avoided. Splices, where required, shall provide sufficient lap to transfer the stress between bars by bond and shear, or by a mechanical connection such as a screw coupling.

64. Offsets in Column Reinforcement.—Vertical reinforcement shall be offset in a region where lateral support is afforded when changes in column cross-section occur and the vertical reinforcement bars are not sloped for the full length of the column.

65. Future Bonding.—Exposed reinforcement bars intended for bonding with future extensions shall be protected from corrosion.

B. Concrete Covering over Metal

66. Moisture Protection.—Metal reinforcement in wall footings and column footings shall have a minimum covering of 3 in. of concrete.

67. Fire Protection.—Metal reinforcement in fire-resistive construction shall be proected by not less than 1 in. of concrete in slabs and walls, and not less than 2 in. in beams, girders and columns, provided aggregate showing an expansion not materially greater than that of limestone or trap rock is used; when impracticable to obtain aggregate of this grade, the protective covering shall be 1 in. thicker and shall be reinforced with metal mesh not exceeding 3 in. in greatest dimensions, placed 1 in. from the finished surface.

The metal reinforcement in structures containing incombustible materials and in bridges where the fire hazard is limited, shall be protected by not less than ¾ in. of concrete in slabs and walls and of not less than 1½ in. in beams, girders and columns.

68. Plaster.—Plaster finish on an exposed concrete surface may be allowed to reduce the thickness of concrete protection required in Sec. 67 by one-half the thickness of the plaster, but the protection shall not be less than that specified in Secs. 66 and 67.

C. Joints

69. Construction Joints.—Construction joints not indicated on the plans nor specified shall be located and formed so as to least impair the strength and appearance of the structure. Horizontal construction joints shall be formed by embedding stones projecting above the surface or by roughening the surface in contact, or by mortises or keys formed in the concrete. Sufficient section shall be provided in horizontal as well as vertical keys to resist shear.

70. Joints in Columns.—Construction joints in columns shall be made at the underside of the floor. Haunches and column capitals shall be considered as part of and built monolithic with the floor construction.

[1] Many conditions affect the hardening of concrete and the proper time for the removal of the forms should be determined by a competent and responsible person.

71. Joints in Floors.—Construction joints in floors shall be located near the center of spans of slabs, beams, and girders, unless a beam intersects a girder at this point, in which case the joints in the girders shall be offset a distance equal to twice the width of the beam. Adequate provision shall be made for shear either by sufficient reinforcement, or by sloping the joint so as to provide an inclined bearing.

72. Monolithic Construction.—Girders and beams designed to be monolithic with walls and columns shall not be cast until 2 hours after the completion of the walls or columns.

73. Construction Joints in Long Buildings.—Construction joints made crosswise of a building 100 ft. or more in length, shall have special reinforcement placed at right angles to the joint and extending a sufficient distance on each side of the joint to develop the strength of the reinforcement by bond. This reinforcement shall be placed near the opposite face of the member from the main tension reinforcement; the amount of such reinforcement shall be not less than 0.5 per cent of the section of the members cut by the joint.

74. Expansion Joints.—Expansion joints shall be so detailed that the necessary movement may occur with the minimum of resistance at the joint. The structure adjacent to the joint shall preferably be supported on separate columns or walls. Reinforcement shall not extend across an expansion joint. The break between the two sections shall be complete, and may be effected by a coating of white lead and oil, asphalt paint or petrolatum, or by building paper, placed over the entire surface of the hardened concrete. Exposed edges of expansion joints in walls or abutments shall be bonded. Exposed expansion joints formed between two distinct concrete members shall be filled with an elastic joint filler of approved quality.

75. Expansion Joints in Long Buildings.—Structures exceeding 200 ft. in length and of width less than about one-half the length, shall be divided by means of expansion joints, located near the middle, but not more than 200 ft. apart, to minimize the destructive effects of temperature changes and shrinkage. Structures in which marked changes in plan section take place abruptly, or within a small distance, shall be provided with expansion joints at the points where such changes in section occur.

76. Sliding Joints.—The seat of sliding joints shall be finished with a smooth troweled surface and shall not have the superimposed concrete placed upon it until it has thoroughly hardened. In order to facilitate sliding, two thicknesses of building paper shall be placed over the seat on which the superimposed concrete is to be deposited.

77. Water-tight Joints.—When it is not possible to finish a section of the structure in one continuous operation and water-tight construction is required, the joints shall be prepared as follows: The surface of the first section of concrete shall be provided with continuous keyways. All laitance and other foreign substances shall be removed from the surface of the concrete first placed; this surface shall then be thoroughly saturated with water and given a heavy coating of neat cement. The next section of concrete shall be placed in such manner as to insure an excess of mortar over the entire surface of the joint. Where shown on the plans, the joint shall be so constructed as to permit of its being caulked with oakum.

IX. Waterproofing and Protective Treatment

A. *Waterproofing*

78. General.—The requirements for quality of concrete in Sec. 28 shall be strictly followed. Particular attention shall be given to workmanship.

79. Integral Compounds.—Integral compounds shall not be used.

80. Membrane Waterproofing.—Membrane waterproofing shall be used in basements, pits, shafts, tunnels, bridge floors, retaining walls and similar structures, where an added protection is desired.

81. Water-tight Joints.—See Sec. 77.

B. *Oilproofing*

82. Oilproofing.—Concrete structures for containing light mineral oils, animal oils, certain vegetable oils and other commercial liquids shall be given a special coating which shall be applied immediately after construction. Floors or other surfaces exposed to heavy concentrations of such oils or liquids shall be similarly protected. The treatment to be applied shall be approved by the engineer.

C. *Concrete in Sea Water*

83. Proportions.—Plain concrete in sea water or exposed directly along the sea coast shall contain not less than 1½ bbl. (6 bags) of Portland cement per cubic yard in place; concrete from 2 ft. below low water to 2 ft. above high water, or from a plane below to a plane above wave action, shall be made of a mixture containing not less than 1¾ bbl. (7 bags) of Portland cement per cubic yard in place. Slag, broken brick, soft limestone, soft sandstone or other porous or weak aggregates shall not be used.

84. Depositing.—Concrete shall not be deposited under sea water unless unavoidable, in which case it shall be placed in accordance with the methods described in Secs. 48 to 51. Sea water shall not be allowed to come in contact with the concrete until it has hardened for at least 4 days. Concrete shall be placed in such a manner as to avoid horizontal or inclined seams or work planes. The placing of concrete between tides shall be a continuous operation, in accordance with the methods described in Sec 42; where it is impossible to avoid seams or joints proceed as in Sec. 43.

85. Protection.—Metal reinforcement shall be placed at least 3 in. from any plane or curved surface, and at corners at least 4 in. from all adjacent surfaces. Metal chairs, supports, or ties shall not extend to the surface of the concrete. Where unusually severe conditions of abrasion are anticipated, the face of the concrete from 2 ft. below low water to 2 ft. above high water, or from a plane below to a plane above wave action, shall be protected by creosoted timber, dense vitrified shale brick, or stone of suitable quality, as designated on the plans.

86. Consistency.—The consistency shall be such as to produce concrete which for mass work shall give a slump of not more than 2 in., and for reinforced concrete a slump of not more than 4 in.

D. Concrete in Alkali Soils or Water

87. Proportions.—Concrete below the ground-line shall contain not less than 1¾ bbl. (7 bags) of Portland cement per cubic yard in place.

88. Consistency.—The consistency of the concrete shall be such as to produce a slump of not more than 2 in., and for small members in which aggregates coarser than ⅜ in. cannot be used, a slump of not more than 6 in.

89. Placing.—Concrete should be placed in such a manner as to avoid horizontal or inclined seams, or work planes; where this is impossible the requirements of Sec. 69 shall be followed.

90. Curing.—Concrete shall be kept wet with fresh water for not less than 7 days following placing.

91. Protection.—Metal reinforcement or other corrodible metal shall not be placed closer than 2 in. to the faces of members exposed to alkali soil or water.

X. SURFACE FINISH

92. General.—Concrete to have exposed surfaces with specified finish shall be mixed, placed and worked to secure a uniform distribution of the aggregates, and insure uniform texture of surface.[1] Placing shall be continuous throughout each distinct division of an area. Joint lines shall be located at indicated points. Voids which appear upon removal of the forms shall be drenched with water and be immediately filled with material of the same composition as that used in the surface, and smoothed with a wood spatula or float. Fins or offsets shall be neatly removed. The work shall be finished free from streaks.

93. Top Surfaces not Subject to Wear.—Top surfaces not subject to wear shall be smoothed with a wood float and be kept wet for at least 7 days. Care shall be taken to avoid an excess of water in the concrete, and to drain off or otherwise promptly remove any water that comes to the surface. Dry cement, or a dry mixture of cement and sand, shall not be sprinkled directly on the surface.

A. Wearing Surfaces

94. One-course Work.—Aggregates for the wearing surface shall have a high resistance to abrasion They shall be carefully screened and thoroughly washed. The least quantity of mixing water that will produce a dense concrete shall be used. The mix shall not be leaner than 1 part of Portland cement and 2½ parts of aggregate. The surface shall be screeded even and finished with a wood float. Excess water shall be promptly drained off or otherwise removed. Overtroweling shall be avoided.

95. Two-course Work.—In two-course work the wearing surface shall be placed within ½ hour after the base course.

If the wearing surface is required to be applied to a hardened base course, the latter shall be prepared by roughening with a pick or other effective tool, thoroughly drenching with water until saturated and covered with a thin layer of neat cement immediately before the wearing surface is placed.

The finished wearing course in two-course work shall not be thinner than 1 in.

96. Curing.—Concrete wearing surfaces constructed in accordance with Secs. 94 and 95, shall be kept wet[2] for at least 10 days in the case of floors and 21 days in the case of roads and pavements.

97.[3] Terrazzo Finish.—Terrazzo finish shall be constructed by mixing 1 part of Portland cement, 2½ parts of crushed marble which will pass through a ½-in. screen and is free from dust, and sufficient water to produce a dense concrete, which shall be spread on the base course and worked down to a thickness of 1 in. by patting or rolling and troweling.

The surface shall be kept wet for not less than 10 days and after thoroughly curing shall be rubbed to a plane surface with a stone or a surfacing machine. Hardened concrete to which a terrazzo finish is to be applied shall be prepared as prescribed in Sec. 95.

97.[3] Terrazzo Finish.—Terrazzo finish shall be constructed by mixing 1 part of Portland cement, 2 parts of sand and sufficient water to produce a plastic mortar, which shall be spread on the base course to a depth of 1 in. Crushed marble, which will pass through a ¼-in. screen and is free from dust, shall be sprinkled over the surface of the fresh mortar and pressed or rolled in.

[1] This is accomplished by uniform proportioning of ingredients, and thorough mixing with the proper amount of water; after placing, the concrete should be thoroughly rodded or forked to force the aggregate against the face forms and prevent the formation of voids.

[2] Prevention of premature drying during the early hardening of concrete is essential to the development of high resistance to abrasion. The surface may be covered with a layer of burlap, earth or sand, kept wet, or it may be divided into small areas by dikes and flooded with water to a depth of 2 or 3 in.

[3] The engineer should strike out one of the two Sections numbered 97.

The surface shall be kept wet for not less than 10 days and after thoroughly curing shall be rubbed to a plane surface with a stone or a surfacing machine. Hardened concrete to which a terrazzo finish is to be applied shall be prepared as prescribed in Sec. 95.

B. Decorative Finishes

98. Rubbed Finish.—Concrete shall be wetted immediately after the forms are removed and rubbed even and smooth with a carborundum brick, or other abrasive, and to uniform appearance without applying any cement or other coating.

99. Scrubbed Finish.—The face forms shall be removed as soon as the concrete has hardened sufficiently. Voids shall be immediately filled with mortar of the same composition as that used in the face. Fins and other unevennesses shall be rubbed off and the whole surface be scrubbed with fiber or wire brushes, using water freely, as the degree of hardness may require, until the aggregate is uniformly exposed; the surface shall then be rinsed with clean water. The corners shall be sharp and unbroken. If portions of the surface have become too hard to scrub in uniform relief, dilute hydrochloric acid (1 part of acid to 4 parts of water) may be used to facilitate scrubbing of hardened surfaces. The acid shall be thoroughly washed off with clean water.

100. Sand Blast Finish.—Immediately following removal of forms, voids shall be filled with mortar of the same composition as that used in the face and allowed to harden. Unevennesses and form marks shall be removed by chipping or rubbing; the face shall then be cut with an air blast of hard sand with angular grains until the aggregate is in uniform relief.

101. Tooled Finish.—The surface shall be permitted to become hard and dry before tooling. The cutting shall remove the entire skin and produce a uniform surface true to lines.

102. Sand Floated Finish.—The forms shall be removed before the surface has fully hardened; the surface shall be rubbed with a wooden float by a uniform circular motion, using fine sand until the resulting finish is even and uniform.

103. Colored Aggregate Finish.—Colored or other special aggregate used for finish shall be exposed by scrubbing as provided in Sec. 99. Facing mortar of 1 part of Portland cement, 1½ parts of sand, and 3 parts of screenings or pebbles shall be placed against the face forms to a thickness of about 1 in. sufficiently in advance of the body concrete to prevent the latter coming in contact with the form.

104. Colored Pigment Finish.—Mineral pigment shall be thoroughly mixed dry with the Portland cement and fine aggregate; care shall be taken to secure a uniform tint throughout.

XI. Design

A. General Assumptions

105. General Assumptions.—The design of reinforced concrete members under these specifications shall be based on the following assumptions:

(*a*) Calculations are made with reference to working stresses and safe loads rather than with reference to ultimate strength and ultimate loads.

(*b*) A plane section before bending remains plane after bending.

(*c*) The modulus of elasticity of concrete in compression is constant within the limits of working stresses; the distribution of compressive stress in beams is therefore rectilinear.

(*d*) The values for the modulus of elasticity of concrete in computations for the position of the neutral axis, for the resisting moment of beams and for compression of concrete in columns, are as follows:

(1) One-fortieth that of steel, when the compressive strength of the concrete at 28 days is below 800 lb per sq. in.;
(2) One-fifteenth that of steel, when the compressive strength of the concrete at 28 days lies between 800 and 2,200 lb. per sq. in.;
(3) One-twelfth that of steel, when the compressive strength of the concrete at 28 days lies between 2,200 and 2,900 lb. per sq. in.
(4) One-tenth that of steel, when the compressive strength of the concrete at 28 days is higher than 2,900 lb. per sq. in.;
(5) One-eighth that of steel for calculating the deflection of reinforced concrete beams which are free to move longitudinally at the supports, and in which the tensile resistance of the concrete is neglected.

(*e*) In calculating the moment of resistance of reinforced concrete beams and slabs the tensile resistance of the concrete is neglected.

(*f*) The adhesion between the concrete and the metal reinforcement remains unbroken throughout the range of working stresses. Under compression the two materials are therefore stressed in proportion to their moduli of elasticity.

(*g*) Initial stress in the reinforcement due to contraction or expansion of the concrete is neglected, except in the design of reinforced concrete columns.

B. Flexure of Rectangular Reinforced Concrete Beams and Slabs

106. Flexure Formulas.—Computations of flexure in rectangular reinforced concrete beams and slabs shall be based on the following formulas:

(a) *Reinforced for Tension Only.*

Position of neutral axis.

$$k = \sqrt{2pn + (pn)^2} - pn \tag{1}$$

Arm[1] of resisting couple,

$$j = 1 - \frac{k}{3} \tag{2}$$

Compressive unit stress[1] in extreme fiber of concrete,

$$f_c = \frac{2M}{jkbd^2} = \frac{2pf_s}{k} \tag{3}$$

Tensile unit stress[1] in longitudinal reinforcement,

$$f_s = \frac{M}{A_s jd} = \frac{M}{pjbd^2} \tag{4}$$

Steel ratio for balanced reinforcement,

$$p = \frac{1}{2} \frac{1}{\dfrac{f_s}{f_c}\left(\dfrac{f_s}{nf_c} + 1\right)} \tag{5}$$

For formulas on shear and bond, see Sec. 120 and 140.

(b) *Reinforced for Both Tension and Compression.*

Position of neutral axis,

$$k = \sqrt{2n\left(p + p'\frac{d'}{d}\right) + n^2(p + p')^2} - n(p + p') \tag{6}$$

Position of resultant compression,

$$z = \frac{\frac{1}{2}k^3d + 2p'nd'\left(k - \frac{d'}{d}\right)}{k^2 + 2p'n\left(k - \frac{d'}{d}\right)} \tag{7}$$

Arm[1] of resisting couple,

$$jd = d - z \tag{8}$$

Compressive unit stress[1] in extreme fiber of concrete,

$$f_c = \frac{6M}{bd^2\left[3k - k^2 + \frac{6p'n}{k}\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)\right]} \tag{9}$$

Tensile unit stress[1] in longitudinal reinforcement,

$$f_s = \frac{M}{pjbd^2} = nf_c\frac{1 - k}{k} \tag{10}$$

Compressive unit stress[1] in longitudinal reinforcement,

$$f'_s = nf_c\frac{k - \frac{d'}{d}}{k} \tag{11}$$

107. Notation.—The symbols[2] used in formulas (1) to (23) are defined as follows:

A_s = effective cross-sectional area of metal reinforcement in tension in beams;
b = width of rectangular beam or width of flange of T-beam;
d = depth from compression surface of beam or slab to center of longitudinal tension reinforcement;
d' = depth from compression surface of beam or slab to center of compression reinforcement;
f_c = compressive unit stress in extreme fiber of concrete;
f_s = tensile unit stress in longitudinal reinforcement;
f'_s = compressive unit stress in longitudinal reinforcement;
h = unsupported length of column;
I = moment of inertia of a section about the neutral axis for bending;
j = ratio of lever arm of resisting couple to depth d;
k = ratio of depth of neutral axis to depth d;
l = span length of beam or slab (generally distance from center to center of supports—see Sec. 108);
M = bending moment or moment of resistance in general;
n = $\frac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to that of concrete;
p = ratio of effective area of tension reinforcement to effective area of concrete in beams = $\frac{A_s}{bd}$;

[1] For f_s = 16,000 to 18,000 lb. per sq. in. and f_c = 800 to 900 lb. per sq. in., j may be assumed as 0.86. For values of pn varying from 0.04, to 0.24, jk is approximately equal to 0.67 $\sqrt[3]{pn}$

[2] For illustration of notation as applied to typical beams or slabs, see Figs. 1 and 2.

p' = ratio of effective area of compression reinforcement to effective area of concrete in beams;
w = uniformly distributed load per unit of length of beam or slab;
z = depth from compression surface of beam or slab of resultant of compressive stresses.

108. Span Length.—The span length, l, of freely supported beams and slabs, shall be the distance between centers of the supports, but shall not exceed the clear span plus the depth of beam or slab. The span length for continuous or restrained beams built monolithically with supports shall be the clear distance between faces of supports. Where brackets having a width not less than the width of the beam and making an angle of 45 deg. or more with the axis of a restrained beam are built monolithic with the beam and the support, the span shall be measured from the section where the combined depth of the beam and bracket is at least one-third more than the depth of the beam. Maximum negative moments are to be considered as existing at the ends of the span, as above defined. No portion of a bracket shall be considered as adding to the effective depth of the beam.

109. Moments in Freely Supported Beams of Equal Span.—The following moments at critical sections of freely supported beams and slabs of equal spans carrying uniformly distributed loads shall be used:

(*a*) Maximum positive moment in beams and slabs of one span,

$$M \qquad wl^2 \tag{12}$$

(*b*) Center of slabs and beams continuous for two spans only,

(1) Positive moment at the center,

$$M = \frac{wl^2}{10} \tag{13}$$

(2) Maximum negative moment,

$$M = \frac{wl^2}{8} \tag{14}$$

(*c*) Slabs and beams continuous for more than two spans,

(1) Center and supports of interior spans,

$$M = \frac{wl^2}{12} \tag{15}$$

(2) Center and interior support of end spans,

$$M = \frac{wl^2}{10} \tag{16}$$

(*d*) Negative moment at the supports of slab or beam built into brick or masonry walls in a manner that develops partial end restraint,

$$M = \text{not less than } \frac{wl^2}{16} \tag{17}$$

110. Moments in Beams Monolithic with Supports.—The following moments at the critical sections of beams or slabs of equal spans cast monolithic with columns or similar supports and carrying uniformly distributed loads shall be used:

(*a*) Supports of intermediate spans,

$$M = \frac{wl^2}{12} \tag{18}$$

(*b*) Center of intermediate spans,

$$M = \frac{wl^2}{16} \tag{19}$$

(*c*) Beams in which $\frac{I}{l}$ is less than twice the sum of the values of $\frac{I}{h}$ for the exterior columns above and below which are built into the beam.

(1) Center and first interior support,

$$M = \frac{wl^2}{12} \tag{20}$$

(2) Exterior supports,

$$M = \frac{wl^2}{12} \tag{21}$$

(*d*) Beams in which $\frac{I}{l}$ is equal to, or greater than, twice the sum of the values of $\frac{I}{h}$ for the exterior columns above and below which are built into the beam,

(1) Center of span and at first interior support of end span,

$$M = \frac{wl^2}{10} \tag{22}$$

(2) Exterior support,

$$M = \frac{wl^2}{16} \tag{23}$$

111. Moment Coefficients of Continuous Beams.—Continuous beams with unequal spans, whether freely supported or cast monolithic with columns, shall be analyzed to determine the actual moments under the given conditions of loading and restraint. Provision shall be made for negative moment occurring in short spans adjacent to longer spans when the latter only are loaded.

C. Flexure of Reinforced Concrete T-Beams

112. Flexure Formulas.—Computations of flexure in reinforced concrete T-beams shall be based on the following formulas:

(*a*) *Neutral Axis in the Flange.*

Use formulas for rectangular beams and slabs in Sec. 106.

(*b*) *Neutral Axis below the Flange.*[1]

Position of neutral axis,

$$kd = \frac{2ndA_s + bt^2}{2nA_s + 2bt} \tag{24}$$

Position of resultant compression,

$$z = \left(\frac{3kd - 2t}{2kd - t}\right)\frac{t}{3} \tag{25}$$

Arm of resisting couple,

$$jd = d - z \tag{26}$$

Compressive unit stress in extreme fiber of concrete,

$$f_c = \frac{Mkd}{bt(kd - \frac{1}{2}t)jd} = \frac{f_s}{n}\left(\frac{k}{1 - k}\right) \tag{27}$$

Tensile unit stress in longitudinal reinforcement,

$$f_s = \frac{M}{A_s jd} \tag{28}$$

Formulas (24), (25), (26), (27) and (28) neglect compression in the stem.[2]

113. Notation.—The symbols[3] used in formulas (24) to (28) are defined in Sec. 107, except as follows:

b' = width of stem of T-beam;
t = thickness of flange of T-beam;

114. Flange Width.—Effective and adequate bond and shear resistance shall be provided in beam-and-slab construction at the junction of the beam and slab; the slab shall be built and considered an integral part of the beam; the effective flange width shall not exceed one-fourth of the span length of the beam, and its overhanging width on either side of the web shall not exceed 8 times the thickness of the slab nor one-half the clear distance to the next beam.

115. Flange Length.—The unsupported length of the compression flange of a T-beam shall not exceed 36 times the least width of the beam.

116. Transverse Reinforcement.—Where the principal slab reinforcement is parallel to the beam, transverse reinforcement, not less in amount than 0.3 per cent of the sectional area of the slab, shall be provided in the top of the slab and shall extend over the beam and into the slab not less than two-thirds of the effective flange overhang. The spacing of the bars shall not exceed 18 in.

[1] For approximate results the formulas for rectangular beams, Sec. 106, may be used.

[2] The following formulas take into account the compression in the stem; they are recommended where the flange is small compared with the stem:

Position of neutral axis,

$$kd = \sqrt{\frac{2ndA_s + (b - b')t^2}{b'} + \left(\frac{nA_s + (b - b')t}{b'}\right)^2} - \frac{nA_s + (b - b')t}{b'} \tag{24a}$$

Position of resultant compression,

$$z = \frac{(kdt^2 - \frac{2}{3}t^3)b + [(kd - t)^2(t + \frac{1}{3}(kd - t))]b'}{t(2kd - t)b + (kd - t)^2b'} \tag{25a}$$

Arm of resisting couple (see footnote Sec. 106),

$$jd = d - z \tag{26a}$$

Compressive unit stress in extreme fiber of concrete,

$$f_c = \frac{2Mkd}{[(2kd - t)bt + (kd - t)^2b']jd} \tag{27a}$$

Tensile unit stress in longitudinal reinforcement,

$$f_s = \frac{M}{A_s jd} \tag{28a}$$

For illustration of certain symbols as applied to typical T-beams, see Fig. 3.

117. Compressive Stress in Supports.—Provision shall be made for the compressive stress at the support in continuous T-beam construction.

118. Shear.—The flange of the slab shall not be considered as effective in computing the shear and diagonal tension resistance of T-beams.

119. Isolated Beams.—Isolated beams in which the T-form is used only for the purpose of providing additional compression area, shall have a flange thickness not less than one-half the width of the web and a total flange width not more than 4 times the web thickness.

E. Diagonal Tension and Shear

a. Formulas and Notation

120. Formulas.—Diagonal tension and shear in reinforced concrete beams shall be calculated by the following formulas:

Shearing unit stress,[1]

$$v = \frac{V}{bjd} \tag{29}$$

Stress[1] in vertical web reinforcement,

$$f_v = \frac{V's}{A_vjd} \tag{30}$$

121. Notation.—The symbols used in formulas (29) to (36) are defined in Sec. 107, except as follows:

a = spacing of web reinforcement bars measured perpendicular to their direction;
A_v = total area of web reinforcement in tension within a distance of a (a_1, a_2, a_3, etc.) or the total area of all bars bent up in any one plane;
α = angle between web bars and longitudinal bars;
f_v = tensile unit stress in web reinforcement;
o = perimeter of bar;
Σo = sum of perimeters of bars in one set;
r = ratio of cross-sectional area of negative reinforcement which crosses entirely over the column capital of a flat slab or over the dropped panel, to the total cross-sectional area of the negative reinforcement in the two column strips;
s = spacing of web members, measured at the neutral axis and in the direction of the longitudinal axis of the beam;
u = bond stress per unit of area of surface of bar;
v = shearing unit stress;
V = total shear;
V' = external shear on any section after deducting that carried by the concrete.

b. Beams without Web Reinforcement

122. Bars not Anchored.—The shearing unit stress in beams in which the longitudinal reinforcement is designed to meet all moment requirements, but without special anchorage, shall not exceed $0.02f'_c$, but in no case shall it exceed 40 lb. per sq. in. Adequate reinforcement shall be provided at all sections where negative moment occurs in beams continuous over supports or built into walls or columns at their ends. (For typical design, see Fig. 4.)

123. Bars Anchored.—The shearing unit stress in beams in which longitudinal reinforcement is anchored by means of hooked ends or otherwise, as specified in Sec. 130, shall not exceed $0.03f'_c$. Adequate reinforcement for both positive and negative moment shall be provided at all sections where maximum moment exists. (For typical design, see Fig. 5.)

c. Beams with Web Reinforcement

124. With Web Reinforcement.—When the shearing unit stress calculated by formula (29) exceeds the values specified in Secs. 122 and 123, web reinforcement shall be provided by one or more of the following methods:

(*a*) Series of vertical stirrups or web bars;
(*b*) Series of inclined stirrups or web bars;
(*c*) Series of bent-up longitudinal bars;
(*d*) Longitudinal bars bent up in a single plane.

Provision against bond failure of the web reinforcement shall be as specified in Sec. 131. (For typical designs, see Figs. 6 and 7. For typical detail of anchorage of longitudinal bars and vertical stirrups, see Fig. 8.)

125. Web or Bent-up Bars.—Where web reinforcement is present and where longitudinal reinforcement is provided to meet all moment requirements, the concrete may be assumed to carry a shearing unit stress not greater than $0.02f'_c$ and not greater in any case than 40 lb. per sq. in. In the case where a series of web bars or bent-up longitudinal bars is used, the web reinforcement shall be designed according to the formula:

$$A_v = \frac{V'a}{f_vjd} \quad \frac{V's\sin\alpha}{f_vjd} \tag{31}$$

(For typical design, see Fig. 9.)

[1] Approximate results may be secured by assuming $j = 0.875$.

126. Bars Bent Up in Single Plane.—Where the web reinforcement consists of bars bent up in a single plane at an angle so as to reinforce all sections of the beam in which the shearing unit stress on the web concrete exceeds $0.02f'_c$, the concrete may be assumed to take a shearing unit stress not greater than $0.02f'_c$, and not greater than 40 lb. per sq. in.; the remainder of the shear shall be carried by the bent-up bars designed according to the formula:

$$A_v = \frac{V'}{f_v \sin \alpha} \tag{32}$$

In case the web reinforcement consists solely of bent bars, the first bent bar shall bend downward from the plane of the upper reinforcement at the plane of the edge of the support or between that plane and the center of the support. (For typical design, see Fig. 10.)

127. Combined Web Reinforcement.—Where two or more types of web reinforcement are used in conjunction, the total shearing resistance of the beam shall be taken as the sum of the shearing resistance as computed for the various types separately.[1]

128. Maximum Shearing Unit Stress.—Where there is no special mechanical anchorage of the longitudinal reinforcement, the shearing unit stress shall not exceed $0.06f'_c$ irrespective of the web reinforcement used.

129. Special Mechanical Anchorage.—Where special mechanical anchorage of the longitudinal reinforcement as prescribed in Sec. 130 is provided, the shearing unit stress as computed by formula (29) may be greater than $0.06f'_c$, but in no case shall it exceed $0.12f'_c$.[2] In this case the concrete may be assumed to take a shearing unit stress of not more than $0.025f'_c$, but not more than 50 lb. per sq. in.

130. Anchorage of Longitudinal Reinforcement.—Special mechanical anchorage of the longitudinal reinforcement for positive moment may consist of carrying the bars beyond the point of inflection of restrained or continuous members a sufficient distance to develop by bond between the point of inflection and the end of the bar a tensile stress equal to one-third the safe working stress in the reinforcement. If such a bar is straight, it shall extend to within 1 in. of the center of the support, or in the case of wide supports shall extend not less than 12 in. beyond the face of the support. Special mechanical anchorage may also be secured by bending the end of the bar over the support in a full semicircle to a diameter not less than 8 times the diameter of the bar, the total length of the bend being not less than 16 diameters of the bar. Any other mechanical device that secures the end of the bar over the support against slipping without stressing the concrete in excess of $0.5f'_c$ in local compression may be used, provided such device does not tend to split the concrete. Negative reinforcement shall be thoroughly anchored at or across the support or shall extend into the span a sufficient distance to develop by bond the tensile stresses due to negative moment. In the case of freely supported ends of continuous beams, special mechanical anchorage shall be provided, which is capable of developing at the end of the span a tensile stress which is not less than one-third of the safe tensile stress of the bar at the point of maximum moment. (For illustrative design, see Fig. 11.)

131. Anchorage of Web Reinforcement.—Anchorage of the web reinforcement shall be by one of the following methods:

(*a*) Continuity of the web bar with the longitudinal bar

(*b*) Carrying the web bar around at least two sides of a longitudinal bar at both ends of the web bar; or

(*c*) Carrying the web bar about at least two sides of a longitudinal bar at one end and making a semicircular hook at the other end which has a diameter equal to that of the web bar.

In all cases the bent ends of web bars shall extend at least 8 diameters below or above the point of extreme height or depth of the web bar. In case the end anchorage of the web member is not in bearing on other reinforcement, the anchorage shall be such as to engage an adequate amount of concrete to prevent the bar from pulling off a portion of the concrete. In all cases the stirrups shall be carried as close to the upper and lower surfaces as fireproofing requirements will permit. (For typical designs, see Figs. 8 and 12.)

132. Size of Web Bars.—The size of web reinforcement bars which are neither a part of the longitudinal bars nor welded thereto, shall be such that not less than two-fifths of the allowable tensile stress in the bar may be developed by bond stresses in a length of bar equal to $0.4d$.[3] The remainder of the tensile stress in the bar shall be provided for by adequate end anchorage as specified in Sec. 131.

133. Breadth of Beams in Shear.—Shearing unit stress shall be computed on the full width of rectangular beams, on the width of the stem of T-beams, and on the thickness of the web in beams of I-section.

134. Shear in Beam-and-tile Construction.—The shearing stress in tile-and-concrete-beam construction shall not exceed that in beams or slabs with similar reinforcement. The width of the effective

[1] In such computation the shearing value of the concrete in the web shall be included once only.

[2] The limit $0.12 f'_c$ is based on the ultimate bearing unit stress of $0.5f'_c$ at which beams reinforced with vertical stirrups fail due to diagonal compression in the webs. A higher value than $0.12f'_c$ may be permitted in beams with inclined web reinforcement, but it is not thought necessary to allow such higher limit to meet the needs of design practice.

[3] This condition is satisfied for plain round stirrups when the diameter of the bar does not exceed $\frac{d}{50}$.

section for shear as governing diagonal tension shall be taken as the thickness of the concrete web plus one-half the thickness of the vertical webs of the tile. (For typical design, see Fig. 13.)

135. Spacing of Web Reinforcement.—The spacing, a, of web reinforcement bars shall be measured perpendicular to their direction and in a plane parallel to the longitudinal axis of the beam. The spacing shall not exceed $\frac{3}{4}d$ in any case where web reinforcement is necessary. Where vertical stirrups are used, or where inclined web bars make an angle more than 60 deg. with the horizontal, the spacing shall not exceed $\frac{1}{2}d$. Where the shearing unit stress exceeds $0.06f'_c$, the spacing of the web reinforcement shall not exceed $\frac{1}{2}d$ in any case, nor $\frac{1}{3}d$ for vertical stirrups or web reinforcement making an angle more than 60 deg. with the horizontal. The first shear reinforcement member shall cross the neutral axis of the member at a distance from the face of the support, measured along the axis of the beam, not greater than $\frac{1}{4}d$, nor greater than the spacing of web members as determined for a section taken at the edge of the support. Web members may be placed at any angle between 20 and 90 deg. with the longitudinal bars, provided that if inclined they shall be inclined in such a manner as to resist the tensile stress in the web.

d. Flat Slabs

136. Shearing Stress.—The shearing unit stress shall not exceed the value of v in the formula,

$$v = 0.02f'_c\,(1 + r) \tag{33}$$

nor in any case shall it exceed $0.03\,f'_c$.

The unit shearing stress shall be computed on

(a) A vertical section which has a depth in inches of $\frac{7}{8}\,(t_1 - 1\frac{1}{2})$ and which lies at a distance in inches of $t_1 - 1\frac{1}{2}$ from the edge of the column capital; and

(b) A vertical section which has a depth in inches of $\frac{7}{8}\,(t_2 - 1\frac{1}{2})$ and which lies at a distance in inches of $t_2 - 1\frac{1}{2}$ from the edge of the dropped panel.

In no case shall r be less than 0.25. Where the shearing stress on section (a) is being considered, r shall be taken as the proportional amount of reinforcement crossing the column capital; where the shearing stress at section (b) is being considered, r shall be taken as the proportional amount of reinforcement crossing entirely over the dropped panel. (For typical flat slab and designation of principal design sections, see Figs. 14 and 15.)

e. Footings

137. Shear and Diagonal Tension in Footings.—The shearing stress shall be computed by formula 29. When so computed the stress on the critical section defined below, or on sections outside of the critical section, shall not exceed $0.02f'_c$ for footings with straight reinforcement bars, nor $0.03f'_c$ for footings in which the reinforcement bars are anchored at both ends by adequate hooks or otherwise as specified in Sec. 130.

138 Critical Section for Soil Footings.—The critical section for diagonal tension in footings bearing directly on the soil shall be taken on a vertical section through the perimeter of the lower base of a frustum of a cone or pyramid which has a base angle of 45 deg. and has for its top the base of the column or pedestal and for its lower base the plane of the center of longitudinal reinforcement.

139. Critical Section for Pile Footings.—The critical section for diagonal tension in footings bearing on piles shall be taken on a vertical section at the inner edge of the first row of piles entirely outside a section midway between the face of the column or pedestal and the section described in Sec. 138 for soil footings, but in no case outside of the section described in Sec. 138. The critical section for piles not grouped in rows shall be taken midway between the face of the column and the perimeter of the base of the frustum described in Sec. 138.

E. Bond

140. Formula.—Bond between concrete and reinforcement bars in reinforced concrete beams and slabs shall be computed by the formula,

$$u = \frac{V}{\Sigma o\, jd} \tag{34}$$

141. Working Stress.—Unless otherwise specified, the reinforcement shall be so proportioned that the bond stress between the metal and the concrete shall not exceed the following:

(a) Plain bars,

$$u = 0.04f'_c \tag{35}$$

(b) Deformed bars, meeting the requirements of Section 23,

$$u = 0.05f'_c \tag{36}$$

142. Bond in Footings.—The bond stress on a section of a footing shall be computed by formula (34.) Only the bars counted as effective in bending shall be considered in computing the number of bars crossing a section. The bond stress computed in this manner on sections at the face of the column or outside the column shall not exceed the value specified in Sec. 141. Special investigation shall be made of bond stresses in footings with stepped or sloping upper surface; maximum stresses may occur at sections near the edges of the footings.

143. Reinforcement in Two or More Directions.—The permissible bond stress given by formulas (35) and (36) for footings and similar members where reinforcement is required in more than one direction shall be reduced as follows:

(*a*) For two-way reinforcement, 25 per cent.

(*b*) For each additional direction, 10 per cent.

144. Anchored Bars.—The bond stresses for bars adequately anchored at both ends by hooks or otherwise, as provided in Sec. 130, may be 1½ times the values specified in Sec. 141. Hooks in footings shall be effectively positioned to insure that they engage a mass of concrete above the plane of the reinforcement.

F. Flat Slabs[1]

145. Moments in Interior Panels.—The symbols used in formulas (37) to (42) are defined in Sec. 107 except as indicated in Secs. 145, 148 and 158. In flat slabs in which the ratio of reinforcement for negative moment in the column strip is not greater than 0.01, the numerical sum of the positive and negative moments in the direction of either side of the panel shall be taken as not less than

$$M_0 = 0.09\,Wl\left(1-\frac{2}{3}\frac{c}{l}\right)^2$$

where M_0 = sum of positive and negative bending moments in either rectangular direction at the principal design sections of a panel of a flat slab;

c = base diameter of the largest right circular cone, which lies entirely within the column (including the capital) whose vertex angle is 90 deg and whose base is 1½ in. below the bottom of the slab or the bottom of the dropped panel (see Fig. 14);

l = span length[2] of flat slab, center to center of columns in the rectangular direction in which moments are considered; and

W = total dead and live load uniformly distributed over a single panel area.

146. Principal Design Sections.—The principal design sections for critical moments in flat slabs subjected to uniform load shall be taken as follows:

(*a*) Negative moment in middle strip: extending in a rectangular direction from a point on the edge of panel $\frac{l_1}{4}$ from column center a distance $\frac{l_1}{2}$ toward the center of adjacent column on the same panel edge.

(*b*) Negative moment in column strip: extending in a rectangular direction along the edge of the panel from a point $\frac{l_1}{4}$ from the center of a column to a point $\frac{c}{2}$ from the center of the same column and thence one-quarter circumference about the column center to the adjacent edge of the panel.

TABLE III.—MOMENTS TO BE USED IN DESIGN OF FLAT SLABS

Strip	Flat slabs without dropped panels		Flat slabs with dropped panels	
	Negative	Positive	Negative	Positive
SLABS WITH 2-WAY REINFORCEMENT				
Column strip............	$0.23M_0$	$0.11M_0$	$0.25M_0$	$0.10M_0$
2 column strips..	$0.46M_0$	$0.22M_0$	$0.50M_0$	$0.20M_0$
Middle strip.............	$0.16M_0$	$0.16M_0$	$0.15M_0$	$0.15M_0$
SLABS WITH 4-WAY REINFORCEMENT				
Column strip...	$0.25M_0$	$0.10M_0$	$0.27M_0$	$0.095M_0$
2 column strips..	$0.50M_0$	$0.20M_0$	$0.54M_0$	$0.190M_0$
Middle strip....	$0.10M_0$	$0.20M_0$	$0.08M_0$	$0.190M_0$

[1] The requirements for flat slabs in Secs. 145 to 162, inclusive, apply to two-way and four-way systems of reinforcement. The Committee is not prepared at this time to submit requirements covering other types of flat slabs.

[2] The column strip and the middle strip to be used when considering moments in the direction of the dimension l are located and dimensioned as shown in Fig. 15. The dimension l_1 does not always represent the short length of the panel. When moments in the direction of the shorter panel length are considered, the dimensions l and l_1 are to be interchanged and the strips corresponding to those shown in Fig. 15 but extending in the direction of the shorter panel length are to be considered.

(c) Positive moment in middle strip: extending in a rectangular direction from the center of one edge of a middle strip a distance $\frac{l_1}{2}$ to the center of the other edge of the same strip.

(d) Positive moment in column strip: extending in a rectangular direction from the center of one edge of a column strip a distance $\frac{l_1}{4}$ to the center of the other edge of the same strip.

147. Moments in Principal Design Sections.—The moments in the principal design sections shall be those given in Table III, except as follows:

(a) The sum of the maximum negative moments in the two column strips may be greater or less than the values given in Table III, by not more than $0.03M_0$.

(b) The maximum negative moment and the maximum positive moments in the middle strip and the sum of the maximum positive moments in the two column strips may each be greater or less than the values given in Table III, by not more than $0.01M_0$.

148. Thickness of Flat Slabs and Dropped Panels.—The total thickness,[1] t_1, of the dropped panel in inches, or of the slab if a dropped panel is not used, shall be not less than:

$$.0382\left(1 - 1.44\frac{c}{l}\right)l \sqrt{Rw'\frac{l_1}{b_1}} + 1\tfrac{1}{2} \qquad (38)^2$$

where R = ratio of negative moment in the two column strips to M_0, and w' uniformly distributed dead and live load per unit of area of floor.

For slabs with dropped panels the total thickness[1] in inches at points away from the dropped panel shall be not less than:

$$t_2 = 0.02l\sqrt{w'} + 1 \qquad (39)$$

The slab thickness t_1 or t_2 shall in no case be less than $\frac{l}{32}$ f floor slabs, and not less than $\frac{l}{40}$ for roof slabs. In determining minimum thickness by formulas (38) and (39), the value of l shall be the panel length center to center of the columns, on long side of panel, l_1 shall be the panel length on the short side of the panel, and b_1 shall be the width or diameter of dropped panel in the direction of l_1 except that in a slab without dropped panel b_1 shall be $0.5l_1$.

149. Minimum Dimensions of Dropped Panels.—The dropped panel shall have a length or diameter in each rectangular direction of not less than one-third the panel length in that direction, and a thickness not greater than $1.5t_2$.

150. Wall and Other Irregular Panels.—In wall panels and other panels in which the slab is discontinuous at the edge of the panel, the maximum negative moment one panel length away from the discontinuous edge and the maximum positive moment between shall be taken as follows:

(a) Column strip perpendicular to the wall or discontinuous edge, 15 per cent greater than that given in Table III.

(b) Middle strip perpendicular to wall or discontinuous edge, 30 per cent greater than that given in Table III.

In these strips the bars used for positive moments perpendicular to the discontinuous edge shall extend to the exterior edge of the panel at which the slab is discontinuous.

151. Panels with Wall Beams.—In panels having a marginal beam on one edge or on each of two adjacent edges, the beam shall be designed to carry the load superimposed directly upon it. If the beam has a depth greater than the thickness of the dropped panel into which it frames, the beam shall be designed to carry, in addition to the load superimposed upon it, at least one-fourth of the distributed load for which the adjacent panel or panels are designed, and each column strip adjacent to and parallel with the beam shall be designed to resist a moment at least one-half as great as that specified in Table III for a column strip.[3] If the beam used has a depth less than the thickness of the dropped panel into which it frames, each column strip adjacent to and parallel with the beam shall be designed to resist the moments specified in Table III for a column strip. Where there are beams on two opposite edges of the panel, the slab and the beam shall be designed as though all the load was carried to the beam.

152. Discontinuous Panels.—The negative moments on sections at and parallel to the wall, or discontinuous edge of an interior panel, shall be determined by the conditions of restraint.[4]

153. Flat Slabs on Bearing Walls.—Where there is a beam or a bearing wall on the center line of columns in the interior portion of a continuous flat slab, the negative moment at the beam or wall line in the middle strip perpendicular to the beam or wall shall be taken as 30 per cent greater than the

[1] The thickness will be in inches regardless of whether l and w' are in feet and pounds per square foot or in inches and pounds per square inch.

[2] The values of R used in this formula are the coefficients of M_0 for negative moment in the column strip in Table III.

[3] In wall columns, brackets are sometimes substituted for capitals or other changes are made in the design of the capital. Attention is directed to the necessity for taking into account the change in the value of c in the moment formula for such cases.

[4] The committee is not prepared to make a more definite recommendation at this time.

moment specified in Table III for a column strip. The column strip adjacent to and lying on either side of the beam or wall shall be designed to resist a moment at least one-half of that specified in Table III for a column strip.

154. Point of Inflection.—The point of inflection in any line parallel to a panel edge in interior panels of symmetrical slabs without dropped panels shall be assumed to be at a distance from the center of the span equal to $\frac{3}{16}$ of the distance between the two sections of critical negative moment at opposite ends of the line; for slabs having dropped panels, the coefficient shall be $\frac{1}{4}$.

155. Reinforcement.—The reinforcement bars which cross any section and which fulfill the requirements given in Sec. 156 may be considered as effective in resisting the moment at the section. The sectional area of a bar multiplied by the cosine of the angle between the direction of the axis of the bar and any other direction may be considered effective as reinforcement in that direction.

156. Arrangement of Reinforcement.—The design shall include adequate provision for securing the reinforcement in place so as to take not only the critical moments but the moments at intermediate sections. All bars in rectangular or diagonal directions shall extend on each side of a section of critical moment, either positive or negative, to points at least 20 diameters beyond the point of inflection as specified in Sec. 154. In addition to this provision, bars in diagonal directions used as reinforcement for negative moment shall extend on each side of a line drawn through the column center at right angles to the direction of the band at least a distance of 0.35 of the panel length, and bars in diagonal directions used as reinforcement for positive moment shall extend on each side of a diagonal through the center of the panel at least a distance equal to 0.35 of the panel length; no splice by lapping shall be permitted at or near regions of maximum stress except as just described. At least two-thirds of all bars in each direction shall be of such length and shall be so placed as to provide reinforcement at two sections of critical negative moment and at the intermediate section of critical positive moment. Continuous bars shall not all be bent up at the same point of their length, but the zone in which this bending occurs shall extend on each side of the assumed point of inflection, and shall cover a width of at least $\frac{1}{15}$ of the panel length. Mere sagging of the bars shall not be permitted. In four-way reinforcement the position of the bars in both diagonal and rectangular directions may be considered in determining whether the width of zone of bending is sufficient.

157. Reinforcement at Construction Joints.—See Sec. 73.

158. Tensile Stress in Reinforcement.—The following formula shall be used in computing the tensile stress f_s in the reinforcement in flat slabs; the stress so computed shall not at any of the principal design sections exceed the values specified in Sec. 205:

$$f_s = \frac{RM_0}{A_s jd} \tag{40}$$

where RM_0 = moment specified in Sec. 147 for two column strips or for one middle strip; and
A_s = effective cross-sectional area of the reinforcement which crosses any of the principal design sections and which meets the requirements of Sec. 156.

159. Compressive Stress in Reinforcement.—The following formulas shall be used in computing maximum compressive stress in the concrete in flat slabs; and the stress so computed shall not exceed $0.4f'_c$:

(*a*) Compression due to negative moment, RM_0, in the two column strips,

$$f_c = \frac{3.5\,RM_0}{kjb_1d^2}\left(1 - 1.2\frac{c}{l}\right) \tag{41}$$

(*b*) Compression due to positive moment, RM_0, in the two column strips or negative or positive moment in the middle strip,

$$f_c = \frac{6RM_0}{kjl_1d^2} \tag{42}$$

160. Shearing Stress.—See Sec. 130.

161. Unusual Panels.—The moment coefficients, moment distribution and slab thicknesses specified herein are for slabs which have three or more rows of panels in each direction, and in which the panels are approximately uniform in size. For structures having a width of one or two panels, and also for slabs having panels of markedly different sizes, an analysis shall be made of the moments developed in both slab and columns, and the values given herein modified accordingly. Slabs with paneled ceiling or with depressed paneling in the floor shall be considered as coming under the requirements herein given.

162. Bending Moments in Columns.—See Sec. 173.

G. Reinforced Concrete Columns

163. Limiting Dimensions.—The provisions in the following sections for the carrying capacity of reinforced columns are based on the assumption of a short column. Where the unsupported length is greater than 40 times the least radius of gyration (40*R*), the carrying capacity of the column shall be determined by formula (48) for slender columns. Principal columns in buildings shall have a width of not less than 12 in. Posts that are not continuous from story to story shall have a width of not less than 6 in.

164. Unsupported Length.—The unsupported length of a column in flat-slab construction shall be taken as the clear distance between the under side of the capital and the top of the floor slab below. For beam-and-slab construction the unsupported length of a column shall be taken as the clear distance between the under side of the shallowest beam framing into it and the top of the floor slab below; where beams run in one direction only the clear distance between floor slabs shall be used. For columns supported laterally by struts or beams only, two struts shall be considered an adequate support, provided the angle between the two planes formed by the axis of the column with the axis of each strut is not less than 75 deg. nor more than 105 deg. The unsupported length for this condition shall be considered the clear distance between struts. When haunches are used at the junction of beams or struts with columns, the clear distance between supports may be considered as reduced by two-thirds of the depth of the haunch.

165. Safe Load on Spiral Columns.—The symbols used in formulas (43) to (50) are defined in Sec. 107, except as indicated in Secs. 165, 167, 170, 172, 180 and 188. The safe axial load on columns reinforced with longitudinal bars and closely spaced spirals enclosing a circular core shall be determined by the following formula:

$$P = A_c f_c + n f_c p A \qquad (43)$$

where

A = area of the concrete core enclosed within the spiral;

P = total safe axial load on column whose $\frac{h}{R}$ is less than 40;

p = ratio of effective area of longitudinal reinforcement to area of the concrete core; and

A_c = $A(1 - p)$ = net area of concrete core.

The allowable value of f_c for use in this type of column shall be determined by the following formula:

$$f_c = 300 + (0.10 + 4p)f'_c \qquad (44)$$

The longitudinal reinforcement shall consist of at least six bars of minimum diameter of ½ in., and its effective cross-sectional area shall not be less than 1 per cent nor more than 5 per cent of that of the enclosed core.

166. Spiral Reinforcement.—The spiral reinforcement shall be not less in amount than one-fourth the volume of the longitudinal reinforcement. It shall consist of evenly spaced continuous spirals held firmly in place and true to line by at least three vertical spacer bars. The spacing of the spirals shall be not greater than one-sixth of the diameter of the core and in no case more than 3 in. The lateral reinforcement shall meet the requirements of the Tentative Specifications for Cold-drawn Steel Wire for Concrete Reinforcement. Reinforcement shall be protected everywhere by a covering of concrete cast monolithic with the core, which shall have a minimum thickness of 1½ in. in square columns and 2 in. in round or octagonal columns.

167. Safe Load on Columns with Lateral Ties.—The safe axial load on columns reinforced with longitudinal bars and separate lateral ties shall be determined by the following formula:

$$P = A'_c f_c + A_s n f_c \qquad (45)$$

where A'_c = net area of concrete in the column (total column area minus steel area); and A_s = effective cross-sectional area of longitudinal reinforcement.

The value of f_c for this type of column shall not exceed $0.20f'_c$. The amount of longitudinal reinforcement considered in the calculations shall be not more than 2 per cent nor less than 0.5 per cent of the total area of the column. The longitudinal reinforcement shall consist of not less than 4 bars of minimum diameter of ½ in., placed with clear distance from the face of the column not less than 2 in.

168. Lateral Ties.—Lateral ties shall be not less than ¼ in. in diameter, spaced not farther than 8 in. apart.

169. Bending Stress in Columns.—Reinforced concrete columns subjected to bending stresses shall be treated as follows:

(a) *Spiral Column.*—The compressive unit stress on the concrete within the core area under combined axial load and bending shall not exceed by more than 20 per cent the value given for axial load by formula (40).

(b) *Columns with Lateral Ties.*—Additional longitudinal reinforcement not to exceed 2 per cent shall be used if required and the compressive unit stress on the concrete under combined axial load and bending may be increased to $0.30 f'_c$.

Tension due to bending in the longitudinal reinforcement shall not exceed 16,000 lb. per sq. in.

170. Composite Columns.—The safe carrying capacity of composite columns in which a structural steel or cast-iron column is thoroughly encased in a spirally reinforced concrete core shall be based on a certain unit stress for the steel or cast-iron core plus a unit stress of $0.25f'_c$ on the area within the spiral core. The unit compressive stress on the steel section shall be determined by the formula:

$$f_r = 18{,}000 - 70\frac{h}{R} \qquad (46)$$

but shall not exceed 16,000 lb. per sq. in. The unit stress on the cast-iron section shall be determined by the formula:

$$f_r = 12{,}000 - 60\frac{h}{R} \qquad (47)$$

but shall not exceed 10,000 lb. per sq. in. In formulas (46) and (47), f_r = compressive unit stress in metal core, and R = least radius of gyration of the steel or cast-iron section.

The diameter of the cast-iron section shall not exceed one-half of the diameter of the core within the spiral. The spiral reinforcement shall be not less than 0.5 per cent of the volume of the core within the spiral and shall conform in quality, spacing and other requirements to the provisions for spirals in reinforced concrete columns. Ample sections of concrete and continuity of reinforcement shall be provided at the junction with beams or girders. The area of the concrete between the spiral and the metal core shall be not less than that required to carry the total floor load of the story above on the basis of a stress in the concrete of 0.35 f'_c, unless special brackets are arranged on the metal core to receive directly the beam or slab loads.

171. Structural Steel Columns.—The safe load on a structural steel column of a section which fully encloses or encases an area of concrete, and which is protected by an outside shell of concrete at least 3 in. thick, shall be computed in the same way as in the columns described in Sec. 170, allowing 0.25 f'_c on the area of the concrete enclosed by the steel section. The outside shell shall be reinforced by wire mesh or hoops weighing not less than 0.2 lb. per sq. ft. of surface of the core and with a maximum spacing of strands or hoops of 6 in. Special brackets shall be used to receive the entire floor load at each story. The working stress in steel columns shall be calculated by formula (16), but shall not exceed 16,000 lb. per sq. in.

172. Long Columns.—The permissible working unit stress on the core in axially loaded columns which have a length greater than 40 times the least radius of gyration of the column core (40R) shall be determined by the formula:

$$\frac{P'}{P} = 1.33 - \frac{h}{120\,R} \tag{48}$$

where P' = total safe axial load on long column;

P = total safe axial load on column of the same section whose $\frac{h}{R}$ is less than 40, determined as in Sec. 167; and

R = least radius of gyration of column core.

173. Bending Moments in Columns.—The bending moments in interior and exterior columns shall be determined on the basis of loading conditions and end restraint, and shall be provided for in the design.[1] The recognized standard methods shall be followed in calculating the stresses due to combined axial load and bending.

H. Footings

174. Types.—Various types of reinforced concrete footings are in use depending on conditions. The fundamental principles of the design of reinforced concrete will generally apply to footings as to other structural members. The requirements for flexure and shear in Secs. 112 to 139, inclusive, shall govern the design of footings, except as hereinafter provided.

175. Distribution of Pressure.—The upward reaction per unit of area on the footing shall be taken as the column load divided by the area of base of the footing.

176. Pile Footing.—Footings carried on piles shall be treated in the same manner as those bearing directly on the soil, except that the reaction shall be considered as a series of concentrated loads applied at the pile centers.

177. Sloped Footing.—Footings in which the depth has been determined by the requirements for shear as specified in Sec. 137 may be sloped between the critical section and the edge of the footing, provided that the shear on no section outside the critical section exceeds the value specified, and provided further that the thickness of the footing above the reinforcement at the edge shall not be less than 6 in. for footings on soil nor less than 12 in. for footings on piles.

178. Stepped Footing.—The top of the footing may be stepped instead of sloped, provided that the steps are so placed that the footing will have at all sections a depth at least as great as that required for a sloping top. Stepped footings shall be cast monolithically.

179. Critical Section for Bending.—In a concrete footing which supports a concrete column or pedestal, the critical section for bending shall be taken at the face of the column or pedestal. Where steel or cast-iron bases are used, the moment in the footing shall be calculated at the edge of the base and at the center. In calculating this moment, the column or pedestal load shall be assumed as uniformly distributed over its base.

180. Square Column on Square Footing.—For a square footing supporting a concentric square column, the bending moment at the critical section is that produced by the upward pressure on the trapezoid bounded by one face of the column, the corresponding outside edge of the footing, and the portions of the two diagonals. The center of application of the reaction on the two corner triangles of this trapezoid shall be taken at a distance from the face of the column equal to 0.6 of the projection of

[1] The Committee is not prepared to make more definite recommendations at this time.

the footing. The center of the application of the reaction on the rectangular portion of the trapezoid shall be taken at its center of gravity. This gives a bending moment expressed by the formula:

$$M \qquad (a + 1.2c)c^2 \tag{49}$$

where M = bending moment at critical section of footing;
a = width of face of column or pedestal;
c = projection of footing from face of column; and
w = upward reaction per unit of area of base of footing.

(For typical footing designs, see Figs. 16 to 18.)

181. Round Column on Square Footing.—Square footings supporting a round or octagonal column shall be treated in the same manner as for a square column, using for the distance a the side of a square having an area equal to the area enclosed within the perimeter of the column.

182. Reinforcement.—The reinforcement necessary to resist the bending moment in each direction in the footing shall be determined as for a reinforced concrete beam; the effective depth of the footing shall be the depth from the top to the plane of the reinforcement. The required area of reinforcement thus calculated shall be spaced uniformly across the footing, unless the footing width is greater than the side of the column or pedestal plus twice the effective depth of the footing, in which case the width over which the reinforcement is spread, may be increased to include one-half the remaining width of the footing. In order that no considerable area of the footing shall remain unreinforced, additional bars shall be placed outside of the width specified, but such bars shall not be considered as effective in resisting the calculated bending moment. For the extra bars a spacing double that used for the reinforcement within the effective belt may be used.

183. Concrete Stress.—The extreme fiber stress in compression in the concrete shall be kept within the limits specified in Sec. 108. The extreme fiber stress in sloped or stepped footings shall be based on the exact shape of the section for a width not greater than that assumed effective for reinforcement.

184. Irregular Footings.—Rectangular or irregular-shaped footings shall be calculated by dividing the footings into rectangles or trapezoids tributary to the sides of the column, using the distance to the actual center of gravity of the area as the moment arm of the upward forces. Outstanding portions of combined footings shall be treated in the same manner. Other portions of combined footings shall be designed as beams or slabs.

185. Shearing Stresses.—See Secs. 137 to 139.

186. Bond Stresses.—See Secs. 142 to 144.

187. Transfer of Stress from Column Reinforcement.—The compressive stress in longitudinal reinforcement in columns or pedestals shall be transferred to the footing by one of the following methods:

(*a*) By metal distributing bases having a sufficient area and thickness to transmit safely the load from the longitudinal reinforcement in compression and bending. The bases shall be accurately set and provided with a full bearing on the footing.

(*b*) By dowels, at least one for each bar and of total sectional area not less than the area of the longitudinal column reinforcement. The dowels shall project into the columns or into the pedestal or footing a distance not less than 50 times the diameter of the column bars.

188. Pedestals without Reinforcement.—The allowable compressive unit stress on the gross area of a concentrically loaded pedestal without reinforcement shall not exceed $0.25f'_c$. If the column resting on such a pedestal is provided with distributing bases for the longitudinal reinforcement, the permissible compressive unit stress under the column core shall be determined by the following formula:

$$r_a = 0.25f'_c\sqrt[3]{\frac{A}{A'}} \tag{50}$$

where r_a = permissible working stress over the loaded area;
A = total net area of the top of pedestal; and
A' = loaded area of pedestal.

189. Pedestals with Reinforcement.—Where the permissible load at the top of a pedestal, determined by formula (50), is less than the column load to be supported, dowels shall be used as specified in Sec. 187. If the height of the pedestal is not sufficient to give the required embedment to the dowels, they shall extend into the footing to a point 50 diameters below the top of the pedestal for plain bars and 40 diameters for deformed bars. If the column load divided by the cross-section of the pedestal exceeds $0.25f'_c$ the pedestal shall be considered as a section of a column and spiral reinforcement shall be provided accordingly.

190. Permissible Load at Top of Footings.—Where distributing bases are used for transferring the stress from column reinforcement directly to the footing, the permissible compressive unit stress shall be determined by formula (50). This formula may be applied by using A as the area of the top horizontal surface of the footing or with the following modifications:

(*a*) In footings with sloping or stepped top in which a plane, drawn from the edge of the base of the column so that it makes the greatest possible angle with the vertical but remains entirely within the footing, has a slope with the horizontal not greater than 0.5, the total bearing area of the footing may be used for A.

(b) In footings in which the slope of the plane referred to is greater than 0.5 but not greater than 2.0, the permissible compressive unit stress at the top shall be determined by direct proportion, in terms of the slope, between the value found for a slope of 0.5 and the value of $0.25f'_c$ for a slope of 2.0. For a slope of 2.0 or greater the compressive unit stress at the top shall not exceed $0.25f'_c$. (For typical footing designs, see Figs. 16 and 18.)

191. Pedestal Footings.—Pedestal footings may be designed as pedestals, that is, without reinforcement other than that required to transmit the column load, except that when supported directly on driven piles, a mat of reinforcing bars consisting of not less than 0 20 sq. in. per foot of width in each direction shall be placed 3 in. above the top of the piles. The height of a pedestal footing shall be not greater than 4 times the average width.

I. Reinforced Concrete Retaining Walls

192. Types of Retaining Walls.—Reinforced concrete retaining walls may be of the following types.

(a) Cantilever;
(b) Counterforted;
(c) Buttressed;
(d) Cellular.

193. Loads and Unit Stresses.—Reinforced concrete retaining walls shall be designed[1] for the loads and reactions, and shall be so proportioned that the permissible unit stresses specified in Secs. 196 to 208 are not exceeded. The heels of cantilever, counterforted and buttressed retaining walls shall be proportioned for the maximum resultant vertical loads to which they will be subjected, but the sections shall be such that the normal permissible unit stresses will not be increased by more than 50 per cent when the reaction from the foundation bed is neglected.

194. Details of Design.—The following principles shall be followed in the design of reinforced concrete retaining walls:

(a) The unsupported toe and heel of the base slabs shall be considered as cantilever beams fixed at the edge of the support.

(b) The vertical section of a cantilever wall shall be considered as a cantilever beam fixed at the top of the base.

(c) The vertical sections of counterforted and buttressed walls and parts of base slabs supported by the counterforts or buttresses shall be designed in accordance with the requirements specified herein for the continuous slab.

(d) The exposed faces of walls without buttresses shall preferably be given a batter of not less than ¼ in. in 12 in.

(e) Counterforts shall be designed in accordance with the requirements specified for T-beams. Stirrups shall be provided in the counterforts to take the reaction from these spans when the tension reinforcement of the face walls and heels of bases is designed to span between the counterforts. Stirrups shall be anchored as near the exposed faces of the face walls, and as near the lower face of the bases, as practicable.

(f) Buttresses shall be designed in accordance with the requirements specified for rectangular beams.

(g) The shearing stress at the junction of the base with counterforts or buttresses shall not exceed the values specified in Secs. 120 to 135.

(h) Horizontal metal reinforcement shall be well distributed and of such form as to develop a high bond resistance. At least 0.25 sq. in. of horizontal metal reinforcement for each foot of height shall be provided near exposed surfaces not otherwise reinforced, to resist the formation of temperature and shrinkage cracks.

(i) Provision for temperature changes shall be made by grooved lock joints spaced not over 60 ft. apart.

(j) Counterforts and buttresses, where used, shall be located under all points of concentrated loading, and at intermediate points spaced 8 to 12 ft. apart.

(k) The walls shall be cast monolithically between expansion joints, unless construction joints made in accordance with Secs. 69 and 73 are provided.

195. Drains.—Drains or "weep holes" not less than 4 in. in diameter and not more than 10 ft. apart, shall be provided. In counterforted walls there shall be at least one drain for each pocket formed by the counterforts.

J. Floor Slabs Supported on Four Sides.[2]

K. Shrinkage and Temperature Stresses.[2]

[1] In proportioning retaining walls consideration shall be given to the following:
(a) Maximum bearing pressure of soil;
(b) Uniformity of distribution of foundation pressure on yielding soils;
(c) Stability against sliding;
(d) Minor increase of the horizontal forces may seriously affect (a) and (b).

[2] The Committee is not now ready to report on these subjects.

L. Summary of Working Stresses.

196. Notation.—f'_c = ultimate compressive strength of concrete at age of 28 days, based on tests of 6 by 12 -in. or 8 by 16-in. cylinders made and tested in accordance with the Standard Methods of Making and Storing Specimens of Concrete in the Field and the Tentative Methods of Making Compression Tests of Concrete.

a. Maximum Direct Stresses in Concrete

197. Direct Compression.
(*a*) Columns whose length does not exceed 40 *R*:
(1) With spirals..................varies with amount of longitudinal reinforcement
(2) Without spirals..$0.20f'_c$
(*b*) Long columns..see Sec. 172
(*c*) Piers and Pedestals:
(1) Without reinforcement ....$0.25f'_c$
(2) Special cases..see Sec. 188

198. Compression in Extreme Fiber.
(*a*) Extreme fiber stress in flexure..$0.40f'_c$
(*b*) Extreme fiber stress adjacent to supports of continuous beams ....$0.45f'_c$

199. Bearing Compression.—Anchorage of reinforcement..$0.50f'_c$

200. Tension.—All concrete members..none

b. Maximum Shearing Stresses in Concrete

201. Beams without Web Reinforcement.
(*a*) Longitudinal bars anchored..$0.03f'_c$
(*b*) Longitudinal bars not anchored..$0.02f'_c$

202. Beams with Reinforcement.
(*a*) Beams with stirrups..see Secs. 125 and 128
(*b*) Beams with bars bent up in several planes..see Sec. 125
(*c*) Beams with bars bent up in a single plane:
(1) Longitudinal bars anchored..$0.12f'_c$
(2) Longitudinal bars not anchored..$0.06f'_c$

203. Flat Slabs.
(*a*) Shear at distance *d* from capital or dropped panel ....$0.03f'_c$
(*b*) Other limiting cases in flat slabs..see Sec. 136

204. Footings.
(*a*) Longitudinal bars anchored..$0.03f'$
(*b*) Longitudinal bars not anchored..$0.02f'$

c. Maximum Stresses in Reinforcement

205. Tension in Steel.
(*a*) Billet-steel bars:
(1) Structural steel grade.......................... 16,000 lb. per sq. in.
(2) Intermediate grade.......................... 18,000 lb. per sq. in.
(3) Hard grade.......................... 18,000 lb. per sq. in.
(*b*) Rail-steel bars.......................... 16,000 lb. per sq. in.
(*c*) Structural steel .... 16,000 lb. per sq. in.
(*d*) Cold-drawn steel wire:
(1) Spirals..........................stress not calculated
(2) Elsewhere..........................18,000 lb. per sq. in.

206. Compression in Steel.
(*a*) Bars.......................... same as Sec. 205 (*a*) and (*b*)
(*b*) Structural steel core of composite column..........................18,000 lb. per sq. in., reduced for slenderness ratio
(*c*) Structural steel column..........................16,000 lb. per sq. in., reduced for slenderness ratio

207. Compression in Cast Iron.
Composite columns with spirals..........................10,000 lb. per sq. in.

d. Maximum Bond between Concrete and Steel

208. Bond.
(*a*) Beams and slabs, plain bars..$0.04f'_c$
(*b*) Beams and slabs, deformed bars..$0.05f'_c$
(*c*) Footings, plain bars, one-way..$0.04f'_c$
(*d*) Footings, deformed bars, one-way..$0.05f'_c$
(*e*) Footings, two-way reinforcement....................(*c*) or (*d*) reduced by 25 per cent
(*f*) Footings, each additional direction of reinforcement....(*c*) or (*d*) reduced by 10 per cent.

TABLE IV.—PROPORTIONS[1] FOR CONCRETE OF GIVEN COMPRESSIVE STRENGTH AT 28 DAYS

The table gives the proportions in which Portland cement and a wide range in sizes of fine and coarse aggregates should be mixed to obtain concrete of compressive strengths ranging from 1,500 to 3,000 lb. per sq. in. at 28 days. Proportions are given for concrete of four different consistencies.

The purpose of the table is twofold:

(1) To furnish a guide in the selection of mixtures to be used in preliminary investigations of the strength of concrete from given materials.

(2) To indicate proportions which may be expected to produce concrete of a given strength under average conditions where control tests are not made.

If the proportions to be used in the work are selected from the table without preliminary tests of the materials, and control tests are not made during the progress of the work, the mixtures in bold-face type shall be used.

The use of this table as a guide in the selection of concrete mixtures is based on the following:

(1) Concrete shall be plastic;

(2) Aggregates shall be clean and structurally sound;

(3) Aggregates shall be graded between the sizes indicated;

(4) Cement shall conform to the requirements of the Standard Specifications and Tests for Portland Cement (Serial Designation: C 9–21) of the American Society for Testing Materials.

The plasticity of the concrete shall be determined by the slump test carried out in accordance with the Tentative Specifications for Workability of Concrete for Concrete Pavements (Serial Designation: D 62–20 T) of the American Society for Testing Materials.

Apply the following rules in determining the size assigned to a given aggregate:

(1) Not less than 15 per cent shall be retained between the sieve which is considered the maximum size[2] and the next smaller sieve.

(2) Not more than 15 per cent of a coarse aggregate shall be finer than the sieve considered as the minimum size.[2]

(3) Only the sieve sizes given in the table shall be considered in applying rules (1) and (2).

(4) Sieve analysis shall be made in accordance with the Tentative Method of Test for Sieve Analysis of Aggregates for Concrete (Serial Designation: C 11–21 T) of the American Society for Testing Materials.

Proportions may be interpolated for concrete strengths, aggregate sizes and consistencies not covered by the table or determined by test.

[1] Based on the 28-day compressive strengths of 6 by 12-in. cylinders, made and stored in accordance with the Tentative Methods of Making Compression Tests of Concrete (Serial Designation: C 39–21 T) of the American Society for Testing Materials.

[2] For example: a graded sand with 16 per cent retained on the No. 8 sieve would fall in the 0–No. 4 size; if 14 per cent or less were retained, the sand would fall in the 0–No. 8 size. A coarse aggregate having 16 per cent coarser than 2-in. sieve would be considered as 3-in. aggregate.

TABLE IV.—PROPORTIONS FOR 1,500 LB. PER SQ. IN. CONCRETE.—(*Continued*)

Proportions are expressed by volume as follows: Portland Cement: Fine Aggregate: Coarse Aggregate.

Thus 1 : 2.6 : 4.6 indicates 1 part by volume of Portland cement, 2.6 parts by volume of fine aggregate and 4.6 parts by volume of coarse aggregate.

Size of coarse aggregate	Slump, in.	Size of fine aggregate				
		0–No. 28	0–No. 14	0–No. 8	0–No. 4	0–⅜ in.
None	½ to 1	1:2.8	1:3.2	1:3.8	1:4.4	1:5.1
	3 to 4	1:2.4	1:2.8	1:3.3	1:3.8	1:4.5
	6 to 7	**1:1.9**	**1:2.2**	**1:2.6**	**1:3.0**	**1:3.6**
	8 to 10	1:1.4	1:1.6	1:1.8	1:2.1	1:2.5
No. 4 to ¾ in	½ to 1	1:2.6:4.6	1:2.9:4.3	1:3.4:4.1	1:3.9:3.6	1:4.6:3.1
	3 to 4	1:2.3:4.0	1:2.6:3.8	1:2.9:3.6	1:3.4:3.2	1:4.1:2.8
	6 to 7	**1:1.8:3.4**	**1:2.0:3.2**	**1:2.3:3.1**	**1:2.6:2.8**	**1:3.1:2.5**
	8 to 10	1:1.1:2.5	1:1.3:2.4	1:1.5:2.4	1:1.7:2.2	1:2.1:2.0
No. 4 to 1 in.	½ to 1	1:2.4:5.3	1:2.7:5.2	1:3.1:5.0	1:3.5:4.7	1:4.3:4.3
	3 to 4	1:2.1:4.7	1:2.4:4.5	1:2.7:4.4	1:3.1:4.1	1:3.7:3.7
	6 to 7	**1:1.6:3.9**	**1:1.8:3.8**	**1:2.1:3.7**	**1:2.4:3.5**	**1:2.9:3.3**
	8 to 10	1:1.1:2.9	1:1.2:2.8	1:1.4:2.8	1:1.6:2.7	1:1.9:2.5
No. 4 to 1½ in	½ to 1	1:2.4:6.0	1:2.7:5.9	1:3.1:5.8	1:3.5:5.4	1:4.1:5.1
	3 to 4	1:2.0:5.4	1:2.3:5.3	1:2.7:5.2	1:3.0:5.0	1:3.5:4.6
	6 to 7	**1:1.6:4.4**	**1:1.8:4.3**	**1:2.0:4.3**	**1:2.3:4.1**	**1:2.7:3.9**
	8 to 10	1:1.0:3.3	1:1.1:3.2	1:1.3:3.2	1:1.5:3.1	1:1.8:2.9
No. 4 to 2 in	½ to 1	1:2.2:6.9	1:2.4:6.8	1:2.8:6.8	1:3.1:6.6	1:3.7:6.4
	3 to 4	1:1.8:6.2	1:2.0:6.1	1:2.1:6.1	1:2.7:6.0	1:3.1:5.7
	6 to 7	**1:1.4:5.1**	**1:1.6:5.0**	**1:1.8:5.0**	**1:2.0:5.0**	**1:2.4:4.8**
	8 to 10	1:0.9:3.8	1:1.0:3.8	1:1.1:3.8	1:1.3:3.8	1:1.5:3.7
⅜ to 1 in.	½ to 1	1:2.8:5.2	1:3.1:5.1	1:3.6:4.8	1:4.2:4.6	1:4.8:4.1
	3 to 4	1:2.4:4.5	1:2.6:4.5	1:3.1:4.3	1:3.6:4.0	1:4.1:3.6
	6 to 7	**1:1.9:3.9**	**1:2.1:3.7**	**1:2.4:3.6**	**1:2.8:3.4**	**1:3.2:3.1**
	8 to 10	1:1.3:2.8	1:1.4:2.8	1:1.6:2.7	1:1.9:2.6	1:2.2:2.4
⅜ to 1½ in.	½ to 1	1:2.8:5.8	1:3.1:5.7	1:3.5:5.5	1:4.1:5.3	1:4.7:4.9
	3 to 4	1:2.4:5.2	1:2.7:5.1	1:3.1:5.0	1:3.5:4.8	1:4.1:4.4
	6 to 7	**1:1.9:4.3**	**1:2.1:4.2**	**1:2.4:4.2**	**1:2.7:4.0**	**1:3.1:3.7**
	8 to 10	1:1.2:3.2	1:1.4:3.2	1:1.6:3.1	1:1.8:3.0	1:2.1:2.9
⅜ to 2 in.	½ to 1	1:2.7:6.6	1:3.0:6.6	1:3.4:6.5	1:3.9:6.4	1:4.4:6.0
	3 to 4	1:2.3:5.9	1:2.6:5.9	1:2.9:5.8	1:3.3:5.6	1:3.7:5.5
	6 to 7	**1:1.8:4.9**	**1:2.0:4.8**	**1:2.2:4.8**	**1:2.6:4.8**	**1:3.0:4.5**
	8 to 10	1:1.2:3.7	1:1.3:3.7	1:1.5:3.7	1:1.7:3.6	1:1.9:3.5
¾ to 1½ in.	½ to 1	1:3.2:5.4	1:3.6:5.3	1:4.1:5.1	1:4.7:4.8	1:5.3:4.4
	3 to 4	1:2.8:4.8	1:3.2:4.8	1:3.6:4.6	1:4.0:4.4	1:4.6:4.0
	6 to 7	**1:2.1:4.0**	**1:2.5:4.0**	**1:2.8:3.9**	**1:3.2:3.7**	**1:3.5:3.4**
	8 to 10	1:1.5:3.0	1:1.7:3.0	1:1.9:2.9	1:2.2:2.8	1:2.5:2.7
¾ to 2 in.	½ to 1	1:3.2:6.2	1:3.6:6.1	1:4.0:6.0	1:4.6:5.8	1:5.2:5.4
	3 to 4	1:2.8:5.5	1:3.1:5.5	1:3.5:5.4	1:3.9:5.2	1:4.5:4.9
	6 to 7	**1:2.1:4.5**	**1:2.4:4.6**	**1:2.7:4.5**	**1:3.1:4.4**	**1:3.5:4.1**
	8 to 10	1:1.4:3.4	1:1.6:3.4	1:1.8:3.4	1:2.1:3.4	1:2.4:3.3
¾ to 3 in.	½ to 1	1:3.2:7.1	1:3.6:7.1	1:4.0:7.0	1:4.6:6.9	1:5.2:6.6
	3 to 4	1:2.7:6.3	1:3.0:6.3	1:3.4:6.3	1:4.0:6.2	1:4.5:5.9
	6 to 7	**1:2.1:5.1**	**1:2.4:5.2**	**1:2.7:5.2**	**1:3.1:6.1**	**1:3.5:4.9**
	8 to 10	1:1.4:3.8	1:1.6:3.9	1:1.8:3.0	1:2.1:3.9	1:2.4:3.8

TABLE IV.—PROPORTIONS FOR 2,000 LB. PER SQ. IN. CONCRETE.—(*Continued*)

Proportions are expressed by volume as follows: Portland Cement: Fine Aggregate: Coarse Aggregate.

Thus 1:2.6:4.6 indicates 1 part by volume of Portland cement, 2.6 parts by volume of fine aggregate and 4.6 parts by volume of coarse aggregate.

Size of coarse aggregate	Slump,	Size of fine aggregate				
		0–No. 28	0–No. 14	0–No. 8	0–No. 4	0–⅜ in.
None.	½ to 1	1:2.2	1:2.6	1:3.0	1:3.5	1:4.1
	3 to 4	1:1.9	1:2.2	1:2.6	1:3.0	1:3.5
	6 to 7	**1:1.5**	**1:1.7**	**1:2.0**	**1:2.3**	**1:2.7**
	8 to 10	1:1.0	1:1.1	1:1.3	1:1.6	1:1.8
No. 4 to ¾ in.....	½ to 1	1:2.1:3.8	1:2.3:3.7	1:2.6:3.5	1:3.0:3.1	1:3.6:2.8
	3 to 4	1:1.7:3.3	1:1.9:3.2	1:2.2:3.1	1:2.6:2.8	1:3.0:2.4
	6 to 7	**1:1.3:2.7**	**1:1.4:2.6**	**1:1.7:2.5**	**1:1.9:2.3**	**1:2.3:2.1**
	8 to 10	1:0.8:1.9	1:0.9:1.9	1:1.0:1.8	1:1.2:1.7	1:1.5:1.6
No. 4 to 1 in	½ to 1	1:1.9:4.5	1:2.2:4.3	1:2.5:4.2	1:2.8:3.9	1:3.4:3.6
	3 to 4	1:1.6:3.9	1:1.8:3.8	1:2.1:3.7	1:2.4:3.5	1:2.8:3.2
	6 to 7	**1:1.2:3.1**	**1:1.3:3.1**	**1:1.5:3.0**	**1:1.8:2.9**	**1:2.1:2.7**
	8 to 10	1:0.7:2.2	1:0.8:2.2	1:1.0:2.3	1:1.1:2.1	1:1.3:2.0
No. 4 to 1½ in ...	½ to 1	1:1.9:5.0	1:2.1:4.9	1:2.4:4.9	1:2.7:4.6	1:3.2:4.4
	3 to 4	1:1.6:4.4	1:1.7:4.3	1:2.0:4.2	1:2.1:4.0	1:2.7:3.8
	6 to 7	**1:1.1:3.5**	**1:1.3:3.5**	**1:1.4:3.5**	**1:1.7:3.4**	**1:2.0:3.2**
	8 to 10	1:0.7:2.5	1:0.8:2.5	1:0.9:2.5	1:1.0:2.4	1:1.2:2.3
No. 4 to 2 in.	½ to 1	1:1.7:5.8	1:1.9:5.7	1:2.1:5.8	1:2.4:5.6	1:2.8:5.5
	3 to 4	1:1.4:5.0	1:1.5:5.0	1:1.8:5.0	1:2.0:4.9	1:2.3:4.7
	6 to 7	**1:1.0:4.1**	**1:1.1:4.1**	**1:1.2:4.1**	**1:1.4:4.1**	**1:1.7:3.9**
	8 to 10	1:0.6:2.9	1:0.7:2.9	1:0.7:3.0	1:0.8:2.9	1:1.0:2.9
⅜ to 1 in	½ to 1	1:2.2:4.4	1:2.5:4.2	1:2.8:4.1	1:3.3:3.8	1:3.8:3.4
	3 to 4	1:1.9:3.8	1:2.1:3.7	1:2.4:3.6	1:2.8:3.4	1:3.2:3.1
	6 to 7	**1:1.4:3.1**	**1:1.5:3.0**	**1:1.8:3.0**	**1:2.1:2.8**	**1:2.4:2.5**
	8 to 10	1:0.9:2.2	1:1.0:2.2	1:1.1:2.2	1:1.3:2.0	1:1.5:1.9
⅜ to 1½ in.....	½ to 1	1:2.2:4.9	1:2.5:4.8	1:2.8:4.7	1:3.2:4.6	1:3.7:4.2
	3 to 4	1:1.9:4.3	1:2.1:4.2	1:2.4:4.1	1:2.7:4.0	1:3.1:3.7
	6 to 7	**1:1.4:3.5**	**1:1.5:3.4**	**1:1.7:3.4**	**1:2.0:3.3**	**1:2.3:3.1**
	8 to 10	1:0.9:2.5	1:1.0:2.5	1:1.1:2.4	1:1.3:2.4	1:1.5:2.3
⅜ to 2 in.	½ to 1	1:2.1:5.6	1:2.3:5.5	1:2.6:5.5	1:3.0:5.4	1:3.5:5.1
	3 to 4	1:1.7:4.8	1:2.0:4.8	1:2.2:4.8	1:2.5:4.7	1:2.9:4.4
	6 to 7	**1:1.3:4.0**	**1:1.4:3.9**	**1:1.6:3.9**	**1:1.8:3.9**	**1:2.1:3.8**
	8 to 10	1:0.8:2.9	1:0.9:2.9	1:1.0:2.9	1:1.2:2.9	1:1.3:2.8
¾ to 1½ in.	½ to 1	1:2.6:4.5	1:2.9:4.5	1:3.3:4.4	1:3.8:4.2	1:4.3:3.9
	3 to 4	1:2.2:3.9	1:2.5:3.9	1:2.8:3.8	1:3.2:3.6	1:3.6:3.3
	6 to 7	**1:1.6:3.2**	**1:1.8:3.2**	**1:2.1:3.1**	**1:2.4:3.0**	**1:2.7:2.8**
	8 to 10	1:1.0:2.3	1:1.2:2.3	1:1.4:2.2	1:1.6:2.2	1:1.8:2.1
¾ to 2 in.	½ to 1	1:2.5:5.2	1:2.8:5.2	1:3.2:5.1	1:3.6:5.0	1:4.1:4.7
	3 to 4	1:2.1:4.5	1:2.4:4.5	1:2.7:4.4	1:3.1:4.3	1:3.5:4.0
	6 to 7	**1:1.6:3.7**	**1:1.8:3.7**	**1:2.0:3.7**	**1:2.3:3.6**	**1:2.6:3.5**
	8 to 10	1:1.0:2.6	1:1.1:2.7	1:1.3:2.6	1:1.5:2.7	1:1.7:2.6
¾ to 3 in.	½ to 1	1:2.5:6.0	1:2.9:5.9	1:3.2:5.9	1:3.6:5.8	1:4.1:5.6
	3 to 4	1:2.1:5.1	1:2.4:5.2	1:2.7:5.2	1:3.1:5.1	1:3.5:4.9
	6 to 7	**1:1.5:4.1**	**1:1.7:4.2**	**1:2.0:4.2**	**1:2.3:4.2**	**1:2.6:4.0**
	8 to 10	1:1.0:2.9	1:1.1:3.0	1:1.3:3.0	1:1.5:3.0	1:1.7:3.0

TABLE IV.—PROPORTIONS FOR 2,500 LB. PER SQ. IN. CONCRETE.—(*Continued*)

Proportions are expressed by volume as follows: Portland Cement: Fine Aggregate: Coarse Aggregate.

Thus 1:2.6:4.6 indicates 1 part by volume of Portland Cement, 2.6 parts by volume of fine aggregate and 4.6 parts by volume of coarse aggregate.

Size of coarse aggregate	Slump, in.	Size of fine aggregate				
		0–No. 28	0–No. 14	0–No. 8	0–No. 4	0–⅜ in.
	½ to 1	1:1.8	1:2.1	1:2.4	1:2.9	1:3.3
None...	3 to 4	1:1.5	1:1.8	1:2.1	1:2.4	1:2.8
	6 to 7	**1:1.1**	**1:1.3**	**1:1.6**	**1:1.8**	**1:2.1**
	8 to 10	1:0.7	1:0.8	1:0.9	1:1.1	1:1.3
	½ to 1	1:1.6:3.2	1:1.8:3.1	1:2.1:3.0	1:2.4:2.7	1:2.9:2.4
No. 4 to ¾ in....	3 to 4	1:1.3:2.8	1:1.5:2.7	1:1.7:2.6	1:2.0:2.4	1:2.4:2.2
	6 to 7	**1:1.0:2.2**	**1:1.1:2.2**	**1:1.3:2.1**	**1:1.5:2.0**	**1:1.8:1.8**
	8 to 10	1:0.5:1.4	1:0.6:1.4	1:0.7:1.4	1:0.8:1.4	1:1.0:1.3
	½ to 1	1:1.5:3.7	1:1.7:3.7	1:2.0:3.5	1:2.2:3.4	1:2.7:3.1
No. 4 to 1 in	3 to 4	1:1.2:3.3	1:1.4:3.2	1:1.6:3.1	1:1.9:3.0	1:2.2:2.7
	6 to 7	**1:0.9:2.6**	**1:1.0:2.5**	**1:1.1:2.5**	**1:1.3:2.4**	**1:1.6:2.3**
	8 to 10	1:0.5:1.7	1:0.6:1.7	1:0.6:1.7	1:0.7:1.6	1:0.9:1.5
	½ to 1	1:1.4:4.2	1:1.6:4.1	1:1.9:4.1	1:2.2:4.0	1:2.5:3.8
No. 4 to 1½ in .	3 to 4	1:1.2:3.7	1:1.3:3.6	1:1.5:3.6	1:1.8:3.5	1:2.1:3.3
	6 to 7	**1:0.9:2.9**	**1:0.9:2.8**	**1:1.1:2.8**	**1:1.3:2.8**	**1:1.5:2.6**
	8 to 10	1:0.5:1.9	1:0.5:1.9	1:0.6:1.9	1:0.7:1.8	1:0.8:1.8
	½ to 1	1:1.3:4.9	1:1.4:4.8	1:1.6:4.9	1:1.9:4.8	1:2.2:4.7
No. 4 to 2 in	3 to 4	1:1.1:4.3	1:1.2:4.2	1:1.3:4.3	1:1.6:4.2	1:1.8:4.1
	6 to 7	**1:0.7:3.3**	**1:0.8:3.3**	**1:0.9:3.4**	**1:1.1:3.3**	**1:1.2:3.3**
	8 to 10	1:0.4:2.2	1:2.4:2.2	1:0.5:2.2	1:0.6:2.2	1:0.6:2.2
	½ to 1	1:1.8:3.7	1:2.0:3.6	1:2.3:3.5	1:2.6:3.3	1:3.0:2.9
⅜ to 1 in .	3 to 4	1:1.4:3.2	1:1.6:3.1	1:1.9:2.9	1:2.2:2.9	1:2.5:2.6
	6 to 7	**1:1.0:2.5**	**1:1.2:2.5**	**1:1.3:2.4**	**1:1.6:2.3**	**1:1.8:2.2**
	8 to 10	1:0.6:1.6	1:0.7:1.6	1:0.8:1.6	1:0.9:1.6	1:1.0:1.5
	½ to 1	1:1.7:4.1	1:1.9:4.1	1:2.2:4.0	1:2.5:3.9	1:2.9:3.6
⅜ to 1½ in	3 to 4	1:1.5:3.6	1:1.6:3.6	1:1.8:3.5	1:2.1:3.4	1:2.3:3.2
	6 to 7	**1:1.0:2.9**	**1:1.2:2.8**	**1:1.3:2.8**	**1:1.5:2.7**	**1:1.8:2.6**
	8 to 10	1:0.6:1.9	1:0.6:1.9	1:0.8:1.8	1:0.9:1.8	1:1.0:1.8
	½ to 1	1:1.7:4.7	1:1.8:4.7	1:2.1:4.7	1:2.4:4.6	1:2.7:4.4
to 2 in.	3 to 4	1:1.4:4.1	1:1.5:4.1	1:1.7:4.1	1:2.0:4.0	1:2.3:3.9
	6 to 7	**1:1.0:3.2**	**1:1.1:3.2**	**1:1.2:3.2**	**1:1.4:3.2**	**1:1.6:3.1**
	8 to 10	1:0.5:2.1	1:0.6:2.1	1:0.7:2.2	1:0.8:2.2	1:0.9:2.1
	½ to 1	1:2.0:3.8	1:2.3:3.8	1:2.6:3.7	1:3.0:3.6	1:3.4:3.3
¾ to 1½ in	3 to 4	1:1.7:3.3	1:2.0:3.3	1:2.2:3.2	1:2.5:3.2	1:2.9:2.9
	6 to 7	**1:1.2:2.6**	**1:1.4:2.6**	**1:1.6:2.6**	**1:1.9:2.5**	**1:2.1:2.3**
	8 to 10	1:0.7:1.7	1:0.8:1.7	1:0.9:1.7	1:1.1:1.7	1:1.2:1.6
	½ to 1	1:2.0:4.4	1:2.2:4.4	1:2.5:4.3	1:2.9:4.3	1:3.3:4.1
¾ to 2 in	3 to 4	1:1.7:3.8	1:1.9:3.8	1:2.1:3.8	1:2.5:3.7	1:2.8:3.6
	6 to 7	**1:1.2:3.0**	**1:1.4:3.0**	**1:1.5:3.0**	**1:1.8:3.0**	**1:2.0:2.8**
	8 to 10	1:0.7:2.0	1:0.8:2.0	1:0.9:2.0	1:1.0:2.0	1:1.2:2.0
	½ to 1	1:2.0:5.0	1:2.2:5.0	1:2.5:5.0	1:2.7:5.0	1:3.2:4.7
¾ to 3 in......	3 to 4	1:1.7:4.3	1:1.9:4.3	1:2.1:4.3	1:2.4:4.3	1:2.7:4.1
	6 to 7	**1:1.2:3.3**	**1:1.4:3.4**	**1:1.5:3.4**	**1:1.8:3.4**	**1:2.0:3.3**
	8 to 10	1:0.7:2.2	1:0.8:2.2	1:0.9:2.2	1:1.0:2.3	1:1.2:2.3

TABLE IV.—PROPORTIONS FOR 3,000 LB. PER SQ. IN. CONCRETE—(*Continued*)

Proportions are expressed by volume as follows: Portland Cement: Fine Aggregate: Coarse Aggregate.

Thus 1:2.6:4.6 indicates 1 part by volume of Portland Cement, 2.6 parts by volume of fine aggregate and 4.6 parts by volume of coarse aggregate.

Size of coarse aggregate	Slump, in.	Size of fine aggregate 0–No. 28	0–No. 14	0–No. 8	0–No. 4	0–3/8 in.
None	1/2 to 1	1:1.5	1:1.7	1:2.0	1:2.3	1:2.7
	3 to 4	1:1.2	1:1.4	1:1.7	1:1.9	1:2.3
	6 to 7	**1:0.9**	**1:1.0**	**1:1.2**	**1:1.4**	**1:1.6**
	8 to 10	1:0.5	1:0.6	1:0.7	1:0.8	1:0.9
No. 4 to 3/4 in.	1/2 to 1	1:1.3:2.7	1:1.5:2.6	1:1.7:2.5	1:1.9:2.4	1:2.3:2.1
	3 to 4	1:1.0:2.3	1:1.2:2.2	1:1.4:2.2	1:1.6:2.0	1:1.9:1.8
	6 to 7	**1:0.7:1.7**	**1:0.8:1.7**	**1:0.9:1.7**	**1:1.1:1.6**	**1:1.3:1.4**
	8 to 10	1:0.3:1.0	1:0.4:1.0	1:0.5:1.0	1:0.5:1.0	1:0.6:0.9
No. 4 to 1 in.	1/2 to 1	1:1.2:3.1	1:1.3:3.1	1:1.5:3.0	1:1.8:2.9	1:2.1:2.7
	3 to 4	1:0.9:2.7	1:1.1:2.6	1:1.2:2.6	1:1.4:2.5	1:1.7:2.3
	6 to 7	**1:0.6:2.0**	**1:0.7:2.0**	**1:0.8:2.0**	**1:0.9:1.9**	**1:1.1:1.8**
	8 to 10	1:0.3:1.2	1:0.3:1.2	1:0.4:1.2	1:0.5:1.2	1:0.6:1.2
No. 4 to 1 1/2 in.	1/2 to 1	1:1.1:3.6	1:1.2:3.5	1:1.5:3.5	1:1.7:3.4	1:2.0:3.2
	3 to 4	1:0.9:3.0	1:1.0:2.9	1:1.2:2.9	1:1.4:2.9	1:1.6:2.7
	6 to 7	**1:0.6:2.2**	**1:0.7:2.2**	**1:0.8:2.2**	**1:0.9:2.2**	**1:1.1:2.1**
	8 to 10	1:0.3:1.4	1:0.3:1.3	1:0.4:1.4	1:0.5:1.4	1:0.5:1.3
No. 4 to 2 in.	1/2 to 1	1:1.0:4.1	1:1.1:4.1	1:1.2:4.1	1:1.4:4.1	1:1.6:4.0
	3 to 4	1:0.8:3.4	1:0.9:3.4	1:1.0:3.5	1:1.1:3.4	1:1.3:3.4
	6 to 7	**1:0.5:2.6**	**1:0.6:2.6**	**1:0.6:2.7**	**1:0.7:2.6**	**1:0.9:2.6**
	8 to 10	1:0.2:1.6	1:0.3:1.6	1:0.3:1.7	1:0.4:1.7	1:0.4:1.7
3/8 to 1 in.	1/2 to 1	1:1.4:3.1	1:1.5:3.0	1:1.8:2.9	1:2.1:2.8	1:2.4:2.6
	3 to 4	1:1.1:2.6	1:1.3:2.6	1:1.5:2.5	1:1.7:2.4	1:2.0:2.2
	6 to 7	**1:0.8:2.0**	**1:0.8:2.0**	**1:1.0:1.9**	**1:1.1:1.9**	**1:1.3:1.8**
	8 to 10	1:0.4:1.2	1:0.4:1.2	1:0.5:1.2	1:0.6:1.2	1:0.7:1.1
3/8 to 1 1/2 in.	1/2 to 1	1:1.4:3.5	1:1.5:3.4	1:1.7:3.4	1:2.0:3.3	1:2.3:3.1
	3 to 4	1:1.1:3.0	1:1.2:2.9	1:1.4:2.9	1:1.6:2.8	1:1.9:2.6
	6 to 7	**1:0.6:2.2**	**1:0.8:2.2**	**1:1.0:2.2**	**1:1.1:2.1**	**1:1.3:2.0**
	8 to 10	1:0.4:1.4	1:0.4:1.4	1:0.5:1.4	1:0.6:1.3	1:0.7:1.3
3/8 to 2 in.	1/2 to 1	1:1.3:4.0	1:1.4:4.0	1:1.6:4.0	1:1.9:3.9	1:2.1:3.8
	3 to 4	1:1.0:3.4	1:1.2:3.4	1:1.3:3.3	1:1.5:3.3	:1.7:3.2
	6 to 7	**1:0.7:2.6**	**1:0.8:2.5**	**1:0.9:2.6**	**1:1.0:2.6**	**1:1.1:2.5**
	8 to 10	1:0.4:1.6	1:0.4:1.6	1:0.5:1.6	1:0.5:1.6	1:0.6:1.6
3/4 to 1 1/2 in.	1/2 to 1	1:1.6:3.2	1:1.8:3.2	1:2.1:3.2	1:2.4:3.1	1:2.7:2.9
	3 to 4	1:1.3:2.7	1:1.5:2.7	1:1.7:2.7	1:2.0:2.6	1:2.3:2.5
	6 to 7	**1:0.9:2.0**	**1:1.0:2.1**	**1:1.2:2.0**	**1:1.4:2.0**	**1:1.5:1.8**
	8 to 10	1:0.5:1.2	1:0.5:1.3	1:0.6:1.3	1:0.7:1.3	1:0.8:1.2
3/4 to 2 in.	1/2 to 1	1:1.6:3.7	1:1.8:3.7	1:2.0:3.7	1:2.4:3.6	1:2.6:3.5
	3 to 4	1:1.3:3.1	1:1.5:3.1	1:1.6:3.1	1:1.9:3.1	1:2.2:3.0
	6 to 7	**1:0.9:2.4**	**1:1.1:2.4**	**1:1.1:2.4**	**1:1.3:2.4**	**1:1.5:2.3**
	8 to 10	1:0.5:1.5	1:0.5:1.5	1:0.6:1.5	1:0.7:1.5	1:0.8:1.5
3/4 to 3 in.	1/2 to 1	1:1.6:4.2	1:1.8:4.2	1:2.0:4.2	1:2.3:4.1	1:2.6:4.0
	3 to 4	1:1.3:3.5	1:1.5:3.6	1:1.6:3.6	1:1.9:3.6	1:2.1:3.5
	6 to 7	**1:0.9:2.6**	**1:1.0:2.6**	**1:1.1:2.6**	**1:1.3:2.6**	**1:1.4:2.6**
	8 to 10	1:0.5:1.6	1:0.5:1.6	1:0.6:1.7	1:0.7:1.7	1:0.8:1.7

STANDARD NOTATION

All symbols used in the Tentative Specifications for Concrete and Reinforced Concrete have been collected here for convenience of reference. The symbols are in general defined in the text near the point where formulas occur. In a few instances the same symbol is used in two distinct senses; however, there is little danger of confusion from this source.

a = spacing of web reinforcement bars measured perpendicular to their direction (see Sec. 135);
a = width of face of column or pedestal;
α = angle between inclined web bars and longitudinal bars;
A = total net area of column, footing, or pedestal, exclusive of fireproofing;
A' = loaded area of pedestal, pier or footing;
A_e = $A(1 - p)$ = net area of concrete core of column;
A'_e = net area of concrete in columns (total column area minus steel area);
A_s = effective cross-sectional area of metal reinforcement in tension in beams or compression in columns; and the effective cross-sectional area of metal reinforcement which crosses any of the principal design sections of a flat slab and which meets the requirements of Sec. 156;
A_v = total area of web reinforcement in tension within a distance of a (a_1, a_2, a_3, etc.) or the total area of all bars bent up in any one plane;
b = width of rectangular beam or width of flange of T-beam;
b' = width of stem of T-beam;
b_1 = dimension of the dropped panel of a flat slab in the direction parallel to l_1;[1]
c = base diameter of the largest right circular cone which lies entirely within the column (including the capital) whose vertex angle is 90 deg. and whose base is 1½ in. below the bottom of the slab or the bottom of the dropped panel (see Fig. 14);
c = projection of footing from face of column;
C = total compressive stress in concrete;
C' = total compressive stress in reinforcement;
d = depth from compression surface of beam or slab to center of longitudinal tension reinforcement;
d' = depth from compression surface of beam or slab to center of compression reinforcement;
E_c = modulus of elasticity of concrete in compression;
E_s = modulus of elasticity of steel in tension = 30,000,000 lb. per sq. in.;
f_c = compressive unit stress in extreme fiber of concrete;
f'_c = ultimate compressive strength of concrete at age of 28 days, based on tests of 6 by 12-in. or 8 by 16-in. cylinders made and tested in accordance with the Standard Methods of Making and Storing Specimens of Concrete in the Field and the Tentative Methods of Making Compression Tests of Concrete.
f_r = compressive unit stress in metal core;
f_s = tensile unit stress in longitudinal reinforcement;
f_s' = compressive unit stress in longitudinal reinforcement;
f_v = tensile unit stress in web reinforcement;
h = unsupported length of column;
I = moment of inertia of a section about the neutral axis for bending;
j = ratio of lever arm of resisting couple to depth d;
jd = $d-z$ = arm of resisting couple;
k = ratio of depth of neutral axis to depth d;
l = span length of beam or slab (generally distance from center to center of supports; for special cases, see Sec. 108 and 148);
l = span length of flat slab, center to center of columns, in the rectangular direction in which moments are considered;[1]
l_1 = span length of flat slab, center to center of columns, perpendicular to the rectangular direction in which moments are considered;[1]
M = bending moment or moment of resistance in general;
M_0 = sum of positive and negative bending moments in either rectangular direction, at the principal design sections of a panel of a flat slab;
n = $\frac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to that of concrete;
o = perimeter of bar;
Σo = sum of perimeters of bars in one set;

[1] In flat slab design, the column strip and the middle strip to be used when considering moments in the direction of the dimension l are located and dimensioned as shown in Fig. 15. The dimension l_1 does not always represent the short length of the panel. When moments in the direction of the shorter panel length are considered, the dimensions l and l_1 are to be interchanged and strips corresponding to those shown in Fig. 15 but extending in the direction of the shorter panel length are to be considered.

p = ratio of effective area of tension reinforcement to effective area of concrete in beams = $\frac{A_s}{bd}$; and the ratio of effective area of longitudinal reinforcement to the area of the concrete core in columns;

p' = ratio of effective area of compression reinforcement to effective area of concrete in beams;

P = total safe axial load on column whose $\frac{h}{R}$ is less than 40;

P' = total safe axial load on long column;

r = ratio of cross-sectional area of negative reinforcement which crosses entirely over the column capital of a flat slab or over the dropped panel, to the total cross-sectional area of the negative reinforcement in the two column strips;

r_a = permissible working stress in concrete over the loaded area of a pedestal, pier or footing;

R = ratio of positive or negative moment in two column strips or one middle strip of a flat slab, to M_0;

R = least radius of gyration of a section;

s = spacing web members, measured at the neutral axis and in the direction of the longitudinal axis of the beam;

t = thickness of flange of T-beam;

t_1 = thickness of flat slab without dropped panels or thickness of a dropped panel (see Fig. 14);

t_2 = thickness of flat slab with dropped panels at points away from the dropped panel (see Fig. 14);

T = total tensile stress in longitudinal reinforcement;

u = bond stress per unit of area of surface of bar;

v = shearing unit stress;

V = total shear;

V' = external shear on any section after deducting that carried by the concrete;

w = uniformly distributed load per unit of length of beam or slab;

w = upward reaction per unit of area of base of footing;

w = uniformly distributed dead and live load per unit of area of a floor or roof;

W = total dead and live load uniformly distributed over a single panel area;

z = depth from compression surface of beam or slab to resultant of compressive stresses.

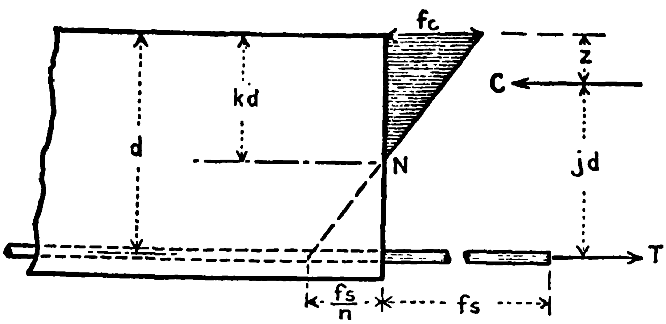


FIG. 1.—Nomenclature for concrete beam reinforced for tension.

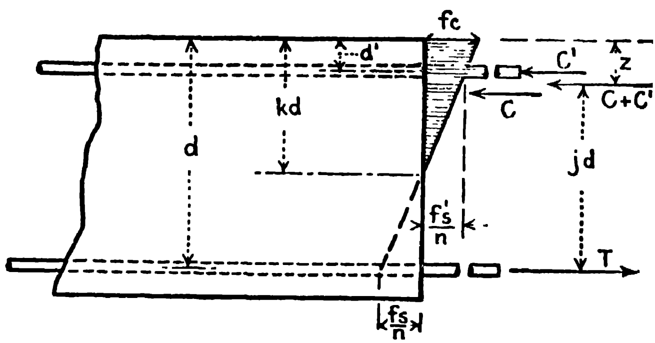


FIG. 2.—Nomenclature for concrete beam reinforced for tension and compression.

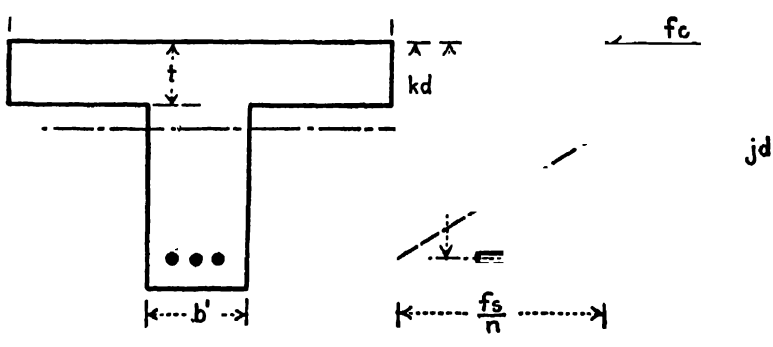


FIG. 3.—Nomenclature for reinforced concrete T-beam.

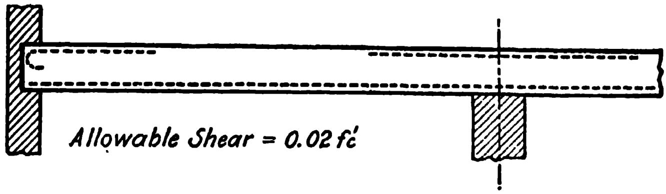


Fig. 4.—Typical reinforced concrete beam; principal longitudinal bars not anchored.

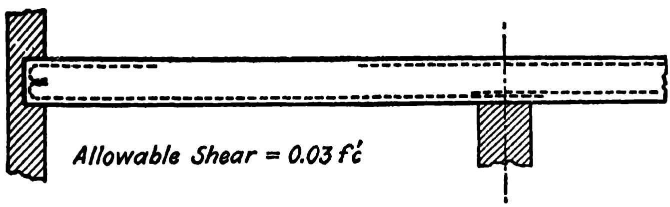


Fig. 5.—Typical reinforced concrete beam; principal longitudinal bars anchored.

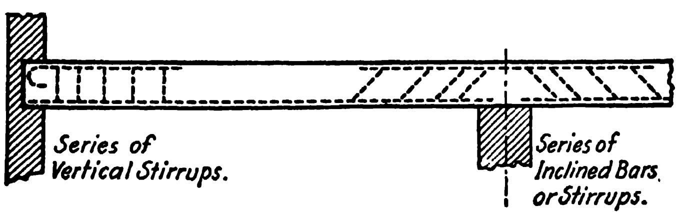


Fig. 6.—Typical reinforced concrete beam; web reinforced by means of series of vertical stirrups or series of inclined bars or stirrups.

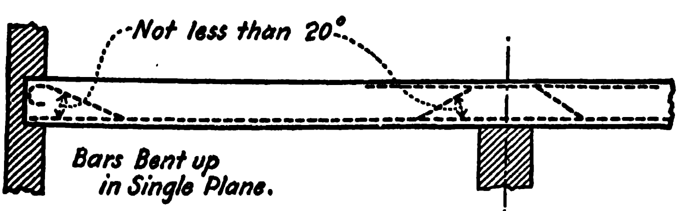


Fig. 7.—Typical reinforced concrete beam; principal longitudinal bars bent up in single plane.

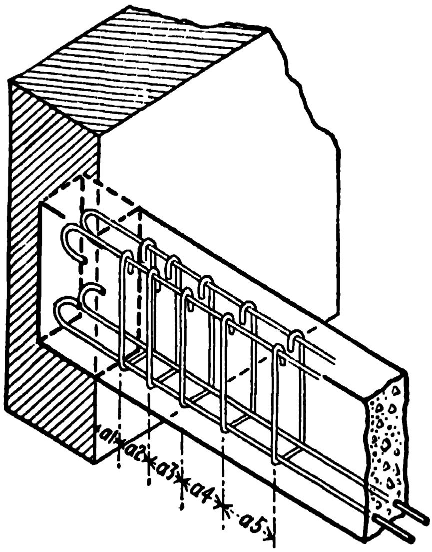


Fig. 8.—Typical reinforced concrete beam with anchored longitudinal bars and vertical stirrups.

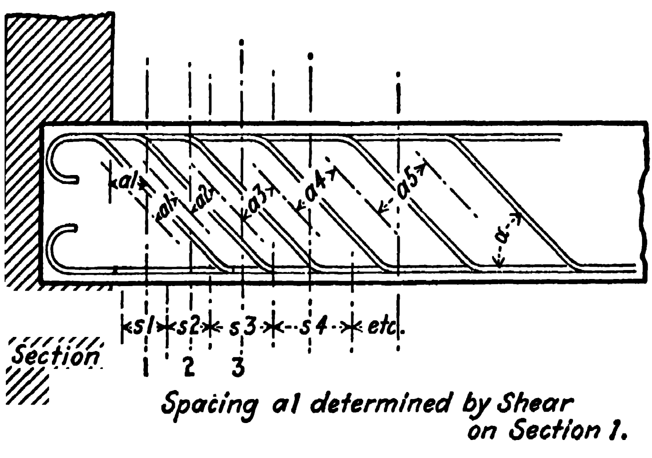


Fig. 9.—Typical beam with web reinforced by means of series of inclined bars.

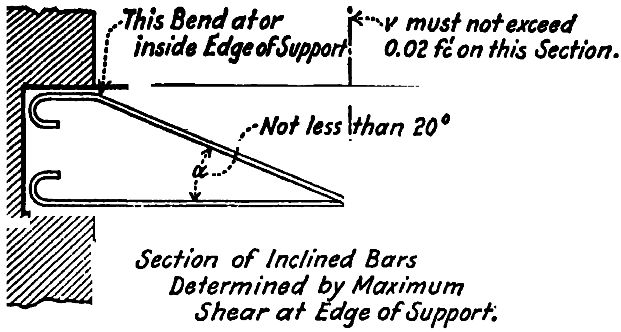


FIG. 10.—Typical beam with web reinforced by means of bars bent up in single plane.

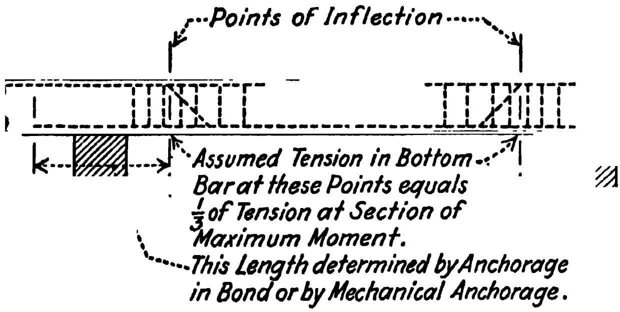


FIG. 11.—Typical web reinforcement for continuous beams.

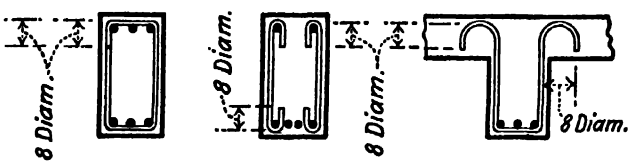


FIG. 12.—Typical methods of anchoring vertical stirrups.

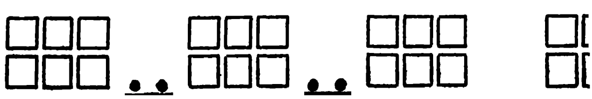


FIG. 13.—Typical reinforced concrete beam-and-tile construction.

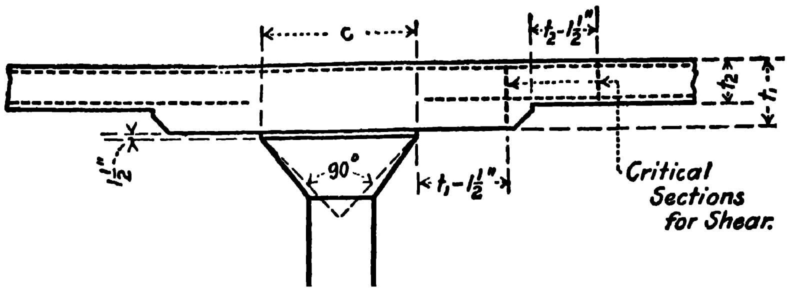


FIG. 14.—Typical column capital and sections of flat slab with dropped panel.

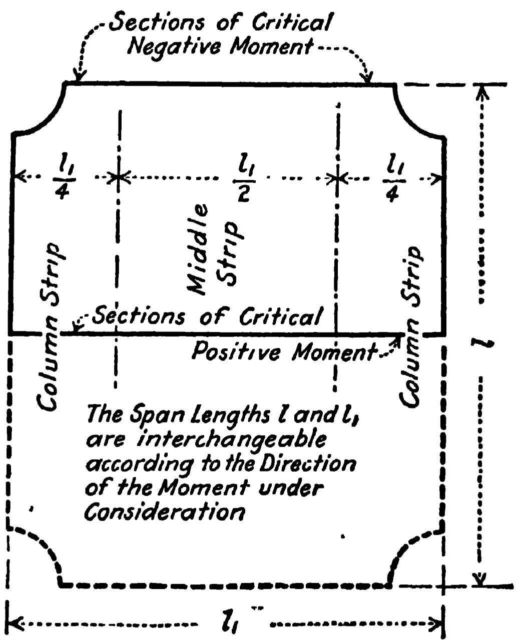


FIG. 15.—Principal design sections of a flat slab.

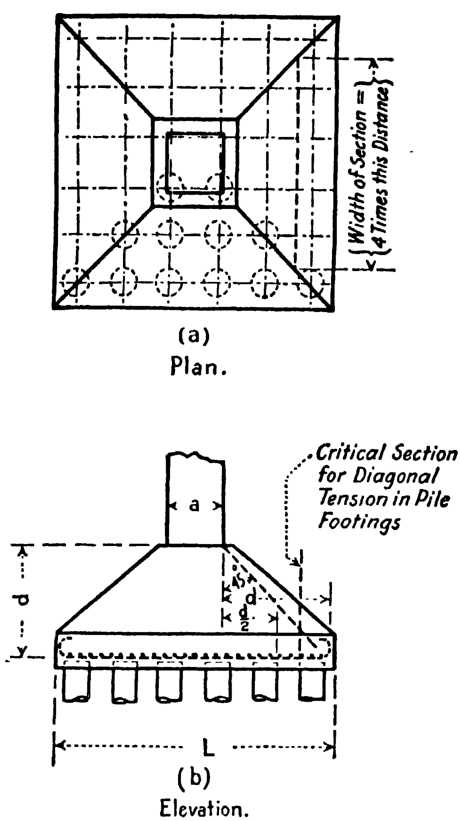


Fig. 16.—Typical sloped reinforced concrete footing on piles.

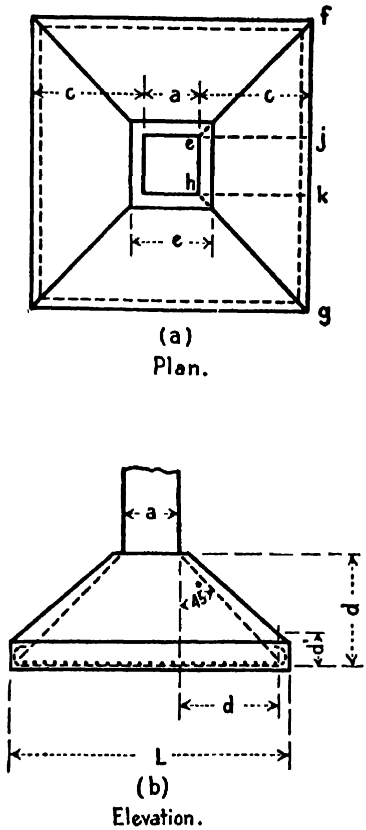


Fig. 17.—Typical sloped reinforced concrete footing on soil.

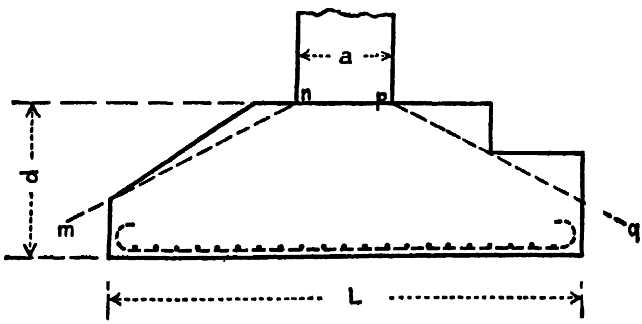


Fig. 18.—Typical sloped or stepped footing.

APPENDIX G

STRENGTH OF STONE MASONRY

With most stones commonly used in building construction, the strength of the stone itself is far greater than any distributed load that will be imposed upon it. Any limitations of strength are therefore imposed either (1) through faulty and uneven bedding, or (2) through the lower strength of mortar in which the stones are set.

Such cases of failure of stone masonry as are on record indicate that failure was by tension of flexure, induced by squeezing out of mortar from joints, rather than in compression. No experimental data has been obtained except on relatively small brick piers, but these tests indicate that the mortar was in each case the weak element of the combination; and that an increase of 50 per cent in the strength of the brick produced no increase of strength in the structure, while substitution of cement for lime mortar increased the strength 70 per cent.

Allowable Pressures on Stone Masonry.—The following data on foundation pressures on masonry are cited in Baker's Treatise on Masonry Construction:

"Early builders used much more massive masonry, proportional to the load to be carried, than is customary at present. Experience and experiments have shown that such great strength is unnecessary. The load on the monolithic piers supporting the large churches in Europe does not usually exceed 30 tons per sq ft. (420 lb. per sq. in.), or about one-thirtieth of the ultimate strength of the stone alone, although the columns of the Church of All Saints at Angers, France, are said to sustain 43 tons per sq. ft. (600 lb. per sq. in.). The stone-arch bridge of 140-ft. span at Pont-y-Pyrdd, over the Taff, in Wales, erected in 1750, is supposed to have a pressure of 72 tons per sq. ft. (1,000 lb. per sq. in.) on hard limestone rubble masonry laid in lime mortar. The granite piers of the Saltash Bridge sustain a pressure of 9 tons per sq. ft. (125 lb. per sq. in.).

"The maximum pressure on the granite masonry of the towers of the Brooklyn Bridge is about 28½ tons per sq. ft. (about 400 lb. per sq. in.). The maximum pressure on the limestone masonry of this bridge is about 10 tons per sq. ft. (125 lb. per sq. in.). The face stones ranged in cubical contents from 1½ to 5 cu. yd.; the stones of the granite backing averaged about 1½ cu. yd., and of the limestone about

In the Rookery Building, Chicago, granite col

(415 lb. per sq. in.) without any signs of weakness.

"In the Washington Monument, Washington, D. C., the normal pressure on the lower joint of the walls of the shaft is 20.2 tons per sq. ft. (280 lb. per sq. in.), and the maximum pressure brought upon any joint under the action of the wind is 25.4 tons per sq. ft. (350 lb. per sq. in.).

"The pressure on the limestone piers of the St. Louis Bridge was, before completion, 38 tons per sq. ft. (527 lb. per sq. in.); and after completion the pressure was 19 tons per sq. ft. (273 lb. per sq. in.) on the piers and 15 tons per sq. ft. (198 lb. per sq. in.) on the abutments.

"The limestone masonry in the towers of the Niagara Suspension Bridge failed under 36 tons per sq. ft., and were taken down,—however, the masonry was not well executed.

"At the South Street Bridge, Philadelphia, the pressure on the limestone rubble masonry in the pneumatic piles is 15.7 tons per sq. ft. (220 lb. per sq. in.) at the bottom and 12 tons per sq. ft. at the top. The maximum pressure on the rubble masonry (laid in cement mortar) of some of the large masonry dams is from 11 to 14 tons per sq. ft. (154 to 195 lb. per sq. in.). The Quaker Bridge dam was designed for a maximum pressure of 16⅗ tons per sq. ft. (230 lb. per sq. in.) on massive rubble masonry in best hydraulic cement mortar.

"In the light of the preceding examples, it may be assumed that the safe load for the different classes of masonry is about as follows, ***provided each is the best of its class***:

	Net Tons per Sq. Ft.	Lb. per Sq. In.
Rubble	10 to 15	140 to 200
Squared-stone	15 to 20	200 to 280
Limestone ashlar	20 to 25	280 to 350
Granite ashlar	25 to 30	350 to 400
Concrete	30 to 40	400 to 550

Allowable Pressures Under Building Codes.—Building codes of various cities in the United States vary widely in regard to pressures allowed on stone masonry. A tabulation of limits and averages permitted by the codes of six cities is as follows:

Kind of stone	Pressures (tons per sq. ft.)		
	High	Low	Average
Granite—cut		43	57.5
Marble and limestone—cut	50	29	38.5
Hard sandstone—cut	30	12	21.0

Mr. Thomas Nolan, in Kidder's Pocket Book, gives the following as allowable loads for different kinds of stonework, and states that "in determining the safe compressive resistance of masonry from tests on the ultimate compressive strength of the same kind, a factor of safety of at least 10 should be allowed for piers and 20 for arches."

	Tons per Square Foot
Rubble walls, irregular stones	3
Rubble walls, coursed, soft stone	2½
Rubble walls, coursed, hard stone	5 to 16
Dimension-stone, squared, in cement mortar	
Sandstone and limestone	10 to 20
Granite	20 to 40
Dressed stone, with ⅜-in. dressed joints, in Portland-cement mortar:	
Granite	60
Marble or limestone, best	40
Sandstone	30

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